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Modified general polynomial transition curves in grade line designing

The article presents a family of modified the so called general polynomial transition curves, determined with the purpose of being applied at the design of the grade line of roads. The designing conditions of those curves have been provided alongside detailed information on determination of particular designing parameters. The application methods of those curves in the process of the grade line designing, based upon economically justified minimization of earth works, have been presented as well.

1. *Introduction*

Formation of curvilinear segments of roads within a location plan and longitudinal section calls for application of appropriate geometrical elements, if it is to meet certain designing criteria. Among those criteria the grade line to be adjusted to the relief is regarded as an elementary one, which means minimization of earth works connected with the accomplishment of the design. Appropriate literature points out the fact that accomplishment of optimum adjustment of the grade line to topographic conditions with the use of traditional geometrical elements in the form of straight lines and circular arcs is considerably difficult. Polynomial aligned functions, once applied, provide much greater possibilities; if so, it is possible to specify (with the use of computers) in an iterative way the optimum values of coefficients and the degree of particular polynomials, to make the course of the grade line possibly least differ from given direction points. Such suggestions have been described for example in [1] and [2]. However, a slightly unclear geometrical form is their shortcoming, which makes them rather "unfriendly" to the designer, even if account is taken of his remarkably reduced participation in the process, due to computerization. Besides, application of the arc radii corresponding to the values resulting from guidelines turns out not to guarantee — in this case — for required visibility distances to be preserved. Due to this designing of the grade line with the use of transition curves may provide an alternative; among the said curves it is possible to find the ones which — to a certain extent — will enable to freely form the curvature thereof, thanks to which the possibility of adjustment to the relief is quite big. In [5] a solution based on the so called general transition curves described in [3] has been presented; it resolves itself down to the grade line in the form of those curves to be immediately adjusted to the relief and makes it

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possible to freely form the curvature of the route in a longitudinal section. A modification of those curves will be presented herein which is meant to simplify the designing process and reduce the range of the necessary calculations.

2. *The modified polynomial general transition curve and the designing conditions thereof*

Beginning P (0, 0) of the general polynomial transition curve to be found will be placed at the beginning of Cartesian coordinate system XOY, end $K(x_K, y_K)$ to be located within the area of positive values of abscissa x, optionally according to axis OX (Fig. 1). Besides, angles of inclination of tangents to the curve at points P and K will be marked as u_1 and u_2, angle of inclination of the chord based on those points will be marked as α.

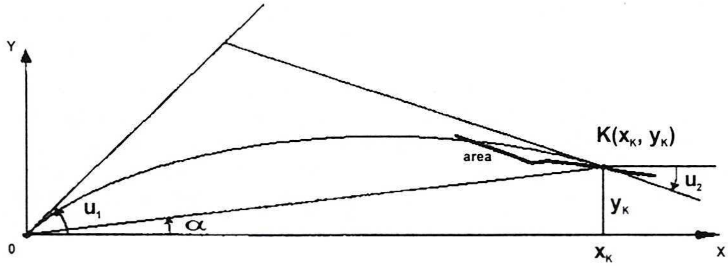

Fig. 1

In accord with [6], search for a general transition curve of an uneven curvature graph, described by means of the following polynomial function

$$y = f(x) = \sum_{i=0}^{i=k} a_i x^i \tag{1}$$

with k = 5, account being taken of the following conditions:

– values of functions at extreme points P and K equal to 0 and y_K respectively;

– inclinations of tangents at points P and K equal to tg α and tg u_2;

– ivalue of curvature at points P and K equals to zero,

leads to an equation of the following form:

$$y = x_K(A_i \operatorname{tg} u_1 + A_2 \operatorname{tg} u_2 + A_3 \operatorname{tg} \alpha) \tag{2}$$

where:

$$A_1 = t - 6t^3 + 8t^4 - 3t^5$$

$$A_1 = -4t^3 + 7t^4 - 3t^5$$

$$A_1 = 10t^3 + 15t^4 - 6t^5$$

Note that $t = \dfrac{x}{x_K}$, $t \in \langle 0;1 \rangle$, and also $\operatorname{tg}\alpha = \dfrac{y_K}{x_K}$

Practical application of the curve described by equation (2) as the curve that satisfies the foregoing conditions calls for the graph thereof not to have inflextion points within interval (0; x_K), and its curvature not to exceed the designed maximum value. According to [6], this will be satisfied if for certain values of tg α, tg u_1, tg u_2 and selected values C and D from interval <–3/2; –2/3>, where:

$$C = \frac{\operatorname{tg} u_1}{\operatorname{tg} u_2} + \frac{C_3}{C_1}\frac{\operatorname{tg}\alpha}{\operatorname{tg} u_2} \tag{3}$$

$$D = \frac{\operatorname{tg} u_1}{\operatorname{tg} u_2} + \frac{D_3}{D_1}\frac{\operatorname{tg}\alpha}{\operatorname{tg} u_2} \tag{4}$$

with

$$C_1 = -36t + 96t^2 - 60t^3$$

$$C_3 = 60t + 180t^2 + 120t^3$$

$$D_1 = -36 + 192t - 180t^2$$

$$D_3 = 60 + 360t - 360t^2$$

the following inequality is satisfied

$$\frac{5}{3} \le \frac{1}{\operatorname{tg}\alpha}\left(\operatorname{tg} u_1 - D \operatorname{tg} u_2\right) \le \frac{5}{2} \tag{5}$$

alongside one of the following inequalities

$$\frac{1}{\operatorname{tg}\alpha}\left(\operatorname{tg} u_1 - C \operatorname{tg} u_2\right) \le \frac{5}{4} \tag{6}$$

for $t < 6/10$ or

$$\frac{1}{\operatorname{tg}\alpha}\left(\operatorname{tg} u_1 - C \operatorname{tg} u_2\right) \ge 5 \tag{7}$$

for $t > 6/10$. Then for appropriate values $t_E = \frac{x_E}{x_K}$ from interval <1/3; 2/3> (x_E – abscissa of the point at which the curvature reaches its extreme) whose range is determined by dependences

$$\frac{1}{\operatorname{tg}\alpha}\left(\operatorname{tg} u_1 - C \operatorname{tg} u_2\right) = -\frac{C_3}{C_1} \tag{8}$$

and

$$\frac{1}{\operatorname{tg}\alpha}\left(\operatorname{tg}u_1 - D\operatorname{tg}u_2\right) = -\frac{D_3}{D_1} \tag{9}$$

diagram of function (2) will not have inflection points, as well as the graph of the curvature of that function will have just one maximum point within interval $< 0; x_K >$.

While applying curve (2) in practice, it is important for maximum value of the curvature of the curve based on two points distant in respect of their abscissas by x_K not to exceed the designed admissible curvature. In order to observe that condition it is necessary to calculate an appropriate radius of the curvature

$$R = \frac{x_K}{\left|C_1\operatorname{tg}u_1 + C_2\operatorname{tg}u_2 + C_3\operatorname{tg}\alpha\right|} \tag{10}$$

argument $t_E = \dfrac{x_E}{x_K}$ to be assumed while determining values C_1, C_2 and C_3; C_1 and C_3 having been determined by dependences mentioned before, whereas

$$C_2 = -24t + 84t^2 + 60t^3$$

For definite values of tg α, tg u_1, tg u_2 and t_E, dependence (10) may be applied at determination of the necessary length of abscissa x_K of the end-point of the curve which secures designed maximum curvature $1/R$ not to be exceeded.

Due to considerably complex relationships between tg α, tg u_1, tg u_2, C, D and t_E, described by the foregoing dependences, the determining manner of value t_E calls for a wider explanation. In accord with [6], for tg $\alpha = 0$, D must equal to C. If so, equation of the curve and designing conditions comply with those in [3] as referred to the curve that is suggested to be applied while designing the grade line in the way described in [5]. If tg $\alpha \neq 0$, it is necessary that $D \neq C$. In accord with [6] the following dependences will be necessary:

$$\frac{\operatorname{tg}u_2}{\operatorname{tg}\alpha} = \frac{\dfrac{D_3}{D_1} - \dfrac{C_3}{C_1}}{D - C} \tag{11}$$

$$\frac{1}{\operatorname{tg}\alpha}\left[\operatorname{tg}u_2(D + C) - 2\operatorname{tg}u_1\right] = \frac{D_3}{D_1} + \frac{C_3}{C_1} \tag{12}$$

and

$$\frac{\operatorname{tg}u_1}{\operatorname{tg}\alpha} = \frac{\dfrac{D_3}{D_1} - \dfrac{D}{C}\dfrac{C_3}{C_1}}{\dfrac{D}{C} - 1} \tag{13}$$

In accord with [6], it is necessary that $C \neq D$. If a set of values of tg α, tg u_1 and tg u_2 is given, dependence (12) might be made use of. On the foregoing basis alongside the course of function

$$G(t) = \frac{D_3}{D_1} + \frac{C_3}{C_1} \tag{14}$$

shown in Fig. 2, one may state that:

– within interval <1/3 ; t_0)

$$-\frac{15}{4} \leq \frac{1}{\operatorname{tg}\alpha}\left[\operatorname{tg} u_2 (D+C) - 2\operatorname{tg} u_1\right] \leq 0$$

– within interval <t_0 ; 6/10)

$$\frac{1}{\operatorname{tg}\alpha}\left[\operatorname{tg} u_2 (D+C) - 2\operatorname{tg} u_1\right] \geq 0$$

– within interval (6/10 ; 2/3>

$$\frac{1}{\operatorname{tg}\alpha}\left[\operatorname{tg} u_2 (D+C) - 2\operatorname{tg} u_1\right] \leq -\frac{20}{3}$$

with to = 0.54901066 ... . Determination of the wanted value of t_E calls for a definite value of sum $D + C$ to be assumed, with $(D + C) \in$ < –3 ; –4/3 >. In order to make that easier Table 1 includes the possible scopes of values and $D + C$, determined for given values of tg α, tg u_1 and tg u_2 on the basis of equation (12), and with regard to the course of function $G(t)$. For the so determined values of $D + C$, there may — it does not have to, though — exist a solution to equation (12) within a certain range of values of t_E. At least partial overlapping of the determined range of $D + C$ with interval < –3 ; –4/3 > is the necessary condition to that.

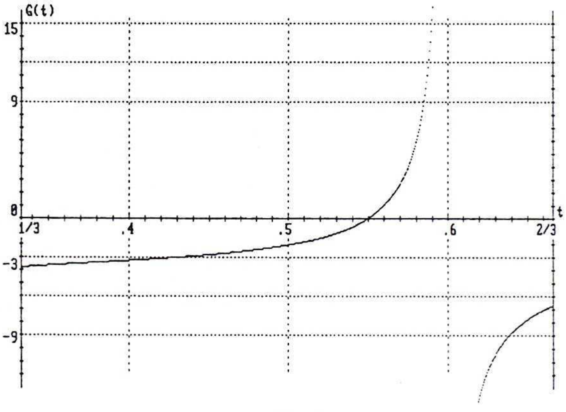

Fig. 2

If only values of tg α tg u_1 were given, according to [6], it would be possible to apply dependence (13). Condition $D \neq C$ means that for $D > C$ the value of proportion D/C should be included in interval $< 4/9; 1)$, while for $D < C$ - in interval $(1; 9/4 >$. Bearing this in mind and taking into account the course of functions $\frac{C_3}{C_1}$, $\frac{D_3}{D_1}$ and function

$$F(t) = \frac{D_3 / D_1}{C_3 / C_1} \tag{15}$$

Table 1

Range of t_E value determined on the basis dependence (12) $t_0 \cong 0.54901...$)

Conditions	Range of t_E	Range of sum $D + C$	
$\operatorname{tg}\alpha > 0$	$\operatorname{tg} u_2 > 0$	$t \in < 1/3; t_0)$	$2\frac{\operatorname{tg} u_1}{\operatorname{tg} u_2} > D + C \geq \frac{1}{\operatorname{tg} u_2}\left(2\operatorname{tg} u_1 - \frac{15}{4}\operatorname{tg}\alpha\right)$
	$\operatorname{tg} u_2 < 0$		$2\frac{\operatorname{tg} u_1}{\operatorname{tg} u_2} < D + C \leq \frac{1}{\operatorname{tg} u_2}\left(2\operatorname{tg} u_1 - \frac{15}{4}\operatorname{tg}\alpha\right)$
$\operatorname{tg}\alpha < 0$	$\operatorname{tg} u_2 > 0$		$2\frac{\operatorname{tg} u_1}{\operatorname{tg} u_2} < D + C \leq \frac{1}{\operatorname{tg} u_2}\left(2\operatorname{tg} u_1 - \frac{15}{4}\operatorname{tg}\alpha\right)$
	$\operatorname{tg} u_2 < 0$		$2\frac{\operatorname{tg} u_1}{\operatorname{tg} u_2} > D + C \geq \frac{1}{\operatorname{tg} u_2}\left(2\operatorname{tg} u_1 - \frac{15}{4}\operatorname{tg}\alpha\right)$
$\operatorname{tg}\alpha > 0$	$\operatorname{tg} u_2 > 0$	$t \in < t_0; 6/10)$	$D + C \geq 2\frac{\operatorname{tg} u_1}{\operatorname{tg} u_2}$
	$\operatorname{tg} u_2 < 0$		$D + C \leq 2\frac{\operatorname{tg} u_1}{\operatorname{tg} u_2}$
$\operatorname{tg}\alpha < 0$	$\operatorname{tg} u_2 > 0$		$D + C \leq 2\frac{\operatorname{tg} u_1}{\operatorname{tg} u_2}$
	$\operatorname{tg} u_2 < 0$		$D + C \geq 2\frac{\operatorname{tg} u_1}{\operatorname{tg} u_2}$
$\operatorname{tg}\alpha > 0$	$\operatorname{tg} u_2 > 0$	$t \in (6/10; 2/3 >$	$D + C \leq \frac{1}{\operatorname{tg} u_2}\left(2\operatorname{tg} u_1 - \frac{20}{3}\operatorname{tg}\alpha\right)$
	$\operatorname{tg} u_2 < 0$		$D + C \geq \frac{1}{\operatorname{tg} u_2}\left(2\operatorname{tg} u_1 - \frac{20}{3}\operatorname{tg}\alpha\right)$
$\operatorname{tg}\alpha < 0$	$\operatorname{tg} u_2 > 0$		$D + C \geq \frac{1}{\operatorname{tg} u_2}\left(2\operatorname{tg} u_1 - \frac{20}{3}\operatorname{tg}\alpha\right)$
	$\operatorname{tg} u_2 < 0$		$D + C \leq \frac{1}{\operatorname{tg} u_2}\left(2\operatorname{tg} u_1 - \frac{20}{3}\operatorname{tg}\alpha\right)$

shown in Fig. 3, appropriate ranges of argument t_E from interval < 1/3; 2/3 > determined for particular tg u_1 and tg α the basis of dependence (13) have been included in Table 2. Determination of the wanted value of t_E requires a certain value of proportion D/C to be assumed according to the signs of tg u_1 and tg α. After it has been determined, it is still necessary to calculate tg u_2, which may be done on the basis of (11), whence the following is implied:

$$\operatorname{tg} u_2 = \frac{1}{C} \operatorname{tg} \alpha \frac{\frac{D_3}{D_1} - \frac{C_3}{C_1}}{\frac{D}{C} - 1} \tag{16}$$

Table 2

Ranges of t_E value determined on the basis of dependence (13) ($t^* \cong 0.40565$ ...)

Conditions			Range of t_E
$D > C \rightarrow \frac{D}{C} - 1 < 0$	$\operatorname{tg} u_1 \geq 0$	$\operatorname{tg} \alpha > 0$	$t \in < 1/3; 1/2)$ or $t \in (1/2; 3/5)$
$D/C \in < 4/9; 1)$		$\operatorname{tg} \alpha < 0$	$t \in (3/5; 2/3 >$
recommended $C = -3/2$	$\operatorname{tg} u_1 < 0$	$\operatorname{tg} \alpha > 0$	$t \in (3/5; 2/3 >$
		$\operatorname{tg} \alpha < 0$	$t \in < 1/3; 1/2)$ or $t \in (1/2; 3/5)$
$D < C \rightarrow \frac{D}{C} - 1 > 0$	$\operatorname{tg} u_1 \geq 0$	$\operatorname{tg} \alpha > 0$	$t \in < 1/3; t^* >$ or $t \in (3/5; 2/3 >$
$D/C \in (1; 9/4 >$		$\operatorname{tg} \alpha < 0$	$t \in < 1/3; 1/2)$ or $t \in (1/2; 3/5)$
recommended $C = -2/3$	$\operatorname{tg} u_1 < 0$	$\operatorname{tg} \alpha > 0$	$t \in < 1/3; 1/2)$ or $t \in (1/2; 3/5)$
		$\operatorname{tg} \alpha < 0$	$t \in < 1/3; t^* >$ or $t \in (3/5; 2/3 >$

It is plain to see that if tg α, t_E and D/C are known, it is necessary to choose a certain value of C from interval < –3/2; –2/3 >. Therefore, before C is calculated, it is necessary to check beforehand whether value D calculated on the basis of (13) as

$$D = C\left(\frac{D_3}{D_1}\frac{\operatorname{tg}\alpha}{\operatorname{tg} u_1} - \frac{D}{C}\frac{C_3}{C_1}\frac{\operatorname{tg}\alpha}{\operatorname{tg} u_1} + 1\right) \tag{17}$$

also includes in interval < –3/2; –2/3 >. In order to secure that it is advisable to choose a possibly biggest value of C if $D < C$, a possibly smallest value of C to be chosen if $D > C$. For example: tg α= 0.01, tg u_1=01 having been given, and D/C = 9/4 having been assumed, t_E = 0.478884 is to be found, whence for C = –2/3 it is implied that D = –3/2 and tg u_2 = 0.02023. For the same tg α, tg u_1 and D/C, yet C = –0.7, value D = –1.575 will be obtained, which exceeds the admissible interval.

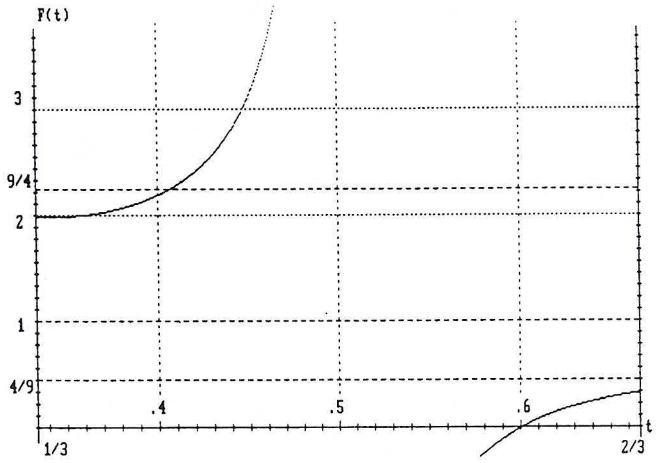

Fig. 3

3. *The curve to be applied at grade line designing*

While designing a grade line by means of curve (2) it is necessary to know the kilometreage and altitudinal ordinates of particular points of a longitudinal section. In the light of grade line/terrain adjustment recommended in the guidelines, in case of curve (2), it is justified to apply a criterion of the following form:

$$F = \sum_{i=1}^{n} (y_i - Y_i)^2 \tag{18}$$

This time Y_i stands for the ordinate of the i-th point of the section of abscissa X_i in the system presented in Fig.1, whereas y_i is the ordinate of the point of curve (2) of abscissa $x_i = X_i$. Due to that, it is necessary to reduce altitudinal ordinates H_i and kilometreage L_i of particular points of the longitudinal section down to a system whose beginning overlaps with beginning P of the curve, which comes about according to the following equations:

$$Y_i = H_i - H_P$$

$$X_i = L_i - L_P$$

where: H_P and L_P – altitude and kilometreage of starting point P.

In the light of the recommended grade line/relief adjustment, the kind of arc (concave/convex) depending on relief should comply with the kind dependent on mutual relationships between tg α, tg u_1 and tg u_2. The required kind of arc conditioned by relief will be defined on the basis of the sign of the following expression:

$$\Sigma\, \Delta h = \Sigma\, (h - H) \tag{19}$$

in which altitude h will be calculated for the i-th point as

$$h_i = H_P + X_i \operatorname{tg} \alpha \tag{20}$$

The kind resulting from mutual relationships between tg α, tg u_1 and tg u_2 will be specified by Table 3.

Table 3

Kind of arc according to relationships between tg α tg u_1 and tg u_2

Conditions		Kind of arc
$\operatorname{tg}\alpha > \operatorname{tg} u_1$	$\operatorname{tg} u_1 > \operatorname{tg} u_2$	-
	$\operatorname{tg} u_1 < \operatorname{tg} u_2$	concave
$\operatorname{tg}\alpha < \operatorname{tg} u_1$	$\operatorname{tg} u_1 > \operatorname{tg} u_2$	convex
	$\operatorname{tg} u_1 < \operatorname{tg} u_2$	-

The final shape of curve (2) is affected by as many as six parameters (tg α, tg u_1, tg u_2, C, D and t_E), which must satisfy the conditions referred to in the preceding subchapter. Strict observance of the minimum condition of function (18), which requires its derivatives to zero according to particular variables of parameters, leads to a system of non-linear equations. Due to this, looking for such an end-point that complies with a curve best adjusted to relief, is rather troublesome. Besides — as experiments presented in [5] as referred to the curve described in [3] imply, practically one should also take into account a case of no solution for a given relief. Thus, due to practical reasons it is proper to simplify the designing process, which most conveniently might be accomplished by means of reduction of the number of variables of parameters of curve (2).

To that end let us take into consideration for example dependence (12), which implies that

$$\operatorname{tg}\alpha = -\frac{2}{\frac{D_3}{D_1}+\frac{C_3}{C_1}}\operatorname{tg} u_1 + \frac{D+C}{\frac{D_3}{D_1}+\frac{C_3}{C_1}}\operatorname{tg} u_2 \tag{21}$$

which in turn makes it possible to write equation (2) as

$$y = x_K\left[\left(A_1 - A_3\frac{2}{\frac{D_3}{D_1}+\frac{C_3}{C_1}}\right)\operatorname{tg} u_1 + \left(A_2 + A_3\frac{D+C}{\frac{D_3}{D_1}+\frac{C_3}{C_1}}\right)\operatorname{tg} u_2\right] \tag{22}$$

or in a simplified way

$$y = x_K\left[W_1 \operatorname{tg} u_1 + W_2 \operatorname{tg} u_2\right] \tag{23}$$

where:

$$W_1 = A_1 - A_3 \frac{2}{\frac{D_3}{D_1} + \frac{C_3}{C_1}}$$

$$W_2 = A_2 + A_3 \frac{D+C}{\frac{D_3}{D_1} + \frac{C_3}{C_1}}$$

If for an optional point of the section as the prospective end of curve (2) a certain value of t_E is chosen from intervals given in Table 1, thus D_3, D_1, C_3 and C_1 having been determined, and if a certain value of $D + C$ from interval < –3; –4/3 > is assumed, the minimum condition of function (18) will depend merely on the zeroing of its derivatives according to angles u_1 and u_2.

If the direction of the tangent at starting point were not given, on the basis of the necessary conditions of the extreme of function (18) to exist and taking (23) into account, the following equations determining tg u_1 and tg u_2 would be obtained:

$$\operatorname{tg} u_1 = \frac{1}{x_K} \frac{\sum_{i=1}^{n} W_2^{(i)^2} \sum_{i=1}^{n} Y_i W_1^{(i)} - \sum_{i=1}^{n} W_1^{(i)} W_2^{(i)} \sum_{i=1}^{n} Y_i W_2^{(i)}}{\sum_{i=1}^{n} W_1^{(i)^2} \sum_{i=1}^{n} W_2^{(i)^2} - \left(\sum_{i=1}^{n} W_1^{(i)} W_2^{(i)} \right)^2} \tag{24}$$

$$\operatorname{tg} u_2 = \frac{1}{x_K} \frac{\sum_{i=1}^{n} W_1^{(i)^2} \sum_{i=1}^{n} Y_i W_2^{(i)} - \sum_{i=1}^{n} W_1^{(i)} W_2^{(i)} \sum_{i=1}^{n} Y_i W_1^{(i)}}{\sum_{i=1}^{n} W_1^{(i)^2} \sum_{i=1}^{n} W_2^{(i)^2} - \left(\sum_{i=1}^{n} W_1^{(i)} W_2^{(i)} \right)^2} \tag{25}$$

At a given direction of the tangent at starting point, i.e. once tg u_1 has been determined (angle u_1 should equal to inclination angle of the tangent (u_2) at the end-point of the preceding curve), the necessary condition of the extreme to exist leads to

$$\operatorname{tg} u_2 = \frac{\sum_{i=1}^{n} Y_i W_2^{(i)} - x_K \operatorname{tg} u_1 \sum_{i=1}^{n} W_1^{(i)} W_2^{(i)}}{x_K \sum_{i=1}^{n} W_2^{(i)^2}} \tag{26}$$

Values of tg u_1 and tg u_2 determined on the basis of the foregoing equations require an appropriate check. First of all they should not exceed the ruling gradients implied by the guidelines. Besides, in the context of the grade line having been adjusted to terrain, it is important that the required kind of arc (concave or convex) conditioned by relief complies with the kind of arc implied by values tg α, tg u_1 and tg u_2.

An optional choice of argument t_E and sum $D + C$ may touch off difficulties in satisfying those conditions. In the foregoing light a completely haphazard choice of te and $D + C$ is inadvisable whatsoever. Gradual searching through the ranges of t_E and $D + C$ is an alternative; this is meant for determination of the optimum values thereof, which considerably augments the cost of calculating work, it is more effective, though.

In the course of further designing process only those pairs of tg u_1 and tg u_2 should be taken into account which comply with given conditions. Next a pair will be chosen from among them for which the value of function (18) is smallest, and at the same time the radius of the curvature determined according to (10) complies with the guidelines. Once those procedures have been repeated for successive points of the longitudinal section assumed as prospective end-point, such point i will be looked for at which the value of expression (18) converted to the number of components is smallest. It becomes the end-point of the designed curve, and at the same time the starting point of a successive segment.

The designing process might be considerably simplified once dependences (12) and (13) have been applied. If there is no given direction of the tangent at starting point, equation (12) may be applied in the way described in chapter 2. Once prospective end of the curve has been assumed, therefore tg α, having been determined, it is necessary — bearing relief in mind — to assume certain tg u_1 and tg u_2, which — after the values of sum $D + C$ have been chosen for them from interval $< -3; -4/3 >$ — makes it possible to solve equation (12) according to t_E. Since in that case it is impossible to explicitly determine the optimum values of tg u_1 and tg u_2, it is advisable to repeat the whole series for different configurations of tg u_1 and tg u_2 taking into account each time at least a few possible values of sum $D + C$. Mind that while assuming the values of tg u_1 and tg u_2, one should take into account the kind of arc they imply (Table 3) as being compliant with the kind conditioned by relief. For further deliberations such a set of tg α, tg u_1 and tg u_2 and $D + C$ will qualify which complies with the minimum value of function (18), as well as the radius of the curvature implied by (10) not exceeding the admissible value. Such point i of the section will eventually become the end of current curve at which the value of expression (18) converted to the number of components is smallest.

At a given direction of the starting curve dependence (13) will be applied, which for given tg α, and tg u_1 and selected value of D/C makes it possible to determine t_E. While designating the interval that the wanted value of t_E should be included in, it is necessary to take into consideration the compliance of the required kind of arc conditioned by relief with the kind implied by mutual relationships between tg α, and tg u_1. According to Fig. 1, tg $\alpha >$ tg u_1 is the necessary condition for the arc to be concave, tg $\alpha <$ tg u_1 — arc to be convex. However, this is not the sufficient condition, this is why — after tg u_2 has been calculated by means of (16) — the actual kind of arc should be checked on the basis of Table 3. As it has been mentioned before, it is advisable to carry out calculations for different proportions of D/C from intervals given in Table 2, as well as different values of C. For given point i assumed as prospective end-point of curve (2), such values of D/C and C will be required for which the value of function (18) is smallest. Both tg u_2 and maximum radius of the curve calculated by means of (10) should not exceed admissible values. Such point i will become the end of current curve which minimum value of expression (18) converted to the number of components corresponds to.

Eventually, after a grade line for the entire length of a longitudinal section has been designed, coordinates of particular curves (2) determined within local systems as in Fig. l, should be converted to the superordinate system in which the longitudinal section is given.

Conclusion: in practice in it quite common that once the direction of starting tangent has been given, the simplified designing way with the use of dependence (13), due to its simplicity, should be regarded as a kind of standard if curves (2) are to be applied.

4. *Final remarks*

The foregoing solution in the form of modified so called general transition curves may provide a useful tool at designing the grade line of routes with regard to optimum adjustment to relief. Its practical application, due to the range of necessary calculations, is possible merely with the use of computer techniques, which is no problem at present whatsoever, as computer assistance to designing has become a standard at all engineering works. In case of road designing the optimum is if the designing process comes about upon the basis of the numerical model of a terrain strip, although as far as the grade line is concerned this is not necessary; it is only enough to specify the longitudinal section along determined route axis.

One more designing aspect, connected with traffic dynamics conditions, should be pointed out herein. The condition of certain actual values of changes of centripetal acceleration not to be exceeded is a significant factor affecting the safety and comfortableness of travelling. This, however, calls for a separate approach.

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Zmodyfikowane ogólne wielomianowe krzywe przejściowe w projektowaniu niwelety

S t r e s z c z e n i e

W pracy przedstawiono rodzinę zmodyfikowanych tzw. ogólnych wielomianowych krzywych przejściowych, wyznaczoną pod kątem jej wykorzystania w projektowaniu niwelety tras drogowych. Podano warunki projektowe tych krzywych oraz szczegółowe wskazówki dotyczące określania poszczególnych parametrów projektowych. Zaprezentowano również metody zastosowania tych krzywych w procesie projektowania niwelety, oparte na uzasadnionej ekonomicznie minimalizacji robót ziemnych.

Анджей Кобрынь

Модифицированные общие многочленные переходные кривые в определении проектной линии

Р е з ю м е

В работе представлено семейство модифицированных кривых, так называемых общих многочленных переходных кривых, которое определённое с точки зрения его использования в определении проектных линий дорожных маршрутов. Представлены проектные условия этих кривых, а также подробные указания относительно определения отдельных проектных параметров. Представлены тоже методы применения этих кривых в процессе определения проектной линии, основаны на экономически обоснованной минимализации земляных работ.