WARSAW UNIVERSITY OF TECHNOLOGY Index 351733
FACULTY OF CIVIL ENGINEERING
COMMITTEE FOR CIVIL AND WATER ENGINEERING
POLISH ACADEMY OF SCIENCES ISSN 1230-2945

DOI: 10.24425/ace.2021.137168

ARCHIVES OF CIVIL ENGINEERING

Vol. LXVII ISSUE 2 2021

s. 275–292

Research paper

Multi-stage analysis of reliability of an example masonry construction

J. Zięba[1], L. Buda-Ożóg[2], I. Skrzypczak[3]

Abstract: The safety of the masonry structure is determined by the value of the partial factor used, which is influenced by many factors. The variability of these factors determines obtaining significant differences in the load levels of various masonry structures. Hence, the analysis of masonry structures should be carried out taking into account a sufficient range of variability of factors affecting its safety. The article presents a multi-stage safety analysis of an exemplary brick masonry column. For the construction, the relationship between partial factors used for interactions in different configurations and factors for the masonry compressive strength was examined. The analyses consisted in determining the reliability index β with the Monte Carlo method. The article presents the results of experimental tests carried out on a real construction, as well as the results of FEM numerical simulations.

Keywords: masonry structures, experimental research, safety, reliability, Monte Carlo simulations

[1] MSc., Eng., Rzeszow University of Technology, Faculty of Civil Engineering, Department of Building Structures , Poznańska 2, Rzeszów 35-084, Poland, e–mail: j.zieba@prz.edu.pl, ORCID: https://orcid.org/0000-0003-1800-5697

[2] DSc., PhD., Eng., Rzeszow University of Technology, Faculty of Civil Engineering, Department of Building Structures , Poznańska 2, Rzeszów 35-084, Poland, e–mail: lida@prz.edu.pl, ORCID: https://orcid.org/0000-0002-1205-1345

[3] DSc., PhD., Eng., Rzeszow University of Technology, Faculty of Civil Engineering, Department of Geodesy and Geotechnics, Poznańska 2, Rzeszów, 35-084, Poland, e-mail: izas@prz.edu.pl, ORCID: https://orcid.org/0000-0003-0978-3040

1. Introduction

Masonry structures are and most likely will continue to be the most-common in general construction. Over the centuries, craftsmanship and masonry have evolved, and the 20th and 21st centuries brought a real revolution in the theory of construction, materials and technology for erecting masonry structures, putting them on par with steel or reinforced concrete. The development of material engineering also in the field of materials used for making masonry elements and mortars made it common to use masonry walls as load-bearing walls. Due to their importance, masonry structures have been the subject of many scientific considerations and research for years. The area of interest are topics related to the use of modern numerical and experimental methods in the analysis of masonry structures. Compressed brick masonry and columns are the most important elements of many buildings made in masonry technology. The popularity of this solution and technological progress oblige to popularize more and more accurate and effective methods of analysing masonry structures. In order to obtain reliable information on material parameters and structure responses to interactions, advanced fully probabilistic analysis methods based on the theory of structure safety and reliability are increasingly being used. When designing a structure, be aware that almost all factors are uncertain and nothing can be predicted with absolute certainty. This basic fact leads to the idea of probabilistic and stochastic treatment of problems in every possible field. In civil engineering, most problems are solved by a deterministic approach that displaces difficult and complex stochastic solutions. A number of simplified methods have also been developed in an attempt to combine deterministic and stochastic approaches, such as the semi-probabilistic concept of structural safety that underlies standard recommendations. Nevertheless, fully probabilistic approaches guarantee the most efficient solutions and provide accurately estimated reliability. Reliability and risk are key issues in a fast-growing world. The safety of the masonry structure is determined by the value of the partial factor used, which is influenced by many factors, including: the category of the masonry element being the indicator of the quality of workmanship; type of mortar, class execution of work. The variability of these factors determines obtaining significant differences in the load levels of various masonry structures. Hence, the analysis of masonry structures should be carried out taking into account a sufficient range of variability of factors affecting its safety.

2. Basis of masonry structural design

2.1. Reliability of construction

Each designed building structure is intended to fulfill specific tasks or perform intended functions. Construction practice requires establishing a reliable border between the allowable and unacceptable implementation of the object. This limit is called the limit state. The most often considered limit state is the ultimate limit state, which, if not exceeded, determines the safety of the structure. The concept directly related to the term 'structural safety' is its 'reliability'. Reliability as opposed to safety is a measurable feature. Correct designing is based on the use of a set of relevant standards, specifying not only the specifics of the type of structure being designed, in this case masonry construction, but also general principles. Traditional structural design uses deterministic values of design parameters. On the other hand, the safety of structures related to the variability of construction parameters is ensured by conservative selection of their values and by the use of safety factors in the equations of limit states. Probabilistic analysis of structures is a kind of extension of deterministic analysis by using random variables to represent the values of structural parameters. Annex C to the PN - EN 1990 [1] standard presents reliable methods for checking the reliability of structures and discusses their statistical and probabilistic algorithms. The most popular methods, simplified level II probabilistic methods, allow quantitative assessment of structural reliability. The most commonly used, popular measure of safety in partial factor method is the reliability index β. In order to determine the reliability index, various methods are used, among which can be distinguished: the FORM (First Order Reliability Method) method and the SORM (Second Order Reliability Method) method [2], referred to in the literature as moment methods. For the FORM method, relationships between the values of the reliability index β and the values of partial safety factors [3] were formulated, which was the basis of design standards. Annex B of the PN-EN 1990 standard gives recommendations for managing the reliability of construction works. In order to differentiate reliability, consequence classes have been established. The defined in [1] consequence classes CC1-CC3 correspond to the reliability classes: RC1 - RC3. Recommended minimum values of the reliability index β, related to reliability classes are also given in PN-EN 1990 [1] – Table 1. The values of the index corresponding to the RC2 class of reliability are significant. Generally, the use of a set of coefficients in force in the Eurocodes determines ensuring structure reliability at the RC2 class level.

Table 1. Minimum values of the reliability index for ultimate limit states and maximum probability of failure [1]

Reliability class	Minimum values β / Maximum values P_f	
	1 year reference period	50 years reference period
RC3	$\beta = 5{,}2$; $P_f \cong 9{,}9 \cdot 10^{-8}$	$\beta = 4{,}3$; $P_f \cong 8{,}5 \cdot 10^{-6}$
RC2	$\beta = 4{,}7$; $P_f \cong 1{,}3 \cdot 10^{-6}$	$\beta = 3{,}8$; $P_f \cong 7{,}1 \cdot 10^{-5}$
RC1	$\beta = 4{,}2$; $P_f \cong 1{,}2 \cdot 10^{-5}$	$\beta = 3{,}3$; $P_f \cong 4{,}8 \cdot 10^{-4}$

In principle, fully probabilistic (level III) methods allow to obtain a correct solution to the problem of assessing the reliability of building structures. However, these methods are not the most popular methods. The reason is the lack of complete statistical data necessary for probabilistic analyses regarding parameters generating load effects and load capacity. Structure reliability in level III methods can be calculated by the following methods: analytical integration, possible in a few simple cases; numerical integration, effective when the number of state variables n ≤ 5 ÷10; Monte Carlo simulation in the random field [4]. The Monte Carlo method is the basic simulation method for determining reliability of a structure. The Polish mathematician S. Ulman worked on developing the scientific basis for this method. In the 1940s, Freudenthal described the assumptions of the Monte Carlo method [5]. Information on the current state of knowledge in this field can be found, among others, in the study [6]. Monte Carlo simulation consists in calculating structure responses to interactions for various values of input variables. If all characteristics and analysis parameters are determined, the simulation is referred to as deterministic. In a situation where at least one of the parameters is variable - random, the analysis is referred to as random simulation, and the analytical model as a stochastic model. The Monte Carlo method is considered to be the technique with the highest accuracy among all methods requiring knowledge of the probability distribution of random variables. The disadvantages of the simulation approach include the considerable time-consuming calculations, little control over the generation product for complex engineering structures, and the difficulty of specifying the optimal, quickly convergent set of generated implementations without using techniques to reduce the population of random variable implementations. In classic construction cases it is recommended to carry out 10^6 do 10^9 simulations in order to obtain satisfactory results of the structure's response to interactions. A significant number of necessary simulations means that this method requires the use along with other methods of analysis that were formulated to reduce the sample population. Reduction techniques save computers time and computing power, which is why they are widely used.

2.2. Analysis of compressed masonry constructions

Masonry is a construction material made of masonry units arranged in a certain way, permanently connected with a mortar. The basic case occurring in masonry constructions is the compressive strength perpendicular to the supporting mortar. The analyses carried out in this article relate to the most common case, i.e. the behavior of the masonry structure under vertical compressive load. The strength of the masonry depends on the strength, deformability and geometry of both masonry units and mortar, which are components of the masonry. An important feature of masonry elements is also much greater compressive strength compared to tensile strength. The relation of the values of these strengths is the basis for the classification of the masonry as a quasi-fragile material. For brittle materials, it can be assumed that the dependence of stresses on deformations has a linear course up to a value of 33% compressive strength and tensile strength [7]. Experiment testing best demonstrates the behavior of the masonry under load. According to the general recommendations of PN-EN 1996-1-1 [8], the characteristic compressive strength of masonry f_k should be determined based on the results of testing of fragment of masonry. In the calculations of the structure, the problem is simplified by treating the masonry as a homogeneous material with equal strength properties at every point. Therefore, analytical relationships estimating the value of masonry compressive strength, taking into account the strength of its components, are introduced. The national annex of PN-EN 1996-1-1 [8] introduces specific values of empirical coefficients and proposes a general formula to present in four variants. The general form of the formula for masonry compressive strength f_k given in the main part PN-EN 1996-1-1 [8] is as follows:

$$f_k = K {f_b}^{\alpha} {f_m}^{\beta} \tag{2.1}$$

where: K, α, β – coefficients determined empirically.

The analysis of masonry structures under compressive load should be carried out according to standard algorithms divided into: masonry loaded mainly vertically and masonry subjected to concentrated loads. Design value of the vertical resistance of a masonry wall or column should be determined from the formula:

$$N_{Rd} = \beta A_b f_d \tag{2.2}$$

where: β - enhancement factor for concentrated loads; A_b – load surface, f_d – design compressive strength of masonry in the direction being considered

The analysis of masonry in the main stage of work is the subject of many domestic and foreign scientific publications: [9; 10; 11]. The value of the vertical resistance of the masonry structure should be determined taking into account all types of unfavorable deviations of the material properties from the characteristic value as well as the uncertainty of the calculation models used in PN-EN 1996-1-1 [8]. The design compressive strength of masonry is therefore determined from the relationship:

$$f_d = {}^{f_k}/_{\gamma_M} \tag{2.3}$$

$$\gamma_M = \gamma_m \cdot \gamma_{Rd} \tag{2.4}$$

where: γ_m – partial factor for materials; γ_{Rd} – partial factor including uncertainties about geometry and modelling.

The basis for determining the γ_m and γ_{Rd} partial factors and their combinations are the recommendations of the PN-EN 1990 [1] standard. Since the masonry is a structural element burdened with many unknowns, determining the value of γ_M for the masonry is particularly difficult and also subject to high uncertainty. The strength of the wall is controlled admittedly on test elements constituting a fragment of the wall, but the basic variable for determining the characteristic strength of the masonry is the compressive strength of the specific shape and dimensions of the masonry elements and cylindrical samples of the mortar. As it is a factor determining, among others, the quality of embedded materials, it is important to properly classify masonry units and mortar as well as the masonry constructions. Masonry units should be primarily assigned to the appropriate category. The category of masonry units is directly related to the control of their production. The belonging of the masonry units to the appropriate category lies with the manufacturer, who is obliged by standard provisions to apply the factory production control and to adopt an appropriate system for assessing the compliance of masonry elements with the declared values. In the case of mortars, from the point of view of the partial factor, the division into designed mortars and prescribed mortars is important. PN-B-03002: 1999 [12] introduced the concept of the class execution of works. Both today's recommendations for determining the computational value of masonry compressive strength are based on the quality of the masonry. The current PN-EN 1996-1-1 [8] standard distinguishes between

2 classes execution of works: class A (rigorous approach to the construction of the wall) and class B. The designer of the structure decides about the choice of the class of masonry works. The values of γ_m adopted in the next, no longer binding, PN-B-03002:2007 [13] standard are also recommended values in the currently binding provisions of PN-EN 1996-1-1 [8]. The values of the partial factor adopted in the National Annex PN-EN 1996-1-1 [8] are the values corresponding to the second and third columns of the table of partial safety factors values given in the main part PN-EN 1996-1-1 [8] - Table 2.

Table 2. The relevant values of the partial factor for materials γ_M according PN-EN 1996-1-1 [8]

Material	Class				
	1	2 (A)	3 (B)	4	5
Masonry made with: units of category I, designed mortar	1,5	1,7	2,0	2,2	2,5
Masonry made with: units of category I, prescribed mortar	1,7	2,0	2,2	2,5	2,7
Masonry made with: units of category II, any mortar	2,0	2,2	2,5	2,7	3,0

The values of γ_M adopted in the main part PN-EN 1996-1-1 [8] depend on five categories execution of works, without specifying the requirements that each of these classes should meet. Annex A gives general recommendations to be followed when class differentiation, e.g. presence of employed by the contractor and independent appropriately qualified personnel to control work, assessment of mortar and filler concrete properties on the construction site, mortar mixing method, dosing of ingredients. Referring to the provision in which the wall strength values are specified in national annexes, publications are known that present the analysis of the wall computational strength on a European scale [14].

3. Case study – reliability determined by the Monte Carlo method

Currently, there are many computer programs that allow the use of the Monte Carlo simulation method in combination with the FEM finite element method. In this work, the Monte Carlo method assumptions were used and simulations were carried out in the Atena, Sara and FREeT program package - programs offered by Cervenka Consulting [15]. Similar analyses for reinforced concrete elements were carried out in [16]. In order to carry out the simulation, work with the program package should be started by making a deterministic numerical model of the analysed structure in Atena. The discrete numerical model generated and calculated is subject to further analysis in SARA and FREeT programs. The FREeT and ATENA programs are integrated into the SARA software package to enable a fully probabilistic non-linear structure analysis. The entire process of non-linear stochastic simulation is user-controlled using commands and interfaces available in SARA. A diagram of the multi-stage reliability analysis of an example masonry structure is shown in the drawing - Fig. 1.

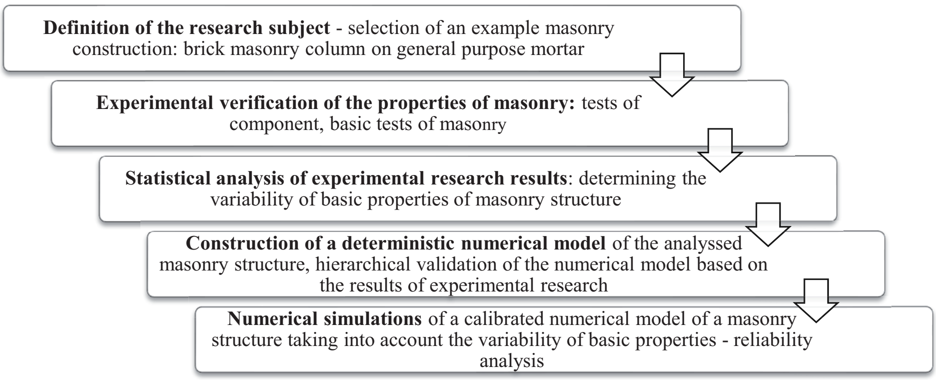

Fig. 1. Multi-stage analysis of reliability of masonry structures by simulation methods - a scheme of conduct

3.1. Experimental verification of properties of masonry

As part of this chapter, the results of experimental tests of brick masonry column on general purpose mortar. The research was carried out in the Construction Laboratory of the Faculty of Civil Engineering of the Silesian University of Technology. The purpose of experimental research was to determine the properties of masonry elements and mortars for hierarchical validation of the numerical FEM model. Tests of structures on a real scale were carried out in accordance with PN-EN 1052-1: 2000 [17]. In the tests of the main part a research model of a full brick masonry column on mortar class M5 with dimensions of 250x250 mm and height of 2615 mm was used - six identical models were tested: five models under uniform load, one model tested cyclically – Fig. 2. In order to obtain the value of the properties of the materials used in the tests, necessary to describe the FEM model of the structure, in the first stage of the experimental tests, compressive and tensile strength of mortar were tested, and masonry elements were tested. Laboratory tests of mortars were carried out in accordance with the requirements of PN-EN 1015-11: 2001 [18].

a) b)

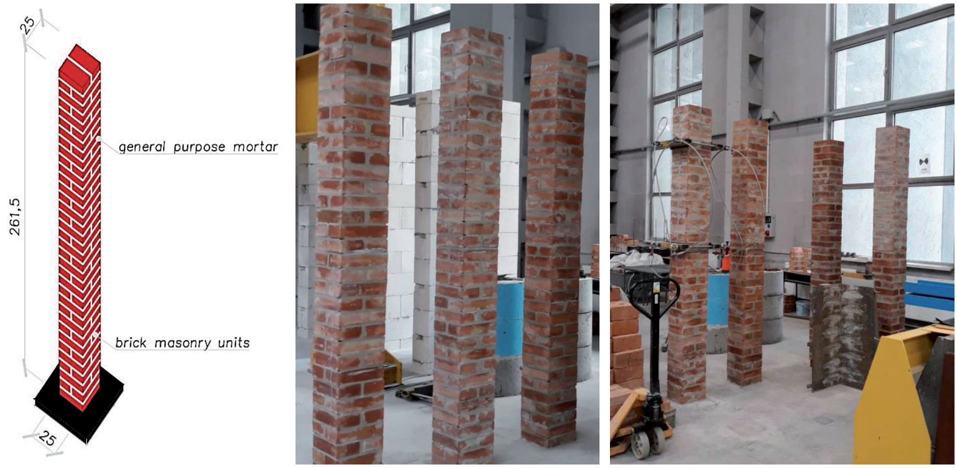

Fig. 2. Shape and dimensions of the brick masonry column model in compression tests: a) shape and dimensions of the column model b) made column models

Compressive strength tests of masonry units were carried out in accordance with the PN-EN 772-1:2003 [19] standard. In the main columns testing, all models were tested in the DrBM-600 testing machine with manual control of load increase and indication accuracy 0.001 kN – Fig. 3. The tests determined the compressive strength of the masonry in a direction perpendicular to the support mortar on the basis of the strength results of test models loaded up to destruction – Table 3.

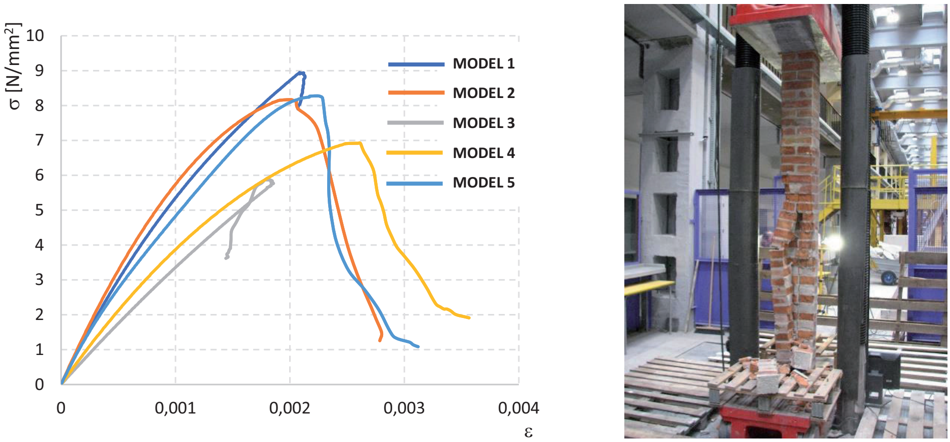

Fig. 3. Relationship stress – strain during compression of solid brick columns under uniform load and damaged column model

Full contact between the models and the testing device was provided with the entire upper and lower surfaces. A modern system for three-dimensional deformation measurements Aramis 6M was used to measure deformations on the surface of samples [20].

Table 3. Summary of test results for brick column models

Brick column model		Maximum destructive force [kN]		Compressive strength of masonry [MPa]		Modulus of elasticity [MPa]		Poisson's ratio [-]	
		$F_{max,i}$	F_m	$f_{k,max,i}$	$f_{k,m}$	$E_{y,max,i}$	$E_{y,m}$	$\nu_{xy,i}$	$\nu_{xy,m}$
1		558,48	477,11	8,94	7,63	6337	5296	0,23	0,27
2		510,58		8,17		6808		0,30	
3		366,78		5,87		3493		0,32	
4		432,41		6,92		4271		0,30	
5		517,28		8,28		5573		0,22	

In the case of a full brick masonry column loaded cyclically, the maximum force reached the value of 430.9 kN. Based on the measured displacement values, the masonry modulus of elasticity was determined, which in this case amounted to 5892 MPa. This value is greater than the average value of 5296 MPa obtained on the basis of test results of the five remaining columns on which uniform load was applied. The results obtained from the tests were subjected to statistical evaluation - Table 4. For this purpose, among others, the classical method was used, in which the average value of the parameter tested and the standard deviation estimator were determined. The application of the classical method required the elimination of questionable results, hence the Q-Dixon test was used to evaluate the data.

Table 4. Value estimators and standard deviation of compressive strength, modulus of elasticity and Poisson's ratio for brick masonry column

Model properties	Average value	Standard deviation	Coefficient of variation
f_k [MPa]	7,63	1,23	16,1 %
E_y [MPa]	5296	1392	26,3 %
ν_{xy} [-]	0,27	0,04	16,1 %

The results of the conducted tests and the observations resulting from them were an impulse to carry out further statistical and reliability analyses.

3.2. Deterministic numerical model of the masonry construction

The next stage of analysis of the selected masonry structure, finally leading to the determination of reliability by simulation methods, was to build a reliable FEM model. The hierarchical validation of the model was based on the results of experimental research, both of the masonry components and the masonry itself, as well as available literature. For the FEM modeling of masonry structures, the ATENA Static + GID software package from Cervenka Consulting was used. This software has already been used in modeling of masonry structures, among others by the authors: Jasiński, Mazur [21; 22]. In the numerical analysis of the masonry, the material model implemented in the program was used: "CC3DNonLinear Cementitious2", described in detail in the literature [23]. For the brick masonry column, a numerical model was built using the exact heterogeneous modeling strategy, which considers the exact modeling of masonry and mortar and their appropriate connection using contact material, so-called interface. The numerical model of the analysed masonry structure - Fig. 4, subjected to hierarchical validation, was used to assess reliability by simulation methods.

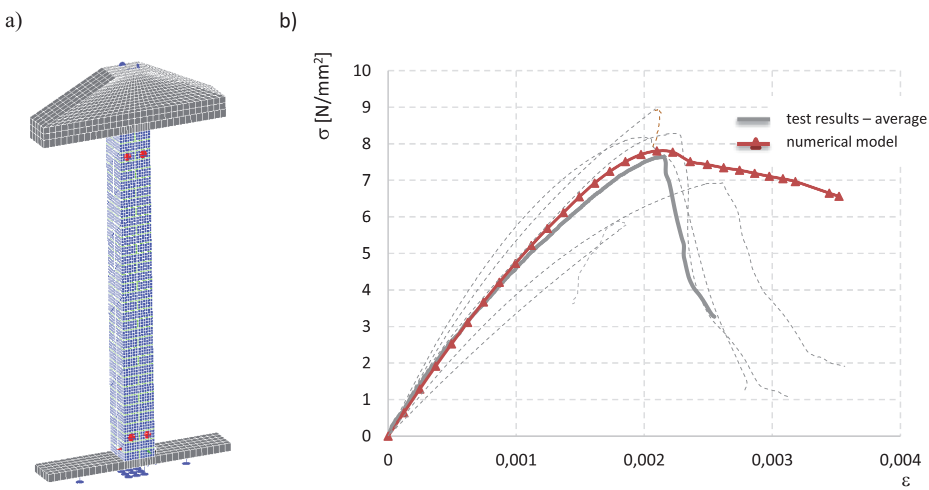

Fig. 4. Deterministic numerical model of a brick column: a) model view, b) relationship stress – strain obtained in tests and numerical calculations of the model

3.3. Estimation of reliability by the Monte Carlo method

Simulation analyses were carried out using the level III fully probabilistic method - Monte Carlo method. The main purpose of the simulations was to approximate the phenomenon or behavior of a given object using its model. This is a procedure where random numbers are generated in probability. Simulations of the column model took into account selected parameters of materials in the whole range of their variability determined on the basis of experimental research or estimated on the basis of literature. The input parameters of the stochastic model were described as the mean, standard deviation and the assumed type of probability density distribution of the random variable. Of all the parameters describing the created numerical model of the masonry structure, key analyses were selected, introducing them into the simulation as random variables. These were properties such as: modulus of elasticity, compressive strength and tensile strength of the masonry units and mortar – Table 5. Coefficients of variation used for these random variables were taken from experimental studies. The log normal distribution LN was chosen to describe the probability density [24; 25].

Table 5. Random variables, their coefficients of variation and probability density functions in the column stochastic model

Random Variable	Coefficient of variation		Probability density function	
	masonry unit	mortar	masonry unit	mortar
E_y	0,26		logarithmic normal LN	
f_c	0,16	0,11	logarithmic normal LN	
f_t	0,16*	0,13	logarithmic normal LN	

* assumed as for compressive strength f_c

Numerical simulations were carried out in the SARA and FREeT software package. An important step in the Monte Carlo analysis was to define the authoritative number of simulations carried out. For this purpose, the Latin hypercube sampling method (LHS) was used, in which representative values of random variables were selected for the declared number of intervals. An analysis was carried out, involving the determination of the range of ranges for which the obtained values of average stress and reliability index stabilize. The values of the output parameters of the column model were compared for the number of intervals: 10, 20, 30, 50, 70 and 100, respectively. The preliminary analyses carried out showed a slight difference in the results obtained when conducting 70 and 100 simulations. Due to the stabilization of results, the analysis included 100 bands in the Monte Carlo method. As a result of 100 simulations carried out for the column model, the structure response was obtained in the form of a probability density function of the average load capacity of the structure. The data set obtained from the simulation was later analysed in the FREeT program. Estimated average value of structure load capacity, variance and approximate form of probability density function were obtained. The histogram together with the approximate function of structure responses for the column model is shown in Figure - Fig. 5.

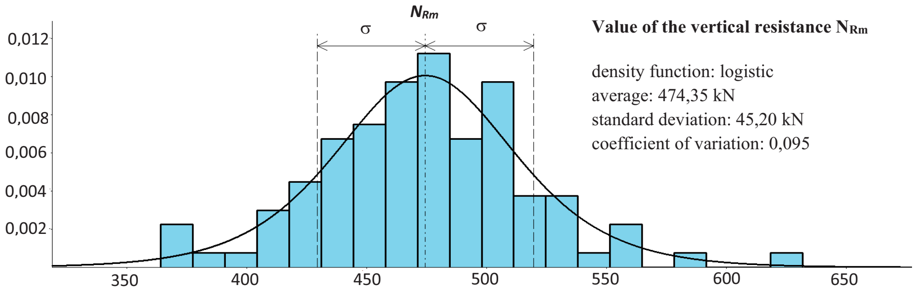

Fig. 5. Histogram together with the approximate function of structure responses for a stochastic brick masonry column model

In the subsequent stages of analysis, the already known structure response was used, with a wide range of variability of its basic parameters. The reliability analysis of the stochastic column model was introduced, introducing permanent load G_k and two variable loads (Q_k and snow S_k) on the effects side. The calculations on the side of actions were carried out assuming that the effect of actions is equal to the design value of the resistance.

$$R_d = E_d \tag{3.1}$$

where:

R_d – design value of the resistance, E_d – design value of effect of action.

The basis for determining the column load capacity was the value of the characteristic compressive strength, which was determined from the value of the average wall strength obtained for 100 simulations. The procedure was carried out in accordance with Annex D of the PN-EN 1990 [1] standard, assuming the distribution of wall strength as logarithmic - normal. Hence the characteristic value should be determined from the formula:

$$X_k = \exp\left[m_y - k_n s_y\right] \tag{3.2}$$

where:

(3.3)
$$m_y = 1/n \sum \ln (x_i)$$

(3.4)
$$s_y = \sqrt{\ln ({v_x}^2 + 1)} \approx v_x$$

k_n - characteristic fractile factor – Table 6; v_x – coefficient of variability of strength obtained from the simulation

Table 6. Values of k_n for the 5% characteristic value according PN-EN 1990

n	1	2	3	4	5	6	8	10	20	30	∞
v_x known	2,31	2,01	1,89	1,83	1,80	1,77	1,74	1,72	1,68	1,67	1,64
v_x unknown	-	-	3,37	2,63	2,33	2,18	2,00	1,92	1,76	1,73	1,64

For the value of 100 simulations, $k_n = 1{,}64$ was assumed and the characteristic value of the masonry compressive strength was determined, which was $X_k = f_k = 6{,}46\ MPa$. Then the computational load capacity of the column was calculated for two cases - class A and B of the structure execution of works, assuming the safety factors for masonry $\gamma_M = 1{,}7$ and $\gamma_M = 2{,}0$, respectively. It was assumed that the designed value of the resistance is equal to the designed effect of interactions and the relation between constant load and the sum of variable loads was defined, as well as the relationship between variable loads according to the formulas:

(3.5)
$$\chi = {}^{Q_k}\!/_{(G_k + Q_k + S_k)}$$

(3.6)
$$k = {}^{S_k}\!/_{Q_k}$$

The calculations on the side of impacts were carried out for the situation of pernament load and two types of variable loads connected by the relationship k = 0.5. Assuming the values of individual load factors belonging to the $\chi \in (0; 1{,}0)$ range, the impact effect was successively transformed into characteristic values for the effect of constant and variable load according to the relationships corresponding to the individual combination patterns of the PN-EN 1990 [1] standard, i.e. formula 6.10a, 6.10b and 6.10.

(3.7) $$E_d = \gamma_G G_k + \gamma_Q \Psi_Q Q_k + \gamma_S \Psi_S S_k \ (6.10a\ [1])$$

(3.8) $$E_d = \xi \gamma_G G_k + \gamma_Q Q_k + \gamma_S \psi_S S_k \ (6.10b\ [1])$$

(3.9) $$E_d = \gamma_G G_k + \gamma_Q Q_k + \gamma_S \psi_S S_k \ (6.10\ [1])$$

where:
G_k - characteristic value of a permanent action, Q_k - characteristic value of a variable action, S_k - snow. Values of partial factors for actions contained in equations, according to EC0 [4] , $\gamma_G = 1{,}35$, $\xi = 0{,}85$, $\gamma_Q = 1{,}5$, $\psi_{0,1} = 0{,}7$.

Formulas for determining individual interactions in the method used are detailed in [26]. Then, statistical parameters (mean, coefficient of variation and probability density function) were determined for random variables. Reliability ratios for individual load factors belonging to the $\chi \in (0; 1{,}0)$ were determined. Table 7 presents parameters of random variables occurring on the side of interactions in the stochastic model of the brick masonry column.

Table 7. Parameters of random variables occurring on the side of interactions in the stochastic model of the brick masonry column

Random Variable	Probability density function	The average value of the variable	Coefficient of variation	Unit
G	rozkład normalny N	G_m	0,10	[kN]
Q	rozkład Gumbela	Q_m	0,20	[kN]
S	rozkład Gumbela	S_m	0,40	[kN]

For example, the safety level of the analysed column was determined in relation to the RC2 reliability class. Based on the prepared database on the effects of impacts and individual reliability indexes according to Cornel determined by the Monte Carlo method, the individual graphs presented were generated.

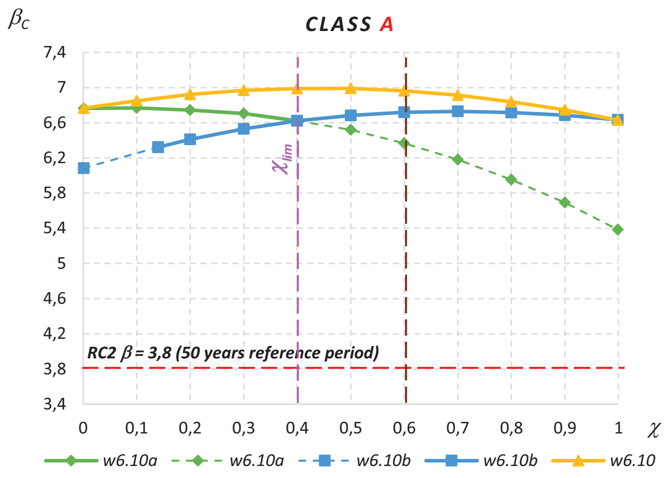

Fig. 6. Relation curves β_C reliability ratio and load factor for construction class A and three different combinations of standard factors (formulas: (3.7), (3.8), (3.9))

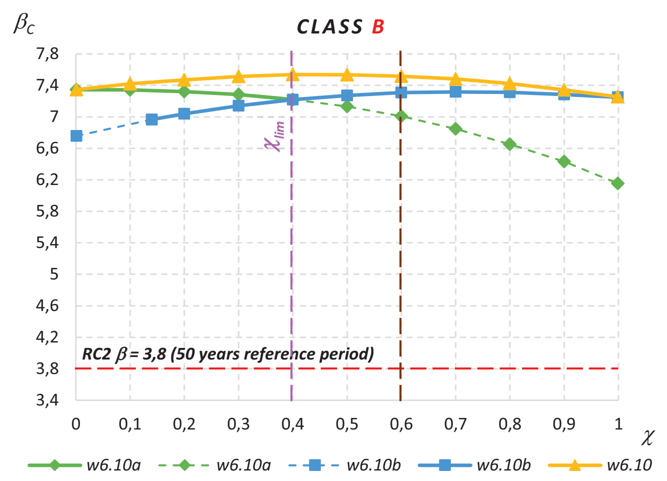

Fig. 7. Relation curves β_C reliability ratio and load factor for construction class B and three different combinations of standard factors (formulas: (3.7), (3.8), (3.9))

4. Conclusions

The analyses carried out in the article consisted of checking whether the specification of the boundary condition, the tested masonry structure, at the level of equality of the calculation values leads to an appropriate level of safety. The most often accepted level of safety of designed structures is the level specified for the RC2 reliability class. With regard to this class, structural safety is ensured using a set of applicable Eurocodes. The minimum estimated values of the reliability index β for each analysed masonry structure exceeded the target value $\beta = 3{,}8$. The graphs presented show that the application of partial coefficients recommended by Eurocodes [1], [8] determines ensuring a sufficient level of safety of masonry structures. Sufficient for this construction would be to use a partial factor corresponding to class A of the structure, regardless of the quality of workmanship. In addition, it should be noted that some of the minimum values of the reliability index β presented in the diagram are for the maximum value of the load factor χ, i.e. $\chi = 1{,}0$. In the literature [27] we can find recommendations suggesting limiting the value of χ to the level of 0,6 – 0,7, because it is the largest practically possible value of the share of variable loads in the total loads occurring in typical constructions.

References

[1] PN-EN 1990, Eurocode – Basis of structural design. 2002 CEN.
[2] Nowak A., Collins K., Reliability of structures. McGraw-Hill, 2000.
[3] Schuëller G.I., Einführung in die Sicherheit und Zuverlessigkeit der Tragwerken, W. Ernst, Berlin, München, 1981.
[4] Bakeer T.: Reliability assessment of vertically loaded masonry walls, Structural Safety 62 (2016) 47–56. https://doi.org/10.1016/j.strusafe.2016.06.004
[5] Freudenthal A.M., The Safety of Structures, Transactions ASCE, Journ. Struct. Div., 1945, 71, No. 8, 112-125.
[6] Winkelmann K., Obliczanie niezawodności konstrukcji inżynierskich metodami symulacyjnymi oraz metodą powierzchni odpowiedzi. Gdańsk University of Technology, Gdańsk, PhD thesis, 2013 (in Polish).
[7] Małyszko L., Modelowanie zniszczenia w konstrukcjach murowych z uwzględnieniem anizotropii, University of Warmia and Mazury Publishing House, Olsztyn, 2005 (in Polish).
[8] PN-EN 1996-1-1+A1:2013 Eurocode 6: Design of masonry structures - Part 1-1: General rules for reinforced and unreinforced masonry structures. 2004 CEN.
[9] Bukowski Ł., Szojda L.: Analysis of brick-wall structural behavior under compression in its plane, Procedia Engineering 193, 329 – 336, 2017. https://doi.org/10.1016/j.proeng.2017.06.221
[10] Matysek P., Identyfikacja wytrzymałości na ściskanie i odkształcalności murów ceglanych w obiektach istniejących, Cracow University of Technology Publisher, Kraków 2014 (in Polish).
[11] Drobiec Ł.: Ściany obciążone głownie pionowo według PN-EN 1996-1 i PN-EN 1996-3, XXVI National Workshops of Design Designer Work, Szczyrk, 9-12 March, 2011 (in Polish).
[12] PN-B-03002:1999. Masonry constructions (in Polish).
[13] PN-B-03002:2007. Masonry constructions (in Polish).
[14] Gajownik R., Sieczkowski J.: Współczynniki bezpieczeństwa i wytrzymałość obliczeniowa muru na ściskanie wg EC6 w wybranych państwach UE, Building Materials, no. 452, 2010 (in Polish).
[15] Cervenka V., Jendele L., and J. Cervenka. 2017.ATENA Program Documentation Part 8 User'sManual for ATENA-GID. Copyright 2000-2017 Červenka Consulting s.r.o., Prague, July 24, 2017.
[16] Buda-Ożóg L., Niezawodność konstrukcji żelbetowych projektowanych metodą Strut and Tie, Rzeszów University of Technology Publishing House, Rzeszów 2019 (in Polish).

[17] PN-EN 1052-1:2000: Methods of test for masonry. Part 1 - Determination of compressive strength;.
[18] PN-EN 1015:2001: Methods of test for mortar for masonry. Part 11: Determination of flexural and
[19] compressive strength of hardened mortar.
[20] PN-EN 772-1:2003: Methods of test for masonry units. Part 1: Determination of compressive strength.
[21] Urbańska K.: Zastosowanie systemu Aramis do pomiarów odkształceń konstrukcji murowych, University of Zielona Góra, Scientific Notebooks no. 166, 2017 (in Polish).
[22] Jasiński R., „Badania i modelowanie murowanych ścian usztywniających", Silesian University of Technology Publishing House, Gliwice 2017 (in Polish).
[23] Mazur W., Drobiec Ł., Jasiński R.: Research and numerical investigation of masonry – AAC precast lintels interaction, Procedia Engineering 193 (2017) 385 – 392. https://doi.org/10.1016/j.proeng.2017.06.228
[24] Cervenka J., Papanikolaou N.K., Three dimensional combined fracture–plastic material model for concrete, International Journal of Plasticity 24, 2192–2220, 2008. https://doi.org/10.1016/j.ijplas.2008.01.004
[25] Schueremans L., Probabilistic evaluation of structural unreinforced masonry. PhD Thesis, Katholieke Universiteit Leuven, 2001.
[26] Glowienka S., Zuverlässigkeit von Mauerwerkswänden aus großformatigen Steinen. PhD Thesis, Technische Universität Darmstadt, 2007.
[27] Zięba J., Skrzypczak I.: Analysis of reliability of compressed masonry structures, 65. Scientific Conference of the Committee on Civil Engineering of the Polish Academy of Sciences and the PZITB Science Committee, Krynica Zdrój, 15-20.09.2019.
[28] Holicky M., Reliability analysis for structural design, Sun Press, 2009.

Wieloetapowa analiza niezawodności przykładowej konstrukcji murowej

Słowa kluczowe: konstrukcje murowe, badania doświadczalne, bezpieczeństwo, niezawodność, symulacje Monte Carlo

Streszczenie:

Bezpieczeństwo konstrukcji murowej determinowane jest wartością zastosowanego współczynnika częściowego, na którą wpływ ma wiele czynników, między innymi: kategoria elementu murowego będąca wyznacznikiem jakości wykonania; rodzaj zaprawy, klasa wykonania robót. Zmienność tych czynników determinuje uzyskanie znacznych różnic w poziomach nośności różnych konstrukcji murowych. Stąd analiza konstrukcji murowych powinna odbywać się przy uwzględnieniu dostatecznego zakresu zmienność czynników wpływających na jej bezpieczeństwo. W artykule przedstawiono wieloetapową analizę bezpieczeństwa przykładowego filara murowanego z cegły pełnej. Dla elementu zbadano zależność pomiędzy współczynnikami częściowymi stosowanych do oddziaływań w różnej konfiguracji oraz współczynnikami do wytrzymałości muru na ściskanie. Analizy polegały na wyznaczeniu wskaźnika niezawodności β metodą Monte Carlo, wskaźnika będącego miarą osiągniętego bezpieczeństwa. W artykule przedstawiono wyniki badań doświadczalnych przeprowadzonych na rzeczywistym elemencie dla wybranych warunków, a także wyniki przeprowadzonych symulacji numerycznych MES. Analizy wykazały poziom bezpieczeństwa jaki zapewniamy projektując konstrukcję w podstawowej klasie niezawodności RC2.

Received: 03.11.2020 Revised: 04.12.2020