Determining horizontal curvature of railway track axis in mobile satellite measurements

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Abstract. The article discusses the applicability of a novel method to determine horizontal curvature of the railway track axis based on results of mobile satellite measurements. The method is based on inclination angle changes of a moving chord in the Cartesian coordinate system. In the presented case, the variant referred to as the method of two virtual chords is applied which consists in manoeuvring with only one GNSS (Global Navigation Satellite System) receiver. The assumptions of the novel method are formulated, and the assessment of its application in the performed campaign of mobile satellite measurements is given. The shape of the measured railway axis is shown in the national spatial reference system PL-2000, and the speed of the measuring trolley during the measurement is calculated based on the recorded coordinates. It has been observed that over the test section, the curvature ordinates differ from the expected waveform, which can be caused by disturbances of the measuring trolley trajectory. However, this problem can easily be overcome by filtering the measured track axis ordinates to obtain the correct shape – this refers to all track segments: straight sections, circular arcs, and transition curves. The virtual chord method can also be the basis for assessing the quality of the recorded satellite signal. The performed analysis has shown high accuracy of the measuring process.

Key words: GNSS measurements; railway track axis; horizontal curvature; moving chord method application; accuracy assessment.

1. INTRODUCTION

Determining main geometric parameters of the railway track in the horizontal plane (positions and lengths of straight track sections, positions, radii and lengths of circular arcs, and positions, types and lengths of transition curves) is a basic operation in the process of railway track shape evaluation. Railway track measurement methods which are in use in different countries [1-7] have a very long tradition, but – despite various innovations introduced – can still be characterised by huge labour consumption, with the associated huge financial expenses.

New possibilities in the field of inventory of engineering objects are created by the development of satellite measurements and an increase in the accuracy of measurements based on the Global Navigation Satellite System (GNSS) [8-12]. In Poland, the method of mobile satellite measurements (Fig. 1) has been developed for over a decade [13-20]. The aim of the ongoing research project BRIK [21-22] is to obtain an implementation solution.

As a result of measurements, a set of figures is obtained which, after relevant postprocessing, compose a set of coordinates in a given Cartesian system (in Poland – with respect to the horizontal plane – the national spatial reference system PL-2000). The collected set of coordinates makes the basis for identifying particular geometric elements of the track. The method traditionally used for this purpose is based on the chart of horizontal sagittas (Fig. 2), being the most

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frequently used tool for assigning track points to sections with defined geometry.

The sagitta chart method is still very popular in railway applications. Since the sagitta diagram is very similar to the curvature diagram, this method is sometimes used for determining the railway track curvature, which seems unjustified from the formal point of view. It is noteworthy that the measurement of sagittas (horizontal and vertical) has been for years the basis of diagnostic methods to evaluate the geometric condition of a railway track [23-27]. The terms “horizontal unevenness chart” and “vertical unevenness chart” used in those measurements mean, in fact, charts of sagittas measured on a given rail. A similar situation can be observed in commercial computer programmes supporting railway track designing, [28-30], inter alia.

It is noteworthy that the measurement of horizontal sagittas is done in the linear coordinate system, while the railway track is described by point coordinates in the Cartesian coordinate system (which results from requirements concerning track axis marking, among other reasons). Transformation from linear to Cartesian coordinates is difficult and may lead to problems in interpretation of the given geometric structure. Therefore, it is advisable to perform railway track shape identification in the Cartesian system. This recommendation would undoubtedly undermine the sense of further use of the sagitta chart method for this purpose, provided that a direct method to determine curvature is available.

2. DETERMINING CURVATURE WITH THE MOVING CHORD METHOD

The definition of curvature imposes the need to operate with inclination angles of tangents to the geometric system. When the analytical description of a given curve is known, this does not pose a problem. However, for the real railway track, most frequently deformed as result of its operation, determining tangents can be troublesome and burdened with relatively large errors, therefore a concept has emerged to operate with chords instead of tangents when determining track curvature. In [31], a theoretical method was proposed which made use of inclination angle changes of a moving chord of given length (so-called moving chord method) to determine curvature. The use of analytical notation enabled precise positioning of chord ends. The method was practically verified on selected geometric layouts. Figure 3 shows a schematic diagram of determining the curvature with the proposed method.

In the moving chord method, it was assumed that for the considered small railway track segment, the tangents (derived at points M and M_i in Fig. 3) and the corresponding chords (sections (i-1) ÷ i and i ÷ (i+1)) are parallel to each other, while the tangency points are projected perpendicularly onto the chord centres. The curvature k_i at point i is calculated from the formula:

\[ k_i = \frac{\Delta \theta_i}{l_i} \]  

where \( l_i \) is the chord length, and the angle \( \Delta \theta_i \) is the difference between the inclination angles of two adjacent chords with common point i, i.e.

\[ \Delta \theta_i = \theta_{i+(i+1)} - \theta_{(i-1)+i}. \]

The use of this procedure requires the information about curve coordinates in the Cartesian reference system, as the values of angles \( \theta_{(i-1)+i} \) and \( \theta_{i+(i+1)} \) correspond to gradients of straight lines describing both chords.

In [31], the proposed method to determine curvature was verified on an unambiguously defined elementary geometric layout of railway track, which consisted of a circular arc and two symmetrically arranged transition curves of the same type and length. The track segment selected for verification had been calculated based on principles of the analytical design method [32]. A number of geometric variants were analysed for different train speeds, types of applied transition curves, and track diversion angles. Full compatibility was observed between the obtained curvature values and those used as the basis for designing the geometry of the analysed track segment, with respect to both the circular arc, and the transition curves. A possibility was also indicated to use the proposed method when determining curvature with respect to both the X-axis in the Cartesian system, and the length parameter in the linear system.

It was also noted that the proposed method has a wide applicability potential. The practical aspect of the present analysis may be identified when geometrical characteristics of the track axis determined from measurements are not known and the basic goal is to determine them. In this situation, the

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Fig. 2. An example of the chart of horizontal sagittas obtained with an electronic track gauge (on the horizontal axis – distance (m), on the vertical axis – sagitta values (mm), the red lines indicate the tolerance limits for the speed 50 km/h).

Fig. 3. Schematic diagram of determining curvature with the moving chord method [26].
The proposed method ideally meets the assumptions of mobile satellite measurements, as these measurements provide a great number of track axis coordinates in the Cartesian coordinate system in a very short time.

In [31], an analytical record of the geometric systems in question was available, so determining the Cartesian coordinates of the ends of the chord directed backwards and forwards was not a problem. In this situation, it was easy to determine the values of the respective slope angles \( \theta_{(i-1)+i} \) and \( \theta_{(i+1)+i} \). When determining the curvature of the track on the basis of measurement data, the coordinates of the track axis are discrete and it would be difficult to create an analytical record of them. Therefore, it is necessary in such a case to adapt the discussed moving chord method to the adopted measurement procedure. This is at the heart of this article.

3. VARIANT OF TWO VIRTUAL CHORDS

A. Applicability of the moving chord method.

In mobile satellite measurements, the characteristics of the considered geometric layout and its mathematical notation are unknown. Hence, the essential problem concerns determining endpoint coordinates of the two adjacent chords, because they cannot be determined analytically. As the measurement points are discrete, the position of the ends of the chords should be found via interpolation in relevant intervals. The basic goal, which is determining the track curvature, can be obtained in various ways, of which two application variants of the moving chord method seem to be most effective:

- variant making use of the fixed base of the measuring trolley, and
- variant of two virtual chords.

When using the measuring platform with two satellite receivers installed at bogie pivot pin points, the positions of the satellite antennas define the so-called fixed base of the trolley. This fixed base can be used as the measuring chord in the moving chord method. The main advantage of operating with fixed base is that the records from both satellite antennas come at the same time, in the same conditions (measuring speed, visibility of satellites). This eliminates situations in which we have to deal with coordinates measured at different times, in the presence of different factors which might affect the accuracy. In the BRIK project [21-22], the fixed base method will be the basic method to determine the railway track axis curvature.

However, in some situations, the need to use data from two satellite antennas may generate certain limitations. Moreover, the operation of the antennas can be affected by disturbances, as a result of which the distance between them calculated from measurements will not be equal to the nominal length of the fixed base (some real situations were recorded where such inequality took place).

Therefore, when the use of coordinates recorded exactly at the same time is not necessary and, simultaneously, we can ensure preserving constant measuring speed, it seems advisable to consider making use of data from only one satellite antenna. This means the necessity to use two virtual chords created forward and backward from point \( i \) (looking in the direction of the measuring run). This solution has an additional advantage which may be of high importance in some situations: different chord lengths can be assumed, which may turn out extremely useful when determining border points between different geometric elements of the track. The fixed base method does not offer such an opportunity.

In this work, intended to analyse the results obtained in the selected measuring campaign, the method based on the use of only one GNSS antenna is applied.

B. Assumptions of the method to determine railway track curvature.

In the conducted research, the full data set for the measuring point comprises:

- time of measurement \( t_i \),
- horizontal eastward coordinate \( Y \) in coordinate system PL-2000,
- horizontal northward coordinate \( X \) in coordinate system PL-2000,
- vertical coordinate \( Z \) in relevant spatial system,
- inclination angle \( \alpha \) in longitudinal profile,
- inclination angle \( \alpha \) in lateral section.

In the first step, the recorded track axis coordinates should be corrected using the angles \( \alpha \) and \( \alpha \) measured by inclinometers [33-34] or by an inertial system. Then, these results can be used for reconstructing the existing position of the railway track axis. When determining the horizontal curvature, this should be done using the corrected horizontal coordinates \( Y(t) \) and \( X(t) \).

All measuring points have their index numbers \( i = 1, 2, \ldots, n \). We start determining the curvature \( k_i \) from point \( i \) situated in such a way that a virtual chord of length \( l \), can be projected backward. Likewise, the calculations should end at point \( i \) from which the virtual chord, of the same length, can still be projected forward.

The basic operation to be performed is finding index numbers of points defining the intervals in which endpoints of virtual chords projected from point \( i \) are situated. For the rear chord, this interval is given by points \( q_i \) (defining the boundary of the interval further from the point \( i \)) and \( q_i+1 \) (defining the boundary of the interval less distant from point \( i \)). In the case of the front chord these are points \( p_i-1 \) (defining the boundary of the interval less distant from point \( i \)) and \( p_i \) (defining the boundary of the interval further from the point \( i \)). These limiting points are found by checking the distances of successive points from point \( i \) in the direction of increasing and decreasing index numbers. For the rear chord these distances are determined from the formula

\[
l_{(i-k)+i} = \sqrt{(Y_i - Y_{i-k})^2 + (X_i - X_{i-k})^2} \tag{3}
\]

and for the front chord from the formula

\[
l_{i+(i+k)} = \sqrt{(Y_i - Y_{i+k})^2 + (X_i - X_{i+k})^2}, \tag{4}
\]

where \( k = N_i \).

After each calculation step, we check whether the condition \( l_{(i-k)+i} \geq l_c \) is met for the rear chord and \( l_{i+(i+k)} \geq l_c \) for the
front chord. The first value $i - k$ for the rear antenna which meets the above condition is marked as $q_i$, while for the front antenna, the first value $i + k$ is marked as $p_i$.

Since, unlike the fixed base method, the considered variant of the moving chord method omits the time parameter and instead makes use of measuring point index numbers, it seems profitable to maintain similar distances between adjacent points. This is equivalent to the requirement to take profitable to maintain similar distances between adjacent points, i.e. point $B_i$, using the formulas:

$$l_{q_i} = \sqrt{(Y_i - Y_{q_i})^2 + (X_i - X_{q_i})^2}, \quad (5)$$

$$l_{q_{i+1}} = \sqrt{(Y_i - Y_{q_{i+1}})^2 + (X_i - X_{q_{i+1}})^2}. \quad (6)$$

This data makes it possible to determine the coordinates of the rear chord endpoint, i.e. point $B_i$, using the formulas:

$$Y_{B_i} = \frac{X_{q_i+1} - X_{q_i}}{Y_{q_i+1} - Y_{q_i}}(Y_{q_i} - Y_{q_i+1}) + Y_{q_i}, \quad (7)$$

$$X_{B_i} = \frac{X_{q_i+1} - X_{q_i}}{Y_{q_i+1} - Y_{q_i}}(X_{q_i} - X_{q_i+1}) + X_{q_i}. \quad (8)$$

The gradient of the straight line passing through points $B_i$ and $i$ and representing the rear chord is:

$$S_{B_{i+1}i} = \frac{X_{B_i} - X_i}{Y_{B_i} - Y_i}. \quad (9)$$

If $s_{B_{i+1}i} > 0$, then the inclination angle of the fixed base is $\theta_{B_{i+1}i} \in (0, \frac{\pi}{2})$ and is given in the $Y$, $X$ coordinate system by the formula:

$$\theta_{B_{i+1}i} = \text{atan} \frac{x_{B_i} - x_i}{y_{B_i} - y_i}. \quad (10)$$

If $s_{B_{i+1}i} < 0$, then the inclination angle of the fixed base is $\theta_{B_{i+1}i} \in (\frac{\pi}{2}, \pi)$ and is given by the formula:

$$\theta_{B_{i+1}i} = \pi + \text{atan} \frac{x_{B_i} - x_i}{y_{B_i} - y_i}. \quad (11)$$

Inclination angle of front chord is determined analogously. Based on the known coordinates $Y_{p_i}, X_{p_i}$ and $Y_{p_{i+1}} - X_{p_{i+1}}$ of points limiting the interval in which the endpoint of the rear antenna is situated, the distances $l_{p_i}$ and $l_{p_{i+1}}$ of these points from point $i$ can be calculated. This data makes it possible to determine the coordinates of the front chord endpoint, i.e. point $F_i$. The gradient of the straight line passing through points $i$ and $F_i$ and representing the front chord is:

$$s_{i+1F_i} = \frac{x_{F_i} - x_i}{y_{F_i} - y_i}. \quad (12)$$

If $s_{i+1F_i} > 0$, then the inclination angle of the fixed base is $\theta_{i+1F_i} \in (0, \frac{\pi}{2})$ and is given by the formula:

$$\theta_{i+1F_i} = \text{atan} \frac{x_{F_i} - x_i}{y_{F_i} - y_i}. \quad (13)$$

If $s_{i+1F_i} < 0$, then the inclination angle of the fixed base is $\theta_{i+1F_i} \in (\frac{\pi}{2}, \pi)$ and is given in the $Y$, $X$ coordinate system by the formula:

$$\theta_{i+1F_i} = \pi + \text{atan} \frac{x_{F_i} - x_i}{y_{F_i} - y_i}. \quad (14)$$

C. Curvature value at point $i$.

The curvature value at point $i$ is given by the formula:

$$k_i = \pm \left[\frac{\theta_{i+1F_i} - \theta_{B_{i+1}i}}{l_i}\right]. \quad (15)$$

The sign “+” in Eq. (15), meaning positive curvature, corresponds to the situation when the convexity along the length of the curve is directed downwards, while the negative value – to the convexity directed upwards.

The virtual chord method is undoubtedly simpler in calculations than that making use of the fixed base of the measuring trolley. However, it should be kept in mind that it is extremely sensitive to measuring speed disturbances and the need to operate on data sets recorded at different times. Therefore, it should be used locally, when the use of the fixed base method turns out ineffective, or when we want to use shorter chords for determining more precisely the border points between geometric elements. In this work, the virtual chord method was also used to assess the efficiency of the applied measuring technique.

4. ASSESSING THE CURVATURE CALCULATION PROCESS IN MOBILE SATELLITE MEASUREMENTS

A. Test segment.

The analysed test segment comprises a fragment of a single-track railway line, of about 4.7 km in length. The measurement was conducted using the GNSS receivers of 100 Hz frequency, installed on a four-axis trolley, at front bogie pivot point (looking in the direction of motion). The trajectory of this railway line fragment recorded in mobile satellite measurements is shown in Fig. 4, in the coordinate system PL-2000.

For further analysis, we should move to the linear system, i.e. calculate the distances of successive measuring points from a given starting point – the so-called $L$ variable. The starting point will be denoted as $i = 1$.

The distance between two adjacent measuring points is:

$$\Delta L_{i+1i} = \sqrt{(Y_{i+1} - Y_i)^2 + (X_{i+1} - X_i)^2}. \quad (16)$$

The linear coordinate $L_i$, being the distance of the given point $i$ from point $(Y_i, X_i)$, is calculated from formula:

$$L_i = \sum_{i=1}^{n-1} \Delta L_{i+1i}. \quad (17)$$
Antenna is situated, the distances of these points from point limiting the interval in which the endpoint of the rear antenna is determined analogously.

If the gradient of the straight line passing through points $i$ is situated are known, the distances of these points from point $i$ are calculated from formula:

$$L_i = \sqrt{(Y_i - Y_1)^2 + (X_i - X_1)^2}. \quad (18)$$

The linear coordinate $L$ enables finding locations of particular geometric elements based on the determined railway track axis curvature, as well as calculating the track mileage. Moreover, in combination with the measured height coordinate $Z$ it can be used for determining the longitudinal profile of the track.

**B. Determining measuring trolley speed during measurement.**

In mobile satellite measurements, the measurements of railway track axis coordinates are taken at fixed time intervals resulting from the frequency of the installed GNSS receivers. For $f = 100$ Hz, the time interval between two successive measurements is 10 ms. Since the distance between two successive measuring points is known, the speed of the measuring trolley in this interval can be directly calculated. This speed (in km/h) is given by the formula:

$$V_{i+1} = 3.6 \cdot f \Delta L_{i+1}. \quad (19)$$

For straight sections, Eq. (19) can be replaced by its modified form:

$$V_{i+1} = 3.6 \cdot f (L_{i+1} - L_i). \quad (20)$$

where the values of $L_i$ and $L_{i+1}$ are calculated from Eq. (18).

Figure 5 shows the measuring speed waveform $V(L)$ covering initial 300 m of the test segment, as calculated from Eq. (20). The assumed starting point had coordinates: $Y_1 = 6473899.914$ m, $X_1 = 5961334.294$ m and was situated on a straight section being the upper part of the trajectory shown in Fig. 4 (the motion of the measuring trolley was in the direction of decreasing $X$ values).

What is noteworthy here is high precision of speed value calculations, which is undoubtedly related with precision in determining distances between successive measuring points. This issue will be discussed in detail in Section D. The very high accuracy of determining the distance $\Delta L_{i+1}$ is indicated by the values of the standard deviation $\sigma_{\Delta L}$ given in Table 1. In the vast majority of cases these values do not exceed 0.5 mm.

A similar situation as that observed in Fig. 5 with respect to the accuracy had place over further 1400 m, until radical deterioration of quality of the recorded GNSS signal was observed. This issue will be given greater attention in the next Section.

**C. Determining horizontal curvature.**

The horizontal curvature $k_i$ at consecutive measuring points was calculated from Eq. (15) assuming the chord length $l_i = 7$ m, equal to the fixed base of the measuring trolley. On straight sections, the linear coordinate was calculated from Eq. (14), while on arcs – from Eq. (13). The results of these calculations enabled drawing the curvature waveform over the geometric layout length.

Figure 6 shows the railway track curvature over the initial 10 m of the test segment. The measurement was taken at increasing speed of the measuring trolley (see Fig. 5). As can be observed, the curvature waveform differs radically from $k(l)$ diagrams presented for model systems in [31]. In that case, the curvature waveform has a significantly higher value.
case, full compatibility of the curvature obtained using the moving chord method with the theoretical waveform was observed, while in Fig. 6 the curvature ordinates $k(L)$ differ visibly from the curvature $k = 0$ expected on the straight section. This may be caused by track deformations resulting from its exploitation, but the main cause seems to be the disturbances of measuring trolley trajectory having place during the measurement. Entering of wheelsets into the track generates lateral displacements of the trolley with the satellite receiver installed on its boogie pivot. The recorded deviations are not random in nature, but reveal certain regularity. The diagram in Fig. 6 was obtained based on data from over 400 measuring points.

However, this irregularity is not a problem from the point of view of the basic goal of the performed measurements, which is determining the curvature of the given geometric layout. This can easily be confirmed by analysing the curvature waveform for a longer track segment (Fig. 7).

Figure 7 shows the track curvature over initial 300 m of the test segment. As can be seen, deviations from zero curvature decrease as a result of speed stabilisation at a given level (see Fig. 5). However, from the practical point of view, of highest importance is that it is sufficient to filter curvature ordinates to obtain the correct track shape [35], as in this case there is no doubt that the presented fragment of the test segment is straight and its curvature is $k = 0$. This was confirmed by $k(L)$ diagrams drawn for further fragments of the test segment.

The $k(L)$ ordinates need also filtering on circular arcs and transition curves. It results from the curvature waveform in the transition curve region (Fig. 8) that these ordinates oscillate around some constant value; we can also assume that the transition from straight section to arc is linear. Filtering out high frequencies enables determining endpoints of the transition curve (and, consequently, its length), as well as the radius of the circular arc.

The horizontal curvature waveform determined with the virtual chord method may also be the basis for assessing the quality of the recorded satellite signal. This possibility is illustrated in Fig. 9 and Fig. 10. At some time during the measurement session, for $L \approx 1725$ m, rapid deterioration in satellite signal quality was recorded, which can be observed as visible increase of deviations from the mean value on both the curvature waveform (Fig. 9), and the speed waveform (Fig. 10). Hence, the virtual chord method makes it possible to detect unfavourable situations caused by disturbances in signal recording and, consequently, to indicate the need to introduce corrections to the mobile satellite measurement procedure in a given track fragment.

D. Coordinate determination accuracy.

Increasing the accuracy and availability of GNSS measurements is one of the major study areas. The mobile satellite measurements on the rail track conducted in 2009-2015 focused on the availability assessment for three accuracy levels, which are required for carrying out individual construction and geodetic-related tasks in railway engineering. These include [13]:

- deformation accuracy (dd) – enabling one to identify the place and extent of rail track deformations, for which the...
The values of $\Delta L_{i+(i+1)}$ certainly differ, depending on local speed on the measuring trolley. Therefore, comparing them can only make sense for a given speed interval (class). It has been assumed that the speed class is given by the number of measuring points situated over the virtual chord length (the same for both chords), and the greater the number of these points, the lower the speed on the considered track fragment. For each speed class of $\Delta L_{i+(i+1)}$ values, the mean values and standard deviations were calculated. The results are collated in Table 1, ordered with respect to the decreasing number of measuring points in the speed class. The following symbols

\[ n_c \] number of points defining the speed class, \( L_p \) and \( L_i \) – endpoints coordinates of the given speed interval, \( \Delta L_{p+k} \) – length of the speed interval, \( n_{p+k} \) – number of measuring points in the interval, \( \bar{V}_{i+(i+1)} \) – average speed, \( \sigma_{V} \) – standard deviation of speed, \( \Delta L_{i+(i+1)} \) – average distance between two adjacent measuring points, \( \sigma_{\Delta L} \) – standard deviation of the distance between adjacent points. It results form Table 1 that the lengths of the intervals comprising a given speed class differ much. However, regardless of the length of the given speed interval (corresponding to the number of measuring points composing

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<th>( n_c )</th>
<th>( L_p ) [m]</th>
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it), a very clear view of the situation has been obtained. Although in each speed class, the values of $V_{i+(i+1)}$ and $\Delta L_{i+(i+1)}$ differ, their mean values increase gradually and steadily with decreasing $n_i$.

The most surprising result of this analysis were the obtained values of standard deviation of the distance between two adjacent measuring points. In nearly all cases, these deviation values did not exceed 0.5 mm and 1% of the mean value. From the point of view of GNSS measurement accuracy assessment, this result is of fundamental importance, as it contradicts traditional scepticism concerning the applicability of these measurements. However, one basic condition should be met in the entire measuring procedure – the measurements should be properly planned and conducted. The analysis presented in the paper confirms the potential of the applied measuring method, simultaneously indicating possible threats. Rapid quality deterioration of the received signal, shown in Fig. 9 and Fig. 10, which took place during the measuring session, persisted until the end of measurement and led to the increase of $\sigma_{\Delta L}$ by several times.

5. SUMMARY

In engineering practice, the type and characteristics of the horizontal curvature existing on a given geometric track layout are most frequently identified indirectly – based on the values of sagittas measured from the chord stretched along the track. Radical improvement in this situation is expected to be brought by the method of mobile satellite measurements, developed in Poland for over a decade. The aim of the ongoing research project BRIK is to obtain an implementation solution. Further use of the sagitta chart method would not make much sense if there existed a direct method to determine curvature. In [31], the assumptions of the novel method to determine horizontal curvature were formulated. This method is based on inclination angle changes of a moving chord in the Cartesian coordinate system. It was verified on an unambiguously defined elementary geometric layout of railway track. The proposed method may turn out extremely applicable when geometrical characteristics of the track axis calculated from measurements are not known and the basic goal is to determine them.

In the article, the applicability of the moving chord method was analysed. The applied variant of this method, referred to as the method of two virtual chords, consists in manoeuvring with only one GNSS receiver. The assumptions of the method to determine railway track axis curvature were given, along with the assessment of its application in the performed campaign of mobile satellite measurements. The measured track axis was shown in the PL-2000 coordinate system. Based on the measured coordinates, the speed of the measuring trolley during the measurement was calculated. Basic activities aimed at determining the horizontal curvature of the track. As it turned out, the obtained $k(L)$ waveforms differed radically from those presented for model systems [31] where full compatibility was observed between the obtained curvature values and those used as the basis for designing the geometry of the analysed track segment. This might be caused by track deformations resulting from its exploitation, but the main cause seems to be measuring trolley trajectory disturbances taking place during the measurement. However, in practice, this problem can easily be overcome by filtering the measured track axis ordinates to obtain the correct shape – this refers to all track segments: straight sections, circular arcs, and transition curves.

The virtual chord method can also be the basis for quality assessment of the received satellite signals. During the measuring session, rapid deterioration in satellite signal quality was observed in the form of visible increase of deviations from the mean value on both the curvature waveform and the speed waveform. The procedure adopted for assessing the obtained accuracy involved the analysis of distances between two adjacent measuring points. For the interval of correct operation of satellite receivers, the performed analysis gave a clear and logical view of the existing situation. The most surprising result of this analysis were the calculated values of standard deviation of the distance between points. In nearly all cases, these deviation values did not exceed 0.5 mm and 1% of the mean value. From the point of view of GNSS measurement accuracy assessment, this result is of fundamental importance, as it contradicts traditional scepticism concerning the applicability of these measurements.

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REFERENCES
Determining horizontal curvature of railway track axis in mobile satellite measurements


[28] Rail design in Civil 3D, Autodesk, San Rafael, USA, 2019.


