Metrol. Meas. Syst., Vol. 28 (2021) No. 2, pp. 383–395
DOI: 10.24425/mms.2021.136614

METROLOGY AND MEASUREMENT SYSTEMS
Index 330930, ISSN 0860-8229
www.metrology.wat.edu.pl

OBSERVATION PROBABILITY ESTIMATION OF DEAD-TIME MODELS USING MONTE CARLO SIMULATIONS

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Abstract

One of difficulties of working with pulse mode detectors is dead time and its distorting effect on measuring with the random process. Three different models for description of dead time effect are given, these are paralizable, non-paralizable, and hybrid models. The first two models describe the behaviour of the detector with one degree of freedom. But the third one which is a combination of the other two models, with two degrees of freedom, proposes a more realistic description of the detector behaviour. Each model has its specific observation probability. In this research, these models are simulated using the Monte Carlo method and their individual observation probabilities are determined and compared with each other. The Monte Carlo simulation, is first validated by analytical formulas of the models and then is utilized for calculation of the observation probability. Using the results, the probability for observing pulses with different time intervals in the output of the detector is determined. Therefore, it is possible by comparing the observation probability of these models with the experimental result to determine the proper model and optimized values of its parameters. The results presented in this paper can be applied to other pulse mode detection and measuring systems of physical stochastic processes.

Keywords: Time interval between pulses, nuclear detector, Monte Carlo simulation, dead time model.

1. Introduction

Responses of pulse mode nuclear detectors are discrete in time. Therefore, detection of a radiation particle as an event engages the detector and its electronics for a period of time. The detection events in nuclear measurements are random processes of two types, correlated events like neutrons in a multiplying media, and uncorrelated events like neutrons originated from a decaying neutron source in a non-multiplying media. Each process has its own randomness nature. Measuring a stochastic process utilizing a pulse mode detection system with certain dead time features, distorts the original process [1]. The distortion depends on the detection system and its timing characteristics. In Fig. 1, the block diagram of a conventional detection system is shown. It has two main parts, the nuclear detector and nuclear instrumentation modules. The

Article history: received October 23, 2020; revised December 31, 2020; accepted February 11, 2021; available online March 19, 2021.

input stochastic process, $R(t)$, is measured by the system and the output stochastic process, $C(t)$, is the result. In an ideal case, both input and output processes are the same. But in a realistic one, $C(t)$ is a distorted stochastic process of $R(t)$.

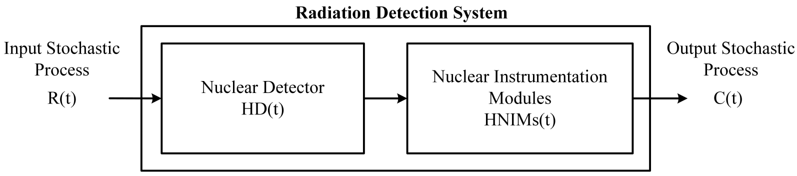

Fig. 1. Symbolic demonstration of a conventional nuclear detection system.

A detection system based on a Geiger–Muller detector is taken as an example, as illustrated in Fig. 2. The detector itself has its own dead time feature. Therefore, after each event the detector cannot register another event until a certain time which is named "G–M Tube Dead Time" in the figure. With the passage of time, the internal detector electrical field is gradually recovered until the time that the amplitude of secondary events can cross the "Discrimination Level" which is determined by the nuclear instrumentation modules of the detection system. This time is defined as "Counting System Dead Time" in Fig. 2. A full recovery of the electrical field of the detector takes a longer time interval which is called "Recovery Time".

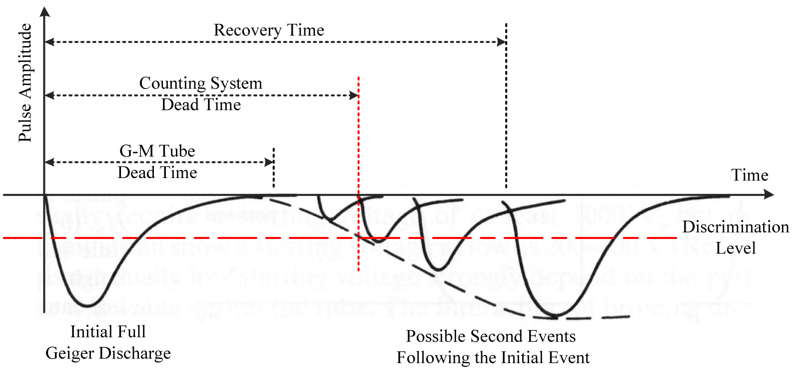

Fig. 2. Timing of a detection event and its following recovery of the detector in a Geiger Muller detection system.

There are three different models proposed in the literature for the dead time of a detection system [2–4], however, there are other methods of describing and correcting dead time losses [5–7]. The first one is called the non-paralizable dead time model. In Fig. 3, the behaviour of this model is shown in a symbolic way.

This figure shows four consecutive events (interaction of the radiation particles with the detector material) of the measuring stochastic process, $R(t)$. After the first event takes place, the detector is not involved in the detection process for a while. This duration is named as non-paralizable dead time, τ_n, as the detector is not responsible for further interactions in this time interval. Any advance events do not affect the detector until the period of dead time is passed. As the first event is the starting event for the dead time process in the detector, it is detected as a valid event, but the two following events are not counted and are lost. After the dead period of time is finished, the fourth event starts another dead time process as only valid events can start the dead

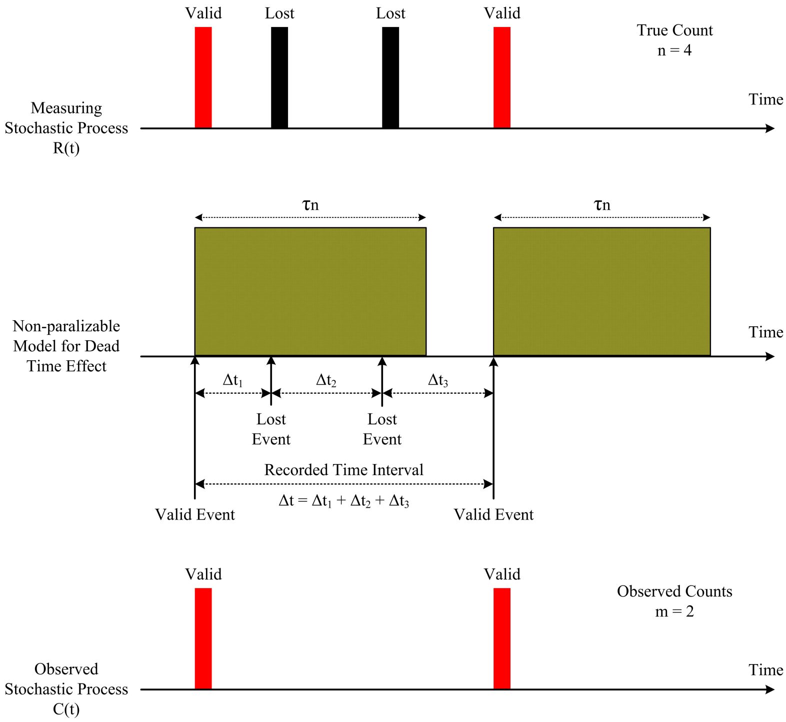

Fig. 3. Non-paralizable dead time model.

time period. In the output of the detection system, $C(t)$, only valid events are observed and events during the dead time are lost. It has two different effects on the observed stochastic process, i. e. reduction in the observed count of the detector (abbreviated as m) and distortion on the measuring stochastic process. The true count of the events, n, is 4, while it is equal to 2 for the observed count. There are three different time intervals between events, while just a time interval is recorded in the detector output causing a distortion on the measuring stochastic process. The relation between true and observed counting rates in non-paralizable dead time model is given as [2]:

$$n = \frac{m}{1 - m\tau_n}\,. \tag{1}$$

The second dead time model for pulse mode radiation detectors is the paralizable one. In Fig. 4, this model is demonstrated representatively. Like the non-paralizable model, four random events are assumed as the true counts of the measuring stochastic process. The first event engages the detector for a period of time during which any secondary event is lost and the dead period is restarted. The second and the third events are lost. As the time interval of the last event, Δt_3, is greater than τ_p, this event is counted as a valid one and, consequently, the dead time of the detection system is also started. The true count (parameter n) in this figure is equal to 4, but the observation (parameter m which is equal to 2) is different. Paralizable dead time is abbreviated as τ_p in Fig. 4. For the paralizable dead time model, the relation between true and observed counting rates is known as [2]:

$$m = ne^{-n\tau_p}\,. \tag{2}$$

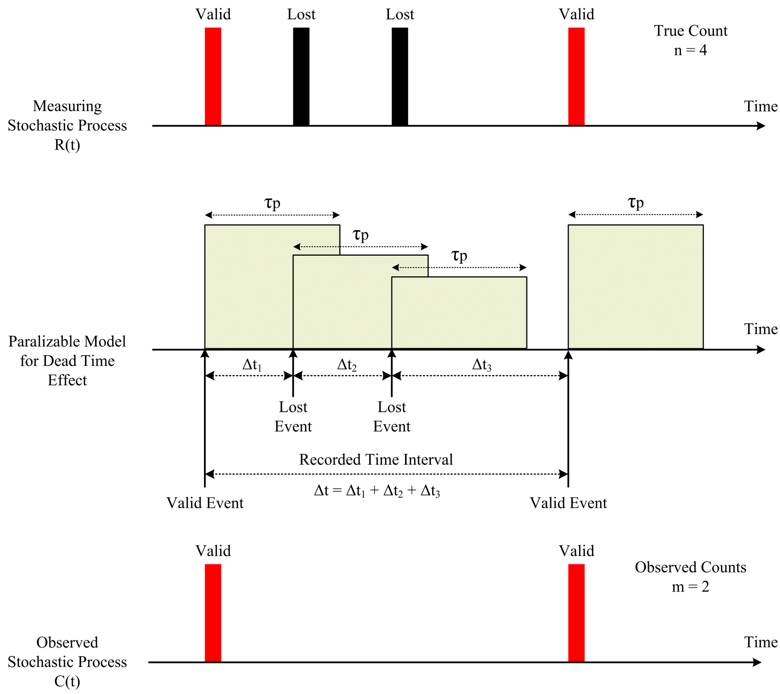

Fig. 4. Paralizable dead time model.

Based on the physical behaviour of the detection system, a combination of the two models introduced above, is also considered. This model which is named the hybrid model [3] is shown in Fig. 5 symbolically. The dead time of the detector is started with non-paralizable dead time and is followed with a paralizable dead time period. Any event during non-paralizable dead time is lost without any effect on the detector behaviour. At the same time, if an interaction is occurred during the paralizable portion of the dead time, the whole dead time process is restarted. The case shown in Fig. 5, consists of six events in total (as $R(t)$, the measuring stochastic process, $n = 6$) in which three of them are lost and the others are counted as valid events ($m = 3$). There are five time intervals due to the occurrence of six interactions, but only two of them are recorded. Lee [3] has investigated and reported the relation between true and observed counting rates in the hybrid dead time model as below:

$$m = \frac{ne^{-n\tau_p}}{1 - n\tau_n}\,. \tag{3}$$

In this research, dead time models are assessed in terms of the distortion caused by their limitations on short time interval pulses. The observation probability of the input pulses in the output of the nuclear detection systems is simulated and evaluated using the Monte Carlo method for different models mentioned above. Comparing the observation probability of the models, with the experimental results, the most consistent model can be determined. The results presented in this paper can be applied to other pulse mode detection and measuring systems based on physical stochastic processes. In the following sections, first, validation of the Monte Carlo simulation is explained. Then, observation probabilities for different models are compared and the effects of counting rates are discussed. Finally, the conclusions are presented.

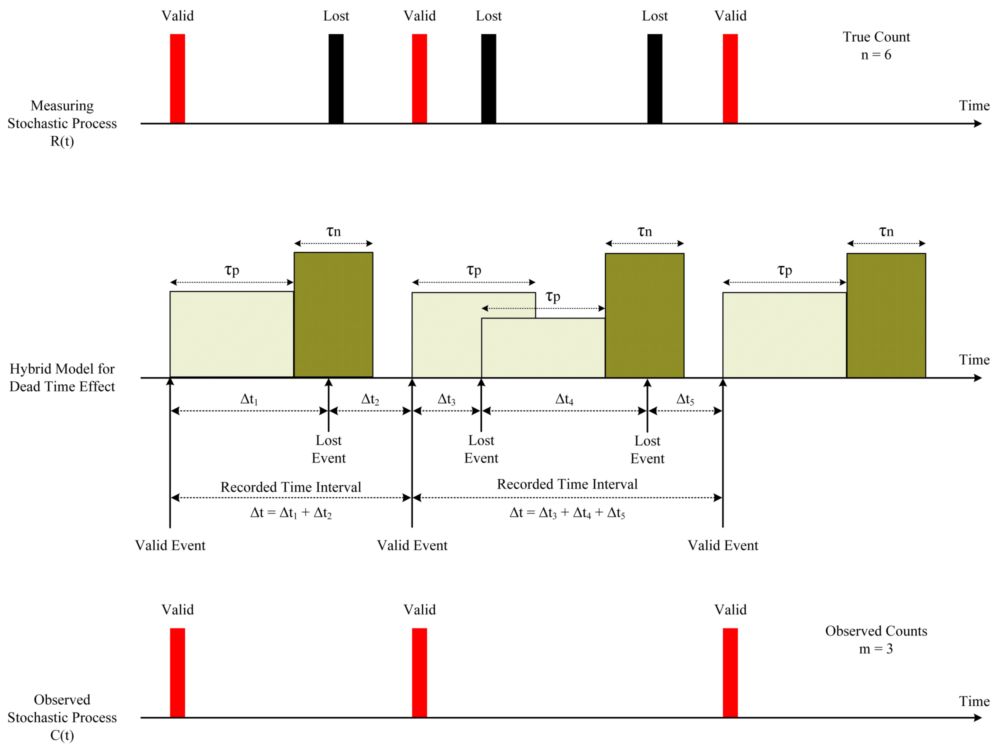

Fig. 5. Hybrid dead time model [3].

2. Monte Carlo simulation and its verification

A useful technique for simulation of stochastic processes is the Monte Carlo method [8–12]. As the Poisson distribution is common in nuclear measurements, it is taken for random generation of time intervals between events caused by nuclear particle interactions. The Poisson probability distribution is given in [2, 13] as:

$$P^k = \frac{(rt)^k e^{-rt}}{k!}\,. \tag{4}$$

In this equation, r is the number of events in unit of time t, and k is an integer value defining the number of expected events, P^k. The observation probability of neighbouring random events in the Poisson distribution is a function of time as below:

$$H_1(t)\,\mathrm{d}t = P^0 \cdot r\,\mathrm{d}t, \tag{5}$$

where $H_1(t)\,\mathrm{d}t$ is the probability of the next event taking place in $\mathrm{d}t$ after the delay of t, $r\,\mathrm{d}t$ is the probability of an event during $\mathrm{d}t$ and by equating $k = 1$ in equation (4), P^0 is obtained as below:

$$P^0 = \frac{(rt)^0 e^{-rt}}{0!} = e^{-rt}. \tag{6}$$

By replacing equation (6) into (5), the definition for $H_1(t)\mathrm{d}t$ is obtained as:

$$H_1(t)\,\mathrm{d}t = re^{-rt}\,\mathrm{d}t, \tag{7}$$

integrating $H_1(t)\,\mathrm{d}t$ in the limits of time interval of two successive events, the result would be the probability of the next event with the time interval of t_{Event}. The cumulative distribution function, ξ is the result of integration as follows:

$$\xi = \int_0^{\Delta t_{Event}} H_1(t')\,\mathrm{d}t' = 1 - e^{-r t_{Event}}. \tag{8}$$

Note that Δt_{Event} is the time at which the next event is seen. Finally, the time interval between successive events in the Poisson distribution probability is expressed by rearrangement of (8) as:

$$\Delta t_{Event} = \frac{\ln\left(\frac{1}{\xi}\right)}{r}. \tag{9}$$

Parameter ξ is a uniform random variable between zero and unity easily generated in MATLAB software engineering tool [14]. In Fig. 6, on the left hand side, the time intervals of 10^4 successive events with the Poisson distribution generated by equation (9) are demonstrated. On the right hand side, the time intervals between observed pulses in the output of a detection system are illustrated. The paralizable dead time and the true counting rate are assumed to be 50 µs and 10^4 pulses per second respectively. Note that computer-based random number generators are called pseudorandom number generators as they are grounded on the algorithms with finite random number generation cycles.

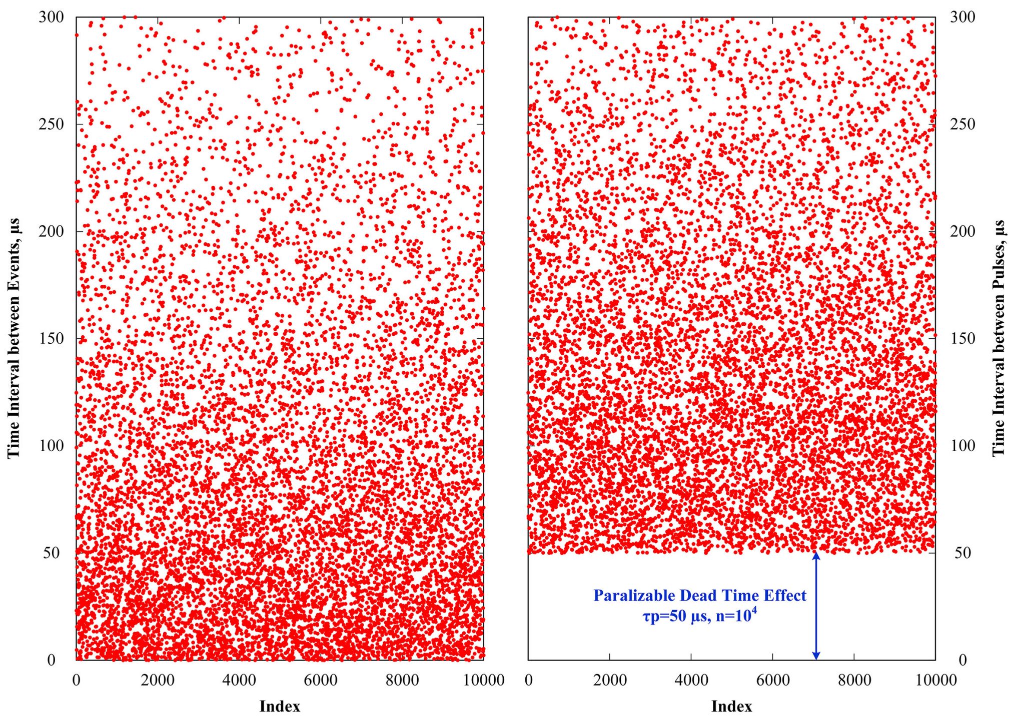

Fig. 6. Pseudorandom Poisson time intervals simulated in the MATLAB software engineering tool [14]. On the left, the generated distribution, on the right, the time intervals between observed pulses in the output of a detection system with 50 µs paralizable dead time are demonstrated. The true counting rate is assumed to be 10^4 pulses per second.

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In equations (1) to (3), the behaviour of the models is defined by analytical formulas. They just show the relation between true and observed counting rates of dead time models. Thanks to using the Monte Carlo simulation, a detailed description can be provided. To validate the Monte Carlo code written in the MATLAB software, the three dead time models are also simulated analytically and the results are compared in Figs. 7 and 8 (true counting rates versus the observed counting

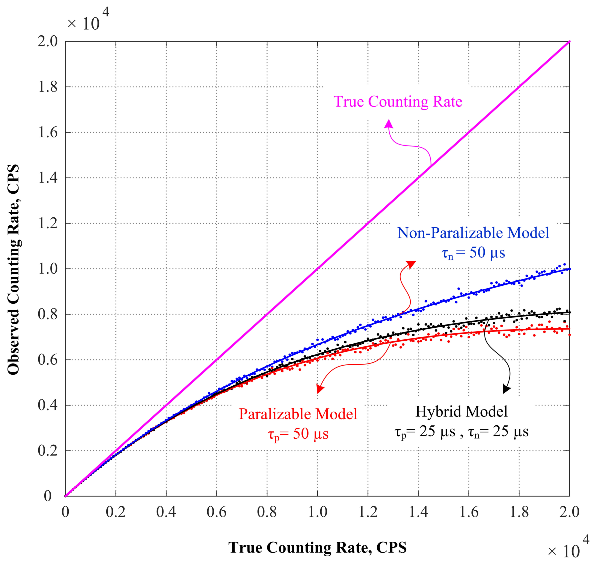

Fig. 7. Analytical and statistical simulation results for dead time models. Parameters of the dead time models are shown on the figure.

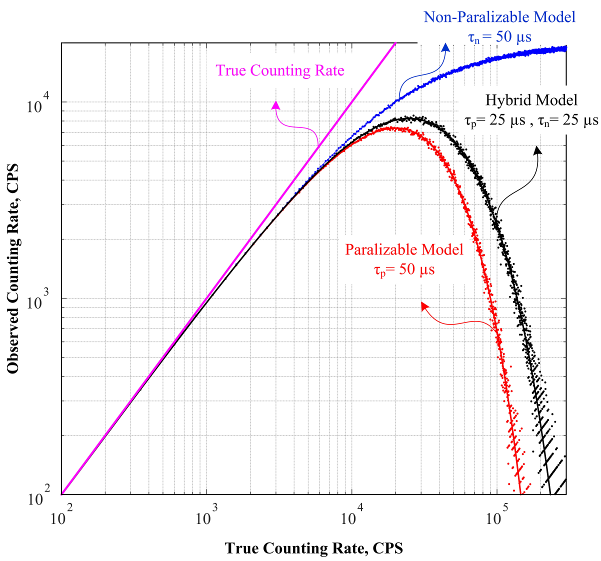

Fig. 8. Analytical and statistical simulation results for dead time models. Parameters of the dead time models are shown on the figure. At high counting rates, the paralizable and hybrid models have decreasing behaviours, while the non-paralizable model is generally increasing.

rate). The behaviour of the three models increases at low counting rates (lower than 2×10^4 CPS of the true counting rate). At the same time, it decreases for the paralizable and hybrid models at high counting rates, as can be seen in Fig. 8 in which the results are demonstrated on logarithmic scales. It should be noted here that parameters of the models assumed in the simulation are mentioned in the figures. A good consistency of the Monte Carlo code with the analytical formulas is obvious.

3. Results and discussion

3.1. Observation probability of different dead time models

Two important dead time effects on a measuring stochastic process, as it was mentioned before, are the losses of counts (as explained in Figs. 7 and 8), and the distortion of the time interval distribution between events. In Fig. 9, the distributions estimated by the dead time models are compared with the pseudorandom Poisson distribution. The shapes of all the dead time models are the same in long time intervals. Yet, they are distorted for time intervals lower than 150 μs. Please, note that values of the parameters are shown in the figure. There are three distortions of the time interval distribution *i.e.*:

– In short time intervals between zero time and the dead time value of the models (for hybrid model this is the summation of the two parameters), no time interval is recorded, as can be seen in Fig. 9, with time intervals lower than 50 μs.
– In the short time interval region, between 50 μs and 150 μs in Fig. 9, distributions obtained from paralizable and hybrid models are distorted severely and no information is gained from this data. There are no distortions for the non-paralizable dead time model in this region.
– In the long time interval region, the shape of the distribution remains the same as the original one (the pseudorandom Poisson distribution). Please, note that at very high counting rates

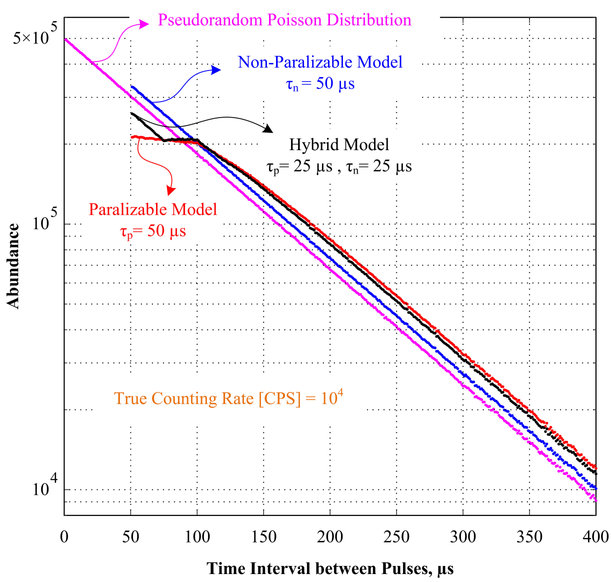

Fig. 9. Comparison of the Poisson distribution (the pseudorandom generated data) of time intervals between pulses with the distributions of different dead time models.

DOI: 10.24425/mms.2021.136614

of the detection system, this region is also distorted (the high counting rate is defined in comparison with the system dead time). Curve fitting to the data in long time interval region is a method that can be used here for estimating the true counting rate. Using the observed counting rate and the estimated true counting rate of the system, the dead time of the system is measured with the method called the time interval distribution (TID) method [1]. Note that the equipment required for measurement of time interval between pulses is explained elsewhere [15].

In Fig. 10, the experimental data and Monte Carlo simulation results for a BF_3 detector are illustrated. The true counting rate is estimated for time intervals longer than 20 μs using the TID method introduced above and is equal to 48580 ± 35 counts per second. Using the paralizable and non-paralizable models, the dead time parameters of these models are easily estimated as 2.4207 ± 0.018 [μs] and 2.2887 ± 0.013 [μs] respectively. More complementary information is mentioned in the figure. Using these experimental data, the Monte Carlo simulation of the detection system is performed and the results are shown on the right hand side of Fig. 10. For hybrid model, it is assumed that paralizable and non-paralizable dead time parameters are the same as 1.2 μs. A comparison of experimental and numerical results (Monte Carlo results) for the varoius time interval distributions shows that:

– In the experiment, the distribution in short time intervals first increases and then decreases after 5 μs. Yet, it is different for all Monte Carlo results.
– Except for the data in the region shorter than 5 μs, the distributions predicted by the dead time models are consistent with the experimental data.

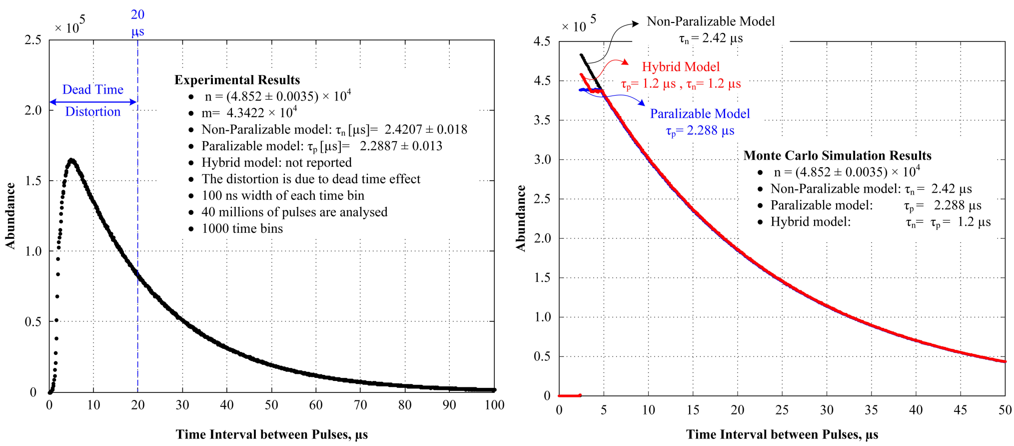

Fig. 10. Comparison of experimental and Monte Carlo results for time interval between pulses of a conventional BF3 neutron detection system.

In Fig. 11, observation probabilities of pulses with different time intervals for the dead time models are demonstrated. For the all cases, below the dead time value of 50 μs, (in the hybrid model this is the summation of both parameters) zero probability is seen. For both paralizable and hybrid models, in the range of 50 μs to 250 μs, the observation probability is an increasing function of the time interval between pulses. Whereas, for the non-paralizable model, all pulses are observed with a hundred percent probability. In the long time interval region, responses of all dead time models are the same and all events are observed as pulses in the output of the detection system.

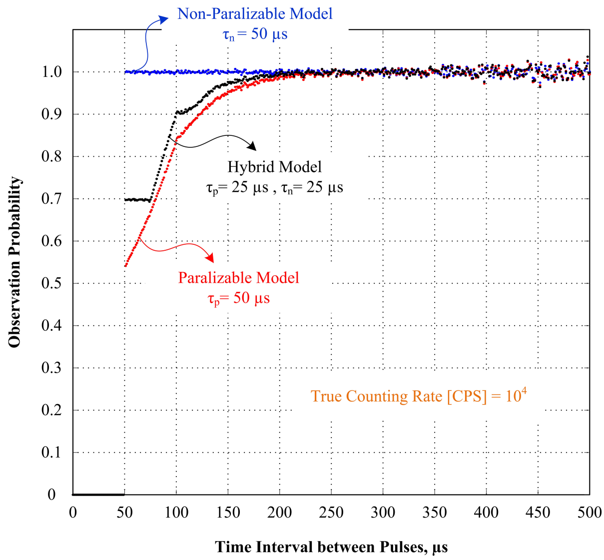

Fig. 11. Observation probability of pulses with different time intervals.

3.2. Observation probability at different counting rates

Due to the dead time effect, particle detection systems have short time memory of previous events occurred in the sensitive detector material. Therefore, behaviour of a detection system is a function of the event rate. More interaction with the detector means more extensive dead time effects on the time interval distribution of pulses. As a result, it is expected that at different event rates, the observation probability might also be altered. In Fig. 12, the observation probability of

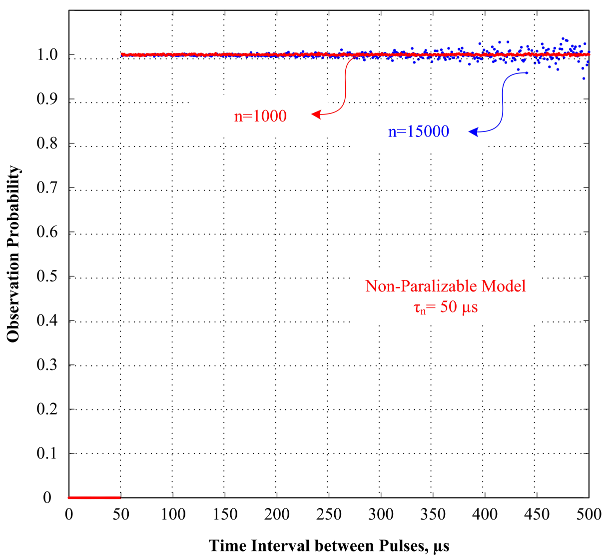

Fig. 12. Observation probability of the non-paralizable dead time model at different event rates.

DOI: 10.24425/mms.2021.136614

the non-paralizable model is depicted. The problem is simulated at two event rates (n = 1000 as the low event rate and n = 15000 as high the event rate of the detector). It obvious that in this model, the event rate parameter does not affect the observation probability of the pulses as the dead time period is not refreshed by the neighbouring events. In Figs. 13 and 14, observation probabilities of both paralizable and hybrid models at different event rates are shown respectively. Similarly to the results explained in Fig. 11, for short time intervals lower than the dead time (for hybrid model this is the summation of the two parameters), zero probability for observation

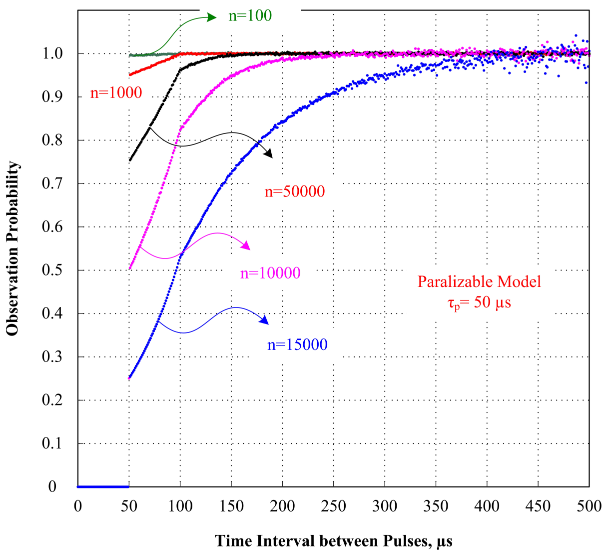

Fig. 13. Observation probability of the paralizable dead time model at different event rates.

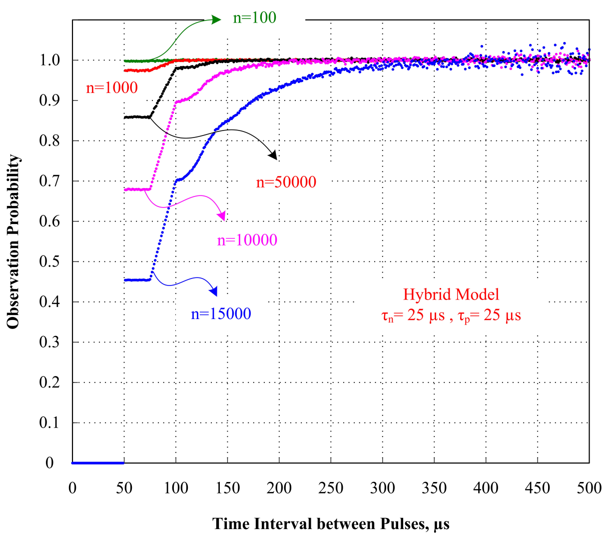

Fig. 14. Observation probability of the hybrid dead time model at different event rates.

of events is seen. Beyond this threshold, the observation probabilities grow with longer time intervals between pulses. At very low event rates (the results for $n = 100$), the behaviour of the two models is nearly the same as that of the non-paralizable model. At the sime time, for higher event rates the results are different. A higher event rate means more extensive distortion of the observation probability of pulses.

4. Conclusions

Dead time is a feature inherent to pulse mode radiation detection systems. It effect interferes with the response of particle detection systems especially at high event rates. That is why the observed counting rate is less than true counting rate of the detection system. Losses of events affect the statistical features of the measuring stochastic process which is distorted as neighbouring events are lost. As pulses with short time intervals are not observed in the output of the detection system, the time interval distribution of the observed stochastic process differs from the measured one. There are three models proposed for description of dead time effect in the literature with analytical formulas describing the relation between true and observed counting rates of a detection system. To investigate the statistical distortions caused by the dead time in detail, a computer code for simulation of dead time models was written based on the Monte Carlo method. It is validated by the analytical formulas at different true counting rates. The following are the conclusive points of this research.

- In applications where distortion of the measuring stochastic process is the main concern, the proper model to minimize the distortion can be determined by comparing the observation probability of dead time models with the measuring results of the detection system.
- For paralizable and non-paralizable models, no observation probability of the time intervals below paralizable and non-paralizable dead times respectively is seen. For hybrid model it is equal to the summation of the parameters (paralizable and non-paralizable dead times). Whereas in the experiment, the time interval distribution can start from nearly zero. It is obvious that the theoretical models do not adequately describe the dead time distortion of the time interval distribution in the short time interval region.
- It is obvious that the observed stochastic process differs from the measuring stochastic process due to the dead time distortion. This effect is investigated in this research using the Monte Carlo simulation. It is suggested that scientists should pay attention to the distortion especially for measurements and analyses concerning statistical features of physical processes.
- It is proved that all dead time models are not able to estimate the time interval distribution in the short time interval regions. But, for long time interval region, their predictions are consistent with the experimental data.

Due to the dead time effect, the distortion on the measuring stochastic process and observation probability of events depend on the event rate itself for paralizable and hybrid models. In other words, the distortion on the measuring stochastic process varies at different event rates in these models.

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