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THE GOLF-BALL MODEL AND THE PURPOSE OF DRUMLIN FORMATION

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Abstract

The purpose of drumlin formation is to facilitate glacier flow. Drumlins form in a deforming layer between ice and ground, they produce a pimpled ground surface which causes less drag in the flowing system, after the fashion of the Prandtl effect which reduces boundary layer detachment (as in the flying golf ball). This pimpled surface has self-organising properties and this causes the development of a low drag situation. The drumlin field is the critical phenomenon; the formation of individual drumlins is a small part of the overall effect.

Key words: Drumlin formation, sub-glacial deforming layer, golf-ball model, self-organisation, sand heap analogy, Turing instability.

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INTRODUCTION

This paper puts forward three related proposals: 1) that drumlins form in a deforming bed layer which separates glacier ice from the ground, 2) that something akin to the Prandtl effect is operating in a drumlin field and the pimpled landscape has the effect of reducing flow stresses at the glacier base and this promotes glacier flow and efficiency, and 3) the drumlin field organises itself to promote this thermodynamically desirable state of affairs; the drumlins form a flow-affecting system by some sort of self-organising process. The three proposals become more speculative with numerical progression, but they are linked in a developmental sequence.

It might be that the study of whole field phenomena is the next logical step in the investigation of drumlin forming processes and effects. It was a logical step to progress to a close look at the soil mechanics of individual drumlin formation (Smalley, Unwin 1968) and it might be logical to look at the whole field, and try to explain whole field characteristics. But at the moment it is necessarily a speculative look; in the sequence of investigative levels proposed by Smalley (1981) we are at the conjecture level. But the idea is attractive, it gives purpose to the drumlin field, which becomes more than just a residue of glacierization. Also we can begin to think about the Yatsu (1966 p.13) 'why' question; we have wrestled with the 'how' question since 1968, perhaps now we should tackle the 'why' question.

DEFORMING BED SYSTEMS

There is considerable support for the idea that drumlins form in the deforming layer that separates glacier ice from the ground proper. Eyles (1993) and Hart (1995) gave elegant illustrations of the concept (Fig. 1).

Within the deforming layer impediments to flow cause drumlins to form and the system adapts to flowing around the obstruction. Smalley and Unwin (1968) explored this idea, and proposed that obstructions might occur when the relatively close packing of till clasts causes increases in shear strength in the flow zone, and the ambient flow stresses are not sufficient to remobilise the obstruction. The dilatancy of the till material, the requirement to expand before remobilisation, made for stable obstructions. The best energy option for the flow system was to adjust the flow pattern rather than remobilise the till.

Within the deforming layer there will be many variations of stress, of strength, of water content, of pore pressure, of material nature (*i.e.* of mineralogy and particle size distribution *etc.*) but the critical property variation must remain within certain limits over wide geographic areas for large drumlin fields to form. Smalley and Piotrowski (1987) suggested that a key parameter was the stress/strength ratio (**R**). The truly critical parameter has to vary in such a way that limits to the field are established. This can be achieved if the **R** ratio is critical, it certainly establishes the upstream and downstream limits (Fig. 2).

THE GOLF-BALL HYPOTHESIS

This was tentatively outlined by Smalley and Warburton (1994). It relates to the observations made by Prandtl in 1914, on boundary layer separation. Prandtl observed the flow of a fluid past a smooth object and then introduced a flow interrupter near to the surface so that the boundary layer was disturbed. He observed that the flow was more efficient when the boundary layer was disturbed; in these conditions the

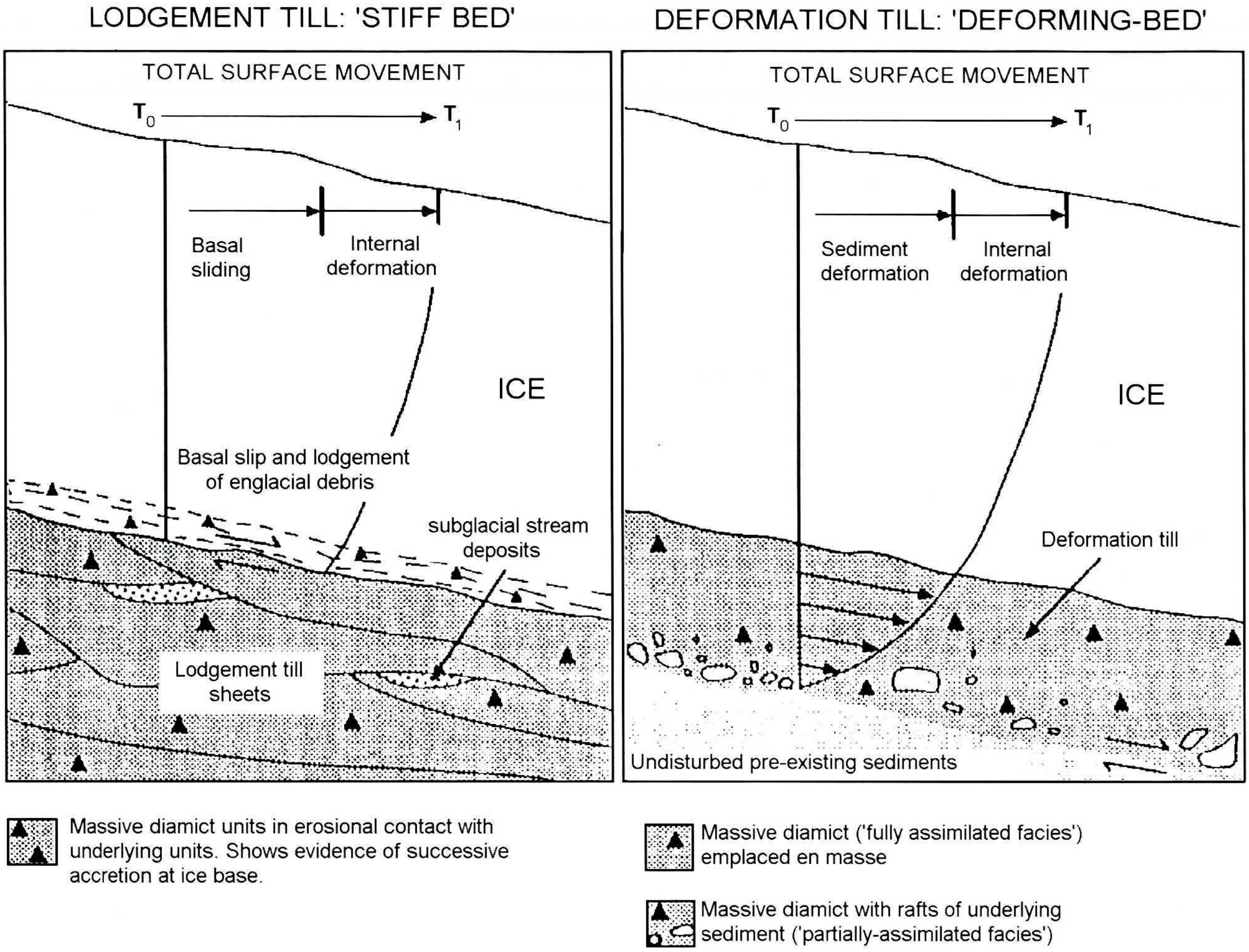

Fig. 1. The deforming bed system (after Eyles 1993). Lodgement till requires a still resistant bed; deformation till results from shearing of deformable substrate sediments or weak rocks.

boundary layer remained attached longer, the wake effect was decreased, and flow was more efficient. He was working with fluids of high Reynolds number, and we have to transfer his findings to a very low Reynolds number system- the flowing glacier and sub-glacial deforming zone. The classic illustration of the Prandtl effect is the modern golf ball in flight. The dimpled surface of the golf ball improves boundary layer attachment and thus reduces air resistance. The dimples golf ball flies further than a smooth ball when both are struck with equal force.

If the rough surface of a golf ball affects fluid flow then the rough ground surface of a drumlin field could have a similar effect. The glacier flowing over the drumlin field flows more efficiently than when it flows over a smooth till plain. The golf ball effect operates, Nature achieves efficiency.

SELF-ORGANISATION

The idea of self-organisation within a dynamic system appears to have been first proposed by Turing (1952) – as described by Coventry and Highfield (1990 p.188). Turing was interested in furnishing a chemical basis for the means by which shape, structure and function arise in living things, a process known as morphogenesis; stretching the meaning slightly we are interested in the morphogenesis of a drumlin field. The more recent interest in self-organising criticality (SOC) has developed largely from the paper by Bak *et al.* (1988), and it spread to geomorphology and concepts of landform organisation (Hallett 1990, Anon. 1994, Rodriguez-Iturbe *et al.* 1994). Bak *et al.* wanted to demonstrate that dynamical systems with extended spatial degrees of freedom in two or three dimensions naturally evolve into self-organised critical states. By self-organised they mean that the system naturally evolves to the state without detailed specification of the initial conditions. Moreover, the critical state is robust with respect to variations of parameters. They suggested that this self-organising criticality is the common underlying mechanism behind various natural phenomena.

They offered an attractive sand-heap model which has a certain resonance with landform studies. Build the heap by randomly adding sand, a grain at a time. The heap will grow and the slope will increase. Eventually, the slope will reach a critical value, called the angle of repose; if more sand is added it will slide off. Alternatively, if we start from a situation where the heap is too steep, the heap will collapse until it reaches the critical state, such that it is just barely stable with respect to future perturbations. Fig. 3 shows the Bak *et al.* (1988) one-dimensional sand heap of length N. The bound-

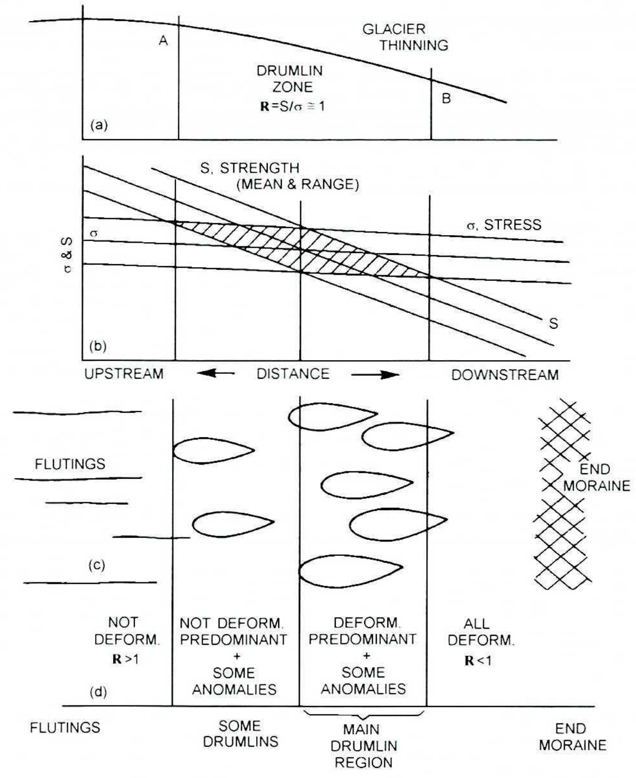

Fig. 2. General conditions for drumlin formation (after Smalley, Piotrowski 1987): (a) drumlin zone occurs towards the edge of the ice sheet, where the ice is thinning. (b) strength S is greater than stress far under the glacier but towards the ice margin the stress and strength curves cross. (c) drumlins are concentrated in the downstream part of the field. (d) drumlin formation is favoured in the 'deforming' part of the field.

ary conditions are such that sand can leave the system at the right hand side only. The numbers Z_n represent height differences $Z_n \equiv h(n) - h(n-1)$ between successive positions along the sand heap. The dynamics are very simple. From Fig. 3 it can be seen that sand is added at the n'th position by letting

$$Z_n \rightarrow Z_n + 1 \qquad (1)$$

When the height difference becomes higher than a fixed critical value Z_0 one unit of sand tumbles to a lower level, *i.e.*

$$Z_n \rightarrow Z_n - 2$$
$$Z_{n\pm1} \rightarrow Z_{n\pm1} + 1 \; for \; Z_n > Z_c \qquad (2)$$

Closed and open boundary conditions are used for the left and right boundaries respectively (see Fig. 3).

$$Z_0 = 0 \; for \; Z_N \rightarrow Z_{N0} - 1 \qquad (3)$$
$$Z_{N-1} \rightarrow Z_{N-1} + 1 \; for \; Z_N > Z_c$$

Equation 2 is a nonlinear discretized diffusion equation; it is nonlinear because of the threshold condition. The process continues until all of the Z_n are below threshold, at which point another sand grain is added, at a random site via equation 1. Bak *et al.* (1988) describe this model as a cellular automaton where the state of the discrete variable Z_n at time t + 1 depends on the state of the variable and its neighbours at time t.

We envisage a critical state existing in the sub-glacial environment in the drumlin forming region demarcated by Smalley and Piotrowski (1987), where **R** is around unity; the stress and strength parameters are about the same. Within this critical region self-organisation takes place as the drumlins form. The analogy has to be stretched a bit; the drumlin forming region moves into the critical state, similar to the critical sand heap state; the sand heap responds to perturbations by local collapses, the till system responds by local structural collapses into drumlin formation. Drumlin formation locally accelerates the flow rate and pushes the system away from criticality.

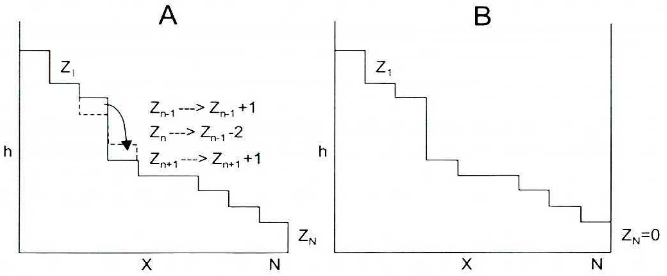

Fig. 3. The Bak *et al.* (1988) one-dimensional sand-heap model. The state of the system is specified by an array of integers representing the height difference between neighbouring plateaux: A – there is a wall at the left and sand can exit only at the right, B – there are walls at both ends.

DISCUSSION

There are various short definitions of SOC:

"This idea …propounds that complicated interactive systems can evolve towards a wobbly condition in which the slightest disturbance may elicit a major disaster. The pedagogic example typically invoked is the building of a sandpile grain by grain. One the pile rises to a certain height- it avalanches." (Anon. 1994)

"…under the pressure of outside stimuli, a system of many complicated interacting parts – anything from a sandpile to the stockmarket – will organise into a precarious state, far from stability, that is prone to unpredictable fluctuations. In the case of a sandpile, the theory predicts that as sand is added to the pile, it will slough off not in regular catastrophic avalanches but in an unpredictable series of small and large ones." (Glanz 1995)

The deforming layer is certainly a system of many complicated interacting parts, so the concept should apply. It is also at, or near, an interesting form of equilibrium, from which it could depart into drumlin formation. The sand-heap model is important because it has already been suggested that landscape erosion is analogous to the sand avalanches (Rodriguez-Iturbe *et al.* 1994, Anon. 1994) and this suggests some link between SOC and drumlin field formation. Glanz (1995) has hinted that the sand-heap model may not be as perceptive as generally believed, but it is the most widely discussed of the SOC models.

If the sub-glacial region was organising itself to reduce energy requirements it would attempt to reduce the flow stress required to ensure glacier movement. It is reasonable to assume a critical state in the deforming layer – in fact it will be akin to the critical state described in soil mechanics by Schofield and Wroth (1969). It is essentially an unstable state in an open system. With SOC operating a landform event occurs which reduces required flow stress, the drumlin field forms. A 'Turing instability' develops which precipitates the system change – which could be from a non-drumlin state to a drumlinized state. A Turing instability is defined as the point where the system is driven far enough from equilibrium for a patterned response to appear (see Coventry and Highfield 1990, p. 190). We are probably using the term in an eccentric way but a term which indicates the transformation of a critical unstable state system into a patterned system is attractive in terms of drumlin field formation. Drumlin field formation is an attempt to sustain the flowing critical state for as long as possible; the ground system organises to move the glacier as far as possible.

CONJECTURES

Drumlins form in the deforming layer when that layer reaches a certain critical state, and most drumlins consist of collapsed till structures. The developing flow pattern in the drumlin field can also promote local erosion and erosional drumlins could also be formed, to participate in shaping the overall field flow structure. To force some collapsed till into motion a large shear stress is required and this must cause the expansion of the close-packed mass – it must expand to the critical flowing state. In simple packing model terms an expansion from 204 to 600 is required (see appendix). As the system expands to 600 it reaches the dilatancy limit and further expansion of the granular part pushes the deformation style out of solid mechanics and into fluid mechanics. The 204 to 600 expansion involves a change in packing density from 0.74 to 0.52; a difficult change to accomplish in the sub-glacial environment.

The concept of dilatancy onset was discussed by Onoda & Liniger (1990, see Dijkstra *et al.* 1995, Dijkstra 2000, p. 95). It is a useful concept demarcating the transition from fluid to granular behaviour. It was designed to apply to systems at extreme dilution, not perhaps the situation to be found in the deforming layer beneath a substantial glacier. But the deforming layer is a minimum density region which cannot become less dense; it is at a critical density to promote glacial flow and motion.

The actual growth of the field can be seen as a nucleation and growth process. Actual observations are impossible so a thought experiment must suffice. The critical state becomes more difficult to sustain, one drumlin forms in the deforming layer; it is a close-packed 600 to 204 drumlin, made of glacial till. The stress pattern around this drumlin changes, it creates a disturbance in the flow system. Adjacent to the drumlin there is high pressure and fast flow but at some critical distance away the pressure drops, the flow speed falls, the system packs into another drumlin. If a flow pattern is imposed on the deforming layer the self-organising characteristics become more developed; flow stress distribution can cause erosion of pre-existing ground to produce non-depositional drumlins. If each drumlin is surrounded by a defining stress field then the drumlin arrangement could be seen as a relatively simple two-dimensional packing; simple landscape ordering rules (see Smalley 1996, Aste, Weire 2000, p. 97) could apply – but in fact the flow system is so complex that a minimum of order will be observed in a real drumlin field.

If drumlin growth is initiated by a whole series of Turing instabilities then the whole field will consist of a set of subfields which will meet and merge in a fairly chaotic manner. When drumlin formation can no longer cause sufficient flow modification the system in effect shuts down. Once past, and fairly close to, the downstream field limit end moraine material starts to be deposited.

The conjectural model requires the drumlin field to form to allow glaciers to continue to flow when the flow causing parameters are dropping below the necessary critical levels. The ground/interface/glacier system organises a flow-promoting ground geometry, which we call a drumlin field.

APPENDIX: PARTICLE PACKING TERMINOLOGY

There is a large gap between the idea of packings and the reality of granular geo-materials. But some simple models are developing; Dijkstra *et al.* (1995) and Dijkstra (2000) have looked at problems of packing transition and their representation. An interesting and possibly relevant packing transition is the 600 to 204 transition. This goes from the most open simple packing to the most compact simple packing. The 600 packing is the simple cubic packing; 204 is the so-called rhombohedral packing. The symbols indicate the nature of the unit cell which defines the packing; the 204 packing has 2 square A sides, no intermediate B sides, and 4 rhombus C sides, hence 204. Three cell side types are defined. The simple cubic 600 cell has six square A sides. It is a rather makeshift system; see Allen (1982, pp. 137–178) for a more detailed explanation. The 600 to 204 transition models, in a very crude way, soil structure collapse in a collapsing soil, and has thus received some attention in soil mechanics and engineering geology. It might be useful to study clast interaction and drumlin formation from a particle packing viewpoint.

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