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# An approach for surface matching using image and object parametric line features in frequency domain

The surface matching is useful to solve the problem of comparing two surfaces obtained from two data sets of the same terrain. The data sets of points could be get from different sources such as Aerial and Satellite Imagery, SAR, IFSAR, LIDAR etc.

The solution of surface matching problem is based on the some characterized contours existed on the surfaces as line features, which have to be expressed by parametric representation in frequency domain. Line features could be determined in the 2-D image or object space. For finding corresponding pair of point belonging to corresponding interest lines on the both surfaces, the image and object line features-based matching method has been used. Surface matching accuracy is estimated basing on the coordinates differences between the original and transformed surface.

#### INTRODUCTION

Nowadays various data sets of the same physical terrain are achieved with different sources as Aerial, Satellite Imagery, SAR, IFSAR (InSAR), LIDAR etc. Integration of these data for concrete task is one of the important directions of investigation.

The DEMs can be extracted by LIDAR, SAR [1, 2, 3]. The building detection and extraction have been also carried out with SAR and LIDAR [8, 11]. Fusing SAR data and leveling measurements for determination of land subsidence was introduced in the work [15]. IFSAR data derived from different satellites were used to investigate terrain deformation caused by earthquake [17]. One of the main goals of using LIDAR datum is to improve automatic generation of Digital Surface Model (DSM) or enhance surface extraction in difficult terrain [6, 7, 12]. Other integrations of SAR/INSAR data for object extraction were published in [5]. The main shared features of IFSAR and LIDAR are discribed in [7, 19].

In general, the two data point sets of same physical surface are in different local systems, different density and points are not correspondingly identical and usually irregularly

distributed. Let  $S1 = \{P_i\}$  i = 1, 2, 3,...n be a first surface described n discrete points and  $S2 = \{Q_j\}$  j = 1, 2, 3, ...m be a second surface described m discrete points. Assuming that two point sets in the first and second image will be correspondingly  $s_1 = \{p_i\}$  and  $s_2 = \{q_i\}$ .

Two tasks of surface matching having to be solved are the correspondence and transformation. These can be performed separately or simultaneously, dependently on the chosen matching approaches.

The first group of approaches for surface matching is based on the simultaneous implementation of correspondence and transformation. In these approaches the points and surface patches are treated as the features. For this goal the first surface will be generated, for example, in TIN models. Then, the correspondence task is to identify the triangle in the first surface to which a point in the second surface belongs. A transformation task is simultaneously established to determine its parameters. The transformation parameters between the two data sets of the same physical surface were proposed [18]. The first is based on the target function of minimizing differences in elevation between two surfaces. The second is based of minimizing distances along normal to surface.

The second group of approaches for surface matching is based on the extracted features in the both surfaces. Since the procedure of extracting features is highly sensitive to random errors and the sudden variation of surface and the time of process need for features extraction makes it computationally inefficient [16]. But this approach has powerful advantage that it is capable of dealing with large orientation differences between surfaces.

The proposed approach in this paper is based on the image and object line features in parametric representation for surface matching. The characterized lines existed on the two images (two surfaces) such as terrain contours, polygon nets along communication ways etc. are the line features, which are, at first, used to solve correspondence task. Then, transformation task would be established. As the points of line in the two images (or two surfaces) are differently distributed and not correspondingly identical, wherever, corresponding points of same lines are difficult to define, due to the differences of scale and starting point. In order to overcome this problem the line matching process in the frequency domain will be used. For this goal, the line equation has to be, at first, determined in the parametric representation by solving space resection task. Next, corresponding parametric lines on two images (or two surfaces) become transformed into frequency domain by Fourier descriptors. Image and object parametric line feature-based matching in frequency domain will be carried out.

### 1. Image parametric line feature-based matching in frequency domain

In this section we present how to determine image line feature in parametric representation and to transform it into frequency domain is presented. Then, the procedure of matching process of parametric line in frequency domain will be described.

## 1.1. Determination of image parametric line equation

The line in 2-D image spatial domain y = f(x) can be written in parametric representation as follows:

$$x = g_0 + g_1 t + g_2 t^2 + g_3 t^3 + \dots$$
  

$$y = h_0 + h_1 t + h_2 t^2 + h_3 t^3 + \dots$$
(1)

For each point i = 1, 2, 3,... with given coordinates  $(x_i, y_i)$  laying on a line, the value of parameter  $t_i$  will be computed by following description (Fig. 1):

$$t_i = 2\pi l_i / L$$
 for close line,  
 $t_i = \pi l_i / L$  for open line. (2)

where:  $l_i$  – the line segment from starting point s to point i (i = 1, 2, 3,...), L – the perimeter of line.

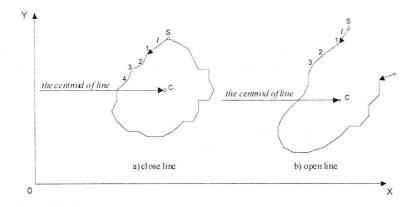


Fig. 1. The elements of line: s – start point; c – centroid; l – line segment; 1, 2, 3... points of line

Suppose the corresponding image and object points can be established by collinearity condition:

$$x = -f_c \frac{a_{11}(X - X_0) + a_{12}(Y - Y_0) + a_{13}(Z - Z_0)}{a_{31}(X - X_0) + a_{32}(Y - Y_0) + a_{33}(Z - Z_0)} = -f_c \frac{u}{w}$$

$$y = -f_c \frac{a_{21}(X - X_0) + a_{22}(Y - Y_0) + a_{23}(Z - Z_0)}{a_{31}(X - X_0) + a_{32}(Y - Y_0) + a_{33}(Z - Z_0)} = -f_c \frac{v}{w}$$
(3)

where:  $a_{11}$ ,  $a_{12}$ ,  $a_{13}$ ;  $a_{21}$ ,  $a_{22}$ ,  $a_{23}$ ;  $a_{31}$ ,  $a_{32}$ ,  $a_{33}$  – the elements of rotation matrix, which are the functions of the image exterior orientation angles  $(\varphi, \omega, \chi)$ ;  $X_0$ ,  $Y_0$ ,  $Z_0$  – the coordinates of image exposure station;  $f_c$  – camera focal length.

Substituting the left side of the Eq. (3) by right side of Eq. (1) we have new equation in the general form:

$$f_1(g_0, g_1, g_2, g_3, \varphi, \omega, \chi, X_0, Y_0, Z_0) = 0$$
  

$$f_2(h_0, h_1, h_2, h_3, \varphi, \omega, \chi, X_0, Y_0, Z_0) = 0$$
(4)

To determine the parameters in Eq.(4) we have to write Equation (4) into Eq. (5) by using Taylor's expansion

$$\frac{\partial f_{1}}{\partial g_{0}}dg_{0} + \frac{\partial f_{1}}{\partial g_{1}}dg_{1} + \frac{\partial f_{1}}{\partial g_{2}}dg_{2} + \frac{\partial f_{1}}{\partial g_{3}}dg_{3} + \frac{\partial f_{1}}{\partial \varphi}d\varphi + \frac{\partial f_{1}}{\partial \omega}d\omega + \frac{\partial f_{1}}{\partial \chi}d\chi + \frac{\partial f_{1}}{\partial X_{0}}dX_{0} + \frac{\partial f_{1}}{\partial Y_{0}}dY_{0} + 
+ \frac{\partial f_{1}}{\partial Z_{0}}dZ_{0} + f_{10} = \upsilon_{1}$$

$$\frac{\partial f_{2}}{\partial g_{0}}dh_{0} + \frac{\partial f_{2}}{\partial g_{1}}dh_{1} + \frac{\partial f_{2}}{\partial g_{2}}dh_{2} + \frac{\partial f_{2}}{\partial g_{3}}dh_{3} + \frac{\partial f_{2}}{\partial \varphi}d\varphi + \frac{\partial f_{2}}{\partial \omega}d\omega + \frac{\partial f_{2}}{\partial \chi}d\chi + \frac{\partial f_{2}}{\partial \chi}d\chi + \frac{\partial f_{2}}{\partial Y_{0}}dX_{0} + \frac{\partial f_{2}}{\partial Y_{0}}dY_{0} + 
+ \frac{\partial f_{2}}{\partial Z_{0}}dZ_{0} + f_{20} = \upsilon_{2}$$
(5)

For each image line feature the number of unknowns in Eq. (5) carries out 14. It means the needed minimum number of points belonging to a line is 7. At a result of solving Eq.(5) we can simultaneously create the image parametric line feature and compute the elements of image rotation matrix for next steps. After having found image exterior orientation we can determine any parametric line features in an image (1).

## 1.2. Least-squares parametric line matching in frequency domain using centroid-based transformation

Centroid-based transformation in frequency domain is proposed to use for the sake of two reasons:

1. In conventional 2D spatial transformation, the translation parameters  $\Delta x$ ,  $\Delta y$  (shifts of the two origins of coordinate system) were unchangeable under the effect of scaling s and rotation  $\Theta$ . By spatial transformation the positional change of the transformed line feature is not corresponding with translation parameters  $\Delta x$ ,  $\Delta y$ , because the centroid of the feature (geometric center of a line feature) is changed by scaling s and rotation  $\Theta$ . In order to obtain explicit form of transformation parameters  $(\Delta x, \Delta y, s, \Theta)$  the change of centroid should be isolated from scaling and rotation. This could be accomplished by mean of transforming a feature about the centroid, which is so-called centroid-based transformation (Fig. 2)

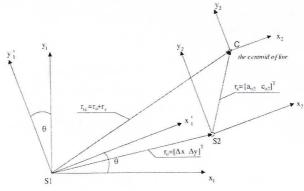


Fig. 2. Similarity centroid-based transformation in 2-D spatial domain

2. In spatial domain corresponding points of same line belonging in two surfaces are difficult to define, due to the differences of scale and starting point. In order to overcome this problem, an algorithm to perform the matching process in the frequency domain is proposed, where the Fourier descriptors of line are matches.

The content of this subsection is briefly taken out of the work [13].

#### 1.2.1. Representation of parametric line in frequency domain

Equation (1) is now transformed into frequency domain by Fourier descriptors as follows:

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} a_0 \\ c_0 \end{bmatrix} + \sum_{k=1}^{\infty} \begin{bmatrix} a_k & b_k \\ c_k & d_k \end{bmatrix} \cdot \begin{bmatrix} \cos kt \\ \sin kt \end{bmatrix}$$
 (6)

The coefficients for close line are:

$$a_{0} = \frac{1}{2\pi} \int_{0}^{2\pi} x(t)dt \qquad c_{0} = \frac{1}{2\pi} \int_{0}^{2\pi} y(t)dt$$

$$a_{k} = \frac{1}{\pi} \int_{0}^{2\pi} x(t)\cos ktdt \qquad b_{k} = \frac{1}{\pi} \int_{0}^{2\pi} x(t)\sin ktdt \qquad (7)$$

$$c_{k} = \frac{1}{\pi} \int_{0}^{2\pi} y(t)\cos ktdt \qquad d_{k} = \frac{1}{\pi} \int_{0}^{2\pi} y(t)\sin ktdt$$

and for open line:

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} a_0 \\ c_0 \end{bmatrix} + \sum_{k=1}^{\infty} \begin{bmatrix} a_k \cos kt \\ c_k \cos kt \end{bmatrix}$$
 (8)

where:

$$a_{0} = \frac{1}{\pi} \int_{0}^{2\pi} x(t)dt \qquad c_{0} = \frac{1}{\pi} \int_{0}^{2\pi} y(t)dt$$

$$a_{k} = \frac{2}{\pi} \int_{0}^{2\pi} x(t)\cos ktdt \qquad b_{k} = \frac{2}{\pi} \int_{0}^{2\pi} x(t)\sin ktdt$$
(9)

with  $a_0$ ,  $c_0$  – the coordinates or zero harmonics of line centroid;  $a_k$ ,  $b_k$ ,  $c_k$ ,  $d_k$  – the higher harmonics; x(t) and y(t) in Eq. (7), (9) will be taken out of Eq. (1); k – natural number, in practice k = 3.

#### 1.2.2. Centroid-based transformation in frequency domain

The starting point can be chosen anywhere along the line, we also consider that a change of the starting point can be interpreted as a phase shift  $\Delta t$ . The  $\Delta t$  can be an arbitrary value between  $\Theta$  and  $2\pi$  for close line, but for open line  $\Delta t$  takes one of two value  $\Theta$  and  $\pi$ , because the starting is either one of the two end points.

The transformation in frequency domain is a transformation directly operated on the given harmonic coefficients  $(a_0, c_0, a_k, b_k, c_k, d_k)$  into new harmonic coefficients  $(a'_0, c'_0, a'_k, b'_k, d'_k)$ . Coordinates of the centroid  $(a_0, c_0)$  are the zero harmonics and others coefficients  $(a_k, b_k, c_k, d_k)$  of higher harmonic are independent of the centroid translation. Therefore centroid-based affine transformation in frequency domain can be divided into parts as follows:

$$\begin{bmatrix} a_0' \\ c_0' \end{bmatrix} = \begin{bmatrix} a_0 \\ c_0 \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} a'_k & b'_k \\ c'_k & d'_k \end{bmatrix} = \begin{bmatrix} e & f \\ r & z \end{bmatrix} \begin{bmatrix} a_k & b_k \\ c_k & d_k \end{bmatrix} \begin{bmatrix} \cos k\Delta t & -\sin k\Delta t \\ \sin k\Delta t & \cos k\Delta t \end{bmatrix}$$
(11a)

For centroid based similarity transformation the affine transformation matrix  $\begin{bmatrix} e & f \\ r & z \end{bmatrix}$  can be substituted by scaling factor s and rotation angle  $\Theta$ :

$$\begin{bmatrix} a'_k & b'_k \\ c'_k & d'_k \end{bmatrix} = s \begin{bmatrix} \cos \Theta & -\sin \Theta \\ \sin \Theta & \cos \Theta \end{bmatrix} \begin{bmatrix} a_k & b_k \\ c_k & d_k \end{bmatrix} \begin{bmatrix} \cos k\Delta t & -\sin k\Delta t \\ \sin k\Delta t & \cos k\Delta t \end{bmatrix}$$
(11b)

#### 1.2.3. Least-squares matching in frequency domain

The problem of finding corresponding points of two lines is realized by least-squares matching using centroid-based transformation in frequency domain. For this goal the transformation parameters can be directly solved in the frequency domain.

For both cases of close and open line the transformation parameters  $\Delta x$ ,  $\Delta y$  can be directly calculated by using the formula (10).

To solve others parameters of similarity transformation as: s,  $\Theta$ ,  $\Delta t$ , the observation equation can be derived from Eq. (11b). We have two cases:

– for close line:

$$\begin{bmatrix} d'_{k} \\ b'_{k} \\ c'_{k} \\ d'_{k} \end{bmatrix} + \begin{bmatrix} v_{a'_{k}} \\ v_{b'_{k}} \\ v_{d'_{k}} \end{bmatrix} = s \begin{bmatrix} a_{k} & b_{k} & -c_{k} & -d_{k} \\ b_{k} & -a_{k} & -d_{k} & c_{k} \\ c_{k} & d_{k} & a_{k} & b_{k} \\ d_{k} & -c_{k} & b_{k} & -a_{k} \end{bmatrix} \begin{bmatrix} \cos \Theta \cos k \Delta t \\ \cos \Theta \sin k \Delta t \\ \sin \Theta \cos k \Delta t \\ \sin \Theta \sin k \Delta t \end{bmatrix}$$

$$(12)$$

The iterative method of least-squares adjustment could be used to determine the unknowns: s,  $\Theta$ ,  $\Delta t$  with given approximations values:  $s_0$ ,  $\Theta_0$ ,  $\Delta t_0$ . First approximations values of unknowns some transformations of computation have been found in [13].

– for open line: with  $\Delta t = 0$  (see Eq. 8) the observation equation will be:

$$\begin{bmatrix} a_k' \\ c_k' \end{bmatrix} + \begin{bmatrix} v_{a_k'} \\ v_{c_k'} \end{bmatrix} = \begin{bmatrix} a_k \\ c_k \end{bmatrix}$$
 (13)

with  $\Delta t = \pi$ :

$$\begin{bmatrix} a'_k \\ c'_k \end{bmatrix} + \begin{bmatrix} v_{a'_k} \\ v_{c'_k} \end{bmatrix} = \begin{bmatrix} a(-1)^k & -c(-1)^k \\ c(-1)^k & a(-1)^k \end{bmatrix} \begin{bmatrix} m \\ n \end{bmatrix}$$
(14)

where:  $m = s.\cos \Theta$ ;  $n = s.\sin \Theta$ .

At last, the unknowns s and  $\Theta$  could be calculated as follows:

$$s = \sqrt{m^2 + n^2}$$

$$\Theta = \arctan\left(\frac{n}{m}\right)$$
(15)

In order to estimate the accuracy of matching process the Mean Square Error is calculated as:

MSE = 
$$\frac{1}{2} \sum_{k=1}^{q} (v_{a'_k}^2 + v_{b'_k}^2 + v_{a'_k}^2 + v_{a'_k}^2)$$
 for close line (16)

In case of open line matching the terms  $v_{b'_i}$   $v_{d'_i}$  in (16) are excluded.

We consider that the determined parameters  $\Delta x$ ,  $\Delta y$ , s,  $\Theta$ ,  $\Delta t$  were affected by camera distortion error, factors of image tilt angles and terrain height differences. To overcome this problem we can choose some lines lying on the four corners of an image and determine simultaneously their equation in the form (1). For each point (x, y) belonging to line there is two observation equations in the formula (5) with 14 unknowns. The needed number of points on a line is desired at least 7. If we have four lines lying on four corners of an image, the number of unknowns being to determine will be 38 (8 coefficients per a line and 6 common image exterior orientation elements), the needed number of points belonging to each line is at least equal to 5.

According to Eq. (12) (Fig. 2) the parameters s,  $\Theta$  are not only the scale factor and rotation angle of line but also are the common parameters for an image transformation, whereas, the parameter  $\Delta t$  is rather different for each line. For k = 3, the number of equation (12) for each line is equal to 12 with 3 unknowns  $(s, \Theta, \Delta t)$ . If we take four lines lying on the four corners of an image to simultaneously least-squares matching the number of equation

in the form (12) will be equal to 48 with 6 unknowns (4 unknowns ( $\Delta t$ ) for four lines and 2 common parameters (s,  $\Theta$ ).

The translation parameters  $(\Delta x, \Delta y)$  are separately computed on the base of (10) for each line feature on an image.

## 1.3. Steps of surface matching using image parametric line feature in frequency domain

Below, we arrange sequences to implement surface matching using image parametric line feature

- 1\* Calculation of line equation in parametric representation and image exterior orientation parameters:
  - a) choose a starting point, compute parametric representation for sequence points lying on the line (Eq.2).
  - b) computation of first approximate coefficients  $g_{00}$ ,  $g_{10}$ ,  $g_{20}$ ,  $g_{30}$ ;  $h_{00}$ ,  $h_{10}$ ,  $h_{20}$ ,  $h_{30}$ :
    - $g_{00}$ ,  $h_{00}$  are denoted as a coordinates of starting point, ie.  $g_{00} = x_s$ ,  $h_{00} = y_s$ .
    - $-g_{10}$ ,  $h_{10}$  and  $g_{20}$ ,  $h_{20}$  can be computed under the condition that three points s, 1, 2, are lain on the line (see Fig.1) as follows:

$$g_{10} = \frac{D_1}{D}$$
;  $g_{20} = \frac{D_2}{D}$ ;  $h_{10} = \frac{D_3}{D}$ ;  $h_{20} = \frac{D_4}{D}$ 

where

$$D = \begin{vmatrix} t_1 & t_1^2 \\ t_2 & t_2^2 \end{vmatrix}; D_1 = \begin{vmatrix} (x_1 - x_s) & t_1^2 \\ (x_2 - x_s) & t_2^2 \end{vmatrix}; D_2 = \begin{vmatrix} t_1 & (x_1 - x_s) \\ t_2 & (x_2 - x_s) \end{vmatrix};$$

$$D_3 = \begin{vmatrix} (y_1 - y_s) & t_1^2 \\ (y_2 - y_s) & t_2^2 \end{vmatrix}; D_4 = \begin{vmatrix} t_1 & (y_1 - y_s) \\ t_2 & (y_2 - y_s) \end{vmatrix}$$

- $-g_{30}$ ,  $h_{30}$  are computed from Eq. (1) when known  $g_{00}$ ,  $g_{10}$ ,  $g_{20}$ ;  $h_{00}$ ,  $h_{10}$ ,  $h_{20}$ .
- c) first approximate image exterior orientation elements are equal to 0, ie.  $\varphi_0 = \omega_0 = \chi_0 = 0$ ;  $X_0 = Y_0 = Z_0 = 0$ .
- d) creating and iteratively solving Eq. (5) to determine the line coefficients  $g_0, g_1, g_2, g_3$ ;  $h_0, h_1, h_2, h_1$  and image angle orientation elements  $\varphi$ ,  $\omega$ ,  $\chi$ ;  $X_0, Y_0, Z_0$ .
- e) compute the elements of the rotation matrix  $a_{11}$ ,  $a_{12}$ ,  $a_{13}$ ;  $a_{21}$ ,  $a_{22}$ ,  $a_{23}$ ;  $a_{31}$ ,  $a_{32}$ ,  $a_{33}$  for next step.
- 2\* Transforming the line in parametric representation (x(t), y(t)) into frequency domain and performing the least-squares matching to determine parameters  $\Delta x$ ,  $\Delta y$ , s,  $\Theta$ ,  $\Delta t$ .
  - a) calculate the coefficients  $a_0$ ,  $c_0$ ,  $a_k$ ,  $b_k$ ,  $c_k$ ,  $d_k$  in frequency domain for close line (Eq. 7) or for open line (Eq. 9). Next, performing (Eq. 6) or (Eq. 8), respectively.

- b) solve least–squares matching for close line (Eq. 10 and Eq. 12) or for open line (Eq. 10, Eq. 13, Eq. 14) to determine transformation parameters  $\Delta x$ ,  $\Delta y$ , s,  $\Theta$ ,  $\Delta t$ .
- c) estimate matching process accuracy by Mean Squares Error (Eq. 16).
- 3\* Transforming point coordinates of any line on the second image  $s_2 = \{q_i\}$  into first image  $s_1 = \{p_i\}$  with help of determined transformation parameters, using centroid-based transformation (Fig. 2):

$$\begin{bmatrix} x_1' \\ y_1' \end{bmatrix} = s \begin{bmatrix} \cos \Theta & -\sin \Theta \\ \sin \Theta & \cos \Theta \end{bmatrix} \begin{bmatrix} x_2 - a_{02} \\ y_2 - c_{02} \end{bmatrix} + \begin{bmatrix} a_{02} \\ c_{02} \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

where:  $a_{02}$ ,  $c_{02}$ ;  $x_2$ ,  $y_2$  – the coordinates of centroid and other points of line in the second image  $s_2 = \{q_i\}$ 

4\* Calculating 3D point coordinates of transformed image coordinates (in first image) with help of exterior orientation of first image:

$$\begin{bmatrix} X_1' \\ Y_1' \\ Z_1' \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + M \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1' \\ y_1' \end{bmatrix}$$

where: M – scale factor calculated on the base of corresponding distances in first image and surface S1.

5\* Finding the corresponding terrain points under the following condition:

$$|\Delta X|; |\Delta Y|; |\Delta Z| \leq |\sigma_T|$$

where:  $\sigma_T$  - the threshold value;  $\Delta X = X_1' - X_1$ ;  $\Delta Y = Y_1' - Y_1$ ;  $\Delta Z = Z_1' - Z_1$ .

6\* Computing RMS errors based on the founded corresponding terrain points:

$$m_X = \sqrt{\frac{[\Delta X \Delta X]}{n}}; \qquad m_X = \sqrt{\frac{[\Delta Y \Delta Y]}{n}}; \qquad m_X = \sqrt{\frac{[\Delta Z \Delta Z]}{n}}$$

where: n – the number of founded corresponding terrain points.

The transformed 3-D point coordinates  $X_1'$ ,  $Y_1'$ ,  $Z_1'$  obtained from the step (4\*) can be treated as approximate values. It means that new surface  $Z_1' = F_1'(X_1', Y_1')$  is approached to first surface S1:  $Z_1 = F_1(X_1, Y_1)$ . For increasing an accuracy and reliability we implement one more time surface matching, basing on the Eq. (18-22) presented in third section.

## 2. Object parametric line feature-based matching in frequency domain

The content of this section is similar to the second section, but the difference between them depends on the fact that parametric line feature of a surface is performed in the 2D object system. Substituting small letters in (Eq. 1) by big letters we obtain the parametric line feature in 2D object system:

$$X = G_0 + G_1 T + G_2 T^2 + G_3 T^3 + \dots$$
  

$$Y = H_0 + H_1 T + H_2 T^2 + H_3 T^3 + \dots$$
(17)

where: T – the parametric representation in object system which is defined by Eq. 2.

After substituting the point planar coordinates X; Y of a line in a surface to Eq. 3 by Eq.17 we receive equation similar to Eq. 4. For finding the coefficients  $G_0$ ,  $G_1$ ,  $G_2$ ,  $G_3$ ,  $H_0$ ,  $H_1$ ,  $H_2$ ,  $H_3$  the equation like as (5) has to be created and solved by iterative least-squares adjustment.

Next step we use formulas presented in section 2. At first, the coefficients  $a_0$ ,  $c_0$ ,  $a_k$ ,  $b_k$ ,  $c_k$ ,  $d_k$ ,  $\Delta x$ ,  $\Delta y$ , s,  $\Theta$ ,  $\Delta t$  are changed by big letters  $A_0$ ,  $C_0$ ,  $A_k$ ,  $B_k$ ,  $C_k$ ,  $D_k$ ,  $\Delta X$ ,  $\Delta Y$ , S,  $\Phi$ ,  $\Delta T$  for object line.

Object parametric line feature-based matching has been done in frequency domain. At the end, we obtain transformation parameters which are used to transform planar coordinates  $(X_2, Y_2)$  of point set S2 into set S1, while the heights of points in S2 have to be enlarged by scale parameter S. Next step, we can use theoretical formulas presented in [16].

We designate  $X_1'$ ,  $Y_1'$ ,  $Z_1'$  – the transformed point coordinates of the set S2 in the set S1. Let  $Z_1 = F_1(X_1, Y_2)$  be first mathematically represented surface. The transformed surface from S2 to S1 by parameters  $\Delta X$ ,  $\Delta Y$ , S,  $\Phi$  is  $Z_1' = F_1'(X_1', Y_1')$ . Two surfaces  $Z_1$ ,  $Z_1'$  have to be matched. Suppose  $P = [X_1 \ Y_1 \ Z_1]^T$  and  $P' = [X_1' \ Y_1' \ Z_1']^T$  are a pair of corresponding points between two surfaces. There is mathematical existence between P and P':

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = R \begin{bmatrix} X_1' \\ Y_1' \\ Z_1' \end{bmatrix} + \begin{bmatrix} E \\ N \\ W \end{bmatrix}$$
(18)

where: R – the rotation matrix of  $\Psi$ ,  $\Omega$ , K angles; E, N, W – translation parameters.

Let point  $P' = [X_0 \ Y_0 \ Z_0]^T$  be the approximate corresponding point, of which  $X_0 = X_1', Y_0 = Y_1'$  and  $Z_0$  is interpolated from  $Z_1 = F_1(X_1, Y_1)$ . Let  $X_1 = X_0 + \Delta X_1$ ,  $Y_1 = Y_0 + \Delta Y_1$ ,  $Z_1 = Z_0 + \Delta Z_1$ . The  $\Delta Z_1$  increment can be computed from  $Z_1 = F_1(X_1, Y_1)$  as follows:

$$\Delta Z_1 = \frac{\partial F_1}{\partial X_1} \Delta X_1 + \frac{\partial F_1}{\partial Y_1} \Delta Y_1 \tag{19}$$

Linearization of Eq. 19 at  $\Psi = \Omega = K = 0$  and E = N = W = 0 gives:

$$\Delta X_{1} = X'_{1} - X_{0} + dE - Y'_{1}dK + Z'_{1}d\Psi$$

$$\Delta Y_{1} = Y'_{1} - Y_{0} + dN + X'_{1}dK - Z'_{1}d\Omega$$

$$\Delta Z_{1} = Z'_{1} - Z_{0} + dW - X'_{1}d\Psi + Y'_{1}d\Omega$$
(20)

In respect of assumption  $X_0 = X_1' Y_0 = Y_1'$  the first two equations of (20) become:

$$\Delta X_1 = dE - Y_1'dK + Z_1'd\Psi$$
  

$$\Delta Y_1 = dN + X_1'dK - Z_1'd\Omega$$
(21)

Basing on (19), (20), (21) the observation equation for corresponding points have the form

$$v_z = Z_0 - Z_1' + (dE - Y_1'dK + Z_1'd\Psi) \frac{\partial F_1}{\partial X_1} + (dN + X_1'dK - Z_1'd\Omega) \frac{\partial F_1}{\partial Y_1} - dW + X_1'd\Psi - Y_1'd\Omega$$
(22)

The parameters  $\Psi$ ,  $\Omega$ , K; E, N, W in Eq. (22) will be solved by least-squares adjustment when first surface S1 have to be generated in DEM models. Since partial differentials  $\partial F_1/\partial X_1$ ,  $\partial F_1/\partial Y_1$  in Eq. (22) have been easily computed.

#### CONCLUSION

The paper presents actual problem of surface matching when we like to compare two surfaces of same scene derived by different sensors. An approach of surface matching is based on the image or object line features which are described in parametric representation.

In the both cases of image or object line feature the number of characterized points of lines is needed to determine their parametric equation. In practice, these lines laying on the surface can be easily chosen, for example, as contour lines, polygon nets along communication ways. Image and object parametric line features would be performed by solving space resection. By this way the two groups of parameters of line equation and image exterior orientation will be simultaneously obtained. As the points of two surfaces are differently distributed and not correspondingly identical, wherever, corresponding points of same line belonging to surfaces are difficult to define. To overcome this difficulty image and object parametric line feature based matching in frequency domain, supporting on the centroid-based transformation, is proposed. By this way the starting point of line can be chosen anywhere along the line. The additional phase shift parameter (a change of starting point) is introduced to matching process.

The accuracy and reliability of surface matching depend on the accuracy and reliability of determined transformation parameters. In order to increase the accuracy and reliability of surface matching some lines features on the four corners of surface (or of image) could be extracted.

Distinct advantage of the presented approach is to solve the problem encountered often in practice while different sensor images of same scene have different grey characteristics and bigger differences of exterior orientations.

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- IAPRS International Archives of Photogrammetry and Remote Sensing.
- PE&RS Photogrammetric Engineering and Remote Sensing.

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### Luong Chinh Ke

Propozycja dopasowania powierzchni ze zastosowaniem parametrycznych cech liniowych na zdjęciu i na powierzchni w częstotliwościowej przestrzeni

#### Streszczenie

Praca przedstawia problem dopasowania powierzchni tego samego terenu przy wykorzystaniu zdjęć wykonanych z pułapu lotniczego i satelitarnego oraz danych, takich jak np.: SAR, IFSAR, LIDAR. Praca

koncentruje się na określeniu możliwości zastosowania liniowych cech, takich jak: poligony wzdłuż sieci komunikacyjnej, czy warstwice terenu, które są przydatne do rozwiązywania problemu dopasowania powierzchni.

Linie określone na zdjęciu lub na powierzchni terenu są przedstawione za pomocą analitycznej funkcji parametrycznej w częstotliwościowej przestrzeni dzięki rozwiązaniu zadania wcięcia przestrzennego i zastosowania wzoru Fourier'a. Proces dopasowania odpowiednich liniowych cech parametrycznych w tej przestrzeni został zrealizowany przy zastosowaniu centroid transformacji. Otrzymane parametry transformacji stanowią podstawowe dane do przekształcenia drugiego zdjęcia (drugiej powierzchni) w pierwsze (pierwszą powierzchnię).

Люонг Чын Ке

Предложение приспособления поверхности с применением параметрических линейных черт на снимк и на поверхности в пространстве частоты

#### Резюме

Представлена проблема приспособления поверхности той-же самой местности с использованием аэро- и космичских снимков, а также таких данных как SAR, IFSAR, LIDAR. Работа сосредоточена на определении возможности применения линейных черт, таких как: полигоны вдоль коммуникационных линий или горизонтальных линий местности, которые являются пригодными для решения проблемы приспособления поверхности.

Линии, определённые на снимке или на поверхности местности представляются при помощи аналитической параметрической функции в пространстве частоты, благодаря решению задачи пространсвенной засечки и применения формулы Фурие. Процесс приспособления соответственных линейных параметрических чертов в этом пространстве был выполнен с применением центроидов преобразования. Полученные параметры преобразования составляют основные данны для преобразования второго снимка (второй поверхности) в первый снимок (первую поверхность).