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# Estimation of post-adjustment correlations between observations on the basis of their topological coexistence in the network

**Mieczysław Kwaśniak**

Department of Engineering Surveying
Warsaw University of Technology
Pl. Politechniki 1, PL-00 661 Warsaw, Poland
e-mail: m.kwasniak@gik.pw.edu.pl

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**Abstract:** The paper presents the results of the studies on the determination of the degree of dependence between the adjusted observations, on the basis of the levels of their co-existence in a network. An approximate model is proposed making it possible to estimate that dependence without the necessity to perform the adjustment procedure. This model can be applied in the procedures of gross error detection in observations. Additionally, a supplementary algorithm to determine the coexistence levels for the observations on the grounds of the matrix of coefficients in the observation equations is presented.

**Keywords:** Adjusted observations, covariance matrix, observation coexistence level

## 1. Introduction

The covariance matrix of adjusted observations $\mathbf{C}_{\hat{\mathbf{L}}}$ (or the covariance matrix of residuals – $\mathbf{C}_{\hat{\mathbf{V}}}$) is commonly used for the evaluation of dependence between the observation values obtained from geodetic network adjustment. Its determination requires, however, performing complicated calculations. In certain geodetic problems it would be more advantageous making possible to determine any element of the matrix $\mathbf{C}_{\hat{\mathbf{L}}}$, even within certain approximation, but without performing complicated calculations. It would be specially useful in detection of gross errors in the observation systems. The iterative Baarda method (Baarda, 1968) is the most often applied approach in practice. This method is based on the results of least squares estimation (LS) and indicates, in a single iteration, an observation suspected of containing a gross error. Such an observation is then removed from the observation system. In case that several gross errors occur simultaneously in the observation system, the process of gross errors elimination requires performing the LS estimation in several iterations. Having the possibility to estimate correlation level for observations suspected to contain gross errors in the given iteration, one could define whether they are encumbered with a single or more gross errors, thus making the process of gross errors elimination more time-saving. Such attitude is represented, among others, in works (Cross and Price, 1985; Ding and

Coleman, 1996), but the determination of correlation level for observations that do not meet the diagnostic criterion is performed on the grounds of covariance matrix for observation corrections $\mathbf{C}_{\hat{\mathbf{V}}}$.

The concept of topological coexistence of observations was applied in the study hereof to define the level of correlation between the adjusted observations. Functional relationship between the observations correlation level after adjustment and the level of observation coexistence was determined on the grounds of numerical examinations on various geodetic networks. It makes possible to simplify the grouping procedure for observations of gross errors suspicion. The algorithm for the determination of observation coexistence levels on the grounds of the matrix of coefficients **A** in observation equations system is also proposed. The algorithm is illustrated on numerical examples.

## 2. Observation coexistence levels

Definition of the observation coexistence level as well as its basic properties are given in the study (Prószyński and Kwaśniak, 2002), where they are referred to local networks with uncorrelated observations. Basic concepts and definitions pertaining to this scope are presented below.

*Direct observation coexistence of* $L_i(P_i)$ and $L_j(P_j)$ exists when the following condition is met:

$$P_i \cap P_j \neq \Phi \tag{1}$$

where

$P_i$, $P_j$ – sets of network points on which observations $L_i$ and $L_j$ are determined;
$\Phi$ – empty set.

The observations $L_i$ and $L_j$ are identified by their entry in observation extra-code, e.g. *centre-left-right* for angular observation.

*Connection path* between observations $L_i$ and $L_j$ presents such a sequence of observations, $L_i, ..., L_k, ..., L_j$, that each two adjacent observations remain in direct coexistence.

*Length of connection path* between observations $L_i$ and $L_j$ is a number of elements in the connection path between $L_i$ and $L_j$, decreased by 1.

*Observation coexistence level* for a pair $L_i$, $L_j$ (hereinafter denoted as $r_{i,j}$) presents the length of the shortest connection path between the observations $L_i$ and $L_j$. Thus, $r_{i,j} = 1$ corresponds to direct coexistence of $L_i$ and $L_j$. In order to provide formal completeness also zero observation coexistence level is introduced, corresponding to observation coexistence with itself, i.e. $r_{i,i} = 0$. From the above definitions it comes that $r_{i,j} = r_{j,i}$.

The concepts presented above are illustrated on the example of levelling network as presented in Figure 1.

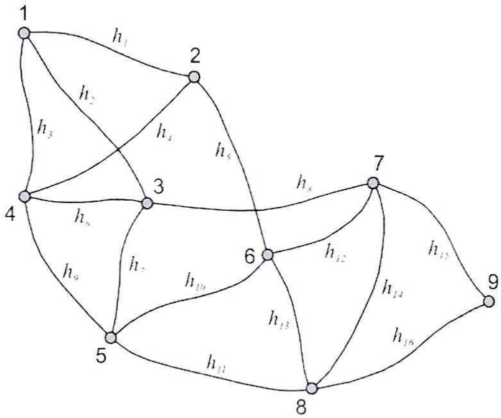


Fig. 1. Exemplary, 9-point levelling network

Let us consider the observations $h_3$ and $h_{14}$. According to the aforesaid definition these observations are not in direct coexistence since

$$P_3 = \{1,4\}; P_{14} = \{7,8\}; P_3 \cap P_{14} = \Phi$$

Exemplary connection paths for this observation pair are as follows:

Path 1: $h_3$, $h_1$, $h_5$, $h_{12}$, $h_{14}$ of the length 4;

Path 2: $h_3$, $h_6$, $h_8$, $h_{14}$ of the length 3;

Path 3: $h_3$, $h_4$, $h_5$, $h_{10}$, $h_{11}$, $h_{14}$ of the length 5.

The shortest connection path for the observations $h_3$ and $h_{14}$ (among all possible paths) is of length 3. Thus, according to the aforesaid definition the observation coexistence level $r_{3,14} = 3$.

The observation coexistence levels define distances (in the meaning of the shortest connection path) between observations in the network's structure. They are easy for determination on the grounds of codes assigned to observations making the network observation system. The relevant algorithm can be found in (Prószyński and Kwaśniak, 2002).

Below a supplementary proposal of algorithm for the determination of the observation coexistence levels is presented, with input data being the matrix of coefficients **A** in observation equations system (linear or linearised).

In this algorithm a special property of power series of modified matrix $\mathbf{AA}^{\mathrm{T}}$ was used. This modification is to replace of the nonzero matrix elements of value 1 (or their absolute values). From the research of this property, carried out by the author of this work, results that

$$\text{for } i \neq j\text{: if } \{(\mathbf{AA}^{\mathrm{T}})^k\}_{i,j} = 0 \text{ then } \mathrm{r}_{i,j} = k$$

where $k > 0$ is an exponent to which a matrix is raised and $i$, $j$ are observation's numbers.

Let us consider a $D$-dimensional geodetic network of $p$ points and $n$ independent observations. Then, the matrix $\mathbf{A}$ of coefficients with unknowns $dx_{1,i}$, $dx_{2,i}$, ..., $dx_{D,i}$ ($i$ – point number) is of dimension $(n \times pD)$. The elements with nonzero values indicate interrelation of two spaces, i.e. the observation space and the unknowns space.

Algorithm

1. Creating the matrix of observation coexistence levels $\mathbf{K}(n \times n)$ and assigning zero-values to its elements

$$\{\mathbf{K}\}_{i,j} = 0 \qquad i, j = 1, 2, ..., n \tag{2}$$

2. Initiating index of current coexistence level and assigning 0 value to it

$$k = 0$$

3. Transforming the matrix $\mathbf{A}$ to binary form according to the formula

$$\{\mathbf{A}'_b\}_{i,j} = \begin{cases} 0 & \text{for } \{\mathbf{A}\}_{i,j} = 0 \\ 1 & \text{for } \{\mathbf{A}\}_{i,j} \neq 0 \end{cases} \qquad i = 1, 2, ..., n \qquad j = 1, 2, ..., pD \tag{3}$$

and then (for $D > 1$)

$$\{\mathbf{A}_b\}_{i,s+1} = \begin{cases} 0 & \text{for } \sum\limits_{j=sD+1}^{(s+1)D} \{\mathbf{A}'_b\}_{i,j} = 0 \\ 1 & \text{for } \sum\limits_{j=sD+1}^{(s+1)D} \{\mathbf{A}'_b\}_{i,j} > 0 \end{cases} \qquad i = 1, 2, ..., n \qquad s = 0, 1, ..., p-1 \tag{4}$$

4. Calculating the matrix product $\mathbf{A}_b\mathbf{A}_b^{\mathrm{T}}$ and transforming it to binary form, saving the results in the matrix $\mathbf{A}_*$

$$\{\mathbf{A}_*\}_{i,j} = \begin{cases} 0 & \text{for } \left\{\mathbf{A}_b\mathbf{A}_b^{\mathrm{T}}\right\}_{i,j} = 0 \\ 1 & \text{for } \left\{\mathbf{A}_b\mathbf{A}_b^{\mathrm{T}}\right\}_{i,j} \neq 0 \end{cases} \qquad i, j = 1, 2, ..., n \tag{5}$$

In the case of $k = 1$, indices $i$, $j$ of $\mathbf{A}_*$ matrix elements with non-zero values indicate observation pairs that remain in direct coexistence, and for $i = j$ we are dealing with a special case when the observation remains in direct coexistence with itself ($r_{i,j} = 0$).

It should be pointed out here that different observational accuracies (weighting) have no impact on the binary form of the matrix $\mathbf{A}$.

5. Making a copy of the matrix $\mathbf{A}_*$ (matrix $\mathbf{A}_*$ shall be needed in consecutive iterations)

$$\mathbf{D}_b = \mathbf{A}_* \tag{6}$$

6. Increasing by 1 the value of the index of current coexistence level

$$k = k + 1 \tag{7}$$

7. Modification of the matrix **K** according to the formula

$$\text{for } i \neq j \text{ if } \{\mathbf{K}\}_{i,j} = 0 \text{ then } \{\mathbf{K}\}_{i,j} = \begin{cases} 0 & \text{for } \{\mathbf{D}_b\}_{i,j} = 0 \\ k & \text{for } \{\mathbf{D}_b\}_{i,j} \neq 0 \end{cases} \qquad i, j = 1, 2, ..., n \tag{8}$$

8. If the off-diagonal elements of zero values do not occur in the matrix **K**, then, exit from the algorithm – otherwise
9. Calculating the matrix product $\mathbf{D}_b\mathbf{A}_*$ and its transformation to binary form

$$\{\mathbf{D}_b\}_{i,j} = \begin{cases} 0 & \text{for } \{\mathbf{D}_b\mathbf{A}_*\}_{i,j} = 0 \\ 1 & \text{for } \{\mathbf{D}_b\mathbf{A}_*\}_{i,j} \neq 0 \end{cases} \qquad i, j = 1, 2, ..., n \tag{9}$$

10. Return to point 6.

After exiting the algorithm, the matrix **K** contains coexistence levels for all possible observation pairs. In respect of the property $r_{i,j} = r_{j,i}$, this matrix is symmetrical. Functioning of the proposed algorithm is illustrated on two examples below.

Example 1

Determine observation coexistence levels for 4-point levelling network as illustrated in Figure 2. In the network we deal with equally precise observations, including repetitions.

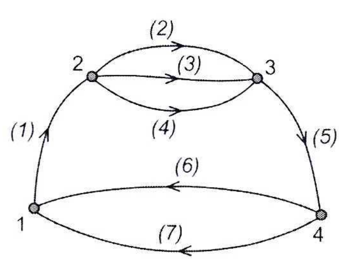


Fig. 2. Exemplary, 4-point levelling network

Realization of the algorithm:

Re. 1: initiating the matrix **K** of zero value elements

$$\mathbf{K} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Re. 2: initiating the index of current coexistence level and attributing $k = 0$ value to it

Re. 3: creating a matrix of coefficients in observation equations system as well as its conversion to binary form

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \end{bmatrix} \qquad \mathbf{A'}_b = \mathbf{A}_b = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Re. 4: creating the matrix product $\mathbf{A}_b\mathbf{A}_b^{\mathrm{T}}$ as well as binary equivalent $\mathbf{A}_*$

$$\mathbf{A}_b\mathbf{A}_b^{\mathrm{T}} = \begin{bmatrix} 2 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 2 & 2 & 2 & 1 & 0 & 0 \\ 1 & 2 & 2 & 2 & 1 & 0 & 0 \\ 1 & 2 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 2 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 2 & 2 \\ 1 & 0 & 0 & 0 & 1 & 2 & 2 \end{bmatrix} \qquad \mathbf{A}_* = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Re. 5: making a copy of the matrix $\mathbf{D}_b = \mathbf{A}_*$

Re. 6: increasing by 1 the value of the index of current coexistence level: $k = 1$

Re. 7: modification of the matrix **K**

$$\mathbf{K} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Re. 8: continuing calculations since off-diagonal elements of zero values still occur in the matrix $\mathbf{K}$

Re. 9: calculating the matrix product $\mathbf{D}_b\mathbf{A}_*$ and its conversion to binary form $\mathbf{D}_b$

$$\mathbf{D}_b\mathbf{A}_* = \begin{bmatrix} 6 & 4 & 4 & 4 & 5 & 3 & 3 \\ 4 & 5 & 5 & 5 & 4 & 2 & 2 \\ 4 & 5 & 5 & 5 & 4 & 2 & 2 \\ 4 & 5 & 5 & 5 & 4 & 2 & 2 \\ 5 & 4 & 4 & 4 & 6 & 3 & 3 \\ 3 & 2 & 2 & 2 & 3 & 4 & 4 \\ 3 & 2 & 2 & 2 & 3 & 4 & 4 \end{bmatrix} \qquad \mathbf{D}_b = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Re. 10: return to point 6

Re. 6: increasing by 1 the value of the index of current coexistence level $k = 2$

Re. 7: modification of the matrix $\mathbf{K}$

$$\mathbf{K} = \begin{bmatrix} 0 & 1 & 1 & 1 & 2 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 2 & 2 \\ 1 & 1 & 0 & 1 & 1 & 2 & 2 \\ 1 & 1 & 1 & 0 & 1 & 2 & 2 \\ 2 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 2 & 2 & 2 & 1 & 0 & 1 \\ 1 & 2 & 2 & 2 & 1 & 1 & 0 \end{bmatrix}$$

Re. 8: since all off-diagonal elements of the matrix $\mathbf{K}$ possess non-zero values we complete determination of coexistence levels.

The determined coexistence levels for all observation pairs are included into the matrix $\mathbf{K}$. Maximum observation coexistence level in this network is 2.

Example 2

Determine observation coexistence levels for angular-linear horizontal structure as presented in Figure 3.

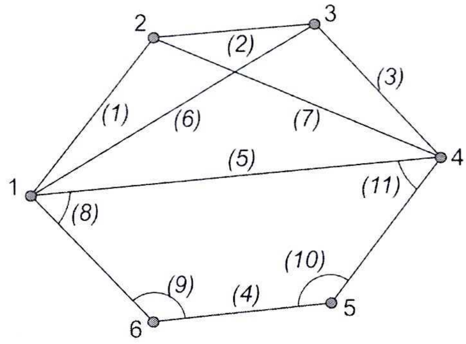


Fig. 3. Horizontal angular-linear structure as well as approximate coordinates of its points

Approximate coordinates:

| No. | *X*[m] | *Y*[m] |
|---|---|---|
| 1 | 55.0 | 10.0 |
| 2 | 120.0 | 55.0 |
| 3 | 125.0 | 115.0 |
| 4 | 70.0 | 160.0 |
| 5 | 10.0 | 120.0 |
| 6 | 2.0 | 55.0 |

Realization of the algorithm:

Re. 1÷2: initiating matrix $\mathbf{K}(11 \times 11)$ of zero value elements as well as the index of current coexistence level $k = 0$

Re. 3: creating matrix of coefficients $\mathbf{A}(11 \times 12)$ (Table 1) as well as its conversion to binary form $\mathbf{A}'_b$, and then to $\mathbf{A}_b$

Table 1. Matrix of coefficients in observation equations system

| $c$ | $l$ | $p$ | $x_1$ | $y_1$ | $x_2$ | $y_2$ | $x_3$ | $y_3$ | $x_4$ | $y_4$ | $x_5$ | $y_5$ | $x_6$ | $y_6$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 0 | -0.82 | -0.57 | 0.82 | 0.57 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2 | 3 | 0 | 0.00 | 0.00 | -0.08 | -1.00 | 0.08 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3 | 4 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.77 | -0.63 | -0.77 | 0.63 | 0.00 | 0.00 | 0.00 | 0.00 |
| 5 | 6 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 0.99 | -0.12 | -0.99 |
| 1 | 4 | 0 | -0.10 | -1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1 | 3 | 0 | -0.55 | -0.83 | 0.00 | 0.00 | 0.55 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2 | 4 | 0 | 0.00 | 0.00 | 0.43 | -0.90 | 0.00 | 0.00 | -0.43 | 0.90 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1 | 4 | 6 | 0.17 | 0.74 | 0.00 | 0.00 | 0.00 | 0.00 | 0.42 | -0.04 | 0.00 | 0.00 | -0.59 | -0.70 |
| 6 | 1 | 5 | -0.59 | -0.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.96 | 0.12 | 1.56 | 0.58 |
| 5 | 6 | 4 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.49 | 0.73 | 1.45 | -0.85 | -0.96 | 0.12 |
| 4 | 5 | 1 | 0.42 | -0.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.07 | -0.69 | -0.49 | 0.73 | 0.00 | 0.00 |

$$
\mathbf{A'}_b = \begin{bmatrix}
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0
\end{bmatrix}
\qquad
\mathbf{A}_b = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 \\
1 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 & 1 & 0
\end{bmatrix}
$$

Re. 4: creating the matrix product $\mathbf{A}_b\mathbf{A}_b^{\mathrm{T}}$ as well as its binary equivalent $\mathbf{A}_*$

$$
\mathbf{A}_b\mathbf{A}_b^{\mathrm{T}} = \begin{bmatrix}
2 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
1 & 2 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 2 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 2 & 2 & 1 \\
1 & 0 & 1 & 0 & 2 & 1 & 1 & 2 & 1 & 1 & 2 \\
1 & 1 & 1 & 0 & 1 & 2 & 0 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 & 1 & 0 & 2 & 1 & 0 & 1 & 1 \\
1 & 0 & 1 & 1 & 2 & 1 & 1 & 3 & 2 & 2 & 2 \\
1 & 0 & 0 & 2 & 1 & 1 & 0 & 2 & 3 & 2 & 2 \\
0 & 0 & 1 & 2 & 1 & 0 & 1 & 2 & 2 & 3 & 2 \\
1 & 0 & 1 & 1 & 2 & 1 & 1 & 2 & 2 & 2 & 3
\end{bmatrix}
\qquad
\mathbf{A}_* = \begin{bmatrix}
1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{bmatrix}
$$

Further procedure is identical as in the Example 1. Finally, the determined matrix of observation coexistence levels is of the following form

$$
\mathbf{K} = \begin{bmatrix}
0 & 1 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 2 & 1 \\
1 & 0 & 1 & 3 & 2 & 1 & 1 & 2 & 2 & 2 & 2 \\
2 & 1 & 0 & 2 & 1 & 1 & 1 & 1 & 2 & 1 & 1 \\
2 & 3 & 2 & 0 & 2 & 2 & 2 & 1 & 1 & 1 & 1 \\
1 & 2 & 1 & 2 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 2 & 1 & 0 & 2 & 1 & 1 & 2 & 1 \\
1 & 1 & 1 & 2 & 1 & 2 & 0 & 1 & 2 & 1 & 1 \\
1 & 2 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
1 & 2 & 2 & 1 & 1 & 1 & 2 & 1 & 0 & 1 & 1 \\
2 & 2 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 0 & 1 \\
1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0
\end{bmatrix}
$$

## 3. Observation coexistence levels and relations between observations after adjustment

The degree of dependence between the adjusted observations is determined by the covariance matrix for these observations. For standardised observation system this matrix is calculated as follows

$$\mathbf{C}_{\hat{\mathbf{L}}} = \mathbf{A}\left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-}\mathbf{A}^{\mathrm{T}} \tag{10}$$

where $\mathbf{A}$ is the matrix of coefficients in standardised observation equations, $\hat{\mathbf{L}}$ is the vector of adjusted observations, and $(\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-}$ is a reflexive g-inverse of $\mathbf{A}^{\mathrm{T}}\mathbf{A}$.

The considerations hereof regarding the degree of dependence between the adjusted observations and observation coexistence levels (hereinafter referred to as: "L-K dependencies") relate to coherent geodetic networks, taking no account of special cases such as: networks without redundant observations, networks with unlimitedly high redundancy level as well as networks combining both the structures. In such cases the dependencies either have or asymptotically tend to zero level (Prószyński and Kwaśniak, 2002).

Determination of exact post-adjustment "L-K dependencies" in a theoretical way is impossible since the observation coexistence levels are topological characteristics of the geodetic network structure being not related to the adjustment process, whilst the correlation of observations after adjustment constitutes the result of this process. Therefore, examinations regarding "L-K dependencies" were performed in empirical way for many variants of geodetic networks, both horizontal and levelling, of various geometry and number of points. In respect of the size of this study, the results of these examinations are illustrated with the example of 8-points levelling network (Fig. 4) of maximum observation coexistence level equal to 4.

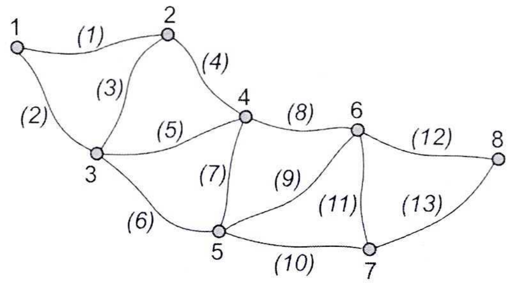


Fig. 4. Exemplary 8-points levelling network; *(·)* – identifies observation number

According to (10) the matrix $\mathbf{C}_{\hat{\mathbf{L}}}$ for the levelling network under analysis is as follows

$$
\mathbf{C}_{\hat{\mathbf{L}}}=\begin{bmatrix}
0.618 & 0.382 & -0.236 & -0.146 & 0.090 & 0.056 & -0.034 & -0.021 & 0.013 & 0.008 & -0.005 & -0.003 & 0.003 \\
0.382 & 0.618 & 0.236 & 0.146 & -0.090 & -0.056 & 0.034 & 0.021 & -0.013 & -0.008 & 0.005 & 0.003 & -0.003 \\
-0.236 & 0.236 & 0.472 & 0.292 & -0.180 & -0.111 & 0.069 & 0.042 & -0.027 & -0.016 & 0.011 & 0.005 & -0.005 \\
-0.146 & 0.146 & 0.292 & 0.562 & 0.271 & 0.167 & -0.103 & -0.064 & 0.040 & 0.024 & -0.016 & -0.008 & 0.008 \\
0.090 & -0.090 & -0.180 & 0.271 & 0.451 & 0.279 & -0.172 & -0.106 & 0.066 & 0.040 & -0.027 & -0.013 & 0.013 \\
0.056 & -0.056 & -0.111 & 0.167 & 0.279 & 0.554 & 0.276 & 0.170 & -0.106 & -0.064 & 0.042 & 0.021 & -0.021 \\
-0.034 & 0.034 & 0.069 & -0.103 & -0.172 & 0.276 & 0.448 & 0.276 & -0.172 & -0.103 & 0.069 & 0.034 & -0.034 \\
-0.021 & 0.021 & 0.042 & -0.064 & -0.106 & 0.170 & 0.276 & 0.554 & 0.279 & 0.167 & -0.111 & -0.056 & 0.056 \\
0.013 & -0.013 & -0.027 & 0.040 & 0.066 & -0.106 & -0.172 & 0.279 & 0.451 & 0.271 & -0.180 & -0.090 & 0.090 \\
0.008 & -0.008 & -0.016 & 0.024 & 0.040 & -0.064 & -0.103 & 0.167 & 0.271 & 0.562 & 0.292 & 0.146 & -0.146 \\
-0.005 & 0.005 & 0.011 & -0.016 & -0.027 & 0.042 & 0.069 & -0.111 & -0.180 & 0.292 & 0.472 & 0.236 & -0.236 \\
-0.003 & 0.003 & 0.005 & -0.008 & -0.013 & 0.021 & 0.034 & -0.056 & -0.090 & 0.146 & 0.236 & 0.618 & 0.382 \\
0.003 & -0.003 & -0.005 & 0.008 & 0.013 & -0.021 & -0.034 & 0.056 & 0.090 & -0.146 & -0.236 & 0.382 & 0.618
\end{bmatrix}
$$

Observation coexistence levels in this network are presented in Table 2.

Table 2. Observation coexistence levels for the network in Figure 4

| *i*/*j* | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | **0** | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 4 |
| 2 | 1 | **0** | 1 | 2 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 |
| 3 | 1 | 1 | **0** | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 |
| 4 | 1 | 2 | 1 | 0 | 1 | 2 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| 5 | 2 | 1 | 1 | 1 | **0** | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| 6 | 2 | 1 | 1 | 2 | 1 | **0** | 1 | 2 | 1 | 1 | 2 | 2 | 2 |
| 7 | 2 | 2 | 2 | 1 | 1 | 1 | **0** | 1 | 1 | 1 | 2 | 2 | 2 |
| 8 | 2 | 2 | 2 | 1 | 1 | 2 | 1 | **0** | 1 | 2 | 1 | 1 | 2 |
| 9 | 3 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | **0** | 1 | 1 | 1 | 2 |
| 10 | 3 | 2 | 2 | 2 | 2 | 1 | 1 | 2 | 1 | **0** | 1 | 2 | 1 |
| 11 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | **0** | 1 | 1 |
| 12 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 1 | 1 | 2 | 1 | **0** | 1 |
| 13 | 4 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | **0** |

Figure 5 illustrates variability of absolute values for non-diagonal elements of $\mathbf{C}_{\hat{\mathbf{L}}}$ (identified with $\left|\left\{\mathbf{C}_{\hat{\mathbf{L}}}\right\}_{i,j}\right|$, where $i \neq j$) for all observations in the network and various coexistence levels.

It comes from Figure 5 that for different observations, the $\left|\left\{\mathbf{C}_{\hat{\mathbf{L}}}\right\}_{i,j}\right|$ values corresponding to a given coexistence level are situated in intervals of different width as well as are of different average values. Thus, the determination of sufficiently precise model for "L-K dependencies" presentation is a very difficult task.

In order to define approximate model of the dependence aforesaid let us to analyse additionally the variability of $\left|\left\{\mathbf{C}_{\hat{\mathbf{L}}}\right\}_{i,j}\right|$ elements related to a single observation (row or column of the matrix $\mathbf{C}_{\hat{\mathbf{L}}}$) versus the coexistence level. Figure 6 presents variability of $\left|\left\{\mathbf{C}_{\hat{\mathbf{L}}}\right\}_{i,j}\right|$ elements for observations No 1, 3 and 7 situated in various points of the levelling network under analysis.

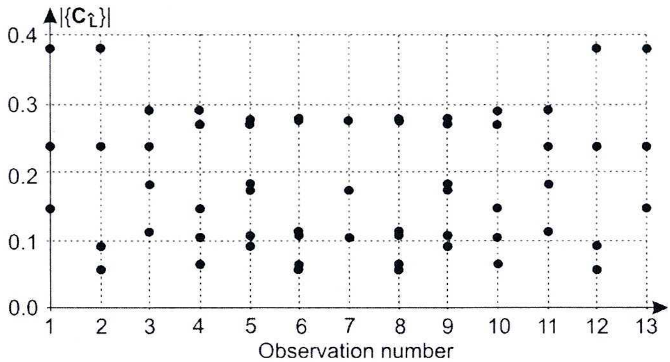


Fig. 5a. Variability of $|\{\mathbf{C}_{\hat{L}}\}_{i,j}|$ for the first coexistence level

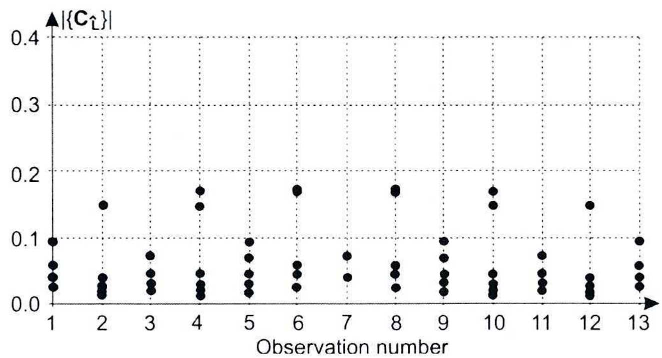


Fig. 5b. Variability of $|\{\mathbf{C}_{\hat{L}}\}_{i,j}|$ for the second coexistence level

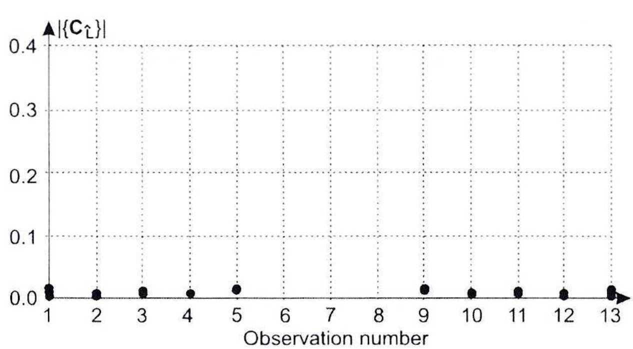


Fig. 5c. Variability of $|\{\mathbf{C}_{\hat{L}}\}_{i,j}|$ for the third coexistence level

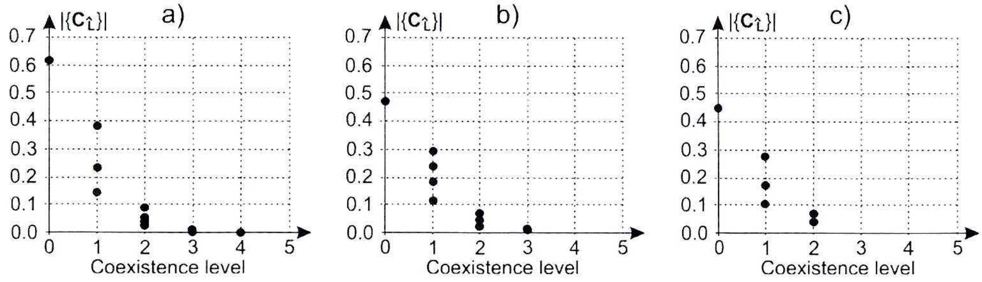


Fig. 6. Variability of elements $\left|\{\mathbf{C}_{\hat{\mathbf{L}}}\}_{i,j}\right|$ a) for observation No 1; b) for observation No 3; c) for observation No 7

As it comes from Figure 6, the degree of dependence of two observations after adjustment is determined by the distance between them expressed by coexistence level for these observations. Similar conclusion was reached in examination of disturbances propagation in geodetic networks depending on point coexistence level (Adamczewski, 1971, 2002).

Taking into account the above conclusion, the proposed approximate model for the determination of the degree of dependence for a pair of observations $(i, j)$ after adjustment, versus their coexistence level is of the following form

$$\left|\left\{\mathbf{C}_{\hat{\mathbf{L}}}\right\}_{i,j}\right| = g e^{-r_{i,j}} \qquad i, j = 1, 2, ..., n \tag{11}$$

where: $g = u/n$

$u$ – number of necessary observations;

$r_{i,j} = \{\mathbf{K}\}_{i,j}$ – coexistence level for observations $i, j$.

The index $g$ is a global measure that represents the average squared standard deviation of standardised observation after adjustment. Its relation to global internal reliability measure of geodetic networks $f$ known from (Caspary, 1988), is as follows

$$g = 1 - f \tag{12}$$

Figure 7 presents results of the approximation for average elements $\left|\left\{\mathbf{C}_{\hat{\mathbf{L}}}\right\}_{i,j}\right|$ values (corresponding to consecutive coexistence levels in which observations No 1, 3, 7 remain in relation to the other observations), after application of the proposed model.

Weak point of the proposed "L-K dependency" model is an invariability of the index $g$ for all observations in the networks whilst internal invariability measures in geodetic networks (and thus also the squares of standard deviation of adjusted observations $\sigma^2_{\hat{L}_{ij}} = g_{ij}$) as calculated for particular observations are more or less differentiated. As it comes from the completed examinations, for levelling networks it is possible to derive in a simple way the approximate formula determining $g_{ij}$ separately for each observation.

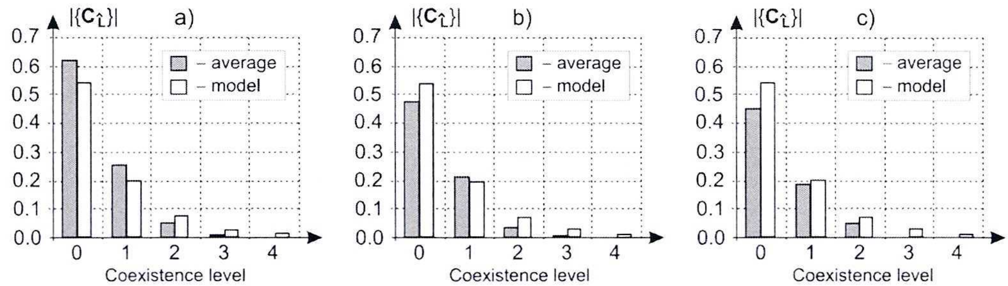


Fig. 7. Variability of average $|\{\mathbf{C}_{\hat{L}}\}_{i,j}|$ elements as well their equivalents obtained from approximation according to the proposed model for: a) obs. No 1; b) obs. No 3; c) obs. No 7

Assuming that the standard deviation of $L_{ij}$ after adjustment is influenced (besides itself) only by observations remaining jointly with it in the first level of coexistence one obtains

$$\sigma^2_{\hat{L}_{ij}} = g_{ij} \approx \frac{1}{\sigma^2_{L_{ij}}} \frac{[p_i] + [p_j] - 2[p_{ij}]}{[p_i][p_j] - [p_{ij}]^2} \tag{13}$$

where $[p_i]$, $[p_j]$ – the sum of weights of observations that come to the point $i$ or $j$, respectively;
$[p_{ij}]$ – the sum of weights of observations that join points $i$ and $j$.

For observations of equal accuracy the formula (13) is reduced to

$$\sigma^2_{\hat{L}_{ij}} = g_{ij} \approx \frac{n_i + n_j - 2n_{ij}}{n_i n_j - n^2_{ij}} \tag{14}$$

where $n_i$, $n_j$ – number of observations that come to the point $i$ or $j$, respectively;
$n_{ij}$ – number of observations that join points $i$ and $j$.

For the network under analysis as shown in Figure 4, the differences between global value $g$ and approximate local values $g_{ij}$ (according to (14)) are shown in Figure 8 against actual values of $\sigma^2_{\hat{L}_{ij}}$.

The determination of $g_{ij}$ as approximation of $\sigma^2_{\hat{L}_{ij}}$ for observations in horizontal net- works is much more difficult because that the coefficients in observation equations make function of azimuths and distances between the network points.

The proposed "L-K dependencies" model was subjected to verification on examples of many different geodetic networks. The verification confirmed that the effect of observation $i$ on $j$ (and vice versa) is practically negligible irrespectively of the network size and maximal observation coexistence level if these observations remain in the coexistence of level $r_{i,j} \geq 3$. This confirmation is consistent with model (11).

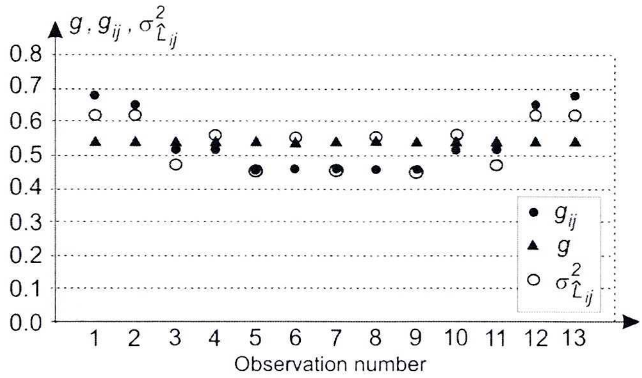


Fig. 8. Discrepancies between values $g$, $g_{ij}$ and $\sigma^2_{\hat{L}_{ij}}$ for the network in Figure 4

## 4. Remarks and conclusions

The observation coexistence level as a topological measure of the distance between two observations in a geodetic network can serve to map intensity level of connections (correlations) between observations after adjustment, resulting from the specified distribution of these observations in the network structure. The higher is the observation coexistence level the weaker is their connection after adjustment. Its determination is intuitively simple and it is possible to make use of the proposed algorithm in the case of more complex geodetic networks.

In respect of complicated nature of theoretical dependency between coexistence level of observation pair and their covariance value after adjustment the model to approximate such dependency is proposed.

It comes from the analysis network of various types and various sizes that two observations do not practically interact in the case their coexistence level is $r_{i,j} \geq 3$. This observation can be applied in the diagnostics of gross errors for simultaneous recognition of many errors and thus to make the process of their removal from the observation system more time-saving.


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## Oszacowanie skorelowania obserwacji po wyrównaniu na podstawie ich koegzystencji topologicznej w sieci

**Mieczysław Kwaśniak**

Zakład Geodezji Inżynieryjnej i Pomiarów Szczegółowych
Politechnika Warszawska
Pl. Politechniki 1, PL – 00 661, Warszawa
e-mail: m.kwasniak@gik.pw.edu.pl

### Streszczenie

W pracy przedstawiono wyniki badań nad określeniem zależności pomiędzy stopniem powiązania obserwacji po wyrównaniu a rzędami koegzystencji tych obserwacji. Zaproponowano przybliżony model tej zależności, pozwalający oszacować stopień powiązania obserwacji bez konieczności realizacji procedury wyrównawczej. Model ten może mieć zastosowanie w procedurach wykrywania błędów grubych w obserwacjach. Podano również uzupełniający algorytm ustalania rzędów koegzystencji obserwacji na podstawie macierzy współczynników równań obserwacyjnych.