



Research paper

Assessment of stresses in reinforcement in the area of joints in composite steel-concrete slabs

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Abstract: The article draws attention to certain aspects of calculating the width of cracks and stresses in composite elements under bending, in which the slab is located in the tension zone. If semi-rigid joints are used in the element, in which the beam is attached to the column by bolts, two types of areas should be distinct in which the reinforcement stresses will be calculated in a different way. The method of calculating stresses in reinforcement will depend on the type of a used joint or on the distance of the considered cross-section from the semi-rigid joint. In order to distinguish the method of calculating stresses in the paper, two areas were introduced: specifically area B and area D. Area B will be the area where the principle of flat sections can be applied, and stresses in the reinforcement are determined using the classical theory by adding the component responsible for the tension stiffening phenomenon. Area D is the area in the vicinity of the semi-rigid joint, where the principle of flat sections cannot be applied. To calculate stresses, consider the balance of joints using the available models of the semi-rigid joint, in particular the spring model. The paper presents the formulas for calculating stresses in the D area for two types semi-rigid joints: joint with a flush end-plate with 2 rows of bolts are used and joint with an extended end-plate with 3 rows of bolts are used.

Keywords: steel-concrete composite structures, composite joints, semi-rigid joints, tension stiffening

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1. Introduction

In a composite steel and concrete structure, in which the concrete is in the tensile zone, the cracks can appear after exceeding tensile strength of concrete. The width of the cracks depends on direct loads (self-weight of components, dead loads, live loads) and indirect loads (shrinkage, temperature changes during hardening of concrete, changes in ambient temperature in the process of construction and use of the structure, subsidence of supports). Occasionally, it can be necessary to check the width of cracks. As it is stated in standard [13], the designers should follow provisions and methods of calculation of the width of cracks in accordance to standard [11].

2. Calculation of the crack width according to the standard PN-EN 1992-1-1

The model used in the theory of cracking is an element subjected to axial tension, which can also be regarded as a model of tension zones in elements subjected to bending and eccentrically loaded elements with a tension zone [5,10]. This theory applies to cracks that appear on a section loaded with a constant bending and it is imperative when the elongation length considered in the study is sufficiently long and the bending moment does not change too quickly on this section.

According to standard [11] the width of cracks is calculated as:

$$(2.1) \quad w_k = s_{r,\max} (\varepsilon_{sm} - \varepsilon_{cm})$$

where: $s_{r,\max}$ is the maximum crack spacing, ε_{sm} is the mean strain in the reinforcement under the relevant combination of loads, including the effect of imposed deformations and taking into account the effects of tension stiffening (only the additional tensile strain beyond the state of a zero strain of the concrete at the same level is considered), ε_{cm} is the mean strain in the concrete between cracks.

The difference in strains is expressed by the formula:

$$(2.2) \quad \varepsilon_{sm} - \varepsilon_{cm} = \max \left\{ \begin{array}{l} \frac{\sigma_s - k_t \frac{f_{ct,\text{eff}}}{\rho_{p,\text{eff}}} (1 + \alpha_e \rho_{p,\text{eff}})}{E_s} \\ (1 - k_t) \frac{\sigma_s}{E_s} \end{array} \right.$$

where: σ_s is a stress in the reinforcing bars, k_t is a coefficient depending on the duration of the load, $k_t = 0.4$ for long term loading and $k_t = 0.6$ for short term loading, $\rho_{p,\text{eff}}$ is the ratio of area of tension reinforcement to the effective tension area of concrete around the reinforcement, α_e is the ratio of the modulus of elasticity of reinforcement to the tangent modulus of elasticity of concrete, a E_s is the modulus of elasticity of reinforcement.

The maximum final spacing is calculated depending on the spacing of the applied reinforcement from the formulas:

- if in the tensile zone the spacing of reinforcement having an adhesion to concrete is not greater than $5(c + \varphi/2)$

$$(2.3) \quad s_{r,\max} = k_3c + k_1k_2k_4 \frac{\phi}{\rho_{p,\text{eff}}}$$

- if in the tensile zone the spacing of reinforcement having an adhesion to concrete is greater than $5(c + \varphi/2)$ or if there is no bonded reinforcement within the tension zone

$$(2.4) \quad s_{r,\max} = 1.3(h - x)$$

where: $k_3 = 3.4$ (recommended value), c – clear cover to the longitudinal reinforcement, k_1 – coefficient which takes account of the bond properties of the bonded reinforcement ($k_1 = 0.8$ for high bond bars, $k_1 = 1.6$ for bars with an effectively plain surface), k_2 – coefficient which is dependent on of the distribution strain: $k_2 = 0.5$ for bending, $k_2 = 1.0$ for pure tension, for cases of eccentric tension or for local areas values should be used which can be calculated from the relation (2.5), $k_4 = 0.425$ (recommended value), ϕ – diameter of the used bars, $\rho_{p,\text{eff}}$ – effective reinforcement ratio for longitudinal reinforcement equal to $\rho_{p,\text{eff}} = \frac{A_s}{A_{c,\text{eff}}}$, A_s – cross sectional area of the applied reinforcement, $A_{c,\text{eff}}$ – effective tensile area surrounding the tensile reinforcement, determined in accordance with [11], h – depth of a cross section, x – depth of concrete in compression.

In the case of an eccentric tensile, the k_2 coefficient is calculated from the formula:

$$(2.5) \quad k_2 = \frac{\varepsilon_1 + \varepsilon_2}{2\varepsilon_2}$$

where: $\varepsilon_1, \varepsilon_2$ shown in Fig. 1.

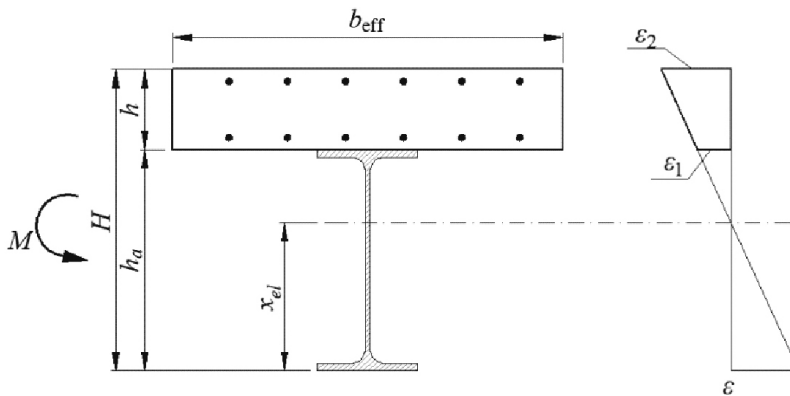


Fig. 1. Distribution of strains in a steel-concrete composite cross-section with a negative bending moment

Eccentric stretching of the concrete slab will occur in a steel-concrete composite beam loaded with a negative moment (Fig. 1). Then, the formula (2.5) can be changed to the form:

$$(2.6) \quad k_2 = 1 - \frac{h}{2(H - x_{el})}$$

When calculating the cracks width in reinforced concrete slabs, the bending plate model and the stresses in the reinforcement can be determined from the classical equations known from the theory of strength of materials. In steel-concrete composite beams, if the tensile slab is not pre-stressed by tendons, it is necessary to take into account the increase of stresses in the reinforcement, in relation to the stresses calculated excluding tensile concrete (“tension stiffening”) [13]. Then, the tensile stresses in the reinforcement from direct loads can be calculated from the formula:

$$(2.7) \quad \sigma_s = \sigma_{s,0} + \Delta\sigma_s$$

and

$$(2.8) \quad \Delta\sigma_s = \frac{k_t f_{ctm}}{\alpha_{st} \rho_s}, \quad \alpha_{st} = \frac{AJ}{A_a J_a}, \quad \rho_s = \frac{A_s}{A_{ct}}$$

where: $\sigma_{s,0}$ – the stress in the reinforcement caused by the internal forces acting on the composite section, calculated neglecting concrete in tension, $\Delta\sigma_s$ – increase of stress in steel reinforcement due to tension stiffening of concrete, k_t – coefficient as in the formula (2.2), f_{ctm} – the mean tensile strength of the concrete, A, J – area and second moment of area, respectively (relates to the effective composite section, neglecting concrete in tension and profiled sheeting), if any, A_a, J_a – the corresponding properties of the structural steel section, A_s – the total area of all layers of longitudinal reinforcement within the effective area A_{ct} , A_{ct} – the effective area of the concrete flange within the tensile zone.

Table 1 presents $\Delta\sigma_s$ values calculated assuming that the steel-concrete composite beam is an IPE 200 or IPE 400 profile made of steel with a yield strength of $f_y = 235$ MPa and $E_s = 210$ GPa, reinforced concrete slab with a thickness of 150 mm and an effective width of 1000 mm made of concrete $f_{ctm} = 2.56$ MPa and $E_{cm} = 31.5$ GPa. The slab is reinforced with steel bars $f_{sk} = 500$ MPa and $E_s = 210$ GPa with the area in Table 1 (Fig. 2).

Table 1. Increase of stresses in steel reinforcement $\Delta\sigma_s$ due to tension stiffening of concrete

Slab	Cross-sectional area of the structural steel section and minimum of reinforcement	Slab reinforcement		
		$A_{s,min}$	7 ϕ 12	7 ϕ 20
			$A_{s1} = 7.92 \text{ cm}^2$	$A_{s1} = 21.22 \text{ cm}^2$
Increase of stresses in steel reinforcement $\Delta\sigma_s$				
RC solid slab	IPE 200, $A_{s,min} = 6.02 \text{ cm}^2$	94.0 MPa	59.6 MPa	9.6 MPa
	IPE 400, $A_{s,min} = 7.56 \text{ cm}^2$	142.7 MPa	134.3 MPa	31.3 MPa
Composite slab with profiled steel sheeting	IPE 200, $A_{s,min} = 4.43 \text{ cm}^2$	94.0 MPa	36.6 MPa	5.9 MPa
	IPE 400, $A_{s,min} = 5.18 \text{ cm}^2$	140.9 MPa	82.4 MPa	19.2 MPa

The slab was adopted in two options – as a solid slab or as a composite slab with a 58 mm profiled steel sheeting height (neglecting profiled sheeting) on the ribs of the sheets perpendicular to the beam. The minimum reinforcement amount was calculated in accordance with section 5.5.1 (5) of standard [13].

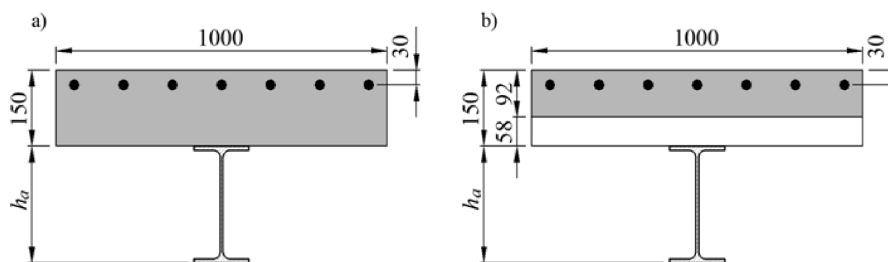


Fig. 2. Two different options for considered sections: a) RC solid slab, b) composite slab with profiled steel sheeting

The increase of stresses in steel reinforcement $\Delta\sigma_s$ due to tension stiffening of concrete can be significant, especially in elements with a low degree of reinforcement. The derivation of the formula (2.7) can be found in [6].

3. Calculating stresses in reinforcement

3.1. Type B and D areas

In the steel-concrete composite structures, the beams can be supported on the columns of the last floor (Fig. 3a), or the beams can be connected to the column of intermediate floor by a rigid joint (Fig. 3b) or a semi-rigid joint (Fig. 3c). The method of calculating stresses in reinforcement will depend on the type of used joint or on the distance of the considered section from the semi-rigid joint. In order to distinguish the method of calculating stresses for the purpose of this paper, two areas can be distinguished, specifically: area B (refers to “Bernoulli”) and area D (refers to “discontinuity”).

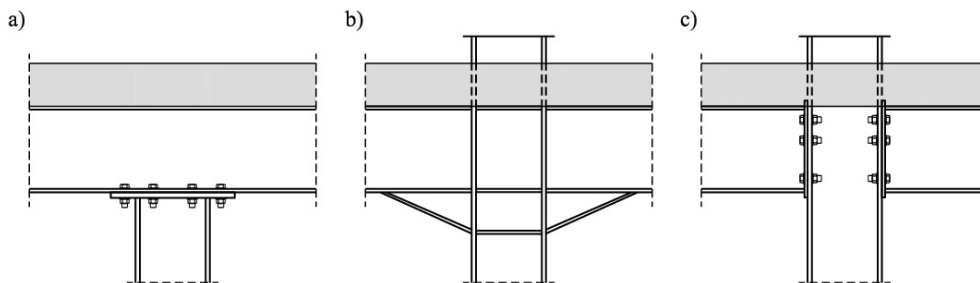


Fig. 3. Sample joints in composite structures (described in text)

Area B will be the area where the principle of flat sections can be applied, and stresses in the reinforcement are determined using the classical theory by adding the component described by the formula (2.7) responsible for the tension stiffening phenomenon. This area will be located on the entire length above the column, when using a continuous beam above the support of

the last floor (Fig. 3a) or the rigid joint (Fig. 3b) and some distance from the semi-rigid joint (Fig. 3c).

Area D is the area in the vicinity of the semi-rigid joint (Fig. 3c), where the principle of flat sections cannot be applied. To calculate stresses, consider the balance of joints using the available models of the semi-rigid joint, in particular the spring model [1, 2, 4, 6, 8, 11, 12].

3.2. The B area

The stresses in the reinforcement can be calculated on the basis of the classical theory (the principle of flat sections can be applied). It is assumed that the cracked concrete element does not transfer tensile loads, and the stresses in the reinforcement are calculated taking into account the phenomenon of “tension stiffening”.

3.3. The D area

The formulas for calculating stresses in reinforcement are provided for two cases, when a semi-rigid joint with a flush end-plate and two rows of bolts (Fig. 4) and a semi-rigid joint with an extended end-plate and three rows of bolts (Fig. 5).

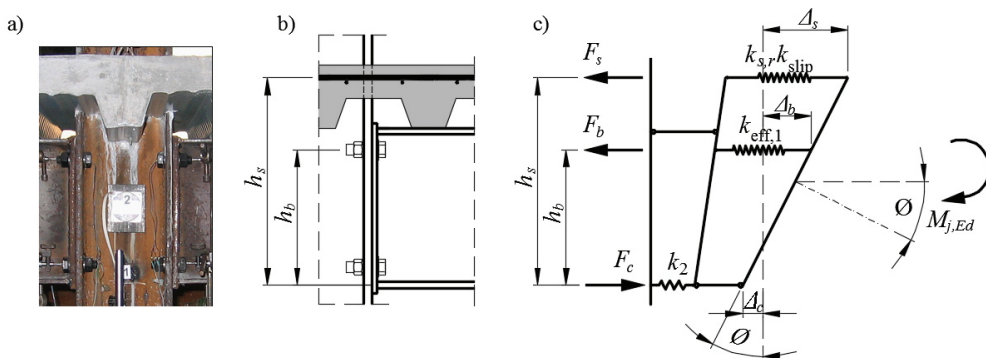


Fig. 4. Semi-rigid joint with a flush end-plate and with a composite slab with profiled steel sheeting a) view on the test stand, b) scheme c) spring model based on [1, 5]

When analyzing a joint, it is assumed that it consists of a specified number of parts (components). Its elements (a column web, bolts or reinforcement bars) are represented by individual springs k_i . The characteristics of these springs are determined by means of stiffness coefficients, which are the basis for determining the stiffness of the joint using the component method [8, 9]. The stiffness coefficients, k_i , are added according to the rules given in the standard [12]. As a result of external loads, individual springs become deformed and the joint rotates by an angle, Φ . When using the component method, the active parts of the joint should be identified first, then their features should be identified and in the last stage individual components should be assembled to obtain the joint characteristics [3, 7].

For a joint with a flush end-plate (see Fig. 4), the equation of force equilibrium in the elastic state, without taking into account the slip between the slab and the beam, is defined as follows:

$$(3.1) \quad F_s + F_b - F_c = 0$$

The displacements can be expressed in equation:

$$(3.2) \quad \frac{\Delta_s + \Delta_c}{h_s} = \frac{\Delta_b + \Delta_c}{h_b} = \phi$$

From Hooke's law, dependencies can be obtained:

$$(3.3) \quad F_s = k_{s,r} \Delta_s E = K_s \Delta_s, \quad F_c = k_2 \Delta_c E = K_c \Delta_c, \quad F_b = k_{\text{eff},1} \Delta_b E = K_b \Delta_b$$

Substituting (3.2) to (3.3) it is found that:

$$(3.4) \quad \frac{F_s}{K_s} + \frac{F_c}{K_c} = \phi h_s$$

$$(3.5) \quad \frac{F_b}{K_b} + \frac{F_c}{K_c} = \phi h_b$$

Finally, three equations can be obtained, as follows:

$$(3.6) \quad \begin{cases} F_s + F_b - F_c = 0 \\ \frac{F_s}{K_s} + \frac{F_c}{K_c} = \phi h_s \\ \frac{F_b}{K_b} + \frac{F_c}{K_c} = \phi h_b \end{cases}$$

According to the provisions of the standard [13], the influence of slip between the concrete slab and the beam caused by the deformability of the connectors can be taken into account by multiplying the stiffness coefficient of reinforcement $k_{s,r}$ by the k_{slip} factor. Finally, an expression describing the force in tensile reinforcement is defined as:

$$(3.7) \quad F_s = \frac{k_{s,r} k_{\text{slip}} (k_{\text{eff},1} h_s + k_2 h_s - k_{\text{eff},1} h_b)}{k_{\text{eff},1} + k_2 + k_{s,r} k_{\text{slip}}} \phi E$$

Figure 5 shows the view on the test stand and the scheme of joints with the extended end-plate and three rows of bolts.

Then,

$$(3.8) \quad F_s + F_{b1} + F_{b2} - F_c = 0$$

$$(3.9) \quad \frac{\Delta_s + \Delta_c}{h_s} = \frac{\Delta_{b1} + \Delta_c}{h_{b1}} = \frac{\Delta_{b2} + \Delta_c}{h_{b2}} = \phi$$

$$(3.10) \quad \begin{aligned} F_s &= k_{s,r} \Delta_s E = K_s \Delta_s & F_c &= k_2 \Delta_c E = K_c \Delta_c \\ [3pt] F_{b1} &= k_{\text{eff},1} \Delta_{b1} E = K_b \Delta_{b1} & F_{b2} &= k_{\text{eff},2} \Delta_{b2} E = K_b \Delta_{b2} \end{aligned}$$

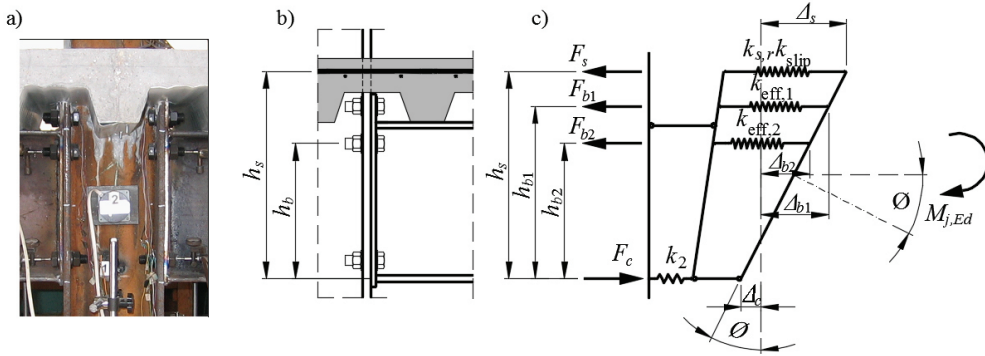


Fig. 5. Semi-rigid joint with an extended end-plate and with a composite slab with profiled steel sheeting:
 a) view on the test stand, b) scheme c) spring model based on [12, 13]

$$(3.11) \quad \begin{cases} F_s + F_{b1} + F_{b2} - F_c = 0 \\ \frac{F_s}{K_s} + \frac{F_c}{K_c} = \phi h_s \\ \frac{F_{b1}}{K_{b1}} + \frac{F_c}{K_c} = \phi h_{b1} \\ \frac{F_{b2}}{K_{b2}} + \frac{F_c}{K_c} = \phi h_{b2} \end{cases}$$

and the expression describing the force in tensile reinforcement is defined as:

$$(3.12) \quad F_s = \left[k_{s,r} k_{slip} h_s - \frac{(k_{s,r} k_{slip})^2 h_s + k_{s,r} k_{slip} k_{eff,1} h_{b1} + k_{s,r} k_{slip} k_{eff,2} h_{b2}}{k_2 + k_{s,r} k_{slip} + k_{eff,1} + k_{eff,2}} \right] \phi E$$

In the formulas (3.3), (3.7), (3.10), (3.12) the stiffness coefficients of individual springs are expressed in the formulas given in [12, 13] and the stiffness coefficients \$k_{eff,1}\$ and \$k_{eff,2}\$ are obtained from the stiffness coefficient individual springs using the formulas in [12].

After taking into account that the initial stiffness of the joint can be expressed by the formula:

$$(3.13) \quad S_{j,ini} = \frac{M_{j,Ed}}{\phi} \quad \rightarrow \quad \phi = \frac{M_{j,Ed}}{S_{j,ini}}$$

Taking into account (3.13), formulas (3.7) and (3.12) can be written:

$$(3.14) \quad F_s = C_s \frac{M_{j,Ed} E}{S_{j,ini}}$$

$$(3.15) \quad \sigma_{s,0} = C_s \frac{M_{j,Ed} E}{A_s S_{j,ini}}$$

In formulas (3.14) and (3.15) \$C_s\$ is described by the expression

- for the semi-rigid joint with the flush end-plate and two rows of bolts (Fig. 4)

$$(3.16) \quad C_s = \frac{k_{s,r} k_{\text{slip}} (k_{\text{eff},1} h_s + k_2 h_s - k_{\text{eff},1} h_b)}{k_{\text{eff},1} + k_2 + k_{s,r} k_{\text{slip}}}$$

- for the semi-rigid joint with the extended end-plate and three rows of bolts (Fig. 5)

$$(3.17) \quad C_s = k_{s,r} k_{\text{slip}} h_s - \frac{(k_{s,r} k_{\text{slip}})^2 h_s + k_{s,r} k_{\text{slip}} k_{\text{eff},1} h_{b1} + k_{s,r} k_{\text{slip}} k_{\text{eff},2} h_{b2}}{k_2 + k_{s,r} k_{\text{slip}} + k_{\text{eff},1} + k_{\text{eff},2}}$$

For stresses calculated from formula (3.15), the component $\Delta\sigma_s$ in the formula (2.8) should be added.

According to the standards, when considering the limit state of cracks, characteristic and long-time loads are taken into account.

3.4. Calculation example

In construction of buildings, the ratio of the characteristic load to long-time load may vary within wide limits depending on such factors as the weight of elements or the size of the variable load and the splitting of long-time loads in the variable load. In realistic case for correctly designed structures, the internal forces relating to the long-term characteristic load can be from 40% to 80% of the load capacity of the design element. The stresses calculations in the tensile reinforcement of the joint using different design solutions of the joint are given below.

A solid slab with a thickness of 15 cm and an effective width of 100 cm reinforced with 8 bars with a diameter of 16 mm made of steel of $f_{sk} = 500$ MPa was used. The slab is attached to a steel beam made of the IPE 300 with $f_{yk} = 235$ MPa with 19 mm diameter and 120 mm high headed studs. It was assumed that in every case the cracking moment was exceeded. Stresses in the reinforcement were calculated using the formulas (2.7) and (2.8).

The first of the considered joints (Fig. 6a) is a rigid connection corresponding to area B, and stresses from external loads σ_s were calculated as for the cracking section.

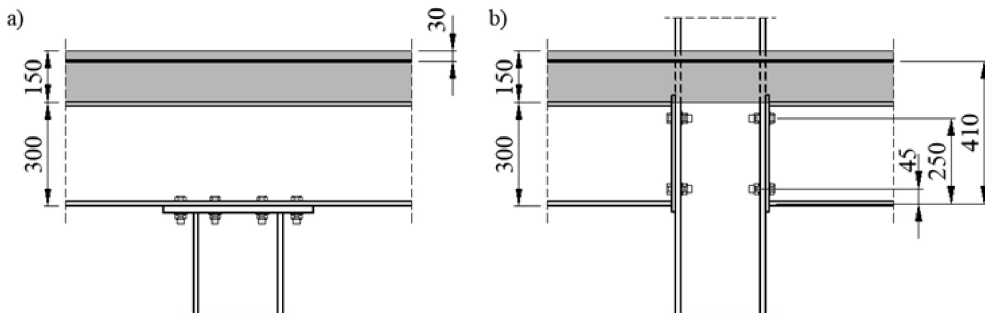


Fig. 6. Schemes of analyzed joints

In the second case (Fig. 6b) the joint equilibrium was considered, and the stresses in the reinforcement was calculated using the formula (3.15). The calculations were carried out

with the assumption that the beam is connected to a column made of the HEB200 section of $f_{yk} = 235$ MPa with the flush end-plate and 4 M20 bolts. Two variants of plate thickness were assumed, in particular: 10 mm and 20 mm. The initial stiffness of the joint, $S_{j,ini}$, and stiffness coefficients of the components of the joint were determined from the formulas given in [12, 13] assuming that at the beam length in the zone of the negative bending moment, there are 15 shear headed studs connecting the concrete slab and the steel beam. The stiffness of a single headed stud is, 100 kN/mm, according to the recommendations of [13].

The calculation results, for a bending moment equal to 200 kN · m (close to the limit value in the Serviceability Limit States (SLS)) are given in Table 2.

Table 2. Stresses values in reinforcement for three types of joint at the bending moment $M_{Ed} = 200$ kN · m

Joint	$\sigma_{s,0}$	$\Delta\sigma_s$	σ_s
	MPa	MPa	MPa
Fig. 6a (the B area)	239.1	35.3	274.4
Fig. 6b (the D area), plate thickness 10 mm	271.1		306.4
Fig. 6b (the D area), plate thickness 20 mm	256.4		291.7

4. Conclusive remarks

The method of calculating stresses in reinforcement will depend on the type of a used joint or on the distance of the considered cross-section from the semi-rigid joint. In order to distinguish the method of calculating stresses in the paper, two areas were introduced: area B and area D, which are described in Section 3.1. In the direct area of a joint, there is the D-type area (relates to “discontinuity”). The stresses in the reinforcement can be determined using, for example, the equation of the joint equilibrium. The joint model can be adopted using the component method presented in [12], or other models described in the literature. The paper presents the derivation of formulas for stresses in reinforcement for two types of joints shown in Figures 4 and 5 using the standard joint model (joint with a flush end-plate with 2 rows of bolts and joint with an extended end-plate with 3 rows of bolt). According to the standard [13], for the calculated stresses, the component $\Delta\sigma_s$ (2.8) should be added. Tensile stresses in reinforcement can be calculated based on the classical theory including tension stiffening phenomenon when using a continuous beam above the support of the last floor (Fig. 3a) or the rigid joint (Fig. 3b) and in some distance from the semi-rigid joint (Fig. 3c). It should be noted that the effect of tension stiffening can play a significant role – especially in the case, when a very small amount of tensile reinforcement for the slab is used (as shown in Table 1). In the example shown in Table 1 for the IPE 400 section and the reinforcement equal to the minimum reinforcement, the increase in stresses caused by the tension stiffening phenomenon was over 140 MPa. The stress increase in the steel reinforcement due tension stiffening phenomenon is directly proportional to the mean tensile strength of the concrete. In addition, when using high tensile strength concrete a significant increase in the stresses should be expected, which is caused by this phenomenon.

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Ocena naprężeń w zbrojeniu w sąsiedztwie węzłów w płytach zespolonych stalowo-betonowych

Słowa kluczowe: konstrukcje zespolone stalowo-betonowe, węzły zespolone, węzły podatne, tension stiffening

Streszczenie:

W artykule zwrócono uwagę na pewne aspekty obliczania szerokości rys i naprężeń w zginanych elementach zespolonych w których płyta znajduje się w strefie rozciąganej. W przypadku zastosowania połączeń podatnych w elemencie, w którym belka mocowana jest do słupa za pomocą śrub, należy

wyróżnić dwa rodzaje obszarów, w których inaczej obliczane będą naprężenia w zbrojeniu. Sposób ich obliczania będzie zależał od rodzaju zastosowanego połączenia lub od odległości rozpatrywanego przekroju od węzła podatnego. W celu wyróżnienia metody obliczania naprężeń w artykule wprowadzono dwa obszary: obszar B i obszar D. Obszar B będzie takim obszarem, w którym można zastosować zasadę płaskich przekrojów, a naprężenia w zbrojeniu wyznacza się korzystając z klasycznej teorii dodając składnik odpowiedzialny za zjawisko tension stiffening. Obszar taki znajdować się będzie na całej długości nad słupem, gdy zastosowano przegubowe oparcie belki ostatniej kondygnacji na słupie (Rys. 3a) czy węzeł sztywny (Rys. 3b) oraz w pewnej odległości od węzła podatnego (Rys. 3c). Obszar D jest to obszar w bezpośrednim sąsiedztwie węzła podatnego (Rys. 3c), gdzie zasada płaskich przekrojów nie ma zastosowania. Aby obliczyć naprężenia należy rozważyć równowagę węzła korzystając z dostępnych modeli węzła podatnego. W artykule przedstawiono wzory do obliczania naprężeń w obszarze D dla dwóch typów węzłów podatnych: gdy blacha jest zlicowana z górną powierzchnią belki i zastosowano 2 rzędy śrub oraz gdy blacha jest wypuszczona powyżej górnej powierzchni belki i zastosowano 3 rzędy śrub. W tabelicy 2 zamieszczono wyniki obliczeń, dla momentu zginającego równego $200 \text{ kN} \cdot \text{m}$ (zblżonego do wartości granicznej w SGU). Obliczenia przeprowadzono przy założeniu, że belka połączona jest ze słupem wykonanym z kształtownika HEB200 $f_{yk} = 235 \text{ MPa}$ za pomocą blachy zlicowanej i 4 śrub M20. Przyjęto dwa warianty grubości blachy: 10 mm i 20 mm. Sztywność początkową węzła $S_{j,ini}$ oraz współczynniki sztywności części składowych węzła wyznaczono ze wzorów zamieszczonych w [12, 13] przy założeniu, że na długości belki w strefie ujemnego momentu zginającego znajduje się 15 sworzni łączących płytę betonową z belką, a sztywność pojedynczego sworznia wynosi, zgodnie z zaleceniami normy [13], 100 kN/mm .

Received: 2021-04-11, Revised: 2021-06-08