State of the art rotordynamic analyses of pumps

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Abstract. State of the art analyses for the rotordynamic assessment of pumps and specific requirements for the simulation tools are described. Examples are a horizontal multistage pump with two fluid film bearings in atmospheric pressure, a horizontal submerged multistage pump with many bearings and a submerged vertical single-stage pump with water lubricated bearings. Due to the seals the rotor of the horizontal pump on two bearings is statically overdetermined and the static bearing forces depend on the deflection in the seals and the bearings. The nonlinear force displacement relation in the bearings hereby is considered. The stability of pumps is assessed by Campbell diagrams considering linear seal and bearing properties. Cylindrical bearings can have a destabilizing effect in case of low loads as in the examples of the submerged pumps. For the pump with many bearings the influence of the bearing ambient pressure and of the bearing specific load on the stability is analyzed. For the vertical pump the limit cycle, i.e. the vibration level of stabilization, is determined with a nonlinear analysis. All examples have a practical background from engineering work, although they do not exactly correspond to real cases. Analyses were performed with the rotordynamic software MADYN 2000.

Key words: rotordynamics; lateral analysis; pumps; fluid-film bearings; pump seals; nonlinear analysis.

1. INTRODUCTION
Fluid-rotor interaction plays a big role in the rotodynamics of pumps. The forces in small clearances, i.e. at the seals and bearings, have the largest impact. The seal forces can have a destabilizing effect, but also a stiffening effect (Lomakin) and thus contribute to carrying the weight load. There is a long history of research on the impact of seals and complete impellers in rotordynamics. They are typically considered by rotordynamic stiffness, damping and mass coefficients. A good summary of activities can be found in the book of Dara Childs [1].

Special requirements for rotordynamic analyses due to the seals are described in this paper for the example of a horizontal pump in atmospheric pressure, Fluid film bearings have a long history in research as well. Famous researchers are Glienicke [2] and Lund [3]. In 1994 2-phase flow was introduced [4] as a new cavitation model. It allows better describing the properties of a mixture of oil and gas as well as considering the influence of elevated environmental pressure on cavitation. At high pressure it is suppressed completely (so called Sommerfeld boundary condition).

In submerged pumps the bearings are lubricated by the process fluid and are in a pressurized environment. Suppression of cavitation therefore plays a big role in these machines. This leads to surprising behavior, different from the normal behavior of bearings in atmospheric pressure environment. Many engineers are not aware of this, since so far it is not described in textbooks. The example of a submerged pump describes this behavior.

Nonlinear fluid film bearing behavior is an issue for vertical pumps, the third example in this paper. They are typically linearly unstable. A nonlinear analysis allows assessing the vibration level by calculating the limit cycle.

2. HORIZONTAL MULTISTAGE PUMP WITH TWO BEARINGS AT ATMOSPHERIC PRESSURE

A. Description of the model.
The model of the centrifugal pump is shown in Fig. 1.

Fig. 1: Horizontal multistage pump on two bearings at atmospheric pressure

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The rotor has a total length of 1'730mm and a bearing span of 1'330mm. The mass of the rotor is 52kg; its nominal speed is 9’000 rpm. It is supported on two tilting-pads bearings with a diameter of 50mm and a clearance of 2.0‰, which are lubricated with oil type ISO VG46 with an inlet temperature of 50°C. The bearings are analyzed as described in [5] with consideration of the 3D distribution of the temperature and viscosity in the fluid film, of the turbulence in the fluid film and of the thermo-elastic deformation of the shaft and pads. The seals at the impeller shrouds and hubs, as well as the balance piston seal are included in the model. The impeller seals are plain annular seals; the balance piston seal is a serrated seal.

The seals are analyzed with the specialized CFD program SEAL 2D/3D, which was originally developed by Nordmann et. al. (see [6]), further improved for industrial use and integrated into MADYN 2000 (see [7]). The code solves the Navier-Stokes equations with the finite difference method. For the balance piston, the radial and tangential forces as a function of the precession frequency at nominal speed are shown in Fig. 2. The calculated points are shown with “Xs”. The speed-dependent linear coefficients, including mass coefficients, are determined by curve fitting (solid lines).

In Fig. 3 the shaft displacements at nominal speed from two different static analyses are shown. The plot on the top is the result of an analysis where the bearings are modelled as rigid supports and the seal forces are not considered. For the bottom plot the analysis was carried out with consideration of the nonlinear bearing characteristics and of the seal forces. The influence of the seals is clearly visible, notably at the balance piston seal, where the shaft is significantly lifted.

**B. Static analysis.**

The static analysis yields the bearing loads, which are needed to determine the load and speed dependent bearing characteristics. The seal forces influence the static equilibrium of the rotor. With a total of 18 seals and 2 bearings the shaft is statically overdetermined. The relation between bearing force and displacement in fluid film bearings is nonlinear. Therefore, a nonlinear static analysis is required. The seal characteristics are assumed as linear, since they have larger clearance and cavities.

On some pumps the bearing housings are raised relatively to the pump casing to reduce the static shaft eccentricity at the seal locations. This can be considered in the analysis. In the present example the seal clearances are sufficiently high to avoid rubbing, and the bearing housings were not lifted. Due to the speed dependence of the seals and bearings the static analysis must be carried out for the whole speed range of interest. The analysis starts with rigid supports instead of the actual bearings (no compliance due to the fluid film), which yields a first estimate of the bearing forces. The journal position in the bearing caused by the bearing force is then calculated yielding a new bearing force and a new position. During each iteration step the bearing code is called with the relevant force and returns the journal position. The iteration stops when the change of force and position in an iteration step is sufficiently small.

Plots of the shaft forces can be found in Fig. 4. With consideration of the seal forces the bearing loads are much lower.
The speed-dependent bearing forces at the non-drive end bearing (NDE bearing) can be seen in the upper left quadrant of Fig. 5. The corresponding displacements are shown in the lower left quadrant. The bearing coefficients can be seen on the right of the figure.

C. Campbell diagram.
The Campbell diagram is calculated with the speed dependent bearing loads and speed dependent linear coefficients for the bearings and the seals. It is shown in Fig. 6 together with the mode shapes at the critical speeds.

In Fig. 7 the assessment of the eigenvalues at nominal speed according to API 610 can be found (see [8]). The modes are well damped and smooth operation is expected for this pump.

3. SUBMERGED HORIZONTAL MULTISTAGE PUMP WITH MANY BEARINGS
Submerged electrical pumps can have many stages and typically have very elastic shafts. For this reason, they have many bearings. In this section of the paper, we look at a sector with a few stages of such a pump to demonstrate some effects, which can occur in such a pump. The whole rotor in such a pump is surrounded by crude oil, which is also used as lubricant for the bearings. In our example the viscosity is 0.1Pa·s at 85°C and the density is 920 kg/m³.

Fig. 5: NDE bearing.
Results of the bearing analysis and linear coefficients.

Fig. 6: Campbell diagram and critical modes.

Fig.7: Assessment of the eigenvalues according to API 610 and mode shapes at 9’000 rpm.
A. Description of the model.
A plot of the analyzed shaft sector can be seen in Fig. 8. The shaft diameter is 36mm; the nominal speed is 4’600rpm. The rotor was cut midway between two bearings. At the ends of this shaft sector the added masses used to model the impellers have half the mass of the actual impellers. Quasi-rigid bending springs have been added at the ends of the rotor to calculate the correct bearing loads in the static analysis. Due to the relatively small impeller head the seal forces are rather small and therefore negligible.
The bearings are closed cylindrical bearings (i.e. cylindrical bearings with one continuous sector between 0° and 360°) with axial oil inlet (pressure difference across the bearing of 2.5bar). The relative bearing clearance is 3‰. The shaft journal is cylindrical, without any axial or helicoidal groove. The bearings are lubricated with the pumped fluid and the ambient pressure increases with each stage (between 107.5bar and 120bar for the present example). The static analysis with weight load was carried out on rigid supports, which is an acceptable simplification for a flexible rotor supported on so many bearings. The bearing specific load is approximately 0.2bar. The bearings were analyzed with consideration of the 2-phase cavitation model described in [5].

Results for the bearing at an ambient pressure of 107.5bar are shown in Fig. 9. Typical of a closed cylindrical bearing, the displacement of the journal is perpendicular to the load. The direct stiffness is very small and the cross-coupling stiffness high, which contributes to destabilizing the rotor.

B. Campbell diagram.
The Campbell diagram can be seen in Fig. 10. The first mode is the forward whirling parallel mode of the shaft. The other modes are the forward bending modes of the rotor with increasing order and frequency. All modes have circular orbits.

For all modes, the natural frequency is approximately half of the rotating speed, which corresponds to the oil whirling frequency. The damping ratio of the 1st mode is slightly negative (i.e. the mode is unstable). The other modes have positive damping ratios. Modes 3 and higher have high damping ratios.

Fig. 8: Horizontal multistage pump with many pressurised bearings.

Fig. 9: Results of the bearing analysis and linear coefficients.

Fig. 10: Campbell diagram and mode shapes at 4’600 rpm.
C. Influence of the bearing ambient pressure when the bearing specific load is constant.

In this section the influence of the ambient pressure on the eigenvalues at nominal speed was analyzed. The bearing specific load is kept constant at 0.2 bar. The ambient pressure is the same for all bearings and is increased from 1 bar (atmospheric pressure) up to 100 bar.

The natural frequencies do not change much with the ambient pressure, whereas it has more influence on the damping ratios. In Fig. 11 the damping ratios of modes 1 and 2 as a function of the ambient pressure are shown. Between 1 bar and approximately 7 bar the damping ratios decrease with increasing ambient pressure. Beyond 7 bar the influence of the ambient pressure is small.

The bearing coefficients at nominal speed as a function of the ambient pressure can be seen in Fig. 12. The direct stiffness coefficients and the cross-coupling damping coefficients causing radial forces decrease with the ambient pressure. At the same time, the cross-coupling stiffness, which tends to destabilize the rotor, increases. The combination of both phenomena contributes to the decrease of damping ratio of the modes when the ambient pressure increases.

D. Influence of the bearing specific load.

In this section the influence of the specific bearing load is shown at constant ambient pressure between 107.5 bar and 120 bar. The damping ratio of the 1st and 2nd modes as a function of the bearing specific load can be seen in Fig. 13. For comparison results with an ambient pressure of 1 bar are shown as well. The actual bearing specific load could be increased for example by reducing the number of bearings, reducing the diameter or the width of the bearings or by applying adequate misalignment between the bearings.

When the bearings are at atmospheric pressure, the damping ratios increase with the specific load slowly up to 2.5 bar of specific load, and then more rapidly. They can even reach very high values for bearing loads around 10 bar. This behavior is expected for rotors on fluid film bearings and reported in many textbooks.

With pressurized bearings, the damping ratio stays constant as long as the bearing load is lower than 0.56 bar and then decreases, which is surprising having the usual behavior of rotors on fluid film bearings in mind.

![Fig. 11: Damping ratio of the 1st and 2nd mode at 4'600 rpm vs. bearing ambient pressure.](image1)

![Fig. 12: Bearing coefficients vs. ambient pressure.](image2)

![Fig. 13: Damping ratio of the 1st and 2nd mode vs. bearing specific load.](image3)
The rotodynamic stiffness and damping coefficients for the two cases are shown in Fig. 14. It can be clearly seen that the pressurized bearing with increasing bearing load continues to behave like an unloaded cylindrical bearing. The direct stiffness remains small and the destabilizing skew symmetric cross coupling stiffnesses are large and increase with the bearing load. The direct damping is also increasing with the load. In contrast to this the direct stiffness increases for the bearing in atmospheric pressure and the stiffness matrix turns into an asymmetric but no longer skew-symmetric matrix.

The behavior can be explained by looking at the pressure distribution and static deflection of the shaft for the two cases: pressurized bearing, bearing in atmospheric pressure.

In Fig. 15 the pressure distribution and the oil ratio in the fluid film are shown for the two cases. The axial pressure difference is 2.5 bar. The results are represented in 2D fields where the x-axis is the circumferential angle (0° is at the bottom of the bearing, i.e., in direction 2 of Fig. 16) and the y-axis is the axial coordinate. For a given location, an oil ratio 1.0 means that the volume is completely filled with oil; an oil ratio equal to 0 means that the volume is completely filled with air.

![Fig. 14: Bearing coefficients vs bearing specific load.](image1)

![Fig. 15: Cavitation area and pressure distribution in the bearing oil film.](image2)
With a low specific bearing load of 0.1 bar the pressure distributions are rather similar for the two cases. There is no cavitation.

With a specific bearing load of 10 bar there is cavitation in the bearing at atmospheric pressure (white area on the oil ratio plot). The pressure peak is narrow in the circumferential direction and centered at about 15°, which is before the angular position of the shaft deflection at about 50° (see Fig. 16). This means the peak is before the narrowest gap in the bearing. Cavitation occurs in the upper part of the bearing from 90° to 270°, i.e., -90°.

In the pressurized bearing there is no cavitation at all, the oil ratio is 1.0 everywhere. The region of cavitation at atmospheric pressure now also contributes to carrying the load due to a pressure below the high ambient pressure. Due to the symmetry of the bearing the pressure increase before the narrowest gap and decrease after the narrowest gap must be symmetric to the horizontal 3-direction to get a resulting vertical force carrying the load. This can only be realized by a horizontal deflection.

In Fig. 16 can be seen, that for the pressurized bearing the deflection is more or less in horizontal direction (perpendicular to the bearing load), whereas for the bearing in atmospheric pressure it resembles the well-known Gumbel curve, i.e. with increasing load the component of the deflection in load direction increases.

### 4. SINGLE STAGE VERTICAL PUMP WITH CYLINDRICAL WATER-LUBRICATED BEARINGS

Fluid film bearings are normally linearized around their static load, which yields the linear stiffness and damping coefficients. The linearized behavior can give good results in a wide range of dynamic loads if the latter do not exceed the static load. If the static load is zero or small, then strictly speaking the behavior is always nonlinear. A linear analysis does not tell at which level the unstable system stabilizes (limit cycle). To get this result, which is essential for an assessment of the rotor behavior, a nonlinear analysis is necessary. In the following example, which is presented more in detail in [9], the rotor is unstable because it has unloaded cylindrical bearings.

### A. Description of the model.

The model of the vertical pump is shown in Fig. 17. The complete length of the assembly is about 12 m. The upper part and the motor are on the left side. The pump has only one impeller at the bottom shown on the right side. The pump rotor is shaded in blue. The casing of the pump, which is a pipe, is modelled as a shaft with zero speed. It is shaded in grey (partly visible as a black line). The pipe is fixed to a foundation, which is denoted as customer support in the model. It consists of a flange of the pipe. The flange is fixed with general springs to the ground. The general springs represent the stiffness of the foundation and introduce anisotropy to the system. The motor rotor is not modelled as part of the shaft since it is coupled to the pump shaft with a flexible coupling. The whole motor including its housing is modelled as a rigid mass (the sphere in Fig. 17) fixed to the flange at the customer support. The distance of the center of gravity to the support is bridged with a rigid element.

The pump shaft is supported in the pipe with an angular contact rolling element bearing, which also carries the axial load, and several bush bearings. The rotor in the pipe is surrounded by pressurized water. Like in the previous example, the elevated ambient pressure and its influence on the cavitation are considered in the bearing analyses with a 2-phase model. The seal located at the impeller is also included in the analysis. Since the relative shaft vibration is relatively small at this location for the dominating modes linear coefficients can be used. The coefficients are applied between the casing and the rotor.
The influence of the water located inside the pipe, which is displaced when the pipe vibrates, is considered by applying an added mass to the pipe (the mass of the enclosed water is added). The water mass displaced by the rotor has a small impact on the dynamics of the shaft line and has been neglected. The influence of the water outside the pipe was not taken into account.

**B. Linear behavior, Campbell diagram.**

The Campbell diagram for a speed range up to 150% speed can be seen in Fig. 18. The first two modes are a cantilever-like bending mode of the pipe and the rotor in two perpendicular directions. There is almost no relative displacement between rotor and pipe. The next modes are bending modes of the rotor with increasing order and increasing relative displacement. The modes appear as elliptically forward and backward whirling modes. They are elliptic due to the anisotropy of the support stiffness. The forward modes with relative displacement become unstable (mode 4, 5, 7, 9) when their frequency is below 50% speed, which is the whirling speed of the fluid in the cylindrical bearings.

**C. Results of a nonlinear run-up analysis.**

The nonlinear models of the fluid film bearings and of the rolling element bearing, as well as the method used to solve the nonlinear equations are presented in detail in [9]. For the present example, a run up with an unbalance of G10 at the impeller has been carried out over a speed range from 10% to 110% speed in 60s, which is almost stationary for this system. Results of this analysis can be seen in the following figures: The absolute displacements of the pipe at the bearing locations in Fig. 19, the orbits of the relative displacements in the fluid bearings in Fig. 20 and the 3D shape at 1500rpm in Fig. 21.
The vibrations of the pipe indeed are huge. At 100% speed (1’500rpm), the level at bush bearing 2 is several mm. The shape at about 1’500rpm in Fig. 21 corresponds to the 2nd bending mode. The by far dominating frequency is about 10Hz, which corresponds approximately to the frequency of this mode. The vibration level at the ball bearing at the top is much lower. At 1’500rpm it is about 300µm, which still corresponds to a high rms value of 33mm/s. This is the location where vibrations of such machines are typically measured. Other locations close to the bearings are difficult to access on such a pump.

The relative vibration level in the fluid bearings at nominal speed in Fig. 20 is about 90% of the bearing clearance for all bearings.

The force at nominal speed in bush bearing 2 is about 2’400N, corresponding to a specific dynamic load of 3bar. The vibration behavior of the pump as presented here is not acceptable or at least at the limit. The surrounding water, which is not considered in the analysis, probably helps attenuating the vibration, especially at such levels as calculated here.

6. CONCLUSIONS

Three examples are presented demonstrating requirements for rotordynamic analyses of pumps and typical results. In a first example the impact of seals on the static deflections and bearing loads as well as on the dynamic behavior is shown. Due to the seals a simple rotor on 2 bearings turns into a statically overdetermined system.

A second example shows the behavior of the sector of a submerged pump. The special phenomenon that higher specific bearing load does not lead to a more stable system is shown and explained.

A third example of a vertical pump with the rotor mounted in a pipe filled with water is shown. Limit cycles of the linearly unstable system during run up are calculated in a nonlinear transient analysis.

REFERENCES