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DYNAMIC STATES EQUATIONS OF TRANSPORT PIPELINE IN DEEP-SEA MINING

The transport pipeline of lifting the underwater minerals to the surface of the water onto the ship during the movement of the vessel takes in the water a curved deformed shape. Analysis of the state of stability of the pipeline showed that if the flow velocity of fluid in the pipeline exceeds a certain critical value V_{kr}, then its small random deviations from the equilibrium position may develop into deviations of large amplitude. The cause of instability is the presence of the centrifugal force of the moving fluid mass, which occurs in places of curvature of the axis of the pipeline and seeks to increase this curvature when the ends of the pipeline are fixed. When the critical flow velocity is reached, the internal force factors become unable to compensate for the action of centrifugal force, as a result of that a loss of stability occurs. Equations describing this dynamic state of the pipeline are presented in the article.

Keywords: deep-sea mining, vertical pipelines, stability of deep-sea pipelines

1. Introduction

In the literature one can find works devoted to solving the problem of vibrations of elastic pipelines with a flowing fluid. Niordson [8] and Heidelman [5] showed that with the flow of a fluid, a loss of stability of the pipeline in the form of buckling is possible, similarly to a loss of stability of a column under the action of a static load. Long [7] first drew attention to the problem of pipeline stability with one free end. Benjamin [1] established the laws that determine the phenomenon of instability of pipeline oscillations with a flowing fluid with one loose end. Gregory and Paidussis [4] considered the question of the instability of oscillations of a cantilever pipeline with liquid flowing through it under the assumption that the pipeline is in a horizontal plane and

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not subject to longitudinal tension. Three-dimensional (3-D), nonlinear, coupled, axial, bending and torsional responses of an 18, OOO-fptipe system are studied with the new nonlinear finite element method (FEM) code presented in Part 1 with an example of the recovery of manganese nodules in the Pacific Ocean. For this Part IIthe, the pipe top is pinned to ship, and it is free and independent of the self-propelled seafloor nodule miner. The pipe system is a vertically varying, current flow when creating the static equilibrium configuration. For dynamic analysis, the pipe is excited by periodic large-amplitude horizontal, as well as vertical, motions, the internal slurry flow, and the external hydrodynamic forces. For torsional coupling, a consistent mass-matrix formulation is used. The external torsional moments induce biaxial bending deflection and vibration in response to a large pipe twist. The axial-to-torsion and axial-to-bending couplings are found to be strong. Response periods to large-amplitude. The upward internal slurry flow reduces the axial stress and increases the horizontal displacements. Numerical stability of the solution is sensitive to the specific sequence of the steps, large passive torque, excitation frequencies, and excessive axial excitation amplitudes. The present response is a part of the pipe restrained.

Chung published a very interesting work with co-authors [2]. The paper presents extensive analyzes of various pipeline systems transporting ocean floor spoil to the ship together with charts illustrating the shape of the pipeline permanently fixed on the ship during its operation. Szelangiewicz et al. [9] presented computer simulation results of vertical deflection and tensions within single and double vertical pipelines with fixed force from the ship's movement (linear movement at constant speed) and regular force from the waves. Yu and Liu [11] analyzed the dynamic characteristics of a vertical pipe under the influence of constant movement, current direction and wave. The simulation results showed that the axial stress is dominant on the vertical pipe, its maximum is located at the pipe top and also all stresses are much less than the allowable value of the vertical pipe. The Yao et al. [10] in recent years has studied the flow characteristics of the mixture in a flexible pipe in an experimental installation.

2. Problem discussion

In previous works, the main focus was usually on determining the impact of mass flow on the natural frequencies of the pipeline and determining the critical frequency of oscillations based on some dimensionless parameters characterizing the inertial properties of the piping system. However, issues related to the establishment of a lower limit of the critical velocity of the mixture at which the pipeline loses its stability have not been studied. The purpose of this work is to obtain the equation of oscillation, which is the starting point to solve the problem of ensuring a stability of underwater pipeline transporting a mixture from the bottom to the surface of the ocean. Fig. 1 shows the diagram of forces applied to the pipeline elements and the mixture flowing in the pipe.

Based on preliminary estimates of the main operational parameters of hydraulic lifting [6], a simplified system of longitudinal and transverse oscillations of the slurry pipeline was obtained:

$$m_0 \frac{\partial^2 u}{\partial t^2} - \frac{\partial N}{\partial s} - \frac{\partial}{\partial s}\left(\frac{\partial M}{\partial s}\frac{\partial v}{\partial s}\right) = q_{nx} + q_{wx} + m_0 g \tag{1}$$

$$m_0 \frac{\partial^2 v}{\partial t^2} + \frac{\partial^2 M}{\partial s^2} - \frac{\partial}{\partial s}\left(N\frac{\partial v}{\partial s}\right) = q_{ny} + q_{wy} \tag{2}$$

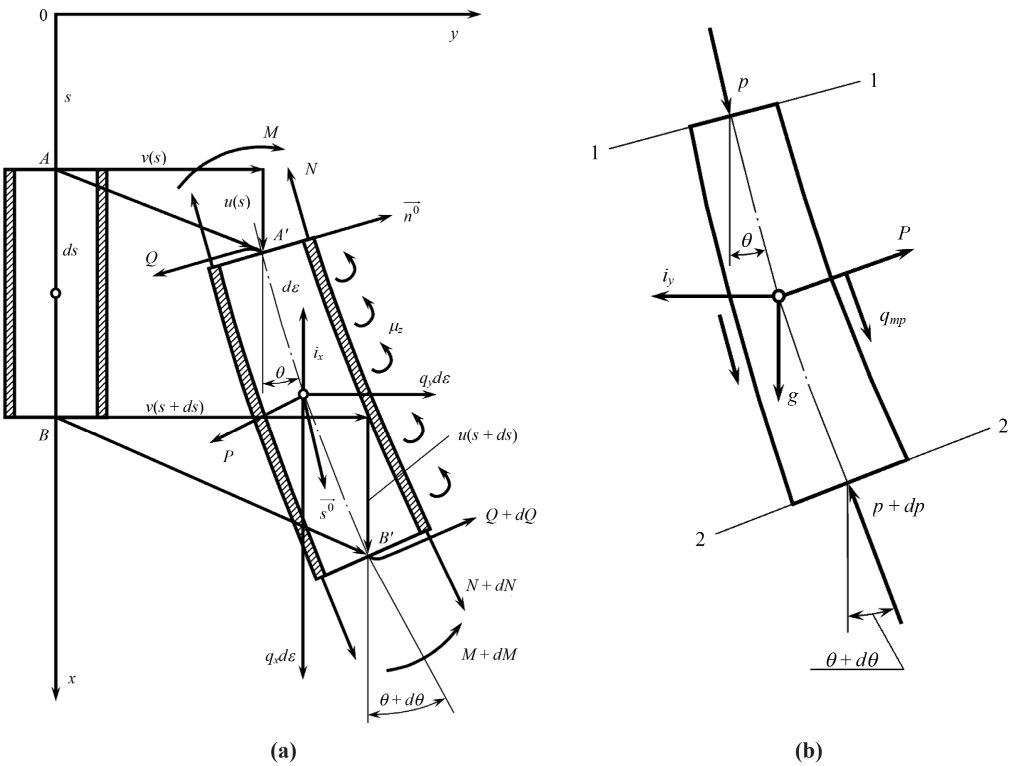

Fig. 1. Diagram of forces and moments acting on pipe element (a) and fluid element in pipe (b)

Along with the desired longitudinal $u(s,t)$ and transverse $v(s,t)$ displacements of pipeline elements, this system of equations contains unknown force factors: axial N and shear Q forces, and torque M, as well as projections of external q_n and internal q_w dynamic loads. In addition, in equations (1) and (2) the following notation is adopted: m_0 – linear mass of the pipeline; g is the acceleration of gravity; t is time; s is the Lagrangian coordinate.

In [3], for the projections of external linear hydrodynamic loads, the expressions were obtained:

$$q_{nx} = -S_n\left(p_n\frac{d\theta}{ds} + \rho_n g\sin\theta\right)\sin\theta - \frac{dR_n}{ds}\sin\theta + \frac{dR_\tau}{ds}\cos\theta + q_{nx}^u \tag{3}$$

$$q_{ny} = -S_n\left(p_n\frac{d\theta}{ds} + \rho_n g\sin\theta\right)\cos\theta + \frac{dR_n}{ds}\cos\theta + \frac{dR_\tau}{ds}\sin\theta + q_{ny}^u \tag{4}$$

where S_n – the area of the pipeline in outer diameter; p_n and ρ_n – pressure and density of external flow; θ – the angle of deviation of the axis of the pipeline from the vertical; dR_n and dR_τ – normal and tangent to the axis of the element running dynamic components of the external total hydrodynamic force; q_n^u – linear inertial unsteady reaction force of external fluid.

Internal hydrodynamic load is caused by friction q_{mp}, hydrostatic pressure q_{gcm} and inertial forces q_{un}, i.e.

$$q_{wx} = q_{xmp} + q_{xgcm} + q_{xun} \tag{5}$$

$$q_{wy} = q_{ymp} + q_{ygcm} + q_{yun} \tag{6}$$

where (see Fig. 1):

$$q_{xmp} = -q_{mp}\cos\theta$$

$$q_{ymp} = -q_{mp}\sin\theta$$

$$q_{mp} = \frac{1}{8}\lambda\pi d_w \rho_w V^2$$

$$q_{xgcm} = S_w\left(p_w\frac{d\theta}{ds} + \rho_w g\sin\theta\right)\sin\theta$$

$$q_{ygcm} = -S_w\left(p_w\frac{d\theta}{ds} + \rho_w g\sin\theta\right)\cos\theta$$

$$q_{xun} = P\sin\theta$$

$$q_{yun} = -P\cos\theta$$

S_w – live section of the pipeline; d_w – inner diameter; p_w and ρ_w – pressure and density of fluid (pulp) inside the pipeline; λ – Darcy-Weisbach coefficient; V is the flow velocity of the pulp.

According to [3], the expressions for the projections of the linear inertial nonstationary force are:

$$q_{nx}^{u} = m_{Np}\left(v_{tt}\cos\theta - u_{tt}\sin\theta\right)\sin\theta \tag{7}$$

$$q_{ny}^{u} = -m_{Np}\left(v_{tt}\cos\theta - u_{tt}\sin\theta\right)\cos\theta \tag{8}$$

where $m_{np} = \frac{1}{4}\pi d_n^2 \rho_n$ – the running added mass; the subscripts below denote the corresponding partial derivatives.

In [3], relationships are also obtained:

$$\frac{dR_N}{ds} = \frac{1}{2}\rho_n C_N d_n\left(u_t^2 + \left(V_\infty - v_t\right)^2\right) \tag{9}$$

$$\frac{dR_\tau}{ds} = \frac{1}{2}\rho_n C_\tau d_n\left(u_t^2 + \left(V_\infty - v_t\right)^2\right) \tag{10}$$

$$P = m_w\left[\left(v_{tt} + 2v_{st}V + v_{ss}V^2\right)\cos\theta - \left(u_{tt} + 2u_{st}V + u_{ss}V^2\right)\sin\theta\right] \tag{11}$$

where: C_n and C_τ – hydrodynamic coefficients of normal and tangential forces; V_∞ – flow velocity; m_w – linear mass of pulp.

In order to reveal in a pure form the instability arising from the internal flow, let us assume that the incident flow is absent ($V_\infty = 0$). Then, assuming that u_t and v_t are small, we obtain that the hydrodynamic forces (9) and (10) are of the second order of smallness. Under the same assumption we have:

$$
\begin{gathered}
\sin\theta \cong v_s \\
\cos\theta = 1 \\
\frac{d\theta}{ds} = v_{ss} \\
M = EJv_{ss} \\
q_{nx} \cong 0 \\
q_{wy} = S_n\left(p_n v_{ss} + \rho_n g v_s\right) - m_{mp} v_{tt} \\
q_{wx} = -q_{mp} \\
q_{wy} = -S_w\left(p_n v_{ss} + \rho_n g v_s\right) - q_{mp} v_s - P \\
P = m_w\left(v_{tt} + 2v_{st}V + v_{ss}V^2\right)
\end{gathered}
\tag{12}
$$

where E is Young's modulus; J is the axial moment of inertia of the section. Moreover, in equation (1) it is possible to neglect the $\frac{\partial M}{\partial s}\frac{\partial v}{\partial s}$ term, as being of the second order volume. In the accepted approximation, it should be assumed that

$$p_n = p_a + \rho_n g\left(s + u\right) = p_a + \rho_n g s \tag{13}$$

where p_a is atmospheric pressure.

Even with the simplifications given, the system of equations (1) and (2) is connected through the axial force N included in both equations. If we neglect the $\frac{\partial}{\partial s}\left(N\frac{\partial v}{\partial s}\right)$ term in Eq. (2), then this equation will be independent of Eq. (1), however, the effect of the longitudinal tension force N is lost.

We assume that the longitudinal vibrations of the pipeline are so small that they can be neglected in comparison with the transverse ones. In this case (1), taking into account (12), reduces to the equation

$$\frac{\partial N}{\partial s} = -m_0 g + q_{mp} \tag{14}$$

determining the static load of the pipeline from the action of gravity and friction of the flowing fluid. If the lower end of the pipeline is free, then the solution to equation (14) will be:

$$N = \left(m_0 g - q_{mp}\right)\left(L - s\right) - p\big|_{s=L} F = N_0 - \left(m_0 g - q_{mp}\right)S \tag{15}$$

where $N_0 = \left(m_0 g - q_{mp}\right)L - p\big|_{s=L} F$ is the weight of the entire pipeline (taking into account the discharge due to the force of Archimedes and the force of internal friction); L is the length of the pipeline.

If there is a concentrated mass at the lower end of the production pipeline (for example, a platform fixing in place the feeding equipment), then its weight in water should be included in the magnitude of the N_0 force.

In this case, equation (2) assumes the form:

$$\left(m_0+m_w+m_{np}\right)\frac{\partial^2 v}{\partial t^2}+EJ\frac{\partial^4 v}{\partial s^4}+\left[m_w V^2+\left(S_w p_w-S_n p_n\right)-\right.$$
$$\left.-N_0+\left(m_0 g-q_p\right)s\right]\frac{\partial^2 v}{\partial s^2}+2m_w V\frac{\partial^2 v}{\partial s\partial t}+$$
$$+\left[m_0 g+g\left(S_w\rho_w-\rho_n S_n\right)\right]\frac{\partial v}{\partial s}=0 \qquad (16)$$

Equation (16) contains pressure $p_w(s)$, density $\rho_w(s)$, and velocity of the mixture V inside the pipeline, which must be taken from the preliminary hydraulic calculation of the airlift or pump lifting variant. In order not to enter in advance into specific law of change in internal pressure, density and velocity and to obtain conditions for the loss of general stability, let us consider equation (16) under the assumption that ρ_w and V is constant along the height of the pipeline. Then for the pumping option

$$p_w=p_a+\left(\rho_w g+\frac{\lambda}{2d_w}\rho_w V^2\right)s \qquad (17)$$

Note that this pressure distribution inside the pipeline will be, when the pump is installed at the lower end of the pipeline and provides there a pressure equal to

$$p_w\big|_{s=L}=p_a+\left(\rho_w g+\frac{\lambda}{2d_w}\rho_w V^2\right)L \qquad (18)$$

which creates a pressure drop $\Delta p=\lambda\frac{L}{d_w}\frac{V^2}{2}\rho_w$ that provides for friction losses.

Using expression (17) for pressure p_w, and assuming that also $\rho_w\equiv\rho_n$ (this condition is fairly well satisfied if the concentration of solids in the stream is insignificant), the coefficient at $\frac{\partial^2 v}{\partial s^2}$ in equation (16) is converted to the form:

$$\left(m_w+\frac{\lambda}{8}\rho_w\pi d_w L\right)V^2-\left(m_0 g-\rho_n gF\right)\left(L-s\right) \qquad (19)$$

where $G_0=m_0 g-\rho_n gF$ – the weight of the meter of the pipeline in the water; F is the area of the metal.

The coefficient at $\frac{\partial v}{\partial s}$ in (16) under specified assumptions is equal to $G_0=(m_0 g-\rho_n gF)$.

In view of the simplifications made, equation (16) takes the form:

$$\left(m_0 + m_w + m_{np}\right)\frac{\partial^2 v}{\partial t^2} + EJ\frac{\partial^4 v}{\partial s^4} + \\ + \left[M_1 V^2 - G_0\left(L - s\right)\right]\cdot\frac{\partial^2 v}{\partial s^2} + 2m_w V\frac{\partial^2 v}{\partial s \partial t} + G_0\frac{\partial v}{\partial s} = 0 \tag{20}$$

where

$$M_1 = m_w + \frac{\lambda}{8}\pi\rho_w d_w L = \frac{\pi d_w^2}{4}\rho_w\left(1 + \frac{\lambda}{2}\frac{L}{d_w}\right) \tag{21}$$

Note that for fairly rough pipes, the second term in this expression can be two orders of magnitude greater than the first.

3. Conclusion

1. On the basis of the calculation scheme developed and taking into account the accepted physically reasonable assumptions, the differential equation of oscillations of an underwater pumping pipeline transporting a two-phase slurry has been obtained.
2. The velocity of the pulp V enters into (20) in an explicit form, therefore further solution of the problem of stability of the pipeline is reduced to determining the magnitude V_{kr} and ensuring the condition $V <. V_{kr}$.

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