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On the validation of CHAMP- and GRACE-type GGMs and the construction of a combined model

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Abstract: A number of new satellite-only Global Gravity Models (GGMs) become progressively available based on the CHAMP and GRACE satellite mission data. These models promise higher (compared to older GGMs) accuracy in the determination of the low and medium harmonics of the Earth's gravity field. In the present study, the latest GGMs generated from CHAMP and GRACE data (namely EIGEN2, EIGEN3p, GGM01C, GGM01S and GRACE01S) have been studied with respect to their accuracy and performance when used in gravity field approximation. A spectral analysis of the new models has been carried out, employing their degree and error-degree variances. In this way, their performance against each other and with respect to EGM96 was assessed, and the parts of the gravity field spectrum that each model describes more accurately have been identified. The results of the analysis led to the development of a combined geopotential model, complete to degree and order 360, whose coefficients were those of CHAMP until degree 5, then GRACE until degree 116, and EGM96 for the rest of the spectrum. Finally, a validation of all models (the combined included) has been performed by comparing their estimates against GPS/levelling data in land areas and TOPEX/Poseidon sea surface heights in marine regions. All tests have taken place over Greece and the eastern part of the Mediterranean Sea. From the results obtained it was concluded that the combined GGM developed provides more accurate results (compared to EGM96), in terms of the differences with the control datasets, at the level of 1-2 cm geoid and 1-2 mGal for gravity (1σ). Furthermore, the absolute geoid accuracy that the combined GGM offers is 12.9 cm (1σ) for n = 120, 25 cm for n = 200 and 33 cm for n = 360, compared to 29 cm, 36 cm and 42 cm for EGM96, respectively.

Keywords: Satellite gravity missions, geopotential model, combined global geopotential model, GGM validation

1. Introduction

The utilization of Global Earth Gravity Models (GGMs) in gravity field and geoid determination was and still is a common practice in geodetic studies during the past two decades. GGMs are mostly used to remove the long-wavelength part of the gravity field spectrum when employing the well-known remove-compute-restore method to determine geoid undulations from, e.g. gravity anomalies and/or altimetric sea surface heights. The internal accuracy of a GGM propagates to the finally estimated geoid heights and thus influences the accuracy of the so-determined geoid model (Tscherning, 2001). Until recently, the main error source in geoid heights determined by the aforementioned method was induced by the GGM used, since the accuracy of the latter reached the level of ±50–60 cm (1σ – standard deviation) for the best available high-resolution model, i.e., EGM96 (Lemoine et al., 1998). The launch of CHAMP and GRACE satellites in July 2000 and March 2002, respectively, signalled a new era in studies related to the estimation of satellite-only and combined GGMs, since they promised enhanced accuracy in the determination of the very-long to long wavelengths of the gravity spectrum. At the present time, four years after the launch of CHAMP, a number of new, satellite-only GGMs have become available based solely on data from CHAMP and GRACE. Most models utilize only a small portion of the data to become available by both satellites in their life span and show already improved accuracies in the low-degree harmonics (see, e.g. Tscherning et al., 2001).

In the present study, a number of those new-generation GGMs are employed to assess the accuracy improvement that they offer in geoid determination. The models derived from CHAMP data are the GeoForschungsZentrum (GFZ) EIGEN2 (Reigber et al., 2003b) and EIGEN3p (Reigber et al., 2005) GGMs, both complete to degree and order 120. The former is determined from about 6 months of CHAMP data compared to three years for the latter. Other CHAMP models are: a) UCPH2003 (Tscherning et al., 2003), derived from one month of the satellite's data and complete to degree and order 90, b) TUM2Sp (Földvary et al., 2005), derived from one year of the satellite's data and complete to degree and order 70, and finally c) ITG_CHAMP01E (Ilk et al., 2005), derived from one year of the satellite's data and complete to degree and order 75. In the GRACE-based GGM front end three models were used, namely the Center for Space Research (CSR) GGM01S (Tapley et al., 2003) and GGM01C (Tapley et al., 2004) both based on 111 days data of the satellite and complete to degree and order 120 and 200, respectively. GGM01S is a satellite only solution, while GGM01C is its combined counterpart. Finally, GRACE01S is a satellite only model based on 49 days of GRACE data and complete to degree and order 140 (Reigber et al., 2003a). Apart from these models, a number of old GGMs, i.e. GGMs compiled during the previous years using satellite tracking methods, altimetry and surface gravity data, were employed as well, to assess the improvement that the latest GGMs offer. The former were EGM96 and EGM96S complete to degree and order 360 and 70, respectively (Lemoine et al., 1998).

2. Spectral analysis of geopotential models

The processing methodology was based on the spectral analysis of the available GGMs from CHAMP and GRACE to determine those that describe more accurately the various frequencies of the gravity field spectrum. Then, a so-called combined model is determined by employing for each degree the coefficients of that GGM which proved to be the most accurate from the previous analysis. All models come as a series of spherical harmonic coefficients, of various degrees and orders, together with the errors associated for each coefficient. Therefore, for all models the harmonic coefficients $\overline{C}^*_{nm}, \overline{S}_{nm}$ and their accuracies $\sigma_{\overline{C}^*_{nm}}, \sigma_{\overline{S}_{nm}}$ are provided. Based on these, the signal and error degree variances for each model, either per degree or cumulatively, can be computed. The models available represent spherical harmonic expansions of the Earth's disturbing potential, therefore the so-determined signal and error degree variances refer to that. Nevertheless, they can easily be converted to represent various quantities related to the Earth's gravity field, such as gravity anomalies, geoid heights, etc. Since the main interest in using a GGM is in geoid or gravity field determination, it has been decided to validate the available models, with respect to the accuracy they provide in geoid heights and gravity anomalies. The signal degree variances represent the amount of the signal contained in each degree or up to a specific degree (if computed cumulatively), while the error degree variances represent the error of the model up to a specific degree.

Since various geopotential models were available and needed to be compared, it was necessary to scale their harmonic coefficients, so that they will all refer to the surface of a sphere of radius R. In that way, the computed signal and error degree variances are comparable. The scaled signal and error degree variances for the various quantities related to the gravity field can be computed as follows (Pavlis, 1998):

a) for the disturbing potential

$$\sigma_n^2 = \left(\frac{GM}{a}\right)^2 \left(\frac{a^2}{R^2}\right)^{n+1} \sum_{m=0}^{n} \left(\overline{C}^{*2}_{nm} + \overline{S}^2_{nm}\right) \tag{1}$$

$$\varepsilon^2_{\sigma_n} = \left(\frac{GM}{a}\right)^2 \left(\frac{a^2}{R^2}\right)^{n+1} \sum_{m=0}^{n} \left(\varepsilon^2_{\overline{C}^*_{nm}} + \varepsilon^2_{\overline{S}_{nm}}\right) \tag{2}$$

b) for gravity anomalies

$$\sigma_n^2 = \left(\frac{GM}{a}\right)^2 (n-1)^2 \left(\frac{a^2}{R^2}\right)^{n+1} \sum_{m=0}^{n} \left(\overline{C}^{*2}_{nm} + \overline{S}^2_{nm}\right) \tag{3}$$

$$\varepsilon^2_{\sigma_n} = \left(\frac{GM}{a}\right)^2 (n-1)^2 \left(\frac{a^2}{R^2}\right)^{n+1} \sum_{m=0}^{n} \left(\varepsilon^2_{\overline{C}^*_{nm}} + \varepsilon^2_{\overline{S}_{nm}}\right) \tag{4}$$

c) for geoid heights

$$\sigma_n^2 = \left(\frac{GM}{\gamma a}\right)^2 \left(\frac{a^2}{R^2}\right)^{n+1} \sum_{m=0}^{n} \left(\overline{C}^{*2}_{nm} + \overline{S}^2_{nm}\right) \tag{5}$$

$$\varepsilon^2_{\sigma_n} = \left(\frac{GM}{\gamma a}\right)^2 \left(\frac{a^2}{R^2}\right)^{n+1} \sum_{m=0}^{n} \left(\varepsilon^2_{\overline{C}^*_{nm}} + \varepsilon^2_{\overline{S}_{nm}}\right) \tag{6}$$

In equations (1)–(6) $\overline{C}^{*}_{nm}, \overline{S}_{nm}$ denote the coefficients of the various GGMs, $\sigma_{\overline{C}^{*}_{nm}}$, $\sigma_{\overline{S}_{nm}}$ and $\varepsilon_{\overline{C}^{*}_{nm}}, \varepsilon_{\overline{S}_{nm}}$ denote their degree and error degree variances respectively, R is a mean Earth radius, GM is the gravitational constant, a – the semi-major axis of the reference ellipsoid, and γ – mean normal gravity on the ellipsoid. Using equations (3)–(6) the signal and error degree variances for the various GGMs from CHAMP and GRACE have been computed (Fig. 1 and Fig. 2). Figure 1 shows that GGM01C has the same power as EGM96 up to its maximum degree of expansion (n = 200), while it retains full power up to about n = 112. Its error is smaller than that of EGM96 up to n = 120 and the accuracy improvement that it offers, compared to the latter, is about 20 times better (see Fig. 2). The GGM01C offers a ±1 cm accuracy up to n = 62 while it reaches the level of ±10 cm at n = 112. The GGM01S model retains full power up to n = 95, while its accuracy improvement, compared to EGM96, is the same as that of GGM01C up to n = 70. The corresponding degrees that GGM01S reaches the 1 and 10 cm level of accuracy are n = 59 and n = 101, respectively. Finally, GRACE01S retains full signal power up to n = 90 and offers an accuracy of 1 cm up to n = 56 while it reaches the level of 10 cm at n = 90.

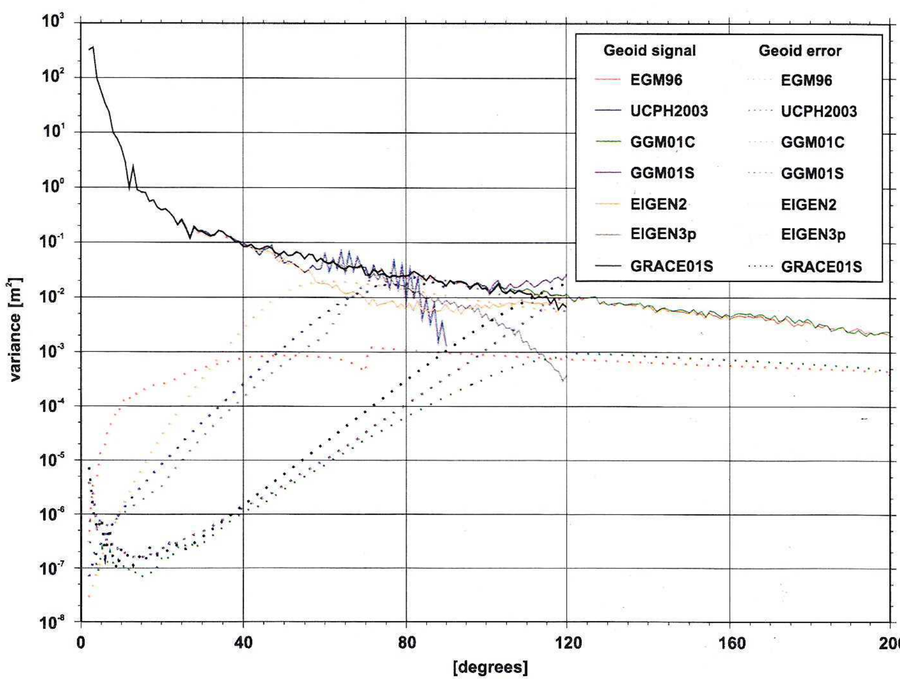

Fig. 1. Geoid signal and error degree variances from the various models

As far as the CHAMP-derived GGMs are concerned, EIGEN2 retains full power up to n = 30 and the improvement in the geoid accuracy that it offers, compared to EGM96, is inferior to the GRACE models. From Figure 2 it can be seen that EIGEN2 is about twice more accurate than EGM96 and gives accuracies of 1 and 10 cm up to

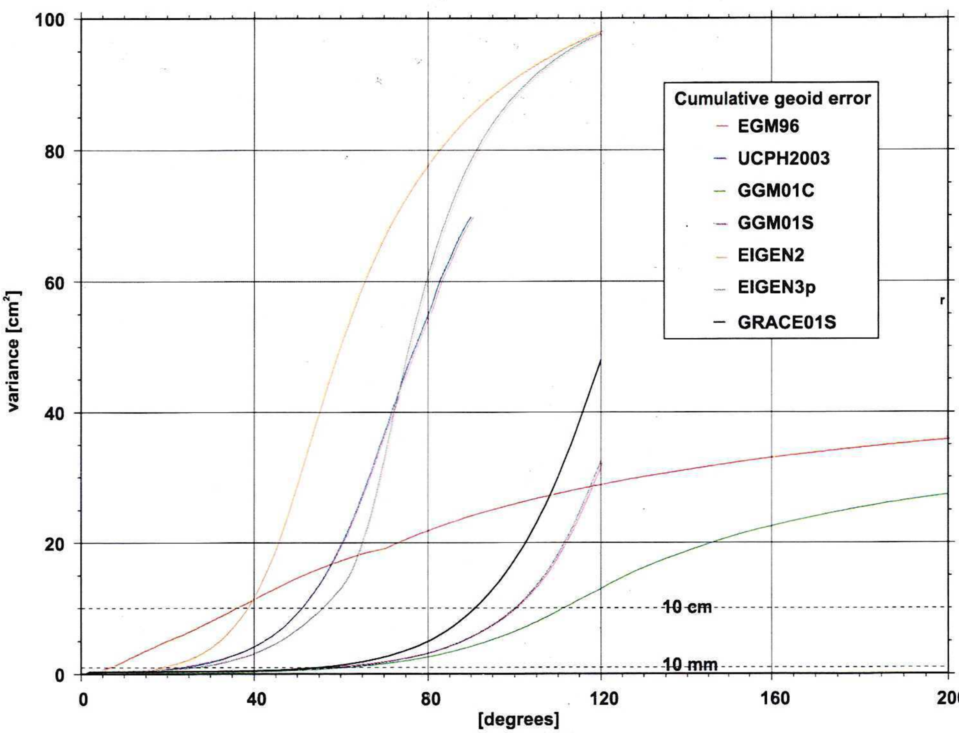

Fig. 2. Cumulative geoid error of the various models

$n = 20$ and $n = 39$, respectively. The EIGEN3p, which is a preliminary model, is about twice more accurate than EIGEN2, which is due to the use of a longer time-series of CHAMP data in its development. It retains full power to $n = 52$ while it reaches the accuracies of 1 and 10 cm to $n = 27$ and $n = 56$, respectively. Finally, the UCPH2003 retains full power to degree $n = 47$ and its accuracy is between the two GFZ models (see Fig. 2). The accuracy of 1 cm is achieved up to $n = 25$ and the 10 cm one up to $n = 51$. From the analysis given so far, the best model that is developed from satellite data alone is GGM01S, while the best combined solution is GGM01C. These models give the accuracy of 1 cm up to wavelengths of 380 km and 319 km, respectively (half wavelength), while the accuracy of 10 cm is retained up to wavelengths as short as 196 km and 176 km, respectively. For EGM96, the corresponding wavelengths are at the 2830 km and 550 km respectively, while for EIGEN3p they reach the 733 km and 354 km. From these results it is clear that the accuracy improvement that the new satellite models offer is significant and taking into account that the development of the CSR GRACE models was based on only 111 days of satellite data, one can expect far better results when new data sets become available. The aforementioned results are summarized for convenience in Table 1.

After that step, a further analysis has been performed by dividing the geoid spectrum in wave bands of 20 degrees from 0 up to 360 (see Table 2).

Table 1. Signal power and accuracy of the various GGMs

GGM	Degree n up to which the model retains full signal power	Degree n up to which the model reaches the accuracy of 1 cm	Degree n up to which the model reaches the accuracy of 10 cm
EIGEN2	30	20	39
EIGEN3p	52	27	56
UCPH2003	47	25	51
GGM01C	112	62	112
GGM01S	95	59	101
GRACE01S	90	56	90
EGM96	360	8	37

Table 2. Cumulative geoid error from the different GGMs in wavebands of 20 degrees

GGM	Harmonic Degrees – Wavebands									
	0–20	0–40	0–60	0–80	0–100	0–120	0–140	0–160	0–200	0–360
EIGEN2	1.04	11.77	50.78	78.12	91.10	98.20				
EIGEN3p	0.63	3.08	13.05	61.04	88.32	97.70				
GGM01C	0.16	0.34	0.93	2.61	6.50	13.00	18.77	22.59	27.44	
GGM01S	0.33	0.48	1.06	3.16	9.97	32.54				
GRACE01S	0.36	0.49	1.35	4.95	17.36	48.01				
EGM96	4.91	11.30	17.22	21.89	26.01	28.99	31.30	33.18	36.08	42.08

In each waveband the accuracy that the different models offer has been assessed and in cases where their performance was ambiguous, the analysis has been contacted by degree. From Table 2 and the spectral analysis performed for all available GGMs in various degrees cumulatively, it was concluded that EIGEN2 provides the most accurate results for degrees 1-5, while the superiority of the GRACE-based models for degrees 6-116 is obvious. This fact signals the scope that each satellite was built for, i.e. that CHAMP intends to accurately map the gravity field at the very low harmonic degrees, while GRACE the long to medium part of the spectrum. From the GRACE models, the one that provides the best accuracy for the degrees 6-116 is GGM01C, while above that degree, the EGM96 model gives the best results.

3. Determination of the combined model

After the spectral analysis of the available GGMs, a so-called combined geoid model has been determined by using for each degree the harmonic coefficients of the CHAMP-or GRACE-type GGM that provided the best accuracy (for the specific degree). Therefore, the combined model was determined as

$$N^{GGM} = N_{CHAMP} + N_{GRACE} + N_{EGM96} \tag{7}$$

where N^{GGM} is the total contribution of the GGMs, i.e., the combined model, and N_i is the contribution of the CHAMP, GRACE or EGM96 geopotential models to specific degrees, correspondingly. It is important to mention that the contribution of EGM96 is needed so as to develop a highly-expanded GGM, since a GGM complete to degree and order, e.g. 200, is of little use for geoid determination because it resolves wavelengths only up to 198 km.

The N_i for each model will successfully provide geoid heights according to the following equations:

$$N_{CHAMP} = \frac{kM}{r\gamma} \sum_{n=2}^{5} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} \left(\overline{C}_{nm}^{*CHAMP} \cos m\lambda + \overline{S}_{nm}^{CHAMP} \sin m\lambda\right) \overline{P}_{nm}(\sin\phi) \tag{8}$$

$$N_{GRACE} = \frac{kM}{r\gamma} \sum_{n=6}^{116} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} \left(\overline{C}_{nm}^{*GRACE} \cos m\lambda + \overline{S}_{nm}^{GRACE} \sin m\lambda\right) \overline{P}_{nm}(\sin\phi) \tag{9}$$

$$N_{EGM96} = \frac{kM}{r\gamma} \sum_{n=117}^{360} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} \left(\overline{C}_{nm}^{*EGM96} \cos m\lambda + \overline{S}_{nm}^{EGM96} \sin m\lambda\right) \overline{P}_{nm}(\sin\phi) \tag{10}$$

Based on this methodology, a combined GGM has been determined and its signal and error degree variances have been estimated. From the analysis performed it was found that the total geoid height error of the new model was at the level of 33 cm (compared to 50 cm for EGM96), while the accuracies of 1 cm and 10 cm were achieved up to degrees $n = 62$ and $n = 112$, respectively (compared to $n = 7$ and $n = 37$ for EGM96). It is obvious from these results that the newly combined GGM, which is based on CHAMP and GRACE data, presents a much more accurate picture of the Earth's gravity field. If the new GGM was truncated only to degree and order 120, then its accuracy would reach the 12 cm only, but a high-degree model is necessary to be used as a reference field for gravity data and altimetric observations for the use in geoid determination.

The so-determined GGM provides a new "combined" geoid model (see Fig. 3) as well as a gravity field for the area under study, with their statistics shown in Table 3. In Table 3 the contributions of a) EIGEN2 to degrees 2-5, b) GGM01C to degrees 6-116 and c) EGM96 to degrees 117-360 are also given. From that Table and studying the cumulative geoid signal up to various degrees, it can be concluded that the main part of the geoid signal is contained in the very-long and long wavelengths (a 44% of the total signal is provided up to $n = 6$) while the short wavelengths contribute only up to 8.7% of the total signal. On the contrary, for gravity anomalies, only 2% of the total signal is contained up to degree $n = 6$ while the main part is provided by the medium wavelengths.

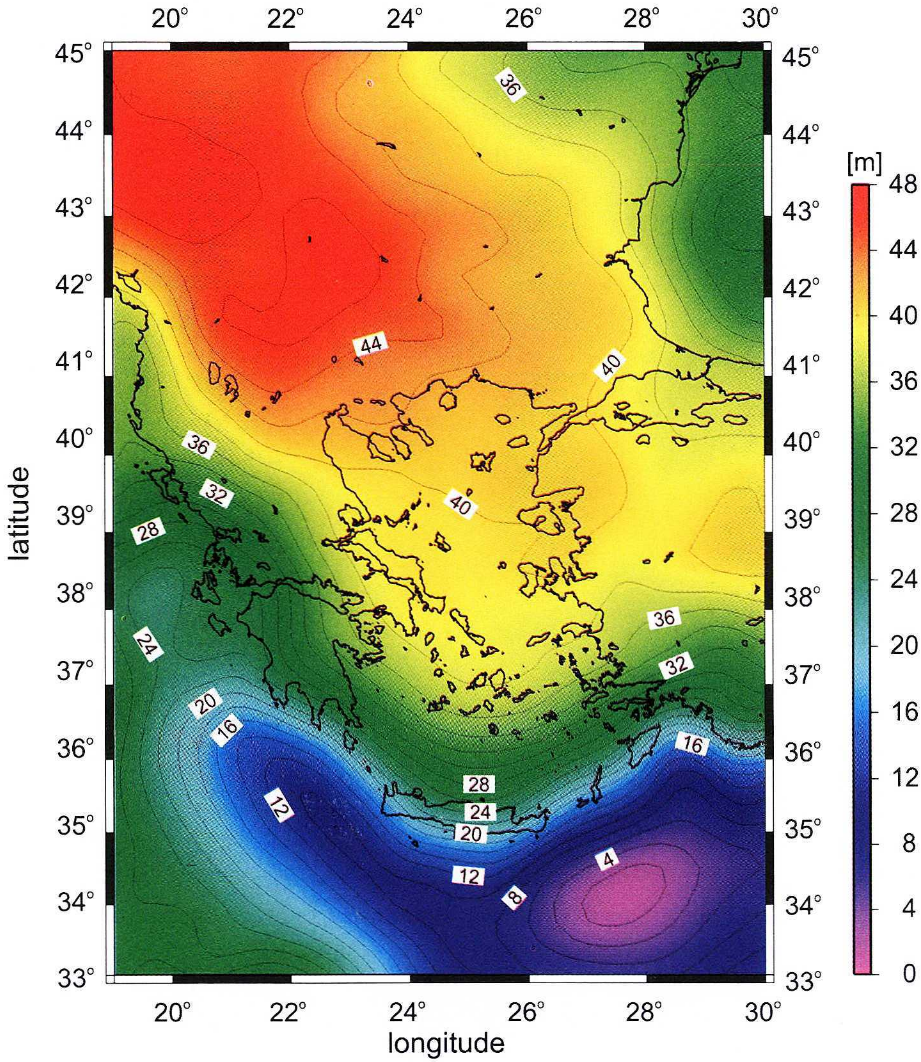

Fig. 3. The new "combined" geoid model in the area under study

Table 3. Statistics of geoid heights and gravity anomalies from the new combined GGM and the contributions of the geopotential models used

	Max	Min	Mean	RMS	Std
N^{Comb} [m]	47.859	0.819	31.467	33.719	±12.117
N^{EIGEN} [m] (n = 2-5)	45.454	21.549	34.453	34.847	±5.299
N^{GGM01C} [m] (n = 6-116)	13.067	−24.611	−2.975	10.673	±10.250
N^{EGM96} [m] (n = 117-360)	4.676	−4.353	−0.011	1.043	±1.043
Δg_f^{Comb} [mGal]	116.147	−198.850	3.471	62.089	±61.992
Δg_f^{EIGEN} [mGal] (n = 2-5)	13.516	7.909	10.650	10.718	±1.201
Δg^{GGM01C} [mGal] (n = 6-116)	112.503	−162.372	-6.788	56.967	±56.561
Δg^{EGM96} [mGal] (n = 117-360)	135.937	−136.327	-0.391	27.105	±27.102

4. Validation of the combined GGM

To assess the accuracy of the CHAMP- and GRACE-derived GGMs as well as that of the combined model, comparisons with 130 GPS/levelling geoid heights and stacked Topex/POSEIDON (T/P) Sea Surface Heights (SSHs) have been performed. Figure 4 shows the distribution of the GPS/levelling benchmarks (BMs), located in Northern Greece in the wider area of Thessaloniki.

For the minimization of the differences between the GGM and GPS/levelling geoid heights different parametric models have been used, namely a) a simple mean removal model (MRM), b) a 1^{st} order polynomial model (1-polyn), c) a 2^{nd} order polynomial model (2-polyn), d) a 3^{rd} order polynomial model (3-polyn), and e) the classic four-parameter transformation model (4-param) corresponding to a datum transformation. First, the performance of the parametric models has been assessed and the one that provides the smallest residual in terms of the standard deviation of the differences (1σ) has been selected. This test has been performed on the geoid height differences between the new combined GGM and the GPS/levelling data employing the aforementioned parametric models. The differences before and after the fit with the use of different parametric models investigated are summarized in Table 4. From Table 4 it is evident that both the standard deviation and the range of the differences between the GPS/levelling and the combined GGM geoid heights are reduced significantly when the 3^{rd} order polynomial model is used. Compared to the differences before the fit, the standard deviation is reduced by 20 cm (48.2%) and the range by 1.148 m (40.8%). Moreover, 3^{rd} order polynomial model compared to the second best which is the 2^{nd}

order polynomial model performs better by 5 cm and 41.1 cm in terms of reducing the standard deviation and range, respectively.

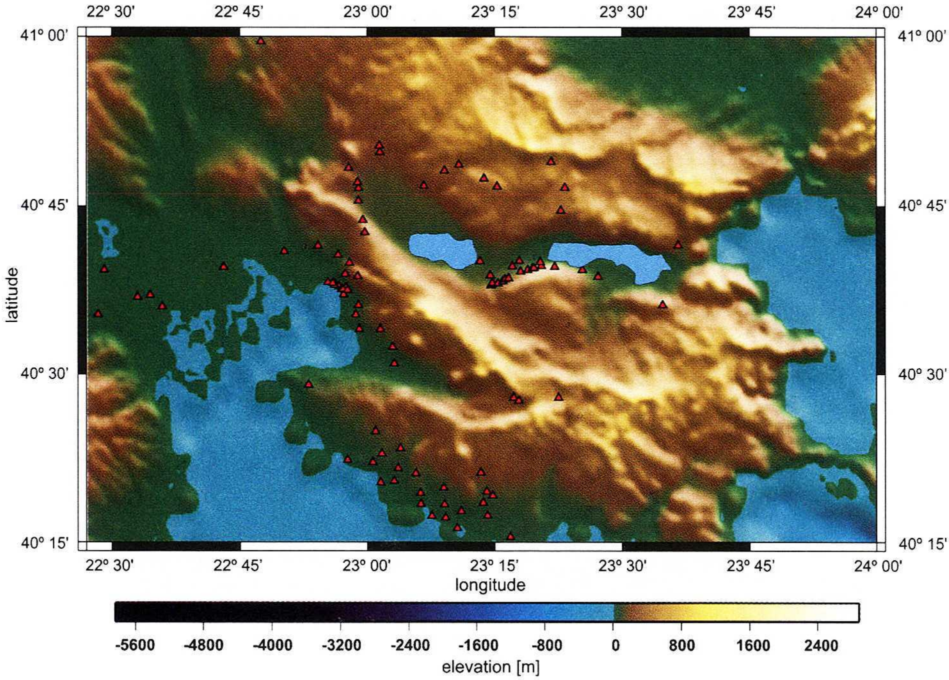

Fig. 4. Distribution of GPS/levelling BMs used for the GGM validation

Table 4. Geoid height differences between $N^{GPS/lev}$ and N^{Comb} before and after the fit of the polynomial models [m]

Param. model	Max	Min	Mean	RMS	Std
before fit	−0.347	−2.469	−1.072	1.150	±0.415
MRM	0.520	−1.321	0.000	0.435	±0.435
1-polyn	0.790	−1.151	0.000	0.283	±0.283
2-polyn	0.797	−1.282	0.000	0.261	±0.261
3-polyn	0.482	−1.186	0.000	0.215	±0.215
4-param	0.828	−1.238	0.000	0.274	±0.274

Using that criterion, the 3rd order polynomial model (11) has been selected as the most appropriate one.

$$N^{GPS/lev} - N^{GGM} = x_1 + x_2\left(\phi_1 - \overline{\phi}\right) + x_3\left(\lambda_1 - \overline{\lambda}\right) + x_4\left(\phi_1 - \overline{\phi}\right)^2 + x_5\left(\lambda_1 - \overline{\lambda}\right)^2 + \\ + x_6\left(\phi_1 - \overline{\phi}\right)\left(\lambda_1 - \overline{\lambda}\right) + x_7\left(\phi_1 - \overline{\phi}\right)^3 + x_8\left(\lambda_1 - \overline{\lambda}\right)^3 + \\ + x_9\left(\phi_1 - \overline{\phi}\right)^2\left(\lambda_1 - \overline{\lambda}\right) + x_{10}\left(\phi_1 - \overline{\phi}\right)\left(\lambda_1 - \overline{\lambda}\right)^2 + v \tag{11}$$

Table 5. Geoid height differences between $N^{GPS/lev}$ and N^{GGM} before (second row for each difference) and after (first row for each difference) the fit of a 3rd order polynomial model [m]

	Max	Min	Mean	RMS	Std
$N^{GPS/lev} - N^{Comb}$	−0.347	−2.469	−1.072	1.150	±0.415
	0.482	**−1.186**	**0.000**	**0.215**	**±0.215**
$N^{GPS/lev} - N^{EGM96}$	−0.821	−2.662	−1.341	1.410	±0.435
	0.483	**−1.185**	**0.000**	**0.221**	**±0.221**
$N^{GPS/lev} - N^{EIGEN2}$	0.630	−2.821	−1.064	1.313	±0.770
	0.491	**−1.234**	**0.000**	**0.221**	**±0.221**
$N^{GPS/lev} - N^{EIGEN3p}$	−0.427	−2.503	−1.202	1.266	±0.397
	0.499	**−1.244**	**0.000**	**0.222**	**±0.222**
$N^{GPS/lev} - N^{UCPH2003}$	−2.213	−4.513	−3.164	3.181	±0.329
	−1.245	**0.501**	**0.000**	**0.223**	**±0.223**
$N^{GPS/lev} - N^{GGM01C}$	−0.046	−2.510	−0.931	1.048	±0.482
	0.481	**−1.226**	**0.000**	**0.219**	**±0.219**
$N^{GPS/lev} - N^{GGM01S}$	−0.169	−3.810	−1.742	1.893	±0.740
	0.508	**−1.254**	**0.000**	**0.224**	**±0.224**
$N^{GPS/lev} - N^{GRACE01S}$	−0.051	−3.225	−1.363	1.484	±0.588
	0.504	**−1.250**	**0.000**	**0.223**	**±0.223**

Then the differences between the GPS/levelling geoid heights and those estimated by the GGMs have been compared, with the results being summarized in Table 5. Before the fit of the parametric model, UCHP2003 and EIGEN3p provide the smallest differences (σ of 33 cm and 40 cm, respectively, compared to 41 cm for the combined model). This can be misleading with respect to the performance of the models if one does not consider the mean value of the differences as well, which for the aforementioned models is about 2 m and 20 cm larger than that of the combined model. Therefore it can be concluded that the new GGM provides more accurate results with respect to UCPH2003 and EIGEN3p. As far as GGM01C is concerned, its differences with the

GPS/levelling data are of 48 cm which is 7 cm worse than that of the new combined model. After the fit it can be seen from Table 5 that the combined GGM provides the best results, i.e. smaller σ and dispersion by about 2% and 3%, respectively, compared to the other models.

The last comparison performed to assess the performance of the combined and the CHAMP and GRACE GGMs was with stacked TOPEX/Poseidon SSHs available for the eastern part of the Mediterranean Sea. Stacking (Knudsen, 1993) refers to the construction of mean sea surface heights along the altimetric satellite tracks employing its repeated passes. As a result a data set free of temporal variations and dynamic ocean effects is constructed from the long series of the satellite data. For the T/P SSHs, data over a ten-year period were available (1992–2002), so that when stacked and given the satellite's repeat orbit of about 10 days, a stacked dataset which was free of temporal and dynamic variations with period longer than 10 days was constructed (Vergos, 2006). This set of data is very useful for validation purposes, since it contains only the geoid height signal and some very small temporal variations (with period smaller than 10 days), which can be regarded as negligible for the eastern part of the Mediterranean.

Table 6. Geoid height differences between T/P^{SSHs} and N^{GGM} before and after the fit of a 3rd order polynomial model [m]

	Max	Min	Mean	RMS	Std
T/P^{SSHs}	49.990	1.068	29.516	32.193	±12.852
$T/P^{SSHs} - N^{Comb}$	1.348	−1.014	0.085	0.330	±0.319
	1.345	**−0.786**	**0.000**	**0.266**	**±0.266**
$T/P^{SSSHs} - N^{EGM96}$	1.026	−1.195	−0.143	0.356	±0.326
	1.157	**−0.719**	**0.000**	**0.269**	**±0.269**
$T/P^{SSHs} - N^{EIGEN2}$	7.063	−8.017	−1.433	3.609	±3.312
	9.414	**−5.981**	**0.000**	**2.973**	**±2.973**
$T/P^{SSHs} - N^{EIGEN3p}$	5.582	−6.041	−0.555	2.354	±2.288
	7.009	**−5.330**	**0.000**	**2.114**	**±2.114**
$T/P^{SSHs} - N^{UCPH2003}$	10.876	−11.627	−2.161	4.694	±4.167
	12.446	**−7.854**	**0.000**	**3.585**	**±3.585**
$T/P^{SSHs} - N^{GGM01C}$	1.600	−1.869	−0.054	0.538	±0.535
	1.739	**−1.419**	**0.000**	**0.481**	**±0.481**
$T/P^{SSHs} - N^{GGM01S}$	2.737	−3.157	−0.052	1.026	±1.025
	1.960	**−3.372**	**0.000**	**0.899**	**±0.899**
$T/P^{SSHs} - N^{GRACE01S}$	2.829	−2.985	−0.008	1.115	±1.115
	2.193	**−3.117**	**0.000**	**0.961**	**±0.961**

As in the case of the GPS/levelling points on land, the differences between the available GGMs and the stacked T/P SSHs have been computed and then minimized using the aforementioned parametric models. Once again, the performance of the parametric models has been assessed and the one that provides the smallest residual in terms of the standard deviation of the differences (1σ) has been selected. That test has been performed on the geoid height differences between the new combined GGM and the stacked T/P SSH data. From that analysis, the 3rd order polynomial model (11) has been selected as the most appropriate one. Table 6 summarizes the differences between the available GGMs and the T/P SSHs before (first row for each model) and after the fit (second row for each model) using the 3rd order polynomial model for the minimization of the differences. From that Table it is evident that the combined GGM provides the smallest differences before as well as after the fit of the parametric model (σ of 31.9 cm and 26.6 cm, respectively). The performance of the combined GGM is almost the same with that of EGM96 (differences at the level of 1 cm in terms of the σ), which can attributed to the good representation of the long wavelengths of the gravity field spectrum by both EGM96 and the new combined GGM. The shape of the parametric surface computed for the fit between the geoid heights from the combined GGM and T/P is presented in Figure 5, while the differences after the fit are depicted in Figure 6.

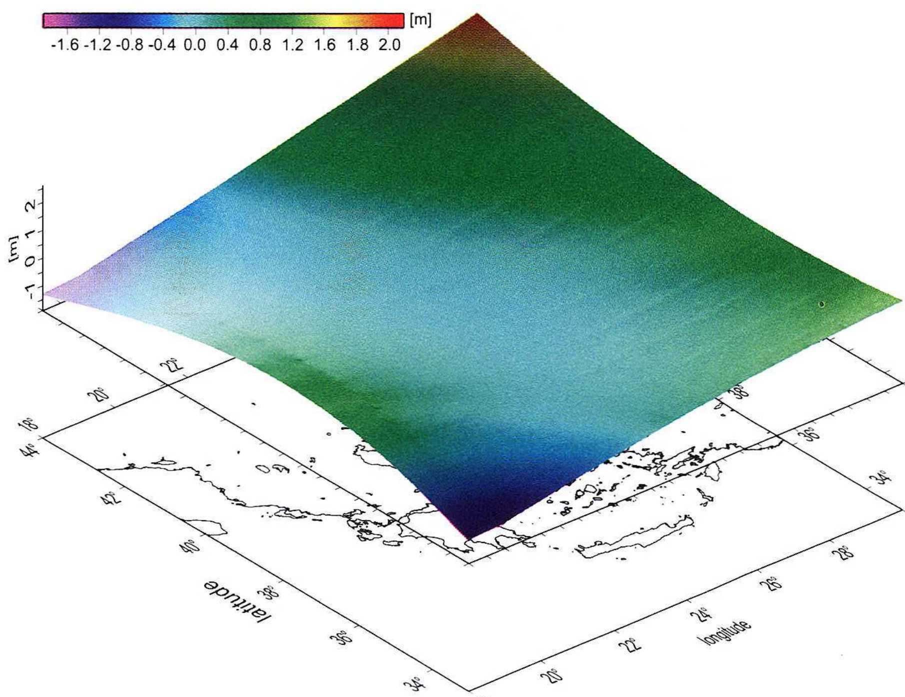

Fig. 5. 3rd order polynomial corrector surface for the fit between the combined GGM and T/P SSHs

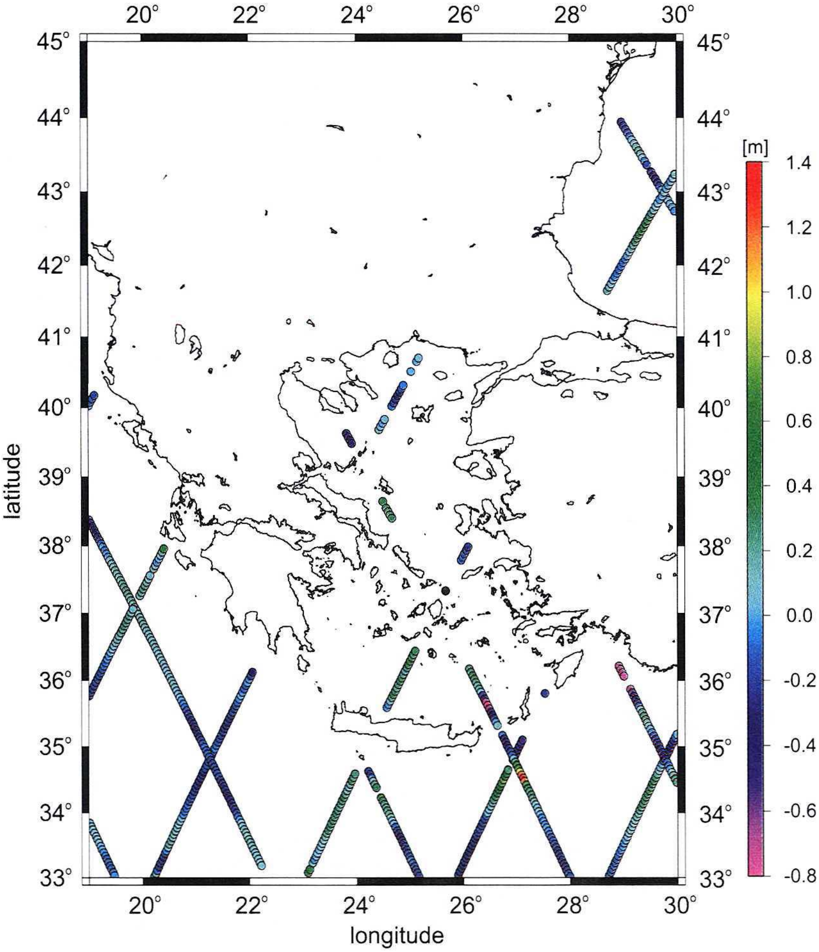

Fig. 6. Differences between stacked T/P SSHs and geoid heights from the combined GGM after the fit of the 3rd order polynomial model

A final point that is quite interesting, refers to the determination of marine geoid models from satellite altimetry data (geodetic and exact repeat mission data) employing the well-known remove-compute-restore method. In that process GGMs are used to remove the long-wavelengths from the SSHs and reduce the mean value of the residual dataset. From Table 6 it can be seen that both the mean value and the standard deviation of T/P SSHs are reduced significantly when referred to the combined GGM, i.e. a 29.1 m (98.5%) reduction of the mean value and a 12.533 m (97.5%) reduction of the σ. The mean value of the reduced to the combined GGM T/P SSHs is 6 cm smaller compared to that of EGM96 (8.5 cm and -14.3 cm, respectively), which provides good evidence that the newly compiled combined GGM performs equally well if not better than EGM96.

5. Conclusions

The analysis of the performance of some CHAMP- and GRACE-based GGMs on geoid and gravity field determination has been presented. From the results obtained it was concluded that the CHAMP models provide the most reliable and accurate results for the very-long wavelengths (up to degree $n = 5$), the GRACE models are superior up to degree $n = 116$, while EGM96 remains the dominant geopotential model for the shorter wavelengths.

From the spectral analysis of the geopotential models coming from the new satellite missions, a new combined GGM was determined using for each degree the coefficients of that GGM which was more accurate. The so-determined combined geopotential model outperforms EGM96 and the other models, at least for the area under study, since it provides the smallest differences when compared with GPS/levelling geoid heights. Before the fit of a parametric model, it provides smaller differences (1σ) by about 2 and 7 cm compared to EGM96 and GGM01C, while the range of the differences was 67 cm smaller than that of EGM96. After the fit, the combined model provides smaller σ by about 2%, compared to the other models, while the magnitude of the differences ranges by about 3% less.

Finally, from the comparison with the stacked T/P SSHs it was concluded that the combined GGM provides the smallest differences before as well as after the fit of the parametric model (σ of 31.9 cm and 26.6 cm, respectively). Moreover, it reduces the mean value of the T/P SSHs by 29.1 m (98.5%) and the standard deviation by 12.533 m (97.5%) significantly, which is of main importance when employing altimetric data for marine geoid modelling.

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The authors extensively used the Generic Mapping Tools (Wessel and Smith, 1998) in displaying the results.

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Ocena wiarygodności globalnych modeli geopotencjału opracowanych z wykorzystaniem danych z misji CHAMP i GRACE oraz utworzenie modelu kombinowanego

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Streszczenie

Progresywnie udostępniane są nowe satelitarne globalne modele geopotencjału (GGMs), opracowane na podstawie danych z misji grawimetrycznych CHAMP i GRACE. Modele te cechują się wzrastającą w porównaniu ze starszymi modelami dokładnością wyznaczenia niskiego i średniego rzędu harmonik pola grawitacyjnego Ziemi. W niniejszej pracy przeanalizowano najnowsze modele wygenerowane w oparciu o dane z misji CHAMP i GRACE, a mianowicie: EIGEN2, EIGEN3p, GGM01C, GGM01S i GRACE01S, w aspekcie ich dokładności i przydatności do badania pola grawitacyjnego Ziemi. Przeprowadzono analizę widmową tych modeli z włączeniem ich wariancji stopnia i błędów wariancji stopnia. Dokonano wzajemnego porównania wyników uzyskanych przy zastosowaniu tych modeli oraz modelu EGM96. Określono najdokładniej opisane przez każdy z modeli części widma pola grawitacyjnego Ziemi. W wyniku opracowano kombinowany globalny model geopotencjału, kompletny do stopnia i rzędu 360, którego współczynniki do stopnia 5 pochodzą z modeli z misji CHAMP, kolejne współczynniki do stopnia 116 – z modeli z misji GRACE, zaś pozostałe – do stopnia 360 – z modelu EGM96. Wysokości geoidy obliczone z badanych modeli, łącznie z modelem kombinowanym, zostały porównane na obszarze lądowym z danymi z pomiarów GPS i niwelacji, zaś na obszarze morskim – z wysokościami morza otrzymanymi z altimetrycznej misji TOPEX/Posejdon. Porównań dokonano na obszarze Grecji i we wschodniej części Morza Śródziemnego. Wyniki porównania wskazują, iż spośród badanych modeli, opracowany kombinowany globalny model geopotencjału dostarcza – w odniesieniu do kontrolnych danych – najdokładniejszych wyników: na poziomie 1–2 cm w wysokości geoidy i 1–2 mGal w anomalii grawimetrycznej (1σ). Co więcej, absolutną dokładność geoidy obliczonej z kombinowanego globalnego modelu geopotencjału oceniono odpowiednio jako 12.9 cm (1σ) dla $n = 120$, 25 cm dla $n = 200$ i 33 cm dla $n = 360$, w porównaniu odpowiednio z 29 cm, 36 cm i 42 cm dla EGM96.