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ON THE COLUMNS BUCKLING LENGTH OF UNBRACED STEEL FRAMES WITH SEMI-RIGID JOINTS

P. KRYSTOSIK [1]

The paper presents the issue of unbraced and semi-rigid steel frames stability with special attention paid to the determination problem of columns buckling length L_{cr} in these frames.

The paper discusses ways of buckling length determination in frames columns with the use of well known, European and American standard procedures, as well as numerical method of stability analysis based on the Finite Element Method (FEM). The presented procedures and analysis methods in calculations of certain steel frames with semi-rigid joints were used. On the basis of obtained results, it has been shown that in many practical cases, the simplified standard procedures of columns buckling length determination can give the results burdened with errors. These errors can have a significant influence on accuracy of columns resistance calculations.

The issues presented in the paper are very important from the practical point of view, and according to the author, they can be used in the practical design of unbraced steel frames.

Keywords: *Stability, unbraced steel frame, buckling length, semi-rigid joints*

[1] PhD., Eng., Koszalin University of Technology, Faculty of Civil Engineering, Environmental and Geodetic Sciences, Śniadeckich 2, 75-453 Koszalin, Poland, e-mail: przemyslaw.krystosik@tu.koszalin.pl

1. INTRODUCTION

Increasing application of semi-rigid joints in steel frames is due to technological aspects, as well as the possibility of more effective resistance use of frames elements, e.g. [16, 20]. It is worth noting that during the design process, structures with semi-rigid joints often require more advanced computational methods than structures with rigid joints, e.g., [3, 4, 5, 13]. This is largely due to the usage of joints with reduced rotational stiffness which causes structure sensitivity increase to deformations, and also greater susceptibility to loss of stability.

The impact significance of joints flexibility on frames stability can be demonstrated on the basis of buckling analysis of braced and unbraced portal frames with semi-rigid joints, which are shown in Fig. 1.

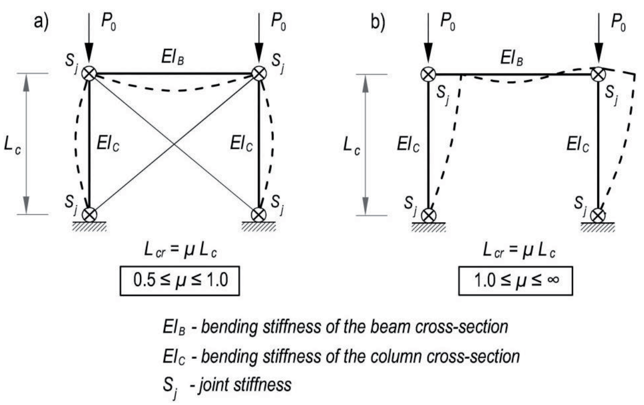

Fig. 1. The analyzed portal frames: a) buckling mode of braced frames, b) buckling mode of unbraced frame

In case of the braced frame (Fig. 1a) with rigid beam (EI_B / EI_C > 5) the value of buckling length coefficient changes from $\mu \approx 0.5$ (rigid joints – $S_j = \infty$) to $\mu = 1$ (hinged joints – $S_j = 0$). Whereas in the unbraced frame (Fig. 1b) the variation range of buckling length coefficient is much larger. Theoretically, it may take the value from $\mu \approx 1.0$ (rigid joints – $S_j = \infty$) to $\mu = \infty$ (hinged joints – $S_j = 0$). However, in practice the appropriate stiffnesses of semi-rigid joints should lead to a significant reduction in variability of that coefficient (e.g. $\mu < 3$).

From the presented analysis of portal frames it is clear that the joints flexibility is much more significant in case of unbraced frames buckling, which are known to be highly susceptible to stability loss.

Many papers have been published so far about the length buckling determination of columns in steel frames. There are a few which may be mentioned, e.g. [2, 10], where relatively simple approaches are presented, as well as papers, e.g. [9, 12, 19], where results are received with the use of more advanced methods.

This paper presents three procedures of buckling length determination of columns in the steel frames. The first two methods are based on European [6] and American [1] recommendations on the assessment of frames columns stability. The third "accurate" method is essentially based on global stability analysis of structure in the FEM (Finite Element Method) approach.

The presented methods were used in calculations of certain steel frames with semi-rigid joints, and afterwards, the results calculated using the above mentioned methods were compared.

The presented solutions clearly indicate that choosing the computation method has a great practical impact on the buckling length value.

According to the author, comments and conclusions presented in the article can be applied in practical design of steel frames with semi-rigid joints.

2. METHOD BASED ON ECCS GUIDELINES

The first method presented in the paper is based on the ECCS [6] recommendations, and it is sort of supplement to the standard [7] in the field of frame design. Determination process of columns buckling length L_{cr} of unbraced frames is performed for a separated part of a load-bearing structure, i.e. the column under study with elements directly connected to it (Fig. 2b). The analyzed part of frame can be replaced with a single element which is supported on both sides by elastic joints.

In the presented procedure the impact of elastic support of column on the buckling length L_{cr} value is defined with the use of η_i coefficients (Fig. 2c). These coefficients determine the degree of rotational flexibility, wherein the minimum value $\eta_i = 0$ means rigid fastening, whereas $\eta_i = 1$ refers to hinged support.

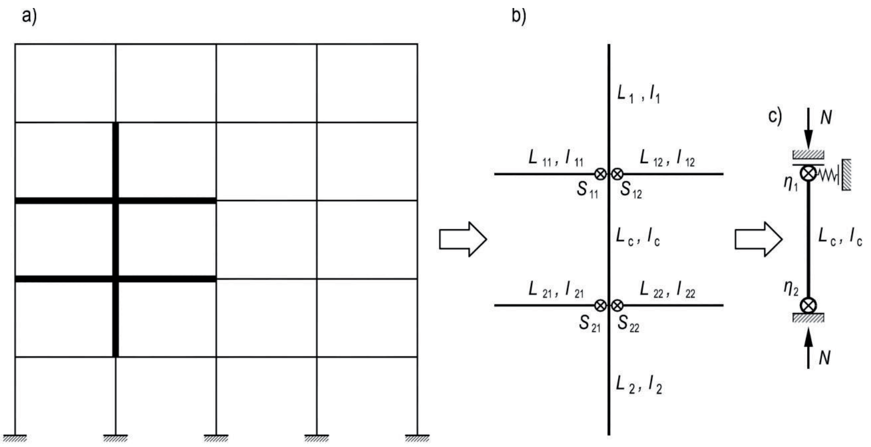

Fig. 2. Computation model: a) example of unbraced steel frame, b) separated part of the load-bearing structure, c) equivalent model of the frame separated part

The intermediate values of η_i coefficients can be calculated with equation [6]:

(1) $$\eta_i = \frac{K_c + K_i}{K_c + K_i + \sum_i \overline{K}_{ij}},$$

where index $i = 1, 2$ is respectively assigned to the upper and the lower joint of the isolated column, whereas index $j = 1, 2$ is associated with elements (beams, alternatively semi-rigid joints) located on the left and right side of the separated system (cf. Fig. 2b). K parameters included in the formula (1) take into account the rotational restraint stiffness of columns:

(2) $$K_c = \frac{EI_c}{L_c},\ K_i = \frac{EI_i}{L_i},$$

and rotational restraint stiffness of beams, in case of their rigid fastening in columns:

(3) $$\overline{K}_{ij} = K_{ij} = \frac{EI_{ij}}{L_{ij}},$$

where I_c, I_i, I_{ij} are respectively moments of cross sections inertia of columns and beams, L_c, L_i, L_{ij} are lengths of columns and beams.

In frames, where internal semi-rigid joints are used with stiffness S_{ij} (beam-to-column joints), K rotational restraints stiffness of columns support by beams should be determined according to the formula:

$$\overline{K}_{ij} = K_{ij}\frac{1}{1+\alpha_{ij}\dfrac{4K_{ij}}{S_{ij}}}, \tag{4}$$

where α parameter for the sway buckling mode of the unbraced steel frame can take the value equal to 1.5, to simplify the calculation [6].

For the adopted computational model of unbraced frame column, the value of buckling length coefficient is calculated according to the relation:

$$\mu = \sqrt{\frac{1-0.2(\eta_1+\eta_2)-0.12(\eta_1\cdot\eta_2)}{1-0.8(\eta_1+\eta_2)+0.6(\eta_1\cdot\eta_2)}}. \tag{5}$$

On the basis of specified η_i parameters the value of μ coefficient can also be obtained with the use of monograms available in literature.

3. METHOD BASED ON AISC GUIDELINES

Designating the buckling length L_{cr} of frames columns according to AISC [1] algorithm, just like in case of ECCS method, is performed for the separated part of frame, which consists of the examined column with elements directly connected to it (Fig. 3a). The analyzed frame part can also be replaced with the use of a single column, which is supported by semi-rigid joints on both ends (Fig. 3b).

The impact of column boundary conditions on its buckling length L_{cr} is also determined, as in the previously described procedure, with the use of η_i coefficients (Fig. 3b).

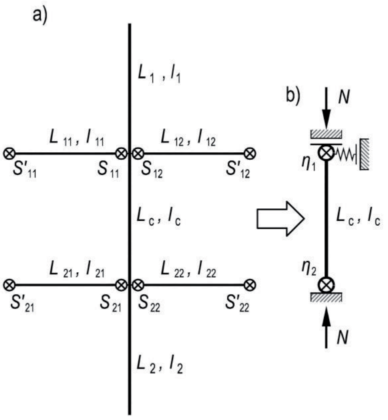

Fig. 3. Computation model: a) separated part of the load-bearing structure, b) equivalent model of the frame separated part

In order to obtain the values of these coefficients the following formula is used [1]:

(6)
$$\eta_i = \frac{K_c + K_i}{\sum_j \kappa_{ij} \cdot K_{ij}},$$

wherein K parameters are calculated according to relationships (2) and (3), whereas the coefficients κ_{ij}, in case of unbraced frames, are obtained from equations [5]:

(7)
$$\kappa_{ij} = \frac{\left(2 + S'_{ij}\right) \cdot S_{ij}}{12 + 4 \cdot \left(S_{ij} + S'_{ij}\right) + S_{ij} \cdot S'_{ij}}.$$

It is easy to notice that in the algorithm described here the κ_{ij} parameters take into account the flexibility of joints connecting beams to columns.

In the presented calculation model, the value of buckling length coefficient of unbraced frame column is calculated acc. to condition [1] :

(8) $$\frac{\eta_1 \cdot \eta_2 \cdot \left(\frac{\pi}{\mu}\right)^2 - 36}{6 \cdot (\eta_1 + \eta_2)} = \left(\frac{\pi}{\mu}\right) \cot\left(\frac{\pi}{\mu}\right).$$

Determination of the μ parameter from the above equation requires using numerical methods of solving nonlinear equations. Alternatively, in order to obtain the buckling length coefficient one can use the formula for the approximate value of this coefficient [15]:

(9) $$\mu = \sqrt{\frac{1.6 \cdot \eta_1 \cdot \eta_2 + 4 \cdot (\eta_1 + \eta_2) + 1.75}{\eta_1 + \eta_2 + 7.5}},$$

or also apply monograms which are found in literature, e.g. [1]. By means of these monograms, and knowing the values of parameters η_1 and η_2, it is easy to obtain the approximate value of coefficient.

4. METHOD BASED ON FEM ANALYSIS

Determining the buckling length L_{cr} can also be performed on the basis of structures stability analysis according to FEM approach. The basis of this type of analysis are the assumptions of the linear stability theory of structures without imperfections (perfect structures). For this reason, in the literature on the subject matter that approach is called the buckling analysis or the linear bifurcation analysis (LBA).
For the purpose of L_{cr} determination a well-known criterion of stability loss can be used:

(10) $$\det\left|\mathbf{K_E} + \lambda \mathbf{K_G}(\mathbf{N}_0)\right| = 0,$$

where $\mathbf{K_E}$ is a global matrix of elastic stiffness, independent from the normal forces, and $\mathbf{K_G(N_0)}$ is a global matrix, linearly dependent on the normal forces. Parameter $\mathbf{N_0}$ is the distribution of normal forces from $\mathbf{P_0}$ load, whereas parameter $\boldsymbol{\lambda}$ is an eigenvalues of equation (10) – the load increase factor defined by the equation:

(11) $$\lambda = \frac{\mathbf{P}}{\mathbf{P}_0} = \frac{\mathbf{N}}{\mathbf{N}_0}.$$

Knowing the eigenvalues **λ** of equation (10) one can easily designate (from equation (12)) the eigen vector **δ** allowing to obtain the modes of structure stability loss:

$$\left|\mathbf{K}_{\mathbf{E}} + \lambda \mathbf{K}_{\mathbf{G}}(\mathbf{N}_0)\right| \boldsymbol{\delta} = 0 . \tag{12}$$

Due to adopting the linear relation between the normal forces in elements and the expressions in the elements geometric stiffness matrixes for the load-bearing structures with greater resistance to instability (i.e. in braced frames) the estimation of critical load $\mathbf{P_{cr}}$ with the use of eigenvalues method of stiffness matrix can be incorrect. Avoiding this kind of computational errors is possible by inserting additional, intermediate nodes in the compressed elements [14].

Among the set of parameters **λ** designated on the basis of criterion (10), the smallest positive value of that parameter has to be chosen. This value is the searched load multiplier which in case of unbraced steel frames corresponds to the sway buckling mode of load-bearing structure. Values of critical forces $\mathbf{N_{cr}}$ in elements determined on that basis, can be used to designate the buckling length coefficients μ in particular columns of frame according to the formula:

$$N_{cr} = \frac{\pi^2 E I_c}{(\mu L_c)^2} \quad \rightarrow \quad \mu = \frac{\pi}{L_c}\sqrt{\frac{E I_c}{N_{cr}}} . \tag{13}$$

This procedure allows to take into account all the factors determining the buckling length of columns in frames. Thanks to this approach, the obtained results may slightly differ from more accurate solutions that can be determined using more precise methods, e.g. determinant method [14].

5. COMPARATIVE ANALYSIS

On the basis of presented methods, the buckling length coefficients μ of columns for a certain frame were designated. The solution of stability issue using the eigenvalue method of stiffness matrix in FEM approach was performed using SOFISTIK program [17].

The subject of the analysis is a two-aisle, three-storey unbraced steel frame (Fig. 4a). It was assumed, that internal joints (beam-to-column joints) are modelled with the use of linear models of semi-rigid joints with certain, initial stiffness S_j, whereas hinged joints were adopted as supports.

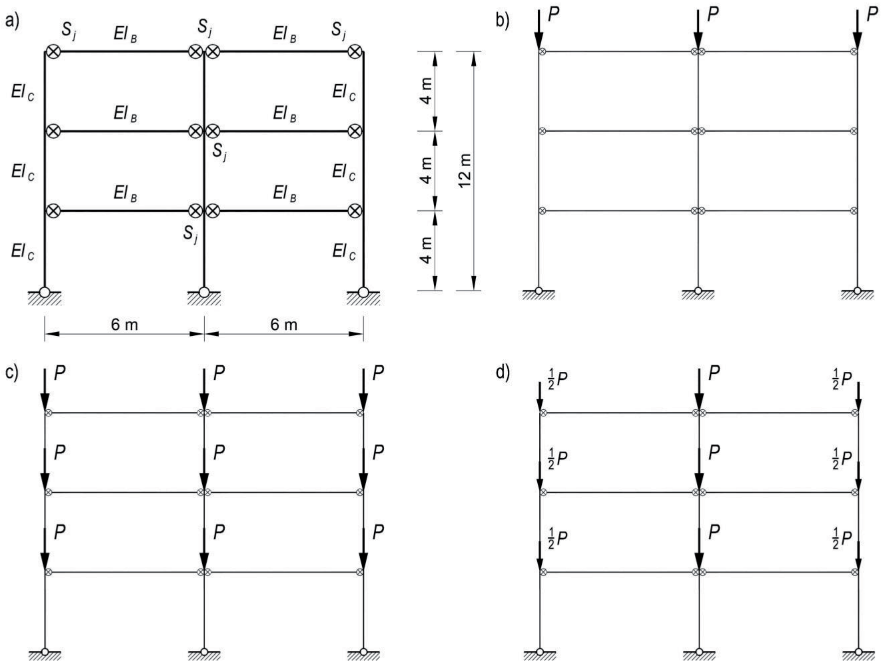

Fig. 4. Analyzed frame: a) frame model, b) load case no. 1, c) load case no. 2, d) load case no. 3

Data: bending stiffness of beams cross sections EI_B = 20000 kN·m^2, bending stiffness of columns cross sections EI_C = 10000 kN·m^2, semi-rigid joints stiffness S_j = 50000 kN·m/rad, P = 100 kN.

The buckling length coefficients μ of columns for the adopted load-bearing structure were designated with the use of formulas (5) and (8), as well as FEM analysis and condition (13). Three different load cases in the form of forces P acting on the frame nodes were assumed (Fig. 4b ÷ d).

The calculation results were presented in Figure 4 as the searched values μ. Symbol μ_{EC} is assigned to values specified according to equation (5), whereas symbol μ_{AISC} refers to the coefficient values determined using formula (8). The coefficients labeled as μ_1, μ_2 and μ_3 present the values determined on the basis of FEM analysis along with equation (13) respectively for 1, 2 and 3 load cases (see Fig. 4b ÷ d).

Based on the results comparison it can be seen that:

- the smallest, average differences appear between the coefficients obtained acc. to ECCS, AISC and the method based on stability analysis in FEM approach for the first load case (Fig. 4b),

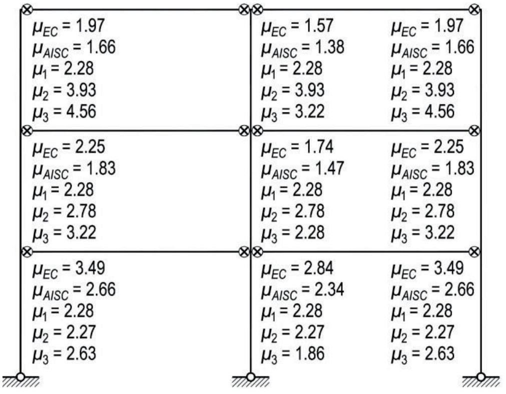

Fig. 5. Results summary of buckling lengths coefficients

- matching the results obtained with the use of the first two methods it is easy to notice that buckling length coefficients determined acc. to ECCS reach larger values than the AISC method. The differences vary in the range from 18% to 28%. However, wherein the larger differences in values refer to the coefficients assigned to the exterior columns of the frame,
- comparing the calculations results according to ECCS and AISC methods with the computations performed on the basis of FEM analysis in the first case (Fig. 4b) it can be seen that the smallest differences in coefficients values were obtained here for the central column of the lowest storey (μ_{EC} = 2.84 and μ_1 = 2.28) and they amount to 20%, and also (μ_{AISC} = 2.34 and μ_1 = 2.28) with 3% of difference. However, the biggest disagreements in analogous comparison refer to the central column of the highest storey. They total respectively 45% and 82%,
- much larger disagreements can be noticed comparing the relevant results from the FEM computation for 2 and 3 load case (Fig. 4c and 4d) with calculations results based on ECCS and AISC guidelines

(this applies especially to results obtained for the columns of the highest storey). From the comparison of appropriate buckling length coefficients of the exterior columns (μ_{EC} = 1.97 and μ_3 = 4.56) we get the difference approx. 135%, and also (μ_{AISC} = 1.47 and μ_3 = 4.56) the disagreement in values equals approx. 210%.

These large differences obtained from comparison of μ coefficients of analyzed frame are mainly determined by the fact that calculations according to ECCS and AISC do not take into account the frame loading, while the procedure based of FEM takes into consideration normal forces in elements caused by current loading, as well as the influence of these forces on members geometrical stiffness.

Furthermore, in methods based on ECCS and AISC guidelines a selected part is analyzed, in some cases a small section of a frame, not the entire structure. This fact causes some inaccuracies in assessing the influence of further located elements, as well as the conditions of mounting the structure in supports on the buckling coefficients.

In summary, it can be said that the buckling length L_{cr} of frames columns obtained on the basis of paragraph 2 and 3 can be burdened with significant errors. It can be particularly well seen comparing the computations results of μ coefficients according to formulas (5) and (8) with results performed with the use of FEM and condition (13) for 3 – i.e. real load case.

6. NUMERICAL EXAMPLE

The subject of the analysis is a three-aisle, three-storey unbraced steel frame, in which internal joints (beam-to-column), as well as external joint (column bases) are semi-rigid joints (Fig. 6). In the presented example, unsymmetrical loading scheme of the frame was adopted in the form of uniformly distributed load of beams with different values.

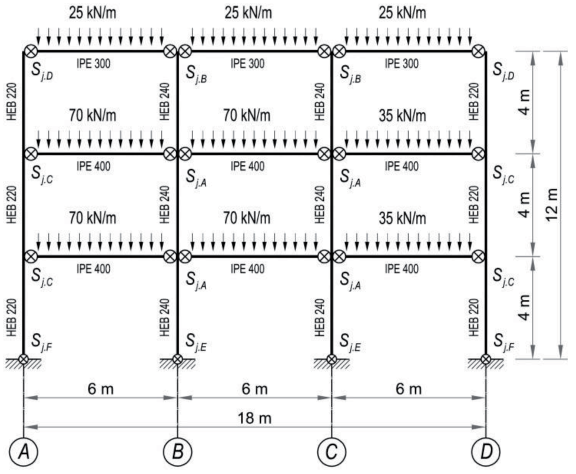

Fig. 6. Static scheme of the analyzed frame

Data: steel S355, steel elasticity modulus $E = 210$ GPa.

Internal and external frame joints, which are shown in Figure 7, were designed according to EC3 [8, 18]. Calculations of these joints in the IdeaStalica software [11] were also performed, in which the initial joints stiffnesses were determined. These stiffnesses are equal respectively: $S_{j.A}$ = 44200 kN·m/rad, $S_{j.B}$ = 33500 kN·m/rad, $S_{j.C}$ = 56500 kN·m/rad, $S_{j.D}$ = 19100 kN·m/rad, $S_{j.E}$ = 11000 kN·m/rad, $S_{j.F}$ = 9800 kN·m/rad. Due to the fact in the analyzed structure the nodes bending moments do not exceed 2/3 of the design resistances of the joints, the appropriate calculations were made with the use of the linear models of semi-rigid joints, with taking into account the initial stiffnesses [8].

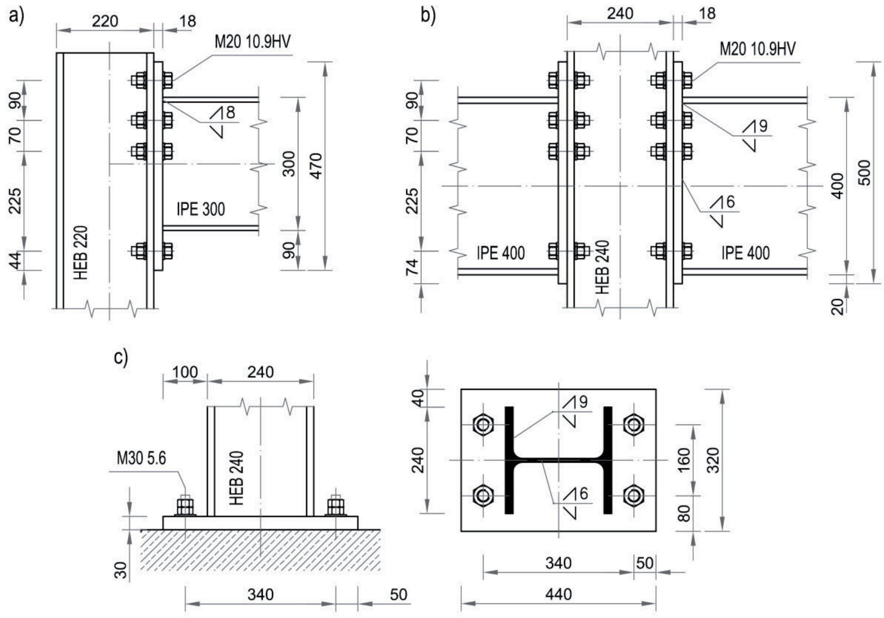

Fig. 7. Joints of the steel frames: a) single-sided joint with stiffness $S_{j.D}$, b) double-sided joint with $S_{j.A}$, stiffness, c) column base with stiffness $S_{j.E}$

For the assumed static scheme of frame, calculations were carried out to determine the columns coefficients μ according to methods described in paragraphs 2, 3 and 4.

The results were presented in Figure 8 as coefficients values μ of the appropriate columns. Symbol μ_{EC} and μ_{AISC} are assigned to values obtained acc. to respective equations (5) and (8), whereas coefficients labelled as μ_{λ} present values determined using the FEM analysis and condition (13).

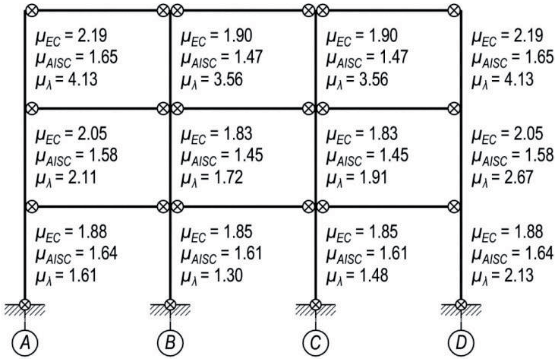

Fig. 8. Results summary of buckling lengths coefficients μ

The calculation results presented for the real steel frame with semi-rigid joints clearly demonstrate that the method type used significantly influences values of the columns buckling length coefficients. On the basis of the obtained data analysis it easy to see, that:

- coefficients calculated acc. to ECCS guidelines reach higher values, than coefficients values obtained according to the AISC algorithm. The differences in these values, on the first storey equal approximately 13%, on the second storey 20-23%, and on the third about 22-25%,
- large disagreements appear between coefficients μ which were determined according to ECCS and AISC methods and coefficients were designated on the basis of frame buckling analysis – for example, differences in the most loaded column of the first storey (column in B axis) reach here respectively (μ_{EC} = 1.85 and μ_{λ} = 1.30) about 30%, and also (μ_{AISC} = 1.61 and μ_{λ} = 1.30) 19%, wherein, coefficients designated here according to equations (5) and (8) have larger values than coefficients determined acc. to formula (13),
- the largest differences in results appear in columns of the highest storey – in combination with coefficients for column in B axis we get the differences reach approx. (μ_{EC} = 1.9 and μ_{λ} = 3.56) 87%, and also (μ_{AISC} = 1.47 and μ_{λ} = 3.56) 142%.

Presented computational results of the analyzed frame confirmed conclusions presented in section 5. On the basis of the analyzed frame example in the paper the practical conclusions can be drawn.

1) Determination of the buckling length coefficients of columns with the use of simplified methods, where a selected part of the static scheme is analyzed and the loading influence is neglected can lead to significant errors.

2) In the presented example of the real frame, calculated according to ECCS and AISC guidelines values of μ coefficients cause a significant reduction of the columns buckling lengths L_{cr} in many places.
3) That type of error is quite dangerous because using smaller than real buckling lengths L_{cr} in resistance calculations leads to unjustifiable overestimation of the column resistance.
4) There is currently a large number of systems supporting the construction design process which allow to perform the stability analysis using FEM. Skillful usage of these programs allows engineers to determine the buckling lengths of steel frame columns more accurately.

7. SUMARY AND CLOSSING REMARKS

The paper presents the issue of determining the buckling length L_{cr} of columns in unbraced steel frames with semi-rigid joints.

In the first part, three different methods of determining μ coefficient were presented. The first one is based on ECCS guidelines - a supplement to European standards concerning steel frames design. On the other hand, the second method is based on AISC outlines and it is a recommended way to determine the L_{cr} length acc. to American standards. Both calculations methods use a similar approach which involves fastening conditions study of the considered column. It is done on the basis of the frame part analysis which completely omits the fact of normal forces occurrence. The third method of coefficients μ calculation presented in the paper uses the results from stability analysis of the whole structure with FEM approach. This way allows performing calculations taking into account the entire structure, as well as the appropriate load model, and thus distribution of normal forces within the structure.

The second part of the paper presents certain computational examples of steel frames, where all three described methods were used to determine the buckling length coefficient of columns. Comparison of the obtained solutions showed large differences between received results. Particularly large divergences were noticed in the list of coefficients obtained according to ECCS and AISC guidelines with the results determined using the method based on global stability analysis.

From among the three ways of coefficient μ determination described in the paper, performing the buckling analysis and using formula (13) allows to obtain the most accurate results. Thus, the mentioned differences in results mean that the methods of determining the μ coefficient according to European and American recommendations are inaccurate. This is due to the simplifications which were made while creating these methods, i.e. omission of axial forces influence and limitation of the structure analysis to a separated section in calculations.

Although the above-mentioned conclusions refer directly to the examples analyzed in the paper, it should be assumed that they are of more general nature and concern most of the unbraced frames used in practice.

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O DŁUGOŚCI WYBOCZENIOWEJ SŁUPÓW NIESTĘŻONYCH RAM STALOWYCH Z WĘZŁAMI PODATNYMI

Słowa kluczowe: *ramy o węzłach podatnych, stateczność, długość wyboczeniowa słupów*

STRESZCZENIE:

W pracy przedstawiono zagadnienie stateczności niestężonych ram stalowych o węzłach podatnych, ze szczególnym zwróceniem uwagi na problem określenia długości wyboczeniowej L_{cr} słupów tych ram.

W pierwszej części pracy przedstawiono trzy metody określania długości wyboczeniowej ($L_{cr} = L_c \cdot \mu$). Pierwsza metoda bazuje na wytycznych ECCS i jest uzupełnieniem europejskich norm w zakresie projektowania ram stalowych. Z kolei druga metoda opiera się na wytycznych AISC i jest zalecanym sposobem określania długości L_{cr} przez amerykańskie normy.

Obydwa sposoby obliczeń wykorzystują bardzo podobne podejście, polegające na analizie warunków zamocowania rozpatrywanego elementu (słupa), przy czym odbywa się to na podstawie analizy wyizolowanego fragmentu ramy, w którym całkowicie pomija się fakt występowania sił normalnych. Trzecia, prezentowana w pracy metoda obliczeń długości wyboczeniowych słupów wykorzystuje wyniki analizy stateczności w ujęciu MES (Metody Elementów Skończonych). W metodzie tej wymagane jest przeprowadzenie globalnej analizy stateczności ustroju, uwzględniając przy tym odpowiedni model obciążenia, a więc i rozkład sił normalnych w konstrukcji.

W drugiej części pracy przedstawiono pewne przykłady liczbowe ram stalowych, w których wyznaczono współczynniki długości wyboczeniowych μ słupów za pomocą wszystkich trzech prezentowanych metod. W pierwszym przykładzie poddano analizie dwunawową, trójkondygnacyjną ramę, uwzględniając w obliczeniach trzy różne przypadki obciążeń. W drugim przykładzie liczbowym wyznaczono poszukiwane wartości współczynników μ słupów dla trójnawowej, trójkondygnacyjnej ramy stalowej, w której węzły wewnętrzne (rygiel-słup), jak również węzły zewnętrzne (podstawy słupów) są węzłami podatnymi.

Porównując ze sobą otrzymane rozwiązania wykazało znaczne różnice pomiędzy uzyskanymi wynikami. Szczególnie duże rozbieżności dostrzeżono przy zestawieniach współczynników otrzymanych wg wytycznych ECCS i AISC z rezultatami określonymi przy użyciu metody opartej na globalnej analizie stateczności. Tak duże rozbieżności otrzymane z porównania wartości współczynników μ analizowanych ram są w głównej mierze spowodowane tym, że w obliczeniach wg wytycznych ECCS i AISC nie uwzględnia się wpływu obciążenia ramy na wartości poszukiwanych współczynników, podczas gdy w metodzie opartej na analizie stateczności wpływ obciążenia ramy jest uwzględniany „automatycznie" (algorytm metody wymaga zdefiniowania obciążenia). A więc, stosowanie modeli obliczeniowych w celu określenia wartości współczynników długości wyboczeniowych, w których pomija się obciążenia ustroju powoduje, że w obliczeniach nie uwzględnia się aktualnej sztywności geometrycznej elementów ramy (w szczególności dotyczy to słupów, w których występują na ogół znaczne siły ściskające). Konsekwencje tego widać w analizowanych przykładach liczbowych, w postaci niedoszacowanej, w wielu miejscach zaniżonej wartości współczynników długości wyboczeniowej słupów.

Ponadto, w metodach opartych na wytycznych ECCS i AISC stosownym obliczeniom poddaje się wydzielony, w pewnych przypadkach niewielki fragment konstrukcji, a nie cały ustrój. Powoduje to pewne niedokładności w ocenie wpływu dalej położonych elementów, jak również warunków zamocowania konstrukcji w podporach na wartość współczynnika μ rozpatrywanego elementu. Spośród trzech opisanych w pracy sposobów określania współczynnika μ, wykonanie globalnej analizy wyboczeniowej oraz użycie wzoru (13) pozwala uzyskać najbardziej dokładne wyniki.

Tak więc wspomniane różnice oznaczają, że sposoby wyznaczania współczynnika μ wg europejskich oraz amerykańskich zaleceń są mało dokładne. Wpływ na to mają uproszczenia, które poczyniono na etapie tworzenia tych metod, a więc całkowite pominięcie w obliczeniach wpływu sił osiowych oraz ograniczenie analizy ustroju do wydzielonego fragmentu konstrukcji.

Chociaż wymienione wnioski bezpośrednio odnoszą się do analizowanych w pracy przykładów, to należy sądzić, że mają one bardziej ogólny charakter i dotyczą większości stosowanych w praktyce ram niestężonych.

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