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*Krzysztof Buczkowski*

Institute of Photogrammetry and Cartography
Warsaw University of Technology
(00-661 Warszawa, Plac Politechniki 1)

# Spatial concentration – generalised measure of testing distribution of objects and spatial phenomena

The paper presents a method of testing trends in spatial distribution of objects and phenomena, which has been called by the author „the spatial concentration method". For those fields of knowledge, where information about trends in spatial distribution of phenomena is important, two meanings of the term „concentration" are used. With respect to functionality it means a mechanism of processes, which lead to diversification of concentration of tested objects or phenomena in space. With respect to the structural aspect, the term „concentration" is understood as description of certain conditions of the tested spatial system, i.e. conditions of higher or lower concentration of the system elements. This paper focuses on concentration with respect to the latter case.

The proposed method of spatial concentration is a tool, which allows for wide research investigations. It has been assumed that concentration is specified with respect to a reference function, which is selected according to the objective of investigations. Therefore it is possible to investigate various aspects of trends in distribution of phenomena. It should be stressed that the proposed method is of general nature, in a sense that it also covers other methods of investigations, which have been based on measures of concentration; the proposed method also allows for searching for trends in spatial distribution of phenomena and objects. The algorithm of determination of spatial concentration, presented in a simple form, is the base for software development and for automation of investigations.

## 1. *The term concentration*

Many researchers, who understand it in many ways, depending on diversified points of view and objectives of investigations (performed for geographic, economic, psychological and other purposes), have used the term „concentration". For those fields of knowledge, where information about trends in spatial distribution of phenomena is important, two meanings of the term „concentration" are used. With respect to functionality it means a mechanism of processes, which lead to diversification of concentration of tested objects or phenomena in space. With respect to the structural aspect, the term

„concentration" is understood as description of certain conditions of the tested spatial system, i.e. conditions of higher or lower concentration of the system elements.

Concentration with respect to the second meaning is the object of cartographic investigations. Conditions, which have been reached by a spatial system, may be characterised and described by means of coefficients, which are based on various measures of concentration; it may be also described graphically by means of a map of concentration. In the first case, an investigated spatial system is considered as a numerical value of the concentration coefficient; in the second case – the basic result of research in a map of concentration. This paper deals with the second way of investigations.

## 2. *Measures and maps of concentration*

Methods of investigations of concentration of spatial objects and phenomena have been widely discussed in scientific literature. At the beginning concentration was specified by means of simple quantitative factors, concerning mainly density or intensity. Maps of density of phenomena, such as map of density of population, of forests, are often a graphic illustration of such simple factors.

Introduction of new methods of concentration investigation is, first of all, connected with development of geography of settlements. Many publications started to appear in the thirties; their authors have been introducing new and more complex measures of concentration, which allow for investigation of various aspects of trends in distribution of settlement networks. Later, coefficients of concentration were introduced to the narrower extent in investigations concerning geography of population, agriculture, industry etc. Review and evaluation of applied measures of concentration has been presented by B.Kostrubiec [8]. It is worth to notice, that some concentration factors have been developed by outstanding cartographers. The factor of F.Uhorczak [15] of 1932 and the factor of A.H.Robinson and J.A.Barnes [13] may be mentioned; their authors were also the authors of first maps of concentration, produced by means of an isarythmic method.

Methods of investigation of concentration based on a method, which was introduced in 1905 by M.O.Lorenz [12] are very popular and have been widely applied. Description of the method, in the Polish language version, was presented by S.Fogelson in 1933 [5]. At present, description of the method, together with many practical examples, have been discussed in some publications related to mathematical statistics, as. For example, by: S.Szulc [14], M.Krzysztofiak and D.Urbanek [11], K.Zając [19]. The Lorenz method has been developed and modified by various authors. R.Jedut [6] attempted to determine relationships between the value of concentration factor and areas of selected territorial units. K.Bromek [1] introduced derivative measures, such as the factor of geometric concentration and eccentricity of concentration. J.Żurkowski [20] presented an analytical method of determination of the concentration curve.

The concentration method in Lorenz approach was introduced to cartography by F.Uhorczak and published in 1948 [16]. Adopting the Lorenz method for cartography, Uhorczak investigated trends in population distribution in Poland. The final result of his work was a map, which has not been, however, called by him „a map of concentration". The

method introduced by F.Uhorczak and called by him later a method of mosaic concentration [17] was then methodologically described by K.Bromek [1], R.Jedut [6] and B.Kostrubiec [9]. The essence of that method concerns determination of concentration with respect to a surface of an assumed system of basic fields. Zones of similar concentration are described on the map basing on a diagram of the Lorenz curve.

The method of mosaic concentration, which is sometimes called „a distributed or localised concentration", has been often applied, mainly for investigations of population distribution. Example of such investigations as well as examples of maps of concentration have been presented, among others, by K.Bromek [1], R.Domański [4], R.Jedut [6], R.Kosiński and A. Wojciechowski [7], B.Kostrubiec [9]. Interesting and untypical utilisation of the discussed method has been proposed by M.Kozak and A.Szromik [10], who investigated the spatial structure of Kraków basing on produced maps of concentration.

In 1971 F.Uhorczak published modification of the method of mosaic concentration, concerning introduction of „compact" zones of concentration. The essence of that modification concerns the consideration of geographic locations of territorial units in the process of determination of zones. That modification has not been widely applied in practice, although, according to F.Uhorczak's opinion, „maps of zone concentration present a better and geographically ordered image of an investigated phenomenon". This results from the fact that the proposed solution leaves too much freedom to the author of a map when zones of concentration are delineated. The discussed method is not an explicit process, what makes certain difficulties concerning introduction of exact mathematical description, and, therefore, automation of map development.

In the majority of cases, units of administrative division or regular geometric fields, are the bases for delineation of concentration zones on a map. However, it is possible to move away from a conventionally understood system of basic fields. An example of such approach is presented by K.Warakomska [18], who produced a map of population concentration, where equidistant lines (buffers), drawn from roads and bus stops, were the basic fields.

Another, unconventional way of development of maps of concentration of point phenomena is presented by K.Buczkowski [2, 3]. The basis for delineation of concentration zones was a set of distances between points. Distances may be measured as straight lines, along roads or in any other way, what creates the possibility to develop many maps of concentration for the same set of points.

## 3. *Spatial concentration*

### 3.1. Initial assumptions

The majority of known and discussed above methods of development maps of concentration, allows to perform investigations only within the frames of a system of surface, regular or irregular reference units. In this case the maximum concentration means location of phenomena or objects in one, and the smallest reference unit, whereas distribution of phenomena within such a unit is not important. This means that such methods allow to learn one of many aspects concerning trends in spatial distribution of

phenomena. A method, which presents more possibilities and which may be applied away from any systems of basic fields, is the proposed method of spatial concentration.

Information on distribution of investigated objects or phenomena in space is the source materials for determination of spatial concentration. The proposed method requires that source data is considered as a finite set of points. It is the apparent limitation only, since, even for phenomena, which are presented by means of linear signatures or ranges on a map, it is also possible to replace a partially continuous image with a discrete set (Fig.1).

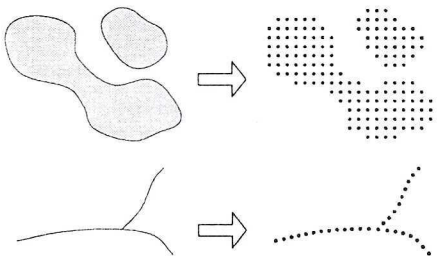


Fig. 1. Replacement of partially continuous images with discrete sets

In the case of the discussed method, concentration of phenomena or objects, is determined with respect to the, so called, reference function. The reference function may be created by arbitrary phenomena, which are determined within the investigated area. This may be population density, a set of distances between points of the investigated set, relative altitudes or absolute altitudes etc. Selection of the reference function depends on objective of investigations.

Determination of spatial concentration means selection of a $k$-element subset, which is characterised by the highest spatial concentration with respect to the assumed reference function, from an $n$-element, initial set of points. For the $n$-element set of points, $n!/(n-k)!k!$ $k$-element combinations may be found, therefore such a subset must be specified in an explicit way.

### 3.2. Definition of spatial concentration

Let us assume that the $n$-element set of points is discussed, which are unequally distributed within the investigated area. Such points correspond to real distribution of investigated phenomena or objects.

Let us further assume that concentration is determined with respect to the reference function, which selection depends on objectives of investigations.

After such initial assumptions, it is proposed to assume the following definition of spatial concentration:

*Let an n-element set of points of co-ordinates: $(X_1, Y_1)$, $(X_2, Y_2)$, ... , $(X_n, Y_n)$ is given within the D area. Let the function of two variables $Z= f(X,Y)$, is determined for the set D, which is further called the reference function. If, from the n-element set a k-element subset is selected, which meets the condition:*

$$\sum_{i=1}^{k} f(X_i, Y_i) = \text{extremum} \tag{1}$$

*Then that subset is characterised by the extreme spatial concentration with respect to the assumed reference function.*

In the particular case, when $f(X_1, Y_1) = f(X_2, Y_2) = f(X_n, Y_n)$, the value of sum is constant for all subset of the same population. Thus we deal with the extreme concentration of the $n$-element set of points.

### 3.3. The reference function

Spatial concentration should be determined with respect to the reference function, selected in accordance with objectives of investigations. This function may present the value of the phenomenon in an absolute way (volume of falls, temperature, altitude above the sea level etc.) or in a relative way (density of population, density of a river network, forestation etc.). This function may be continuous (distribution of temperature, pressure etc.) or discrete (a set of distances between points of an investigated set, the sum of distances to five closest neighbours etc.).

As it turns out from the condition specified in the definition, knowledge of the mathematical equation of the reference function is not required for determination of concentration. Only function values in points of the investigated set are required for performed testing. Those values may be obtained in various ways, depending on characteristics of the reference function. They may be read from the map (e.g. when the reference function is presented by means of the isoline method) or obtained as a result of measurements on a map (e.g. when the reference function is a set of distances assigned to points). They may also be determined by calculations based on various source materials or in another, justified way.

### 3.4. The distribution function and the curve of spatial concentration

The presented definition explicitly specified the condition, which should be met to characterise the selected subset of points by the extreme spatial concentration. The best way to practically implement the condition specified in the definition is to determine the distribution function of concentration.

For that purpose let us consider values of the reference function: $Z_1, Z_2, ..., Z_n$, in points of the investigated set, as a set of elementary events and let us assign identical frequency to each point:

$$p_i = \frac{1}{n} \text{ where } \sum_{i=1}^{n} p_i = 1 \tag{2}$$

Distribution of the random variable, specified in this way, allows to determine the distribution function of that variable:

$$F(Z) = P(Z < z) = \sum p_k \tag{3}$$

The distribution function allows to distinguish subset of a specified population, which are characterised by the extreme concentration, or, in other words, it allows to determine concentration classes. In order to increase perception abilities, step points of the distribution function may be connected by a continuous line, obtaining the function diagram; it is proposed to call this diagram as the spatial concentration curve. The spatial concentration curve may the basis for creation of new statistical measures, which allow to perform the general evaluation of trends in distribution of points of the investigated set. It also allows to develop a transparent legend for the map of concentration.

Practical approach is illustrated in Fig. 2, where all stages of development of concentration maps for 10-element set of points are presented in a model way.

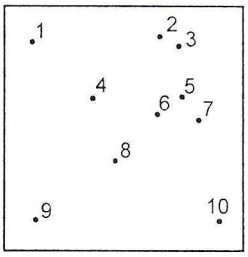
For a large set of points, in the case when concentration is determined with respect to a phenomenon presented by means of isolines, the discussed procedure may be simplified and an approximate shape of the spatial concentration curve may be delineated immediately.

For that purpose, the location of isolin es of the reference function should be delineated on the map; if possible, those isolines should evenly cover the entire investigated area. The next stage is to determine the number of points, separately for each zones, delineated by isolines of neighbouring values. Determined numbers of points should be expressed as parts of the entire set. This allows to find a cumulative series of the number of points and to assign an appropriate isoline value to each element of the series. Thus, pairs of numbers are obtained, which are transferred as points on a plane of a rectangular co-ordinate system; isoline values should be marked on the abscissa and corresponding values of the cumulative series should be marked as ordinates. As a result of graphical approximation, the approximate diagram of the spatial concentration curve is obtained.

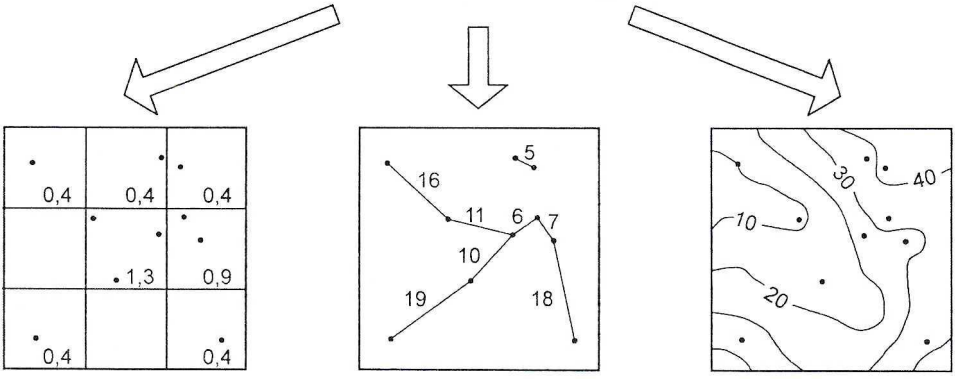
### 3.5. Determination of classes and presentation of spatial concentration on a map

The calculated distribution function or the spatial concentration curve is the basis for determination of concentration classes. Practical approach is presented in Fig. 2F and 2G, where classes covering 50% of all points, which are characterised by the highest spatial concentration with respect to the assumed reference function, have been determined. Only one concentration class has been determined in this case. More classes may be introduced, depending on objectives of investigations.

In the process of determination of classes it is worth to consider the shape of the spatial concentration curve. Using the diagram, points of inflexion, and therefore ranges where the function is convex or concave, may be distinguished. Points of inflexion point to those values, which should be considered in the process of class selection. The expansion of classes is determined by location of the curve. Those parts of the diagram, which are almost parallel to the abscissa, should fall into one class. Convexity of the function leads to the




A Distribution of objects, for which spatial concentration is investigated

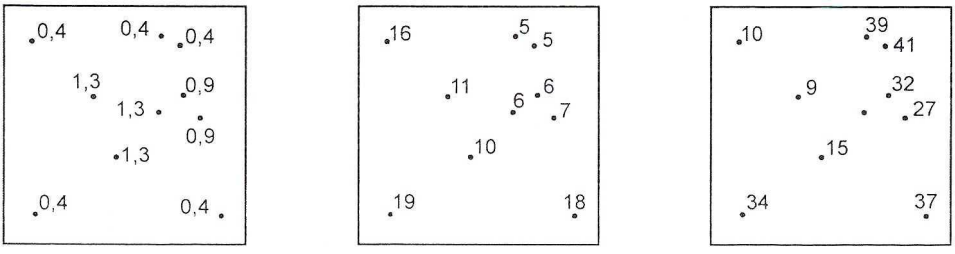



B Selection of the reference function with respect to which spatial concentration is investigated

density for the system of regular basic field

distance to the closest neighbour (straight line)

digital model of a statistical surface




C Specification of the reference function values in points of an investigated set

| nr | z | p |
|---|---|---|
| 1, 2, 3, 9, 10 | 0,4 | 0,5 |
| 4, 5, 6 | | |
| 7 | 0,9 | 0,2 |
| 8 | 1,3 | 0,3 |
| | | 1,0 |

| nr | z | p |
|---|---|---|
| 2, 3 | 5 | 0,2 |
| 5, 6 | 6 | 0,2 |
| 7 | 7 | 0,1 |
| 8 | 10 | 0,1 |
| 4 | 11 | 0,1 |
| 1 | 16 | 0,1 |
| 10 | 18 | 0,1 |
| 9 | 19 | 0,1 |
| | | 1,0 |

| nr | z | p |
|---|---|---|
| 4 | 9 | 0,1 |
| 1 | 10 | 0,1 |
| 8 | 15 | 0,1 |
| 6 | 25 | 0,1 |
| 7 | 27 | 0,1 |
| 5 | 32 | 0,1 |
| 9 | 14 | 0,1 |
| 10 | 37 | 0,1 |
| 2 | 39 | 0,1 |
| 3 | 41 | 0,1 |
| | | 1,0 |

nr - point numbers
z - values of the reference function in points of the investigated set
p - frequency

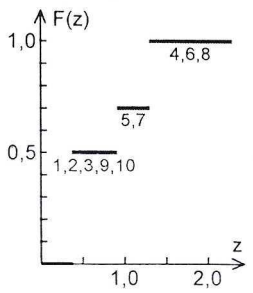
D Statistical distribution of an investigated phenomenon (distribution of random variables)

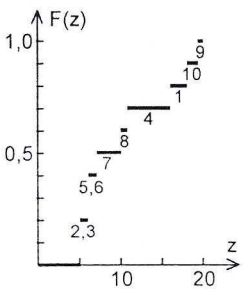
| z | F(z) |
|---|---|
| 0,4 | 0 |
| 0,9 | 0,5 |
| 1,3 | 0,7 |
| | 1 |

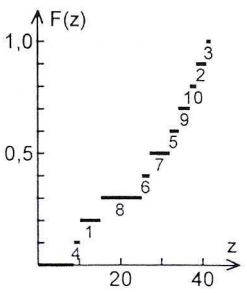
| z | F(z) |
|---|---|
| 5 | 0 |
| 6 | 0,2 |
| 7 | 0,4 |
| 10 | 0,5 |
| 11 | 0,6 |
| 16 | 0,7 |
| 18 | 0,8 |
| 19 | 0,9 |
| | 1 |

| z | F(z) |
|---|---|
| 9 | 0 |
| 10 | 0,1 |
| 15 | 0,1 |
| 25 | 0,1 |
| 27 | 0,1 |
| 32 | 0,1 |
| 14 | 0,1 |
| 37 | 0,1 |
| 39 | 0,1 |
| 41 | 0,1 |
| | 1 |

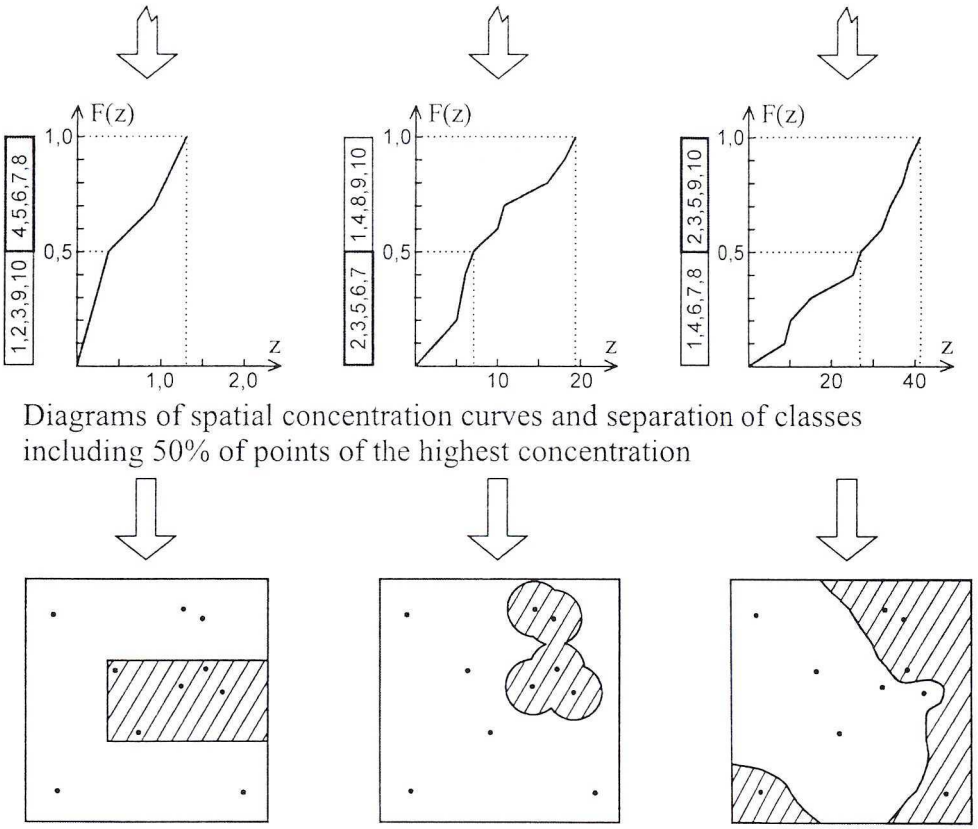
E Distribution functions of spatial concentration in a tabular form




F Diagrams of distribution functions of spatial concentration




G Diagrams of spatial concentration curves and separation of classes including 50% of points of the highest concentration

Fig. 2. Successive stages (the algorithm) of development of spatial concentration maps

necessity of creation of classes of the increasing expansion; its concavity to classes of the decreasing expansion.

Creation of classes is equivalent with division of points of the investigated set into subsets, which are characterised by various levels of spatial concentration. The important element of works is appropriate selection of a method of cartographic presentation, which llows, in a readable way, to present those subsets on a map. It has been stated that the best results are obtained, when the presentation method is connected with the nature of the reference function. This is presented in Fig. 2H, where some possibilities of the proposed method have been also presented.

## CONCLUSIONS

The paper presents new proposals concerning investigations of trends in spatial distribution of phenomena and objects. The proposed method of spatial concentration is

a tool, which allows to perform wide investigations. It has been assumed that concentration is determined with respect to the reference function, selected in accordance with objectives investigations. Therefore it is possible to investigate various aspects of trends of phenomena distribution. It should be stressed that the proposed method is of general nature, in a sense, that it also covers other methods of investigations, which have been based on measures of concentration; it gives practically unlimited possibilities to search for trends in spatial distribution of phenomena and objects.

The algorithm of determination of spatial concentration, presented in a simple form, forms the bases for software development and for combining it with existing systems, which support the process of development of maps, as well as Geographic Information Systems.

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*Krzysztof Buczkowski*

**Koncentracja przestrzenna – uogólniona miara badania rozmieszczenia obiektów i zjawisk przestrzennych**

Streszczenie

W pracy przedstawiona została metoda badania prawidłowości w przestrzennym rozmieszczeniu obiektów i zjawisk, nazwana przez autora metodą koncentracji przestrzennej. W tych dziedzinach wiedzy, dla których istotne jest poznanie prawidłowości w przestrzennym rozmieszczeniu zjawisk, pojęcie koncentracji jest używane w dwojakim znaczeniu. W ujęciu funkcjonalnym oznacza ono mechanizm przebiegu procesów, które prowadzą do zróżnicowania skupienia badanych zjawisk lub obiektów w przestrzeni. W ujęciu strukturalnym natomiast, pojęcie koncentracja rozumiane jest jako opis pewnego stanu jaki osiągnął badany układ przestrzenny, stanu większego lub mniejszego skupienia jego elementów. Przedmiotem rozważań jest koncentracja w tym drugim znaczeniu.

Proponowana metoda koncentracji przestrzennej jest narzędziem, które daje szerokie możliwości prowadzenia badań. Przyjęto założenie, że koncentracja jest wyznaczana względem wybranej zgodnie z celem badań funkcji odniesienia. Istnieje w związku z tym możliwość badania różnych aspektów prawidłowości rozmieszczenia zjawisk. Należy podkreślić, że proponowana metoda ma charakter ogólny, w tym sensie, że obejmuje również inne stosowane dotychczas sposoby badań oparte na miarach koncentracji oraz daje duże możliwości poszukiwań prawidłowości w przestrzennym rozmieszczeniu zjawisk i obiektów.

Podany w prostej formie algorytm wyznaczania koncentracji przestrzennej daje podstawy do przygotowania oprogramowania i automatyzację procesu badań.

*Кшиштоф Бучковски*

**Пространственная концентрация – обобщённая мера исследования размещения пространственных объектов и явлений**

Резюме

В работе представлен метод исследования закономености в пространственном размещении объектов и явлений, названный автором методом пространственной концентрации.

В тех областях знаний, для которых существенным авляется знание закономерности в пространственном размещении явлений, понятие концентрации употребляется в двойном значении. В функциональном смысле оно обозначает механизм хода процессов, которые ведут к дифференцированию сколения исследуемых явлений или объектов в пространстве. Зато в структуральном подходе, понятие концентрации рассматривается как описание некоторого состояния, какого достигла исследуемая прострaнственная система, состояния большего или меньшего скопления её элементов. Предметом наших рассуждений является концентрация в этом другом значении.

Предлагаемый метод пространственной концентрации является орудием, которое даёт широкие возможности проведения исследований. Принято предположение, что концентрация определяется относительно избранной, согласной с целью исследований, функции относимости. Существует в связи с тем возможность исследования разных аспектов закономерности размещения явлений. Следует подчеркнуть, что предлагаемый метод имеет общий характер, в том смысле, что охватывает также другие применяемые до сих пор способы исследований, основанные на мерах концентрации, а также даёт большие возможности поиска закономерности в пространственном размещении явлений и объектов.

Представленный в простой форме алгоритм определения пространственной концентрации даёт основы для подготовки программного обеспечения и автоматизации процесса исследования.