#### GOSPODARKA SUROWCAMI MINERALNYMI - MINERAL RESOURCES MANAGEMENT

021 Volume 37

Issue 2

Pages 63-84

DOI: 10.24425/gsm.2021.137568



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# The Hotelling's Rule in practice – analysis of gold mining sector

# 1. Origins of Hotelling's Rule

A survey of Hotelling's model of resource extraction and tests of that theory have influenced strongly academic discussions on the economics of exhaustible resources for many decades. Harold Hotelling was the first to determine the implications of finite reserves for the evolution of prices and consumption under an optimal plan. He attempted to show that competitive markets might contribute to achieve the planner's solution (Slade and Thille 2010). By exhaustible resources Hotelling means non-renewable resources, as opposed to renewable ones. The expression "exhaustible resource" is somewhat of a misnomer, since both type of resources are exhaustible. Hotelling recognizes that a complete study of the subject would include semi-replaceable assets such as forests and stocks of fish "[renewable resources], but chooses to limit himself to absolutely irreplaceable assets" [non-renewable resources] (Gaudet 2007).

Hotelling concluded that the owner of a finite natural resource is indifferent to either exploiting it or leaving it *in situ* when the marginal profit increases at the prevailing interest

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rate. The rationale behind this statement is that, if the return is lower than this, the resource owner will shift assets to a better performing investment. If the return is greater, the owner will leave the resource where it is as it appreciates faster than other investments. In other words, the resource will not be produced at all unless it can be produced at a rate that returns the prevailing interest rate (Slade and Thille 2010). Hotelling noted that this sets the upper limit on a monopoly producer's profit from production. To explain this, it is necessary to appeal to the law of demand. For a downward sloping demand curve, the monopoly producer can only increase the price by decreasing the production level. Hotelling's Rule means that if the producer decreases production at a rate that makes the marginal profit change by the interest rate, total profit from the resource will be maximized. If the production level is changed in any other way, total profit will be less than the maximum. Hotelling's Rule is often called the r-percent rule and paraphrased as the price must increase by r percent, where r referring to the interest rate (Dixon 2012).

One may analyze this rule from the opportunity cost concept. In fact a firm exploiting mineral (or any other non-renewable) resources incurs not only regular production costs but also, with each one unit of output, reduces an amount of natural resources available for extraction and, in turn, future sales. Thus, in exchange for a financial stream earned today it waives future gains. From a financial standpoint, this opportunity cost is the net present value (NPV) of the future profits to be created if corresponding mineral resources are kept intact (Otto et al. 2006).

It is worth mentioning that the responsibility for naming the Hotelling's observations a rule falls on Robert Solow, who first used the term Hotelling's rule in his lecture given soon after Hotelling's death in 1973 (Solow 1974). In reference to this, Solow rightfully reflects that the Hotelling's concept is not a 'rule' at all in the appropriate sense as it doesn't enjoin anything. Hotelling's principle is a description of what a foresighted competitive market would do, under simple conditions. Nonetheless, the term "Hotelling's Rule" appears in numerous scientific texts and papers and is well-known to generations of natural resource economists.

The consequences of Hotelling's Rule are far reaching. In the area of mineral asset valuation following it would simplify valuation processes enormously. Since their value will be determined solely by the level of current margins no other factors but these margins and recoverable reserves need to be analyzed. The challenge in mineral asset valuation would be reduced to the following items:

- valuator needs to determine a relevant period in history which reflects the "current margins level" for a mineral deposit in consideration;
- all components of full cost need to be recognized which includes the cost of capital employed.

Consequently, many researchers and practitioners have attempted either to prove or to reject Hotelling's Rule. However very few of them took a company based, microeconomic approach based on analyses of annual reports. This work pretends to fill up, at least in a limited way, this gap.

# 2. Testing the Hotelling's Rule and its' subsequent extensions and interpretations

Although Hotelling derived several variants of his model, in particular he solved the monopolist's extraction problem and he considered extraction costs that increase as the resource base is depleted, since that time other researchers have introduced additional factors expanding the original model. Below there is a list of the most significant extensions (Slade and Thiele 2010):

- augmentation of the resource base through discoveries (Pindyck 1978),
- uncertainty about the size of reserves (Gilbert 1979),
- future demand and costs (Pindyck 1980),
- possibility of recycling and stockpiling (Levhari and Pindyck 1981),
- imperfect competition among producers has been considered using a dominant firm or a cartel model (Gilbert 1978; Salant 1976),
- cost-lowering technological improvements (Slade 1982),
- taxation effects (Teece et al. 1993; Dasgupta and Heal 1979),
- impact of natural resources abundance/scarcity on a national economy (Boyce and Emery 2005).

General equilibrium effects have been included by embedding the Hotelling model in a model of aggregate growth (Stiglitz 1976). By the 1990s, however, interest in the subject began to diminish, and by 2000 the flow of new theories and tests had been substantially reduced but still remains on a noticeably high level (Huang 2018).

While undertaking any further analyses of the Hotelling's Rule it is of utmost importance to acquire a clear understanding of what Hotelling meant. He, being a mathematician by background and practicing economic sciences in early 20th century used various terms in a meaning pertaining to his times. In the original phrasing of his rule he used the word "net price" to describe an economic variable to be incremented by r-rate. Immediately an explanation was added that: "p is to be interpreted as the net price received after paying the cost of extraction and placing upon market – a convention to which we should adhere throughout" (Hotelling 1931). Hence, using the notion of price Hotelling meant rather gross margin or profit applying present economic terms. Nordhaus in his already indicated paper understands net price as royalty (Nordhaus 1973).

It is quite surprising that many researchers seem to oversee the issue of proper interpretation of price in the Hotelling Rule and in order to explain a presumed lack of cost consideration they developed, probably unintentionally, their own theories. Such amendments may be considered as a valuable contribution of authors, however the problem starts when they claim to test their models as if they were equal to Hotelling's original one and draw conclusions in respect to its viability and usefulness. Origins of such disputable interpretation of the Hotelling Rule can be traced to the famous article of Krautkraemer who presents it as stating that: "In case of zero marginal extraction cost the price of the resource equals the in situ value and so the resource price also would increase at the rate of interest".

Surprisingly, soon after this sentence one can find a statement: "Of course it is the in situ value of the resource stock, rather than resource price itself, that model implies will be increasing over time (...)" (Krautkramer 1998) recognizing that the former was to be different (usually lower) due to various economic parameters, for example forecasted extraction costs. On the other hand Potocki assumes that the extraction, transport and sale of oil is cost free. He claims that Hotelling could make such supposition due to the importance of easy to explore reservoirs in Saudi Arabia and Kuwait. He states that "according to Hotelling's model oil prices should increase at a rate equal to the interest rate" and analyses nominal and real final prices not corrected for extraction and sales costs, only to find that they definitely deviate from the Hotelling Rule. Only in the final discussion does Potocki (Potocki 2009) indicate that Hotelling's formula needs to be modified, so that marginal profit is to be used instead of price.

Given the far reaching consequences of Hotelling's Rule it is not a surprise that, once "rediscovered", it has been constantly tested by various scholars using different examples of natural resources and methods. These attempts have produced mixed and sometimes contradictory results. The tests that had been performed were once classified by Slade and Thille into two wide groups: descriptive and structural. The first class assesses outcomes that are associated with the market equilibrium without specifying the nature of that equilibrium (Slide and Thiele 2010). They are more assessments of various models developed on the basis of the Hotelling Rule than statistical tests. The second class tests a specific model by estimating structural equations. Therefore, it imposes more structure (e.g., on the cost function and the nature of competition in the market), but then the risk is that a given configuration employed may be inappropriate.

**Descriptive studies** have traditionally focused on mineral commodity prices using long-term data (sometimes spanning over a century or more) on various fuel and non-fuel minerals prices. The study of Barnett and Morse (Barnett and Morse 1963) can be quoted as a classical example of such an approach. They analyzed mineral commodities prices formally, looking at relative price trends in an attempt to uncover evidence of natural-resource scarcity to come to the conclusion that scarcity was not a problem since real prices had fallen over time. Other researchers who have examined price trends presented various findings, however none of them has been able either to prove or to reject completely Hotelling's Rule. Instead they have reached some expansions or corrections:

Several researchers have assessed the possibility that price paths might be U-shaped, starting with Slade (Slade 1982), who based her descriptive tests on the idea that price declines might be caused by a technical change. She found that there were quadratic trends to reveal evidence of upturns in the real prices of mineral commodities if the development of the 1970's and thereafter is taken into account. She has been followed by many other researchers who have applied an array of econometric techniques and reached a variety of suppositions. For example, Berck and Roberts (Berck and Roberts 1996) estimated both difference and trend-stationary models and found evidence of U-shapes under the former but not the latter one. Pindyck (Pindyck 1999)

based his research on 127 years of data identified the marginal extraction costs as a key factor creating parabolic shape of fuel prices.

 Heal and Barrow (Heal and Barrow 1980) related metal price movements to interest rates and stated that changes in interest rates, not interest—rate levels, predict prices.

Structural models contain formal tests of the Hotelling model and usually rely on estimates of some combination of an industry-wide demand function, a production, profit, or cost function for an extractive firm or industry, and a first-order condition (e.g., an Euler equation) that is associated with dynamic-profit maximization (Slide and Thiele 2010). Most frequently quoted examples include works of Halvorsen and Smith (Halvorsen and Smith 1991), Stollery (Stollery 1983) as well as of Slade (Slade 1982). Stollery found support for the Hotelling model and identified the suitable discount rate of 15%. In more rent paper Atewmba and Nkuiya (Atewmba and Nkuiya 2017) calculated the discount rate separately for periods of increasing and decreasing prices for 14 different minerals (mostly fuels and metals). They got mixed results spanning but in most cases the rate was close to zero. What Slide observed was that one could estimate demand and cost, and use those equations to solve for the market equilibrium that is implied by dynamic profit maximization. Then, she recommends to test, if observed price and output paths lie within the confidence intervals that surround the paths predicted by Hotelling's Rule. It was also indicated that many researchers who have estimated cost or profit functions for individual mines or mining industries modelled in fact both mining and refining ore as transferred inside a vertically integrated firm. Here costs (and thus margins) can be approximated by one of two methods. They can be calculated as the difference between price and marginal cost or a shadow price of the unpriced ore to the vertically integrated metal producer was to be developed. These two methods do not measure the same parameter. The first is the shadow price of one unit of contained metal in situ, whereas the second is the shadow price of one unit of ore of the current grade, also in situ. In the following section author took the former approach. As all the selected companies are integrated gold producers, focused mostly on this metal and do not sale ore or concentrate in substantial volume, it was safe to assume that refining was part of the mining operations. This is supported by the fact that in many cases the division between mining and processing is purely academic. A mined raw material is rarely offered for sale, or even transported outside a pit, without at least primary processing.

What is important to indicate, any rejection of the Hotelling model is partial. Such rejection refers to a particular mineral and time. What makes full rejection even more challenging is a number of new formulations, often quite far from the original model. As an example, one may indicate a model proposed and estimated by Miller and Upton (Miller and Upton 1985). They showed, using market valuations of a sample of US oil and gas companies and linking these valuation to value of their mineral reserves, that in a competitive market, the value of reserves in optimally managed mineral deposits should depend solely on the current spot price net of marginal-extraction cost, regardless of extraction time.

# 3. Testing Hotelling's Rule based on corporate operational and financial results – methodology

An approach adopted in this paper is based on an assumption that Hotelling's Rule refers primarily to microeconomics. There are mining enterprises, mainly their management boards to decide upon the initiation and pace of mineral deposits' exploitation. Despite all the circumstances and drivers conditioning their choices (see: Veldhuizen and Sonnemans 2018) at the end their judgments depend on two fundamental factors: expected return and risk involved. Therefore, if a decision to postpone extraction is to be taken, a loss of current benefits has to be appropriately compensated with expected future increase. This fact lays down a foundation for the first hypothesis. The rate of margins' growth (or a discount rate) cannot be a risk-free one. In fact, it has to reflect a risk associated with the delay. It's hard to indicate/calculate such parameter for all decisions across one industry. Therefore, a rate of industry WACC was adopted for subsequent calculations. This is an aggressive assumption since individual discount rates should be higher.

The second hypothesis refers to the volume of the minerals produced. Shall Hotelling's Rule hold, output needs to be reduced by individual companies not just globally. Decreasing prices typically forces the weakest players out of the market therefore a global fall in output cannot be associated with mechanism under analyses. There are minerals which, due to their importance, either globally or for an individual country fall under strong governmental influence. The most prominent case is that of crude oil. But in most cases markets are quite fragmented and global output results from an array of individual decisions. Thus, if the Hotelling Rule holds, the majority of individual companies need to decrease production when the margins fall below expected values and increase if they are above.

The third hypothesis regards the consequences of the second one. If output follows deviations of margins from the growth line set by Hotelling's Rule we shall observe that in periods of higher margins output grows, while when they are lower, it declines, in both situations contradicting the original trend. Therefore, the real margins, in the long term, shall follow the growth line set by Hotelling's Rule.

Consequently, proving Hotelling's Rule requires testing the above three indicated hypotheses:

- Growth rate of margins per unit realized by mining companies must exceed a rate equal to their WACC,
- Output shall follow deviations from the Hotelling growth line,
- Margins shall follow Hotelling growth line.

## 4. Gold miners 2004-2020

The test was run based on the performance of selected gold companies. Their set was determined by the following requirements:

- They have to be listed on one of the leading mining stock exchanges (Toronto, Sydney, London, New York).
- They have to belong to the top gold producers.
- At least 50% of their revenues had to come from gold operations.

Five companies meet the above requirements: Barrick Gold, Newmont, Kinross, AngloGoldAshanti and Newcrest. Although a share of the selected 5 companies in global production decreases gradually they were still responsible for 16% of the global supply of mined gold in 2020. The total volume available was considerably bigger due to an addition of recycled metal. If this is considered, the share of selected companies in the total supply surpasses 10%.

The selected companies output for years 2004–2019/20 was confronted with the share of the remaining suppliers and price development (Figure 1). It is visible that global production, the one of chosen companies and prices evolve differently. World production was in constant growth (actually it decreased for the first time in 2019 just after the last year in analyses), selected companies reported a decline in all years (as related to the preceding one) except for 2011 and 2017. Prices reached the peak in 2012, then fell significantly by 2015, recovered partially in 2016 and maintained a very similar level by 2018.

All the selected companies manage a portfolio of mines and undeveloped deposits in various life-cycle stages. Consequently, they enjoy a certain freedom of decision as they can slow down or speed up developments of new projects in response to a changing environment.

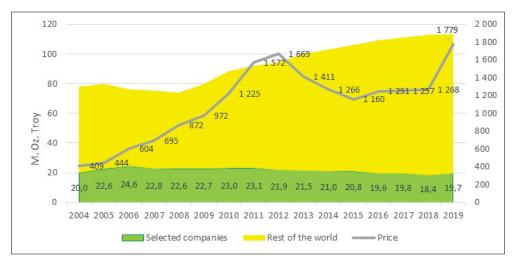


Fig. 1. Share of selected 5 producers in the total world primary gold production (2004–2019)
Source: Own calculations based on companies' reports and World Gold Council Statistics
and American Geological Service Statistics

Rys. 1. Udział analizowanych 5 producentów w światowej produkcji pierwotnej złota

Nearly all the selected companies carry out other activities than gold mining but their impact on the overall results is relatively small. Measured by revenues it varies from 20–30% for Barrick Gold and Newcrest, through 10% in case of Newmont down to near zero for Kinross and AngloGoldAshanti.

# 5. Testing Hotelling's Rule using financial and operational results of selected Gold Mining Companies 2004–2018

#### 5.1. Data retrieval and selection

The primary source of data were annual reports of selected companies. Since all of them were public these documents were available on relevant web pages. Although financial statements were prepared under various accounting rules depending on a company origin and incorporation, it was assumed that differences were not significant enough to disable meaningful comparisons – given the purpose of the analyses. At the time of submitting the manuscript only Barrick Gold, Newmont and AngloGoldAshanti had published their 2020 annual reports. Newcrest follows a different calendar than the reporting year – ending on June 30, therefore it's 2020 annual report in fact covers the 2<sup>nd</sup> half of 2019 and the 1<sup>st</sup> half of the 2020. In the case of Kinross the last report covered 2019. These differences do not significantly affect the subsequent analysis as it is relative and dynamic. The tables below (Table 1–5) present individual data for each company while the last one consists of combined results achieved via summing or a weighted average, whatever was appropriate. Hereby there is explanation for the key items presented:

**Profit on gold** was calculated by the author. Typically reports disclose revenues from the sales of gold but costs had to be estimated based on itemized information. In one case (2004 for Newcrest) a concentrate had to be diluted from its cost base. As far as the cost are concerned, the industry consistently applies a concept of cash costs. Differences relate to the treatment of royalties, mine closing expenses (decommissioning and rehabilitation). There is also a problem of hedging recognition both on the revenue and the cost side.

Gold sales and gold extraction are typically presented outright in an annual report.

**Profit (margin) on gold** sold was calculated by the author and may insignificantly differ from some data shown due to the need to preserve consistence across all five companies. The author applies both terms interchangeably in some contexts. In essence both of them stand for a difference between revenues and costs. Profit is rather a financial accounting term, used in financial statements in relation to a whole reporting entity or a segment formally defined and reported. Margins dominate in managerial accounting and financial analyses. Therefore, where values come directly from financial reports, they are referred to as profits. However, in case they result from the author's own calculations, they are referred to as margins.

 Table 1.
 Barrick Gold – selected financial & operational data, 2004–2020

 Tabela 1.
 Barrick Gold – wybrane dane finansowe i operacyjne, 2004–2020

Gap between real and assumed margin	y USD/oz. Troy	-238.82	-580.37	-254.01	-106.54	-46.25	-360.48	-286.64	-65.23	234.74	328.83	225.61	106.45	66.73	-12.10	-3.68	31.05	I
Net margin if Hotelling rule holds	USD/oz. Troy	300.14	268.00	236.15	213.16	193.10	175.89	160.51	145.80	133.29	121.44	111.09	101.79	92.53	83.85	76.53	70.09	64.59
Av net margin increse to 2004	%	-0.32%	-12.48%	-1.76%	3.93%	7.08%	-13.06%	-11.44%	2.49%	24.30%	31.97%	31.68%	26.38%	25.31%	3.57%	6.20%	%65'95	0.00%
Est. Margin on gold net of capital charge	USD/oz. Troy	61.32	-312.36	-17.86	106.62	146.86	-184.58	-126.13	80.57	368.03	450.27	336.70	208.25	159.26	71.75	72.84	101.14	64.59
Capital charge per unit	USD/oz. Troy	749.51	814.19	453.38	420.54	385.22	333.52	430.24	385.14	503.33	488.27	323.60	306.40	266.02	204.11	170.06	99.43	97.28
Capital charge	USD, millions	3 656.85	4 451.19	2 060.15	2 229.71	2 119.85	2 028.82	2 703.63	2 762.97	3 670.31	3 686.47	2 505.30	2 238.55	2 020.43	1 644.07	1 426.83	528.95	480.19
Capital employed	USD, millions	30 499.15	32 996.20	19 102.00	21 470.50	21 664.30	21 166.66	26 808.44	29 440.24	37 586.38	39 575.66	27 440.35	22 363.16	19 511.62	17 179.46	15 534.37	6 212.00	5 856.00
Av margin increse to 2004	%	10.59%	7.83%	7.33%	9.51%	10.42%	-0.75%	6.51%	12.46%	23.42%	28.54%	26.40%	26.03%	27.31%	19.45%	22.50%	23.90%	0.00%
Profit (margin) on an unit sold	USD/oz troy	810.82	501.83	435.52	527.16	532.07	148.94	304.11	465.71	871.37	938.54	660.29	514.65	425.28	275.85	242.91	200.56	161.87
Gold	Oz. Troy. M	4 760.0	5 465.0	4 527.0	5 323.0	5 517.0	6 117.0	6 249.0	7 166.0	7 421.0	7 676.0	7 765.0	7 423.0	7 657.0	8 060.0	8 643.0	5 460.0	4 958.0
Gold sales	Oz. Troy. M	4 879.0	5 467.0	4 544.0	5 302.0	5 503.0	6 083.0	6 284.0	7 174.0	7 292.0	7 550.0	7 742.0	7 306.0	7 595.0	8 055.0	8 390.0	5 320.0	4 936.0
Profit from gold operations	USD, M	3 956	2 744	1 979	2 795	2 928	906	1161	3 341	6 354	7 086	5 112	3 760	3 230	2 222	2 038	1 067	662
Year		2020	2019	2018	2017	2016	2015	2014	2013	2012	2011	2010	2009	2008	2007	2006	2005	2004
Company		Barrick Gold																

Table 2. Newmont – selected financial & operational data, 2004–2020

Tabela 2. Newmont – wybrane dane finansowe i operacyjne, 2004–2020

Company	Year	Profit from gold operations	Gold sales	Gold extraction	Profit (margin) on an unit sold	Av margin increse to 2004	Capital employed	Capital charge	Capital charge per unit	Est. Margin on gold net of capital charge	Av net margin increse to 2004	Net margin if Hotelling rule holds	Gap between real and assumed margin
		USD, M	Oz. Troy. M	Oz. Troy. M	USD/oz troy	%	USD, millions	USD, millions	USD/oz. Troy	USD/oz. Troy	%	USD/oz. Troy	USD/oz. Troy
Newmont	2020	4 000	5 831.0	5 543.0	682.99	9.45%	34 208.92	4 101.65	703.42	-17.43	-1.50%	138.84	-156.27
Newmont	2019	2 663	6 456.0	6 004.0	412.48	6.43%	34 922.26	4 711.01	729.71	-317.23	-12.58%	123.97	-441.20
Newmont	2018	1 870	5 516.0	5 479.0	338.97	5.42%	18 137.27	1 956.10	354.62	-15.65	-1.56%	109.24	-124.89
Newmont	2017	1 945	5 632.0	5 654.0	345.26	%00.9	18 426.33	1 913.57	339.77	5.49	-17.27%	09:86	-93.11
Newmont	2016	1 744	5 172.0	5 243.0	337.23	6.31%	18 559.41	1 816.04	351.13	-13.90	-1.64%	89.32	-103.22
Newmont	2015	1 952	5 677.0	5 707.0	343.92	7.09%	23 714.00	2 272.99	400.39	-56.46	-5.88%	81.36	-137.83
Newmont	2014	1 856	5 240.0	5 231.0	354.17	8.14%	22 051.69	2 223.91	424.41	-70.24	-7.64%	74.25	-144.49
Newmont	2013	2 304	5 489.0	5 463.0	419.71	11.17%	19 995.80	1 876.61	341.88	77.83	2.09%	67.45	10.38
Newmont	2012	4 460	5 466.0	5 583.0	816.00	22.41%	24 169.09	2 360.11	431.78	384.22	24.97%	99.19	322.56
Newmont	2011	4 755	5 820.0	5 873.0	817.00	26.02%	21 044.62	1 960.31	336.82	480.18	33.19%	56.17	424.00
Newmont	2010	3 847	6 296.0	6 418.0	611.00	24.78%	18 481.04	1 687.32	268.00	343.00	32.09%	51.39	291.62
Newmont	2009	2 971	6 534.0	6 534.0	454.70	22.94%	16 558.84	1 657.54	253.68	201.02	25.49%	47.09	153.93
Newmont	2008	2 057	6 150.0	6 150.0	334.47	19.89%	12 412.44	1 285.31	208.99	125.48	18.06%	42.80	85.68
Newmont	2007	1 304	6 091.0	6 091.0	214.09	9.77%	10 948.20	1 047.74	172.01	42.07	-13.31%	38.79	3.28
Newmont	2006	1 476	7 086.0	7 268.0	208.30	13.44%	11 984.36	1 100.76	155.34	52.95	-9.45%	35.40	17.56
Newmont	2005	1 072	8 215.0	8 240.0	130.49	-19.39%	10 659.37	907.65	110.49	20.01	-69.02%	32.42	-12.41
Newmont	2004	815	6 969.1	6 790.5	117.00	0.00%	7 404.51	607.17	87.12	29.88	0.00%	29.88	Ι

Table 3. Kinross – selected financial & operational data, 2004–2019

Tabela 3. Kinross – wybrane dane finansowe i operacyjne, 2004–2019

Company	Year	Profit from gold operations	Gold sales	Gold extraction	Profit (margin) on an unit sold	Av margin increse to 2004	Capital	Capital charge	Capital charge per unit	Est. Margin on gold net of capital charge	Av net margin increse to 2004	Net margin if Hotelling rule holds	Gap between real and assumed margin
		USD, M	Oz. Troy. M	Oz. Troy. M	USD/oz troy	%	USD, millions	USD, millions	USD/oz. Troy	USD/oz. Troy	%	USD/oz. Troy	USD/oz. Troy
Kinross	2020												
Kinross	2019	987	2 512.8	2 527.8	392.84	%60.9	8 460.50	1 141.32	454.21	-61.38	4.55%	56.22	-117.59
Kinross	2018	579	2 533.0	2 475.0	228.62	2.50%	7 450.01	803.48	317.21	-88.58	-6.36%	49.54	-138.12
Kinross	2017	726	2 622.0	2 698.0	276.96	4.22%	7 571.90	786.34	299.90	-22.94	-2.37%	44.71	-67.65
Kinross	2016	633	2 779.0	2 810.0	227.85	2.89%	7 341.60	718.38	258.50	-30.65	-3.29%	40.51	-71.16
Kinross	2015	320	2 635.0	2 620.0	121.25	-2.59%	7 033.14	674.13	255.84	-134.58	-10.78%	36.90	-171.48
Kinross	2014	620	2 743.0	2 710.0	226.18	3.40%	8 347.00	841.79	306.89	-80.71	-8.45%	33.67	-114.38
Kinross	2013	946	2 669.0	2 631.0	354.55	9.10%	9 573.80	898.50	336.64	17.91	-13.28%	30.59	-12.68
Kinross	2012	1 779	2 654.0	2 678.0	670.46	19.44%	13 576.10	1 325.71	499.51	170.95	12.94%	27.96	142.99
Kinross	2011	1 732	2 701.0	2 703.0	641.36	21.74%	15 713.10	1 463.68	541.90	99.45	6.36%	25.47	73.98
Kinross	2010	1 171	2 537.0	2 528.0	461.37	19.07%	14 983.01	1 367.95	539.20	-77.83	-14.09%	23.30	-101.13
Kinross	2009	918	2 487.0	2 470.0	369.00	17.92%	7 375.20	738.26	296.85	72.15	2.24%	21.35	50.80
Kinross	2008	574	1 889.0	1 997.0	304.08	17.07%	6 836.00	707.87	374.73	-70.66	-20.29%	19.41	-90.07
Kinross	2007	383	1 576.0	1 589.0	243.27	14.54%	6 321.90	605.01	383.89	-140.61	-47.01%	17.59	-158.20
Kinross	2006	316	1 511.0	1 476.0	208.87	13.59%	1 845.80	169.54	112.20	29.96	22.34%	16.05	80.61
Kinross	2005	110	1 628.0	1 609.0	67.38	-58.37%	1 520.20	129.45	79.51	-12.13	-18.78%	14.70	-26.83
Kinross	2004	94	1 575.0	1 654.0	59.87	0.00%	1 658.20	72.96	46.32	13.55	0.00%	13.55	ı

Table 4. AngloGoldAshanti – selected financial & operational data, 2004–2020

Tabela 4. AngloGoldAshanti – wybrane dane finansowe i operacyjne, 2004–2020

-	>																	
Gap between real and assumed margin	USD/oz. Troy	391.82	103.84	42.57	-18.76	31.34	46.26	18.58	355.21	577.16	667.07	420.55	247.84	-165.45	32.96	42.05	-38.48	ı
Net margin if Hotelling rule holds	USD/oz. Troy	62.42	55.74	49.11	44.33	40.16	36.58	33.38	30.32	27.72	25.25	23.10	21.17	19.24	17.44	15.91	14.58	13.43
Av net margin increse to 2004	%	12.97%	6.22%	2.53%	-6.88%	0.85%	2.29%	-2.15%	21.96%	32.26%	40.33%	37.87%	33.02%	-34.41%	-7.94%	-5.27%	-37.01%	0.00%
Est. Margin on gold net of capital charge	USD/oz. Troy	454.24	159.57	91.68	25.57	71.50	82.84	51.96	385.53	604.88	692.33	443.65	269.01	-146.21	50.40	57.96	-23.91	13.43
Capital charge per unit	USD/oz. Troy	209.68	168.32	189.88	176.18	175.07	161.29	189.08	194.22	265.91	212.15	173.06	114.14	64.21	102.60	105.04	88.91	48.57
Capital charge	USD, millions USD, millions	512.59	417.34	645.59	664.54	628.49	639.51	842.90	797.26	1 048.76	918.83	781.35	524.92	319.89	561.95	591.88	548.20	283.10
Capital employed	USD, millions	4 275.14	3 093.73	5 986.00	6 399.00	6 423.00	6 672.00	8 358.00	8 495.00	10 740.00	9 864.00	8 558.00	5 244.00	3 089.27	5 872.00	6 444.00	6 438.00	6 434.14
Av margin increse to 2004	%	9.22%	4.82%	4.03%	1.71%	3.57%	3.81%	4.06%	15.23%	23.41%	27.86%	24.97%	18.81%	-10.79%	-1.86%	0.35%	-59.84%	0.00%
Profit (margin) on an unit sold	USD/oz troy	663.92	327.90	281.56	201.75	246.56	244.13	241.03	579.74	870.80	904.48	616.71	383.15	-82.00	153.00	163.00	65.00	62.00
Gold extraction	Oz. Troy. M	3 050	3 280	3 412	3 755.0	3 628.0	3 947.0	4 436.0	4 105.0	3 944.0	4 331.0	4 515.0	4 599.0	4 982.0	5 477.0	5 635.0	6 166.0	5 829.0
Gold sales	Oz. Troy. M	2 445	2 479	3 400	3 772.0	3 590.0	3 965.0	4 458.0	4 105.0	3 944.0	4 331.0	4 515.0	4 599.0	4 982.0	5 477.0	5 635.0	6 166.0	5 829.0
Profit from gold operations	USD, M	1 623	813	957	761	885	896	1 075	2 380	3 434	3 917	2 784	1 762	-409	838	616	401	361
Year		2020	2019	2018	2017	2016	2015	2014	2013	2012	2011	2010	2009	2008	2007	2006	2005	2004
Company		AngloGoldAshanti																

Table 5. Newcrest – selected financial & operational data, 2004–2020

Tabela 5. Newcrest – wybrane dane finansowe i operacyjne, 2004–2019

Company	Year	Profit from gold operations	Gold sales	Gold extraction	Profit (margin) on an unit sold	Av margin increse to 2004	Capital employed	Capital charge	Capital charge per unit	Est. Margin on gold net of capital charge	Av net margin increse to 2004	Net margin if Hotelling rule holds	Gap between real and assumed margin
		USD, M	Oz. Troy. M	Oz. Troy. M	USD/oz troy	%	USD, millions	USD, millions	USD/oz. Troy	USD/oz. Troy	%	USD/oz. Troy	USD/oz. Troy
Newcrest	2020												
Newcrest	2019	1 041	2 143.0	2 171.0	485.77	7.60%	10 383.96	1 400.80	653.66	-167.89	-8.91%	348.95	-516.84
Newcrest	2018	1 129	2 529.0	2 488.0	446.42	7.52%	9 451.68	1 019.36	403.07	43.35	-2.81%	307.47	-264.12
Newcrest	2017	819	2 328.0	2 346.0	351.80	6.15%	9 178.20	953.16	409.43	-57.63	-5.03%	277.54	-335.17
Newcrest	2016	936	2 377.0	2 381.0	393.77	7.69%	9 582.25	937.62	394.46	-0.68	-0.09%	251.43	-252.11
Newcrest	2015	723	2 452.0	2 439.0	294.86	5.60%	8 953.10	858.15	349.98	-55.12	-5.77%	229.02	-284.14
Newcrest	2014	988	2 433.0	2 423.0	364.16	8.45%	9 436.68	951.69	391.16	-27.00	-3.55%	208.99	-235.99
Newcrest	2013	1 022	2 032.0	2 110.0	502.95	13.42%	13 463.54	1 263.55	621.83	-118.87	-12.30%	189.84	-308.72
Newcrest	2012	1 532	2 333.0	2 286.0	656.70	19.13%	15 682.42	1 531.39	656.40	0.30	-48.99%	173.55	-173.26
Newcrest	2011	2 034	2 474.0	2 527.0	822.00	26.13%	13 683.37	1 274.61	515.20	306.80	24.93%	158.11	148.68
Newcrest	2010	935	1 745.0	1 762.0	536.08	22.09%	4 556.57	416.02	238.40	297.67	29.00%	144.64	153.03
Newcrest	2009	675	1 637.4	1 631.0	412.46	20.57%	4 005.73	400.97	244.89	167.57	21.01%	132.54	35.03
Newcrest	2008	511	1 764.7	1 781.2	289.56	15.65%	3 046.19	315.43	178.74	110.82	14.45%	120.48	99:6-
Newcrest	2007	544	1 627.0	1 617.3	334.45	27.37%	2 381.64	227.92	140.09	194.36	44.37%	109.17	85.18
Newcrest	2006	287	1 498.5	1 529.9	191.52	8.77%	1 969.15	180.87	120.70	70.82	4.72%	99.64	-28.81
Newcrest	2005	293	1 060.2	1 157.5	275.97	70.49%	1 613.72	137.41	129.61	146.36	126.61%	91.26	55.11
Newcrest	2004	59	249.6	761.8	237.43	0.00%	466.67	38.27	153.33	84.10	0.00%	84.10	ı

Table 6. Selected financial & operational data of all 5 selected companies combined, 2004–2020

Tabela 6. Wybrane dane finansowe i operacyjne wszystkich 5 analizowanych producentów razem, 2004–2020

e	roy		0.7															
Gap between real and assumed margin	USD/oz. Troy	18.79	-274.28	-50.64	-26.40	-1.22	-118.22	-93.26	77.47	312.26	409.88	280.67	175.06	35.03	33.19	43.42	13.12	ı
Net margin if Hotelling rule holds	USD/oz. Troy	69.82	62.34	54.93	49.58	44.92	40.91	37.34	33.92	31.01	28.25	25.84	23.68	21.52	19.50	17.80	16.30	15.02
Av net margin increse to 2004	%	2.00%	-10.18%	-17.61%	-7.58%	-3.20%	-7.42%	-6.44%	6.24%	23.22%	31.46%	29.63%	25.21%	-3.27%	-6.56%	-2.64%	-54.44%	0.00%
Est. Margin on gold net of capital charge	USD/oz. Troy	88.61	-211.94	4.29	23.18	43.70	-77.31	-55.93	111.38	343.27	438.13	306.51	198.74	56.55	52.70	61.22	29.42	15.02
Capital charge per unit	USD/oz. Troy	628.76	636.03	350.11	333.10	320.29	311.05	357.50	353.95	458.13	406.71	295.95	246.43	207.72	179.04	143.86	100.57	91.48
Capital charge	USD, millions	8 271.09	12 121.66	6 484.69	6 547.32	6 220.38	6 473.60	7 563.93	7 598.88	9 936.28	9 303.89	6 757.93	5 560.25	4 648.93	4 086.70	3 469.88	2 251.65	1 789.20
Capital employed	USD, millions	68 983.2	89 856.7	60 127.0	63 045.9	63 570.6	67 538.9	75 001.8	80 968.4	101 754.0	8.088 66	74 019.0	55 546.9	44 895.5	42 703.2	7.777 78	26 443.3	21 819.5
Av margin increse to 2004	%	9.75%	6.63%	5.76%	6.26%	%66.9	3.40%	6.42%	12.45%	22.13%	26.62%	24.49%	22.43%	13.04%	12.70%	12.56%	-19.69%	0.00%
Profit (margin) on an unit sold	USD/oz troy	717.37	424.09	354.40	356.28	363.99	233.75	301.57	465.33	801.39	844.84	602.46	445.17	264.27	231.74	205.08	129.99	106.50
Gold extraction	Oz. Troy. M	13 353.0	19 447.8	18 381.0	19 776.0	19 579.0	20 830.0	21 049.0	21 475.0	21 912.0	23 110.0	22 988.0	22 657.0	22 567.2	22 834.3	24 551.9	22 632.5	19 993.3
Gold sales	Oz. Troy. M	13 154.6	19 058.2	18 522.0	19 656.0	19 421.0	20 812.0	21 158.0	21 469.0	21 689.0	22 876.0	22 835.0	22 563.4	22 380.7	22 826.0	24 120.5	22 389.2	19 558.7
Profit from gold operations	USD, M	6 2 2 9	8 248	6 514	7 046	7 127	4 869	6 348	6 663	17 560	19 524	13 849	10 086	5 964	5 292	5 035	2 942	2 129
Year		2020	2019	2018	2017	2016	2015	2014	2013	2012	2011	2010	2009	2008	2007	2006	2005	2004
Company		Sum/average																

**Revenues & hedging impact.** All companies employ various hedging strategies to minimize price risk and provide detailed information on them. Also they report their realization prices benchmarked to current gold market prices. A review of this information revealed that the actual impact of hedging is insignificant for the analyses presented in the article.

**Capital employed** was calculated by the author as total assets net of current liabilities. This method is biased since a portion of short-term liabilities consists of an interest-bearing debt, but giving the purpose of analyses, the resulting error was considered small enough to be omitted.

A capital charge was applied to capital employed at the uniformed rate equal to industry WACC as calculated by Damodaran for the metal industry. This is a simplification but represents a good starting point for a further analyses.

## 5.2. Testing the hypothesis about the pace of margin growth

The growth rate of margins per unit realized by mining companies must exceed a rate equal to their WACC. To test this assumption for each company a string of expected margins was created using 2004 as a starting point and industry WACC. Since after applying a capital charge 2 companies in 2004 demonstrated negative margins (AngloGoldAshanti and Kinross), the risk-free rate was applied in their cases and only for 2004. The resulting calculations are shown below (Figures 2–4).

The results show no evidence of following Hotelling's Rule. Margins deviated strongly from 2009 up to 2012 and then in 2019 from the assumed path. It can be argued though that

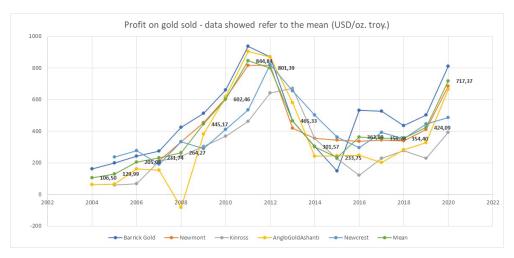


Fig. 2. Operational margins on gold sold of selected companies 2004–2020 Source: own calculations based on companies' reports

Rys. 2. Marże operacyjne na sprzedanym złocie wybranych firm 2004–2020



Fig. 3. Margins net of capital charge on gold sold, 2004–2019/2020 Source: own calculations based on companies' reports

Rys. 3. Marże bez narzutu kapitałowego na sprzedane złoto, 2004–2019/2020

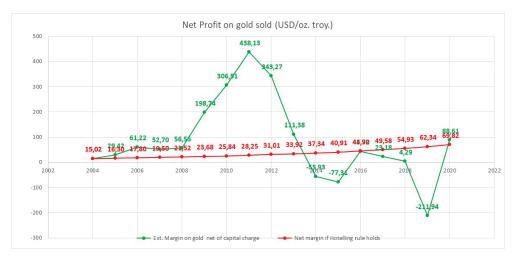


Fig. 4. Real net margins versus Hotelling based margins, 2004–2020 Source: own calculations based on companies' reports

Rys. 4. Realne marże netto a marże hotelowe, 2004–2020

this was an exceptional situation of global crises and demand from investors looking for a "safe heaven". But if the Hotelling rule really worked this shall cause an increase in output to correct the demand based shock. Nothing like this happened. Moreover, these deviations were extremely high (Figure 5).

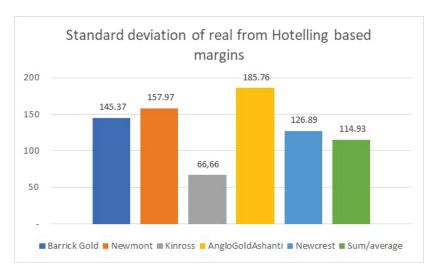


Fig. 5. Standard deviation from Hotelling based margins for the period 2004–2019/2020 Source: own calculations

Rys. 5. Odchylenie standardowe od teoretycznych marż wynikających z twierdzenia Hotellinga

# 5.3. Testing the hypothesis about output development

The second hypothesis defined was that output shall follow deviations from the Hotelling growth line. If the margins are low, production shall be restricted to delay extraction and,



Fig. 6. Number of years when Hotelling's Rule was presumably followed

Rys. 6. Ilość lat, w których decyzje zarządów były zgodne z twierdzeniem Hotellinga

reversely, when margins are high, output shall grow. The author assumed that such a decision could be made within the year as typically managements base their decisions on forecasts and estimated future results. Below (Figure 6) numbers of years when such mechanism could be noticed. In total, this is nearly half of all the observations. This is clearly the lack of any relation.

## 5.4. Testing the hypothesis about the pace of margin growth

The expected margins have been set using industry WACC. This is a biased parameter since it does not recognize individual capital structure and risks of companies under analyses. Therefore, it is possible that applying company specific discount rates would lead to results more supportive to the Hotelling preposition. To check this hypothesis the following procedure was applied. For each company the actual rate of estimated margin growth was calculated for each year. Then a linear growth function was identified using the minimum square root estimation. It was assumed that the correlation of actual results with such a function defines how well an individual company fits" the Hotelling rule. This is a dynamic parameter since individual risk factors changed every year. However mining is a capital intensive, long-term oriented business. As such, individual risks factor cannot vary significantly in the short term. Therefore, it should be possible to determine an easy function describing the individual risk parameter. The results of a linear regression analysis are given below (Table 7).

Table 7. Results of linear regression of individual "fit" to the Hotelling Rule for selected companies

Tabela 7. Wyniki regresji liniowej indywidualnego "dopasowania" do Reguły Hotellinga dla wybranych firm

Company	X(1)	X(0)	R2	R
Barrick Gold	-0.0232	1.2241	0.4277	0.6540
Newmont	0.0193	1.3126	0.1350	0.3674
Kinross	0.0030	1.0060	0.0050	0.0707
AngloGoldAshanti	0.1175	0.4296	0.4130	0.6427
Newcrest	-0.1006	1.4252	0.2121	0.4605

X(1) stands for the slope of a regression line, and X(0) for intercept and R2 and R denote Correlation Coefficient and Coefficient of Determination. The obtained results indicate medium level to low level of the R coefficient. Therefore they cannot be considered as supporting the Hotelling Rule.

## **Conclusions**

Hotelling's Rule implies three fundamental consequences regarding the development of margins in the mining sector. Firstly, that they shall gradually increase following a rate of return compensating delayed benefits from a postponed deposits' exploitation. Second, that decline in margin below a growing path shall cause a reduction in production. And finally, that each producer shall be aware of its individual rate of return as well as follow it in the decision making process. The analyses presented herewith was based on relevant financial and operational results of the top 5 gold miners. It revealed no support for Hotelling's Rule. There's no evidence that over a period of 16–17 years margins follow any exponential line. Neither is there evidence of a relation between changes in output and margins. The period covers a full cycle of gold prices. Certainly, one may try to expand the length of analyses, but this would create another problem. This of economic value of resources in the context of technology and social changes. Although mineral deposits are non-renewable they may lose markets and become obsolete. The closest example is that of coal. Any coal miners slowing down exploitation fifteen years ago gave up margins they could get, forever.

The publishing of this work was supported by the Andrzej Frycz Modrzewski Krakow University WZIKS/EiF/5/2020.

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#### THE HOTELLING'S RULE IN PRACTICE - ANALYSIS OF GOLD MINING SECTOR

## Keywords

valuation, corporate finance, sustainable finance, Hotelling Rule, mineral assets

#### Abstract

The paper presented intends to fill up a gap in surveying the Hotelling Rule by taking a company based, microeconomic approach based on analyses of annual reports. Using selected data three fundamental hyphothesis are tested:

- 1) growth rate of margins ("net margins" including a capital charge) per unit realized by mining companies must exceed a rate equal to their cost of capital,
- 2) output shall follow deviations from the Hotelling growth line,
- 3) margins shall follow a path set by individually defined expected rate of return.

The analysis was based on 5 leading gold producers, responsible for ca 15–20% of global primary production, all of them public and listed on a stock exchange for the entire period of 2004–2019/2020. As margin shall grow at a rate compensating individual risk of a company in consideration, they shall not be homogenous. At 1st step industry WACC was adopted to calculate a normalized capital charge. The calculations revealed no support for Hotelling Rule. There is no evidence that over a period of above 15 years margins follow any path determined by a growing expotential function, following a compound rate. Subsequently it was checked whether output volume is corrected due to development of actual versus expected (resulting from the Hotelling Rule) margin values. Selected companies were near indifferent to this parameter while taking decisions in area of volumes supplied. Neither there is no evidence of relation between changes in output and margins. Finally, it was checked whether differences between expected and actual margins' growth paths could be described by a linear function, resulting from consequent adoption of a risk rate component. Here neither any evidence was found. In conclusion no support for the Hotelling rule was identified.

#### TWIERDZENIE HOTELLINGA W PRAKTYCE – ANALIZA SEKTORA GÓRNICTWA ZŁOTA

# Słowa kluczowe

wycena, finanse przedsiębiorstw, twierdzenie Hotellinga, aktywa geologiczno-górnicze, finanse a zrównoważony rozwój

## Streszczenie

Prezentowany artykuł ma na celu wypełnienie luki w badaniach nad twierdzeniem Hotellinga poprzez przyjęcie podejścia mikroekonomicznego, opartego na analizie sprawozdań finansowych konkretnych firm górniczych. Wykorzystując zawarte w nich dane, poddano sprawdzeniu trzy hipotezy:

- stopa wzrostu marży jednostkowej netto (z uwzględnieniem kosztu kapitału) musi być wyższa niż koszt kapitału,
- zmiany wolumenu produkcji powinny odzwierciedlać odchylenia od stopy wzrostu marż wyznaczonych twierdzeniem Hotellinga,
- marże powinny wzrastać zgodnie z indywidualnie wyznaczoną przez każdego producenta stopą wzrostu.

Analizę przeprowadzono na przykładzie 5 wiodących producentów złota, którzy dostarczali 15-20% podaży pierwotnej (z wyłączeniem odzysku) tego metalu, notowanych na giełdach w całym badanym okresie. Ponieważ wzrost marż powinien kompensować indywidualne ryzyko każdej z firm, nie powinien on być homogeniczny. Jednak w pierwszym kroku średnioważony koszt kapitału (WACC) dla całego górnictwa złota został wykorzystany dla wyznaczenia kosztu kapitału. Obliczenia wykazały brak zgodności z twierdzeniem Hotellinga. Nie znaleziono dowodu, że w 15-letnim badanym okresie wzrost marż odpowiadał jakiejkolwiek wykładniczej funkcji. Następnie zbadano, czy wolumen produkcji był korygowany tak, aby przeciwdziałać odchyleniom od teoretycznej ścieżki wzrostu marż wyznaczonej twierdzeniem Hotellinga. Wykazano, że korporacje podejmowały decyzje o wydobyciu niezależnie od zmian w marżach. Tak samo nie znaleziono dowodu na zbieżność pomiędzy poziomem ich indywidualnej produkcji a wielkością ich marż. W końcu sprawdzono, czy różnice pomiędzy oczekiwanymi a uzyskiwanymi marżami mogą być opisane przez statystyczną zależność w postaci funkcji liniowej mogacej wynikać z zastosowania innej stopy ryzyka niż tej homogenicznej przyjętej pierwotnie do obliczeń. Tu też nie znaleziono żadnej przesłanki potwierdzającej taką zależność. W konsekwencji stwierdzono brak przesłanek uzasadniających stosowanie twierdzenia Hotellinga na poziomie mikroekonomicznym.