10.24425/acs.2022.141717

*Archives of Control Sciences*
*Volume* 32(LXVIII), 2022
*No.* 2, *pages 383–408*

# Fuzzy state feedback with double integrator and anti-windup for the Van de Vusse reaction

C.A. MÁRQUEZ-VERA, M.A. MÁRQUEZ-VERA, Z. YAKOUB, A. MA'ARIF, A.J. CASTRO-MONTOYA and N.R. CÁZAREZ-CASTRO

Chemical processes use to be non-minimum phase systems. Thereby, they are a challenge for control applications. In this paper, fuzzy state feedback is applied in the Van de Vusse reaction that has an inverse response. The control design has an integrator to enhance the control performance by eliminating the steady-state error when a step reference is applied. An anti-windup action is used to reduce the undershoot in the system response. In practice, it is not possible to have always access to all the state variables. Thus, a fuzzy state observer is implemented via LMIs. Frequently, the papers that show similar applications present some comments about disturbance rejection. To eliminate the steady-state error when a ramp reference is used, in this work, a second integrator is aggregated. Now, the anti-windup also reduces the overshoot generated due to the usage of two integrators in the final application.

**Key words:** fuzzy state feedback, LMIs, Van de Vusse reaction, double integrator, anti-windup



C.A. Márquez-Vera (e-mail: carmarquez@uv.mx) is with Universidad Veracruzana, Prolongación Venustiano Carranza S/N, Col. Revolución, Poza Rica 93390, Veracruz, Mexico.

M.A. Màrquez-Vera (corresponding author, e-mail: marquez@upp.edu.mx) is with Polytechnic University of Pachuca, C. Pachuca-Cd. Sahagún Km 20, Ex-Hacienda de Santa Bárbara, Zempoala 43830, Hgo., Mexico.

Z. Yakoub (e-mail: yakoubzaineb@yahoo.fr) is with University of Gabès, National Engineering School of Gabès, Rue Omar Ibn El Khattab, Zrig Eddakhlania, Gabès 6029, Tunisia.

A. Ma'arif (e-mail: alfian.maarif@te.uad.ac.id) is with Universitas Ahmad Dahlan, Jl. Kapas No. 9, Semaki, Kec. Umbulharjo, Yogyakarta 55166, Indonesia.

A.J. Castro-Montoya (e-mail: ajcastrofiq@gmail.com) is with Universidad Michoacana de San Nicolás de Hidalgo, Edif. M, Ciudad Universitaria, Morelia 58030, Michoacán, Mexico.

N.R. Cázarez-Castro (e-mail: nohe@ieee.org) is with Instituto Tecnológico de Tijuana, Calz. Tecnológico S/N, Fracc. Tomás Aquino, Tijuana 22414, BC, Mexico.

Received 5.10.2021. Revised 1.3.2022.

## 1. Introduction

Chemical reactors are often used as examples for non-minimum phase processes. A classical reactor in chemical engineering is the continuous stirred tank reactor (CSTR) that guarantees the same substratum concentration in all places inside the reactor due to the mixer used in these kinds of reactors [1]. In the CSTR the reactants are introduced and removed simultaneously.

There are some control designs developed for chemical reactors. The most common control in the industry is the proportional, integral, and derivative (PID) controller, even when the CSTR shows a nonlinear dynamic. Interesting modelling of the CSTR using recursive least squares and tuning a PID to control the reactor is presented in [2].

The Ziegler-Nichols tuning method is often used in literature. One alternative to propose the PID gains is to apply swarm intelligence. A bio-inspired tuning method was given in [3], where the artificial bee colony was used to find the control gains that optimize some criteria focused on the closed-loop behaviour, and their results were compared with the obtained using genetic algorithms to tune a controller for the CSTR.

The swarm intelligence seems to be a good alternative to tune a PID. An advantage is that the control design is model-free. However, a disadvantage is to simulate the controlled systems many times until a determined criterion is reached. This implementation can be made off-line, and when the desired behavior is found, the control gains computed can be implemented into the inline application [4]. Anyway, swarm intelligence is a good idea to propose the control gains if the approached model is very similar to the actual behavior of the reactor.

The PID control was applied in the production of aluminium chloride and some considerations for the simulation were mentioned in [5]. In [6] different PID tuning methods were used to control a CSTR, and the best results were obtained using a PI controller tuned by the Skogestad Internal Model Control (SIMC). Particle Swarm Optimization (PSO) was used to find a PID capable to control the inverse response shown by the Van de Vusse reaction in [7], where the criterion IST$^3$E was implemented to evaluate the PSO for having the smallest undershoot.

Adaptive control is another technique that can be used to control a CSTR. For example, in [8] a fuzzy model reference, adaptive control was used to approach a linear system behavior as time grows up, in this way, the controlled concentration was evolved in different set-points, reducing each time the error was computed with respect to the linear model reference as its Fig. 3 showed.

The main contribution of [9] was a feedforward design by a stable system inversion. Therefore, the transition problem from a minimum-phase behavior to the non-minimum phase case was solved in an algebraic way. Moreover, the CSTR can be used with the Van de Vusse reaction.

### 1.1. Controlling the Van de Vusse reaction

The Van de Vusse reaction is a nonlinear process with an inverse response and works in an isothermal condition. Isothermal CSTR with the Van de Vusse reaction was used as a challenging benchmark in [10] due to non-minimum phase behavior. The inverse response occurs when initially the output signal in a system responds in the opposite direction of the steady-state value [11].

To propose a PI controller, in [12] was used linear approximation regarding the time-response. Authors used a second-order system with two real poles, and an unstable zero. In Fig. 4 [12] authors showed the approached model that was very similar to the nonlinear system. The process used in [12] was a CSTR with the Van de Vusse reaction, and also some robustness considerations were included using concepts like the maximum sensitivity and the DS-d tuning method shown in [13].

A state feedback controller with a nonlinear observer was presented in [14], where it was also aggregated an integrator to improve the steady-state performance, and the stability of the approach was investigated via an input-to-state Lyapunov function. In Fig. 7 of [14], the simulation results showed an over-shoot for certain conditions in the Van de Vusse reaction.

In [9] two nonlinear controls were presented in open-loop, without taking into consideration if the system has minimum-phase or not to propose a controller. The controller was of the feedforward type, and its difficulty followed to form the system inversion necessity. One way to solve this issue is to implement a fuzzy or neural model. In this way, soft computing is a good option to control nonlinear systems with an inverse response.

### 1.2. Intelligent control for the Van de Vusse reaction

A control application of the Van de Vusse reactor comparing a PID against a type-2 fuzzy controller can be found in [15]. As the first approach, a linearization of the nonlinear process was made to obtain a transfer function. The PID was proposed using the linear model approximation. The main contribution was a fuzzy type-2 version of the PID that was more efficient, having less overshoot in the time response.

In [16] two proposals were given: the internal model control (IMC) and direct inverse control, both using artificial neural networks. One way to propose a controller or even an approached model when the mathematical model of the process is unknown is to implement an artificial neural network. The authors used as a case of study the Van de Vusse reaction and commented on the nonlinearities in the model. Pulses were used in [16] to validate the neural model using three input nodes, four hidden neurons in one layer, and one neuron as output. The best result was obtained using the direct inverse control, which was possible thanks to the neural model inversion. Similarly to swarm intelligence, it was necessary to make 9478 iterations to train the neural network in [16]. The difference is that they simulated the neural network many times, whereas in [3] the entire control process was simulated to evaluate each PID performance for certain gains.

Recently, a neural network controller was developed in [17] using reinforcement learning. The main reason was the difficulty of nonlinear systems modelling and its parameter uncertainty. Therefore, an initial neural network is trained using the first experiment in simulation, and then the network is adapted using the new data available from the process, in this case, the Van de Vusse reaction. The control was tuned according to the current set-point in the process to have the highest expected performance.

A similar application without the fuzzy logic implementation can be found in [18], where a linear-quadratic regulator (LQR) plus an integrator, called LQI, was presented (Fig. 5, [18]). The Van de Vusse reaction was controlled using a PID, sliding modes control, and their LQI proposal (Sec. 3.2, [18]). This last approach gave the best result showing the smallest undershoot.

In this article, the LQI is implemented using a fuzzy model, and different LQIs are obtained, its stability is determined via linear matrix inequalities (LMIs), and a fuzzy observer is used to make the state feedback [19]. The undershoot obtained depended principally on the placement of the poles.

A first approximation of the superiority of fuzzy control was shown in [20] where P, PI, and PID linear controllers were tuned using the Ziegler-Nichols first method and applied to the Van de Vusse reaction. The simulation results were compared with a fuzzy-P controller, which presented a better performance than the linear controllers. However, the fuzzy-P controller was designed to reach a predefined set-point, having problems following different references. For this reason, an integrator is proposed in this article as in [18] was proposed.

The Van de Vusse reaction was modeled in [21] using a fuzzy bilinear system. The LMIs [22] were also used to guarantee stability. The bilinear approximation is similar to the Taylor linearization. If the linearization is made in several points, the linear submodels obtained can be added in a fuzzy way.

Similar to the state-space submodels representation, this paper uses sector nonlinearity to obtain the linear submodels to guarantee global stability [23]. The fuzzy state feedback is obtained using LMIs with extended matrices to compute the integrator gain. The fuzzy observer is also implemented by using LMIs. Thus, the control is based on output feedback, and the placement of the poles uses the observed state feedback.

Several control techniques such as PID, internal model control (IMC), and state feedback, applied in different reactions, also the Van de Vusse reaction were studied in [24]. An interesting topic is the implementation of anti-windup for a conditional integrator. The anti-windup implemented in the present work uses the fuzzy LQI to reduce the undershoot obtained due to the inverse response [18].

The main contribution of this work is the computing of the state feedback with two integrators for each linear submodel named fuzzy LQI$^2$. To guarantee stability, LMIs were used for the calculation of the gain. Tracking properties maybe check if ramp references are used in the Van de Vusse reaction. An anti-windup action is proposed to reduce the undershoot shown by the inverse response system. In the same way, the anti-windup reduces the overshoot when the reference is a step signal because using two integrators to follow a ramp signal deteriorates the performance when a step reference is applied. The linear submodels were obtained using the sector nonlinearity, and they are joint via a fuzzy model. Furthermore, the control signal and the observer used in the estimated state feedback are fuzzy systems.

This article is organized as follows: In Section 2 the Van de Vusse reaction is described, where its mathematical model is shown. The fuzzy control proposed is presented in Section 3, where the subsections describe the fuzzy state feedback, the observer design, the use of two integrators, and the effect of the anti-windup action. Subsequently, the results are presented in Section 4, and conclusions are drawn in Section 5. The Appendix shows the control and observer gains computed using LMIs.

## 2. Van de Vusse reaction

The isothermal series/parallel Van de Vusse reaction is an example of an inverse response process. This process can be described as the non-minimum phase nonlinear process. This reaction gives the desired product that is accompanied by consecutive and parallel reactions that produce unwanted products. The reaction is described by the following equations [14]:

$$\frac{\mathrm{d}C_{\mathcal{A}}(t)}{\mathrm{d}t} = d(t)\left(C_{\mathrm{Ain}} - C_{\mathcal{A}}(t)\right) - k_1 C_{\mathcal{A}}(t) - k_3 C_{\mathcal{A}}(t)^2, \tag{1}$$

$$\frac{\mathrm{d}C_{\mathcal{B}}(t)}{\mathrm{d}t} = -d(t)C_{\mathcal{B}}(t) + k_1 C_{\mathcal{A}}(t) - k_2 C_{\mathcal{B}}(t), \tag{2}$$

$$\frac{\mathrm{d}C_{C}(t)}{\mathrm{d}t} = -d(t)C_{C}(t) + k_2 C_{\mathcal{B}}(t), \tag{3}$$

$$\frac{\mathrm{d}C_{\mathcal{D}}(t)}{\mathrm{d}t} = -d(t)C_{\mathcal{D}}(t) + \frac{1}{2}k_3 C_{\mathcal{A}}(t)^2. \tag{4}$$

where $\mathcal{A}$ is the cyclopentadiene reagent, $\mathcal{B}$ is the cyclopentenyl taken as the process output. Due to the strong reactivity of the reagents $\mathcal{A}$ and $\mathcal{B}$, an undesired product $\mathcal{D}$ is obtained, it is the dicyclopentadiene, and also a consecutive product $C$, the cyclopentanediol. The process input is the dilution rate $d(t)$ which is the quotient $F(t)/V(t)$, being $F(t)$ the inlet flow and $V(t)$ the liquid volume inside the reactor. The parameters $k_1$, $k_2$ and $k_3$ are the reaction rate constants, they determine if a product is generated or consumer into the chemical reaction and the reaction rate obtained. Also, these parameters show the selectivity to have certain product instead of the others. The parameters are not time dependent, however, if the reactor is not isothermal, the parameters can be temperature dependent [25].

The equations (1) and (2) do not depend on $C_C(t)$ and $C_{\mathcal{D}}(t)$. Thus, the process can be represented by using (1) and (2), the reaction scheme is described by:

$$\mathcal{A} \rightarrow_{k_1} \mathcal{B} \rightarrow_{k_2} C, \tag{5}$$

$$2\mathcal{A} \rightarrow_{k_3} \mathcal{D}. \tag{6}$$

The process consists of the production of cyclopentenyl $\mathcal{B}$ from cyclopentadiene $\mathcal{A}$ by using acid-catalyzed electrophilic addition of water [26]. The other subproducts are the dicyclopentadiene $\mathcal{D}$ is produced by a Diels-Alder reaction as a subproduct of reactants $\mathcal{A}$ and $\mathcal{B}$, and the cyclopentanediol $C$ is generated by adding another water molecule [20]. The reaction scheme using the molecules used is:

$$C_5H_6 \longrightarrow_{+H_2O/H+} C_5H_7OH \longrightarrow_{+H_2O/H+} C_5H_8(OH)_2,$$

$$2C_5H_6 \longrightarrow C_{10}H_{12}.$$

The nonlinear model described by (1) and (2) has some characteristics according to [16] as:

- Input multiplicity, thus the process is not control affine.
- Gain sign change producing a non-minimum phase behavior.
- Asymmetric response due to nonlinearities.
- Time lag in measuring instruments often do not taken into account.

A CSTR with the Van de Vusse reaction is shown in Fig. 1, where the control signal $d(t)$ is the dilution rate. The reaction rates $k_1$, $k_2$ and $k_3$ determines is a product is obtained or consumed in the chemical reaction, also these parameters determine the reactor type according to the relation $\theta_1 > \theta_2$, where $\theta_1 = k_3 C_{\mathcal{A}_f}/k_1$ and $\theta_2 = k_2/k_1$, being $C_{\mathcal{A}_f}$ the initial concentration of $\mathcal{A}$. In this way, the reactor can be a Plug Flow Reactor (PFR) or a CSTR as in this case [27].

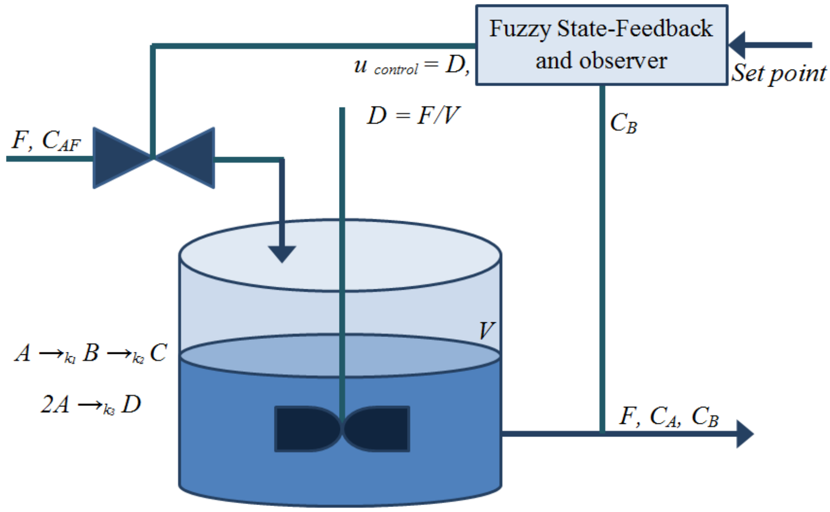


Figure 1: Isothermal CSTR with van de Vusse reaction

## 3. Fuzzy control design

Fuzzy logic has emerged as an alternative to work with incomplete information, parameters uncertainty, and to describe colloquial ideas to denote for example a “big error”, or “low velocity” [28]. When a controller is designed, an approached model of the system to control is often used. The direct control uses the model parameters to tune the control gains [29]. An indirect control can be built using the time or frequency response of the system [30]. Even the control design is achieved by adapting some parameters [31] or taking into consideration some limits for the unknown functions that describe the system dynamics [32].

When the model is not well defined or has parametric uncertainty, fuzzy logic can be used to approximate the model or propose the controller using expert knowledge [33]. In this paper, an approached model is obtained using the sector nonlinearity [23]. Thereby, some linear submodels are obtained, and state feedback is designed for each state-space submodel. However, stability of each closed-loop does not imply that the whole fuzzy system is also stable [34].

Linear matrix inequalities (LMIs) are used to guarantee stability when the fuzzy control is applied. Thereby, state-space submodels are obtained from the nonlinear process using the sector nonlinearity [23]. To build the membership functions, some cotes for the concentrations $C_{\mathcal{A}}(t)$ and $C_{\mathcal{B}}(t)$ could be determined to have two membership functions to fuzzify some terms in the state-space representation. Hence, the submodels are represented by the following system:

$$\begin{pmatrix} \dfrac{\mathrm{d}C_{\mathcal{A}}(t)}{\mathrm{d}t} \\ \dfrac{\mathrm{d}C_{\mathcal{B}}(t)}{\mathrm{d}t} \end{pmatrix} = \begin{pmatrix} -k_1 - k_3 C_{\mathcal{A}}(t) & 0 \\ k_1 & -k_2 \end{pmatrix} \begin{pmatrix} C_{\mathcal{A}}(t) \\ C_{\mathcal{B}}(t) \end{pmatrix} + \begin{pmatrix} C_{A\mathrm{in}} - C_{\mathcal{A}}(t) \\ -C_{\mathcal{B}}(t) \end{pmatrix} d(t). \tag{7}$$

Taking the elements $z_1(t) = -k_1 - k_3 C_{\mathcal{A}}(t)$, $z_2(t) = C_{Ain} - C_{\mathcal{A}}(t)$, and $z_3(t) = -C_{\mathcal{B}}(t)$, it is possible to have eight linear submodels using the two membership functions that fuzzify each $z_i(t)$ element. Thus, the state-space submodels can have the following representation:

$$\begin{pmatrix} \dfrac{\mathrm{d}C_{\mathcal{A}}(t)}{\mathrm{d}t} \\ \dfrac{\mathrm{d}C_{\mathcal{B}}(t)}{\mathrm{d}t} \end{pmatrix} = \begin{pmatrix} z_1(t) & 0 \\ k_1 & -k_2 \end{pmatrix} \begin{pmatrix} C_{\mathcal{A}}(t) \\ C_{\mathcal{B}}(t) \end{pmatrix} + \begin{pmatrix} z_2(t) \\ z_3(t) \end{pmatrix} d(t). \tag{8}$$

The fuzzy partition used to approach the elements $z_1(t)$, $z_2(t)$ and $z_3(t)$ was implemented with two fuzzy sets. Figure 2 shows the $z_3$ fuzzy partition, for the

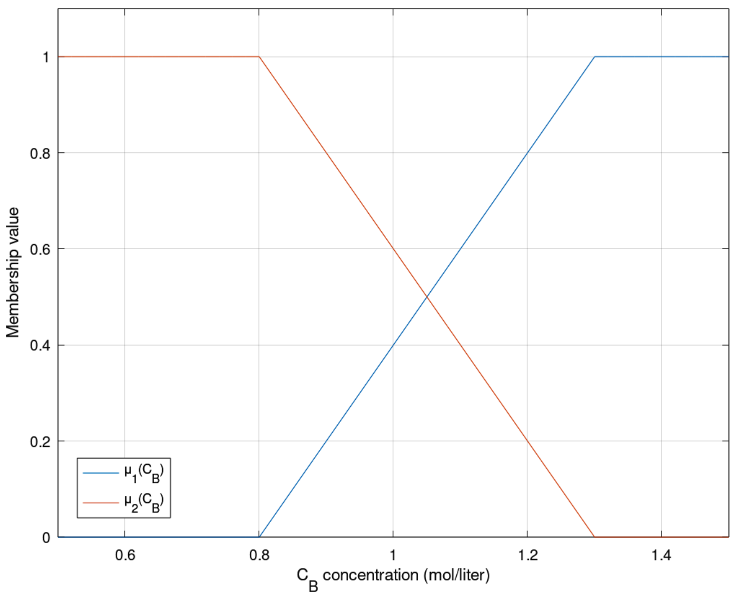


Figure 2: Fuzzy partition for $z_3$

other elements, similar fuzzy sets were defined with values between [2.5 3.4] for $z_1$ and [6.6 7.5] for $z_2$, these values were obtained regarding values near the limits of the elements in the Van de Vusse simulation.

### 3.1. Fuzzy state feedback

To have a stable state feedback it is necessary to guarantee stability not only for each individual fuzzy submodel. Thus, to investigate the global stability, the following Lyapunov function is proposed

$$V(t) = \frac{1}{2}\begin{pmatrix} C_{\mathcal{A}}(t) & C_{\mathcal{B}}(t) \end{pmatrix} P \begin{pmatrix} C_{\mathcal{A}}(t) \\ C_{\mathcal{B}}(t) \end{pmatrix}$$

is defined. If it is possible to find positive definite matrix $P$, then the fuzzy state feedback is stable in the Lyapunov sense [19].

The poles placement allows to locate eigenvalues of the closed-loop system matrix $(A-BK)$ in arbitrary places by a proper choice of the controller gain matrix $K$. The inequalities that represent the state feedback are defined as follows:

$$\left(A_{i,i} - B_i K_i\right)^T P + P\left(A_{i,i} - B_i K_i\right) + 2\alpha P < 0, \tag{9}$$

and for the fuzzy intersection between two submodels is necessary to keep,

$$\left(\frac{\left(A_{i,j} - B_i K_j\right)^T P + P\left(A_{j,i} - B_j K_i\right)}{2}\right)^T P + P\left(\frac{\left(A_{i,j} - B_i K_j\right)^T P + P\left(A_{j,i} - B_j K_i\right)}{2}\right) + 2\alpha P \leqslant 0, \tag{10}$$

where $B_i$ is now the input matrix in the classical state-space representation, $K_i$ is the state feedback gain, and $\alpha > 0$ is the decay rate.

Relations (9) and (10) are not jointly convex in $K_i$ and $P$ [23], thus to have the LMIs the new variables $X = P^{-1}$ and $M_i = K_i X$ [35] are introduced. To eliminate the steady-state error, an integrator is added in a similar way to the shown in [18], thus the following extended matrices are used:

$$\hat{A} = \begin{pmatrix} A & 0 \\ -C & 0 \end{pmatrix}, \tag{11}$$

$$\hat{B} = \begin{pmatrix} B \\ 0 \end{pmatrix}, \tag{12}$$

where $C = \begin{pmatrix} 0 & 1 \end{pmatrix}$ because the concentration $C_{\mathcal{B}}(t)$ is the output of the system.

Hence, the LMIs to be solved are:

$$X > 0, \tag{13}$$

$$-X\hat{A}_{i,i}^T - \hat{A}_{i,i}X + M_i^T\hat{B}_i^T + \hat{B}_iM_i - 2\alpha X > 0, \tag{14}$$

$$\begin{aligned} -X\hat{A}_{i,j}^T - X\hat{A}_{j,i}^T - \hat{A}_{i,j}X - \hat{A}_{j,i}X + M_i^T\hat{B}_j^T \\ + M_j^T\hat{B}_i^T + \hat{B}_iM_i + \hat{B}_jM_j - 4\alpha X \geqslant 0. \end{aligned} \tag{15}$$

Thus, the common matrix found is $X \in \mathbb{R}^{3\times 3}$, the state feedback gain is computed to be $\hat{K}_i = X^{-1}M_i = \left( K_1 \;\; K_2 \,\middle|\, -K_{I1} \right)$, where $K_{I1}$ is the integrator gain.

### 3.2. Fuzzy observer

The estimation error $e(t) = C_{\mathcal{A}}(t) - \widetilde{C}_{\mathcal{A}}(t)$ has its dynamic according to the eigenvalues of $(A - LC)$, where $L$ the observer gain. The fuzzy observer is computed using the following inequality:

$$\left(A_{i,i} - L_iC\right)^T Q + Q\left(A_{i,i} - L_iC\right) + 2\beta Q < 0, \tag{16}$$

and for the intersection between fuzzy sets, it is necessary to maintain:

$$\begin{aligned} \left(\frac{\left(A_{i,j} - CL_j\right)^T Q + Q\left(A_{j,i} - CL_i\right)}{2}\right)^T Q \\ + Q\left(\frac{\left(A_{i,j} - CL_j\right)^T Q + Q\left(A_{j,i} - CL_i\right)}{2}\right) + 2\beta Q \leqslant 0, \end{aligned} \tag{17}$$

where $L_i$ is the observer gain, $\beta > 0$ is the decay rate, and $Q > 0$.

In the same way aforementioned, (16) and (17) are not jointly convex in $L_i$ and $P$, and to have the LMIs it is necessary to introduce the new variables $X = Q^{-1}$ and $N_i = XL_i$.

Now, the LMIs to be solved are:

$$X > 0, \tag{18}$$

$$-XA_{i,i}^T - A_{i,i}X + C^TN_i^T + N_iC - 2\beta X > 0, \tag{19}$$

$$\begin{aligned} -XA_{i,j}^T - XA_{j,i}^T - A_{i,j}X - A_{j,i}X + C^TN_i^T \\ + C^TN_j^T + N_iC + N_jC - 4\beta X \geqslant 0. \end{aligned} \tag{20}$$

A scheme of the fuzzy state feedback and the fuzzy observer is shown in Fig. 3. The application of the state feedback gains $K_i$, the integrator gains $K_I$, and the fuzzy observer $L_i$ is used because the concentration $C_{\mathcal{B}}(t)$ is the only measured variable.

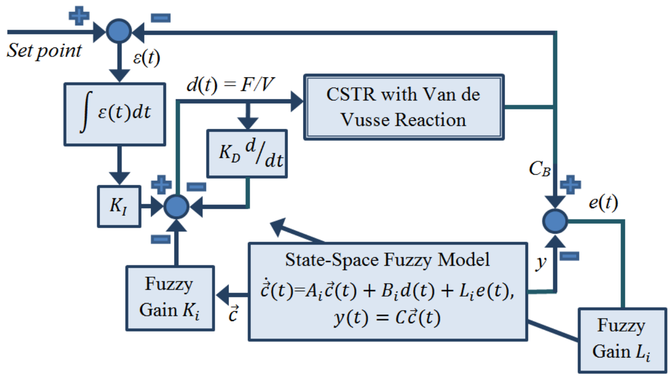


Figure 3: Fuzzy control scheme

### 3.3. Anti-windup proposal

To avoid the undershoot due to the inverse response, an anti-windup element is added to the final scheme. This element used to be a signal saturation of the integral action as shown in Fig. 3 of [36]. Authors of [36] emphasized the difference between the original control signal and the saturated one. However, in this paper, a derivative term is used to reduce the control action $d(t)$ instead of saturating the control signal. The derivative term works as a counterpart of the integrator.

Instead of saturation only [37], the control signal derivative is used to emulate limitation in the control signal variation. Its final value to be reached is reduced as well because the control action increases slowly if its derivative is used to compute the control action. The anti-windup as the saturation in the rate and amplitude of the control action is presented in [38].

Thus, the control signal is formed by:

$$d(t) = K_{I1} \int \varepsilon(t)\mathrm{d}t - K_1\widetilde{C}_{\mathcal{A}}(t) - K_2\widetilde{C}_{\mathcal{B}}(t) - K_D\dot{d}(t), \tag{21}$$

where $\widetilde{C}$ is an estimated concentration computed by the observer, and $K_D$ is a derivative gain used for the anti-windup action, this gain is chosen to be small for stability issues. The error is $\varepsilon(t) = \text{set-point} - C_{\mathcal{B}}(t)$.

The augmented subsystems are now third-order linear subsystems. Thus, the derivative gain $K_D$ must be selected small for stability reasons. The anti-windup term reduces the control signal if this signal increases quickly. For example, if the output diverges from the reference, then its derivative is positive, and the anti-windup action decreases the control signal by subtracting $K_D\dot{d}(t)$.

### 3.4. Double integrator

Finally, an integrator in the controller can be used to have a zero steady-state error when a step reference is applied to a linear system, and a constant error is obtained if the reference is a ramp [39].

Since the controller is computed using linear subsystems, the error constant is different for different operation points. Thus, the steady-state error when the ramp is applied to the closed-loop is not constant for the nonlinear system, this error diverges from the reference.

Taking into consideration the notation $k_i$ for the model parameters, and $K_i$ for the state feedback gain, the analysis is made using at first only the state feedback, then the integrator is added. Finally, the anti-windup proposal is introduced.

Using the original system (8) given by matrices:

$$A = \begin{pmatrix} z_1(t) & 0 \\ k_1 & -k_2 \end{pmatrix}, \qquad B = \begin{pmatrix} z_2(t) \\ z_3(t) \end{pmatrix},$$

and $C = \begin{pmatrix} 0 & 1 \end{pmatrix}$, where the fuzzy terms $z_i$ were used to linearize the process. The final linear subsystem can be represented by transfer functions of the form:

$$\frac{C_{\mathcal{B}}(s)}{D(s)} = \frac{-s+b}{s^2+a_1 s+a_2}, \tag{22}$$

where $b = k_1 z_2 - z_2 z_3$, $a_1 = z_1 + k_2 + K_1 z_2 - K_2 z_3$, $a_2 = k_2 z_1 - K_2 z_1 z_3 + z_2 K_1 k_2 + z_2 k_1 K_2$ and $D(s)$ is the control signal. Here the signs represent the term sign according to the model (8). The state feedback gains $K_1$ and $K_2$ can be chosen to have some locations in the Laplace plane. In this paper they are designed to guarantee stability.

Adding the integrator gives open-loop subsystems expressed as:

$$\frac{C_{\mathcal{B}}(s)}{D(s)} = \frac{-s+b}{(K_{I1} s)\left(s^2+a_1 s+a_2\right)}, \tag{23}$$

as is well known the closed-loop is now:

$$\frac{C_{\mathcal{B}}(s)}{R(s)} = \frac{-s+b}{K_{I1} s^3 + K_{I1} a_1 s^2 + (K_{I1} a_2 - 1)\, s + a_2 b},$$

where $R(s)$ is the reference signal. Taking the limit when $s \to 0$, the rate constant is $1/a_2$, then the steady-state error is $\varepsilon_{ss} = a_2$. Thus, as $a_2$ depends on fuzzy terms $z_1$, the final error is not a constant for each submodel and as it can be appreciated in Fig. 7, the error increases with time.

The undershoot can be reduced using the anti-windup, in this case, the control signal derivative $K_D s D(s)$. Now, the anti-windup is aggregated to the open-loop

(23) that has the state feedback and one integrator. Regarding Fig. 3, the error is sent directly to the system, and this signal is returned after its derivative calculation

$$\frac{D(s)}{\varepsilon(s)} = \frac{1}{K_D s + 1} .$$

The open-loop using the three parts, the integrator $K_{I1}$, the state feedback $K = \begin{pmatrix} K_1 & K_2 \end{pmatrix}$, and the anti-windup action $K_D$, is given by:

$$\frac{C_{\mathcal{B}}(s)}{D(s)} = \frac{-s + b}{(K_D s + 1)(K_{I1} s)\left(s^2 + a_1 s + a_2\right)} . \tag{24}$$

The control action is defined by:

$$d(t) = K_{I1} \int \varepsilon(t)\,\mathrm{d}t + K_{I2} \iint \varepsilon(t)\,\mathrm{d}t - K_1 C_{\mathcal{A}}(t) - K_2 C_{\mathcal{B}}(t) - K_D \dot{d}(t). \tag{25}$$

Using two integrators the open-loop is now:

$$\frac{C_{\mathcal{B}}(s)}{D(s)} = \frac{-s + b}{(K_D s + 1)\left(K_{I1} s + K_{I2} s^2\right)\left(s^2 + a_1 s + a_2\right)} , \tag{26}$$

the rate constant is infinity, then the final error, even with a ramp reference, is zero in steady-state. Due to the fuzzy control, the over and undershoots are different for each set-point, but the error converges in all cases to zero.

As mentioned above, now the subsystems with the anti-windup are fifth-order linear subsystems, and the unstable zero affects the stability in the root-locus sense. Furthermore, the gain $K_D$ is selected to be small for computing issues.

## 4. Results

The Van de Vusse reaction is used as a case of study and conducts a nonlinear process that shows an inverse response. The initial conditions and parameters values are shown in Table 1.

Table 1: Initial conditions for simulation

| Parameter | Value | Unit |
|---|---|---|
| $C_{Ain}$ | 3 | mol/liter |
| $C_{\mathcal{A}}$ | 1.117 | mol/liter |
| $C_{\mathcal{B}}$ | 10 | mol/liter |
| $k_1$ | 5/6 | min$^{-1}$ |
| $k_2$ | 5/3 | min$^{-1}$ |
| $k_3$ | 1/6 | mol/min |

### 4.1. Fuzzy model

The sector nonlinearity is used to meet the fuzzy model composed by state-space submodels as the consequents of the rules using (8). Such the approached model allows to compute the state feedback, and to aggregate integrators to improve the control performance. An interesting point is how to propose the limits to compute the functions $z_i(t)$.

The chemical process has the inverse response, which can be observed in its step response depicted in Fig. 4, where the comparison between the nonlinear model response and the fuzzy model obtained is shown, both simulations use the initial conditions presented in [12, 13, 20, 21] and illustrated in Figs. (2a), (7), (3), and (3) of these references respectively.

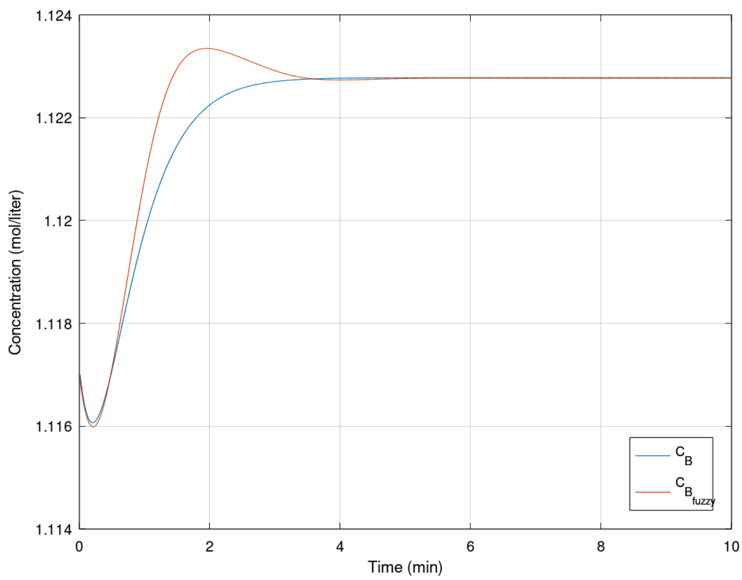


Figure 4: Comparison between the model (1)–(4) and fuzzy model using subsystems of the form (8)

The best approximation to the nonlinear Van de Vusse reaction was presented in [13], where a second-order non-minimum phase linear system was used to approach the nonlinear one, a similar case to the report given by [12]. The fuzzy bilinear system is shown in [21] which is also a good approximation. Five submodels have been used with a common matrix $B$ from the classical notation for state-space systems. Using our methodology, two matrices are obtained for the transfer state matrix $A$ and four for the input matrix $B$ using the classical representation $\dot{x} = Ax + Bu$, and eight fuzzy rules were obtained.

### 4.2. Fuzzy state feedback and observer

The inequalities (13), (14), and (15) were solved using the CVX tool [40] by applying the extended matrices $\widetilde{A}$ and $\widetilde{B}$. Thereby, the state feedback with a double integrator is globally stable. The parameter $\alpha$ is proposed to be a positive variable in [23], this parameter is used to change the decay rate. To modify the speed response is used the largest Lyapunov exponent is obtained by maximizing $\alpha$ subject to (14) and (15). Here, it was used $\alpha = 0.02$, and the matrix $P$ was computed as:

$$P = \begin{pmatrix} 129.9981 & 0.0009 & -0.0006 \\ -0.0009 & 33.2615 & -55.0463 \\ -0.0006 & -55.0463 & 91.9725 \end{pmatrix}.$$

In the same way, the observer gains for each submodel were computed using (18), (19), and (20). The decay rate is regulated using $\beta = 0.2$ being also guaranteed the stability thanks to the possibility of finding a common positive definite matrix $Q$ which is:

$$Q = \begin{pmatrix} 177.9480 & -0.0005 \\ -0.0005 & 200.0079 \end{pmatrix}.$$

The reference is formed for steps of different magnitudes as was shown in Fig. 3 of [14]. The control simulation can be observed in Fig. 5. Due to the

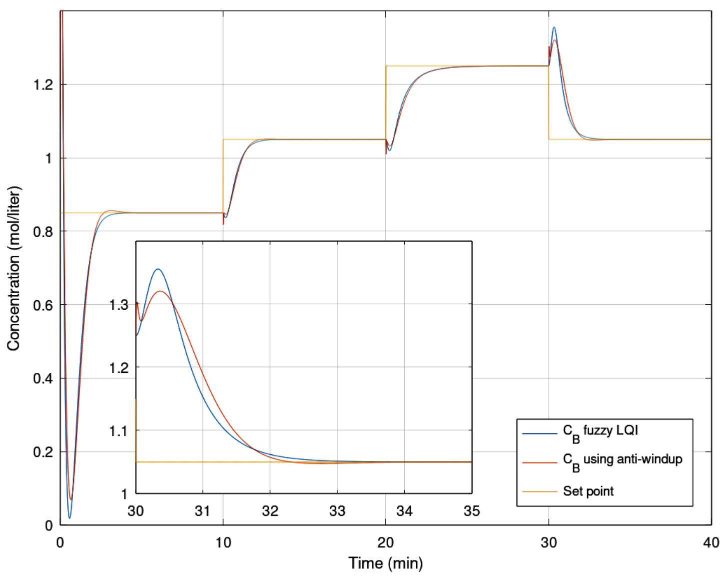


Figure 5: Response obtained with the fuzzy control in the nominal condition

similarity to the control design shown in [18], this proposal is named fuzzy LQI. The LQI is computed for each submodel, and a derivative term is used to limit the control action.

The control proposal in this work is named fuzzy $\text{LQI}^2$ due to the similarity to the proposal shown in [18]. In this case, the control scheme has an anti-windup action and a fuzzy observer as Fig. 3 shows.

The settling-time ($t_s$) for linear models approximated to the Van de Vusse reaction was shown in [7]. the result was obtained by tuning the PID controller with PSO. Also the results reported in [41] and [42] show the $t_s$ obtained around 2.84, 4.96, and 10.43 minutes respectively in the best case.

Some papers with similar results showed the time-base in seconds when the time must be given in minutes as Table 1 shows in the material balances $k_1$, $k_2$ and $k_3$. Initially, the Van de Vusse benchmark described in [9] showed the time-base in minutes (Fig. 3, [9]). In the same way, [12] and [13] showed approached models Figs. 4 and 7 respectively, using minutes as the units for the $x$-axis.

Table 2 shows results concerning time-response and errors obtained with and without the anti-windup proposal. These parameters were averaged because the nonlinear system shows different behavior in each set-point presented in Fig. 5.

Table 2: Control performance

| Control scheme | $t_s$ | Undershoot | IAE | ISE |
|---|---|---|---|---|
| Fuzzy LQI without anti-windup | 2.24 | 39.12% | 1.7697 | 0.8622 |
| Fuzzy LQI with anti-windup | 2.01 | 27.05% | 1.7418 | 0.7704 |

One way to reduce the undershoot due to the inverse response is using a model reference to have a smooth reference. For example, if a filter is applied to the reference before closing the control loop.

The filter used as model reference was proposed to have a time constant of 30 seconds regarding Fig. 4 to have a similar behavior than the process in open-loop. The sample time used to have a discrete time filter was 0.6 seconds, in this way we have 50 samples before reach the time constant. The model reference in discrete time is defined by:

$$\frac{M_{\text{ref}}(z)}{R(z)} = \frac{0.0198}{z - 0.9802}. \tag{27}$$

The control simulation is shown in Fig. 6, where it can be seen a delay due to the inverse response but the undershoot is reduced because the smooth change in the reference makes a small change in the control action instead of limiting this action using the anti-windup proposed. However, Fig. 6 also shows both state feedback with and without anti-windup to see the difference. The undershoot is reduced, and delay even with the model reference follows from the inverse response. Nevertheless, the settling time is around 3 minutes. The last step change has an amplitude of 0.2 lower than the above reference. Thereby, the maximum value reached by the output signal is 53.35% higher than the above reference and 46.65% using the anti-windup.

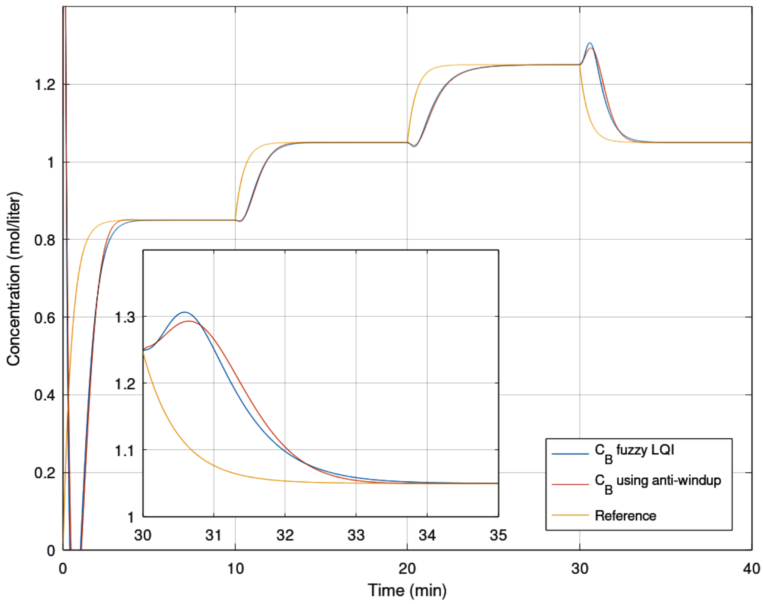


Figure 6: Response obtained using a model reference

A similar result is shown in [15]. Presented there equations (8) and (12) gives the matrices used in the model and a similar open-loop transfer function with numerator $-1.117s + 3.129$ after linearization. And a similar scheme for the anti-windup can be found in [38] (as Fig. 5).

In contrast, the results shown in Fig. 5 present a settling time of 2 minutes approximately, and the undershoots 77.9% without the anti-windup, and 60.4% with the control signal derivative added to the control action.

Using an integrator is possible to eliminate the steady-state error if it is used a step reference signal. A constant error can be expected if a ramp reference is applied instead of the step signal. Applying a ramp signal the error diverges from zero because we have linear submodels with different rate constants. Figure 7 shows a similar simulation to the presented in Fig. 5, but with a ramp reference in the last part of the simulation.

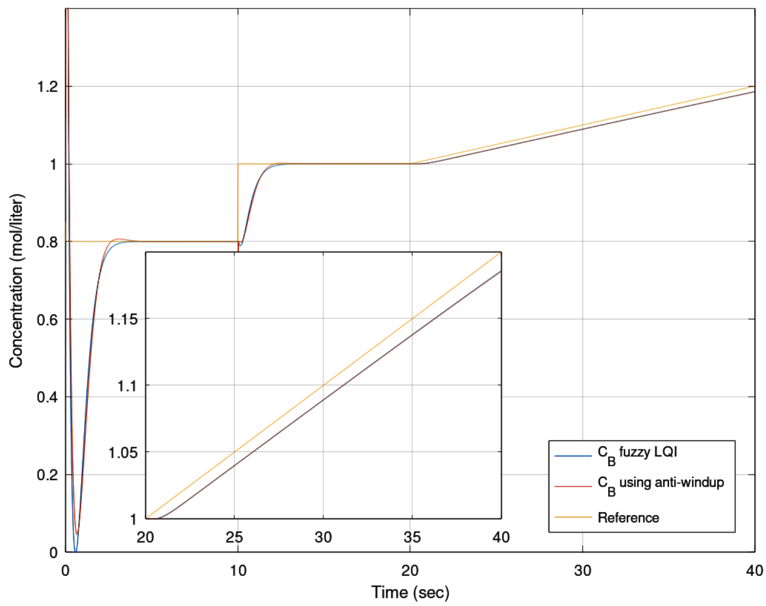


Figure 7: Response obtained using steps and ramp references

### 4.3. Results obtained using the LQI[2]

In some works where the Van de Vusse reaction is controlled, the effect of disturbances is also shown. Here, a servo-system is implemented to follow a higher-order reference, as shown in Fig. 7. It is possible to implement two integrators as [43] explains in a state feedback design: the modification is to augment the matrices used before computing the state feedback.

To follow a ramp reference is required to have two integrators, thus to compute the state feedback with two integrators is necessary to use a extended matrix $\widetilde{\widetilde{A}}$ by aggregating a matrix $I_{\text{ramp}} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ [43], now we have:

$$\widetilde{\widetilde{A}} = \begin{pmatrix} A & 0 & 0 \\ -C & 0 & 0 \\ 0 & I_{\text{ramp}} & 0 \end{pmatrix}, \tag{28}$$

and

$$\widetilde{\widetilde{B}} = \begin{pmatrix} B \\ 0 \\ 0 \end{pmatrix}. \tag{29}$$

Now, the Ackermann formula gives $\widetilde{K}_i = X^{-1} M_i = \left( K_1 \;\; K_2 \,\middle|\, -K_{I1} \;\; -K_{I2} \right)$.

Some previous and related works show comments about disturbance rejection and use step references. In some cases, the set-point changes to evaluate the time response. This paper shows the control performance using two integrators when ramp references are applied to the Van de Vusse reaction. A disadvantage of aggregating another integrator is the overshoot shown in the closed-loop response. Thus, the anti-windup action helps to reduce the over and undershoot. Now, the steady-state error converges to zero when a ramp reference is applied, as is shown in Fig. 8.

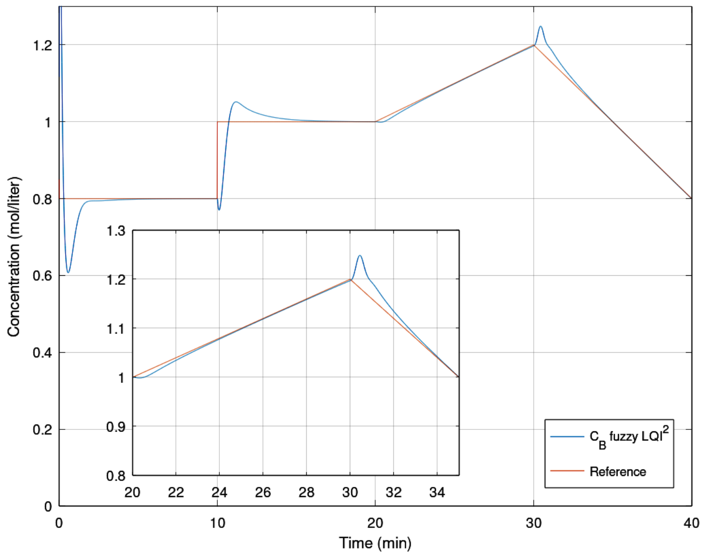


Figure 8: Response obtained using a double integrator and a ramp reference

The state feedback and the integrators' gains were obtained using LMI to guarantee stability. This way is different than poles placement and quite similar to the LQI shown in [18]. The control action is named fuzzy LQI$^2$ due to the double integrator.

The gain matrices $K_i$ and $L_i$ are shown in the appendix. In this case, $\alpha = 0.002$ to solve the LMIs for computing the state feedback with a double integrator.

Now, the common matrix $P \in \mathbb{R}^{4\times4}$ obtained using (15), (16) and (17) with the extended matrices $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ is:

$$P = \begin{pmatrix} 124.9995 & 0.0001 & -0.0002 & 0.0000 \\ 0.0001 & 33.2092 & -55.0641 & 0.2406 \\ -0.0002 & -55.0641 & 91.9684 & 0.1930 \\ 0.0000 & 0.2406 & 0.1930 & 0.9907 \end{pmatrix}.$$

The gain $K_D$ can be selected greater than in single integrator case because a faster change in the control signal is obtained using two integrators. Therefore, the anti-windup action also reduces the overshoot. The $t_s$ is averaged for the same reason mentioned above because, different set-points, different control gains and linear submodels are used to control the chemical reaction. These results are shown in Table 3.

Table 3: Control performance using two integrators

| Control scheme | $t_s$ | Undershoot | IAE | ISE |
|---|---|---|---|---|
| Fuzzy LQI$^2$ with anti-windup | 2.26 | 21.04% | 0.6098 | 0.1632 |

## 5. Conclusions

The benchmark known as the Van de Vusse reaction is used in this paper, which serves as a nonlinear process that shows an inverse response. Some works developed nonlinear and intelligent controllers, and compare their results against the obtained by using a PID control. Also, some articles show how to eliminate the steady-state error when a step reference is applied. Furthermore, some works explain the disturbance rejection for this kind of process.

In the present work, we added two integrators to eliminate the steady-state error when a ramp reference is used. To eliminate the undershoot due to the inverse response of the system, anti-windup action is proposed. It uses the control signal derivativeiedzw to delimitate the control signal variation, which could be bigger than using only one integrator.

An interesting result is that the anti-windup action reduces the undershoot due to the inverse response and the overshoot caused by the double integrator in the controller.

The usage of a filter to have a model reference was not sufficient because the inverse response aggregates a delay, and the undershoot was not reduced in a significant way as shown in Fig. 6.

The state feedback and integrator gains are computed using LMIs to guarantee stability. The linear subsystems in the state-space representation are obtained using the sector nonlinearity to produce a fuzzy system, where each rule consequent is a linear state-space.

An alternative is to compute the state feedback for the resulting state-space fuzzy model, here the system is stable for each point, thereby the Ackermann formula could be computed to obtain the state feedback, the observer, and the integrators' gains. But in other circumstances, if the resulting state-space model is unstable, it is not possible to compute any gain.

Although the state feedback is obtained using LMIs to guarantee stability, some points need to be considered:

- The parameters $\alpha$ and $\beta$ for the decay rate give a different performance. Here the observer is proposed to be faster in convergence than the control dynamic. In [23] these parameters are considered as variables to be maximized.
- The gain $K_D$ needs more clarification to be proposed because a small value guarantees stability, and by increasing this parameter, the over and undershoot are reduced, but the main problem is the computational convergence.

The simulations show the closed-loop performance of the control scheme proposed. It is planned to design a multivariable control to regulate the concentration $C_{\mathcal{A}}(t)$ as future work. Another topic is to show a design procedure to propose the gain $K_D$. Also, an adaptive fuzzy model can be developed instead to approach the nonlinear process by sector nonlinearity.

## Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## A. State feedback gains with one integrator

The state feedback and observer gains are shown in one matrix with delimiter to denote each one of the fuzzy submodels' parameters, being eight state feedback gains. Using two membership functions to get linear subsystems in three locations gives eight fuzzy rules. The rows have the form $K = \begin{pmatrix} K_1 & K_2 & \big| & K_{I1} \end{pmatrix}$.

$$K = \left(\begin{array}{cc|c} 4.582 & 0.285 & -1.279 \\ \hline 4.582 & 0.285 & -1.279 \\ \hline 4.582 & 0.285 & -1.279 \\ \hline 4.143 & 3.913 & -1.279 \\ \hline 4.143 & 3.913 & -3.307 \\ \hline 4.143 & 3.913 & -3.308 \\ \hline 4.582 & 0.285 & -1.279 \\ \hline 4.583 & 0.287 & -1.299 \end{array}\right).$$

### B. Observer gains

In the observer, only two gains $L_1$ and $L_2$ were computed, and they are the following:

$$L = \left(\begin{array}{c|c} 0.8337 & 0.8332 \\ 8.9449 & 8.9435 \end{array}\right).$$

### C. State feedback gains with two integrator

Each gain obtained has four terms, and they are eight linear submodels, thus the gains are shown into a $8 \times 3$ matrix where the last two columns represent the integrator gains. The rows have the form $\widetilde{K} = \left( K_1 \;\; K_2 \,|\, K_{I1} \;\; K_{I2} \right)$

$$\widetilde{K} = \left(\begin{array}{cc|cc} 1.282 & -1.590 & -2.121 & -4.010 \\ \hline 1.282 & -1.590 & -2.121 & -4.010 \\ \hline 1.285 & -1.612 & -2.126 & -4.222 \\ \hline 1.285 & -1.612 & -2.126 & -4.222 \\ \hline 1.205 & -1.409 & -2.011 & -6.111 \\ \hline 1.205 & -1.409 & -2.422 & -6.154 \\ \hline 1.287 & -1.590 & -2.121 & -4.442 \\ \hline 1.285 & -1.595 & -2.422 & -4.255 \end{array}\right).$$

## References

[1] P. Skupin, M. Metzger, P. Laszczyk and M. Niedzwiedz: Experimental and bifurcation analysis of a hybrid CSTR plant. *Chemical Engineering Research and Design*, **148** (2019), 191–201. DOI: 10.1016/j.cherd.2019.06.010.

[2] A. Simorgh, A. Razminia and V. Shiryaev: System identification and control design of a nonlinear continuously stirred tank reactor. *Mathematics and Computers in Simulation*, **173** (2020), 16–31. DOI: 10.1016/j.matcom.2020.01.010.

[3] W. Chang: Nonlinear cstr control system design using an artificial bee colony algorithm. *Simulation Modelling Practice and Theory*, **31** (2013), 1–9. DOI: 10.1016/j.simpat.2012.11.002.

[4] L. Chen, S. Du, Y. He, M. Liang and D. Xu: Robust model predictive control for greenhouse temperature based on particle swarm optimization. *Information Processing in Agriculture*, **5** (2018), 329–338. DOI: 10.1016/j.inpa.2018.04.003.

[5] A. Palencia-Díaz, J. Carpinteiro-Durangoand J. Fabregas-Villegas: Modeling, simulation and control of a reactor for the production of aluminum chloride. *Prospectiva*, **10** (2012), 31–36. DOI: 10.15665/rp.v10i2.230.

[6] M. Kumarand R.S. Singh: Comparison of non-linear, linearized 2nd order and reduced to FOPDT models of CSTR using different tuning methods. *Resource-Efficient Technologies*, **2** (2016), S71–S75. DOI: 10.1016/j.reffit.2016.11.003.

[7] M. Irshad and A. Ali: Optimal tuning rules for PI/PID controllers for inverse response processes. *IFAC PapersOnLine*, **51** (2018), 413–418. DOI: 10.1016/j.ifacol.2018.05.063.

[8] M. Bahita and K. Belarbi: Model reference neural-fuzzy adaptive control of the concentration in a chemical reactor (CSTR). *IFAC PapersOnLine*, **49** (2016), 158–162. DOI: 10.1016/j.ifacol.2016.11.093.

[9] K. Hagenmeyer and M. Zeitz: Van de Vusse CSTR as a benchmark problem for nonlinear feedforward control design techniques. In: *IFAC Nonlinear Control Systems*, Ed. Elsevier, Stuttgart, Germany, (2004), 1123–1128.

[10] H. Perez, B. Ogunnaike and S. Devasia: Output tracking between operating points for nonlinear processes: Van de Vusse example. *IEEE Transactions on Control Systems Technology*, **10** (2002), 611–617. DOI: 10.1109/TCST.2002.1014680.

[11] K. Vu: A model predictive controller for inverse response control systems. *IFAC PapersOnLine*, **48** (2015), 562–567. DOI: 10.1016/j.ifacol.2015.09.02.

[12] C. Mesén, E. Vargas and J. Torres: *Sintonización y análisis del funcionamiento de un sistema CSTR*. Proyecto Final: IE-0431 Sistems de Control, Diagnostic test, Universidad de Costa Rica, 2021.

[13] V. Alfaro, P. Balaguer and O. Arrieta: Robustness considerations on pid tuning for regulatory control of inverse response processes. In: *IFAC Conference on Advanced PID Control*, Ed. Elsevier, Bresia, Italy, (2012), 193–198. DOI: 10.3182/20120328-3-IT-3014.00033.

[14] S. Kuntanapreeda and P. Marusak: Nonlinear extended output feedback control for CSTRs with Van de Vusse reaction. *Computers & Chemical Engineering*, **41** (2012), 10–23. DOI: 10.1016/j.compchemeng.2012.02.010.

[15] A. Bertone, R. da M. Jafelice and B. Goes: Classic and fuzzy type-2 control for the Van de Vusse reactor: A comparative study. In: *Proceeding Series of the Brazilian Society of Computational and Applied Mathematics*, Ed. S. de Matemática Aplicada e Computacional. **6** Sao Carlos, Brazil, (2018), 1–7. DOI: 10.5540/03.2018.006.02.0258.

[16] R. Malar and T. Thyagarajan: Artificial neural networks based modeling and control of continuous stirred tank reactor. *American Journal of Engineering and Applied Sciences*, **2** (2009), 229–235. DOI: 10.3844/ajeassp.2009.229.235.

[17] G. Cassol, G. Campos, D. Thomaz, B. Capron and A. Secchi: Reinforcement learning applied to process control: A Van de Vusse reactor case study. in: *International Symposium on Process Systems Engineering–PSE*, Eds. M.I. Mario and R. Eden, San Diego, California, United States of America, (2018), 1123–1128. DOI: 10.1016/B978-0-444-64241-7.50087-2.

[18] W. Cargua, M. Gallegos, P. Leica, L. Guzmán-Beckmann and O. Camacho: Control schemes comparison for CSTR chemical reactors. *Ciencia e Ingeniería*, **39** (2018), 177–189.

[19] M.A. Márquez-Vera, L.E. Ramos-Velasco and B.D. Balderrama-Hernández: Stable fuzzy control and observer via LMIs in a fermentation process. *Journal of Computational Science*, **27** (2018), 192–198. DOI: 10.1016/j.jocs.2018.06.002.

[20] H. Ojeda-Elizarras, R. Maya-Yescas, S. Hernández-Castro, J. Segovia-Hernández and A.J. Castro-Montoya: Fuzzy control of a nonlinear system with inverse responce: Van de Vusse reaction. *International Journal of Latest Research in Science and Technology*, **2** (2013), 1–5. DOI: 10.1016/j.eswa.2010.09.158.

[21] S. Tsai: Robust $H_\infty$ control for Van de Vusse reactor via T-S fuzzy bilinear scheme. *Expert Systems with Applications*, **38** (2011), 4935–4944. DOI: 10.1016/j.eswa.2010.09.158.

[22] H. Lam, F. Leung and P. Tam: Fuzzy state feedback controller for nonlinear systems: Stability analysis and design. In: *Ninth IEEE International Conference on Fuzzy Systems*, FUZZ- IEEE. Ed. IEEE, San Antonio, United States of America, (2000), 677–681. DOI: 10.1109/FUZZY.2000.839102.

[23] K. Tanaka and H. O. Wang: *Fuzzy control systems design and analysis: A linear matrix inequality approach*. Wiley-Intescience, New York, 2001.

[24] I. Carrera-Flores: *Diseno de sistemas de control para procesos aplicados a reactores continuos tipo tanque agitado (CSTR)*. Master's thesis, Escuela Politécnica Nacional, Quito, Ecuador, 2014.

[25] H. Chen, A. Kremling and F. Allgöwer: Nonlinear predictive control of a benchmark CSTR. In: *Proceedings of 3rd European Control Conference*, Rome, Italy, (1995), 3247–3252.

[26] S. Engell and K. Klatt: Nonlinear control of a nonminimum-phase CSTR. In: *Proceedings of the American Control Conference*, (ed. IEEE), San Francisco, United States of America, (1993), 2941–2945. DOI: 10.23919/acc.1993.4793439.

[27] J.G. van de Vusse: Plug-flow type reactor versus tank reactor. *Chemical Engineering Science*, **19**(12), (1964), 994–996. DOI: 10.1016/0009-2509(64)85109-5.

[28] T. Ross: *Fuzzy Logic with Engineering Applications*. John Wiley & Sons, Ltd., West Sussex, 2008.

[29] P. Woolf: *Chemical Process Dynamics and Controls*. Open textbook library, University of Michigan Engineering Controls Group, 2009.

[30] J. Verhaegh, F. Kupper and F. Willem: Frequency response based multivariable feedback control design for transient RCCI engine operation. *IFAC PapersOnLine*, **53** (2020), 14008–14015. DOI: 10.1016/j.ifacol.2020.12.921.

[31] J. Koo, D. Park, S. Ryu, G. Kim and Y. Lee: Design of a self-tuning adaptive model predictive controller using recursive model parameter estimation for real-time plasma variable control. *Computers & Chemical Engineering*, **123** (2019), 126–142. DOI: 10.1016/j.compchemeng.2019.01.002.

[32] E. J. Herrera-López, B. Castillo-Toledo, J. Ramírez-Córdova and E. Ferreira: Takagi-Sugeno fuzzy observer for a switching bioprocess: Sector nonlinearity approach, In: *New Developments in Robotics Automation and Control*. Ed. Alex Lazinica, Intech, Shangai, China, (2008) 155–180.

[33] E. Chavero-Navarrete, M. Trejo-Perea, J. Jáuregui-Correa, R. Carrillo-Serrano, and G. Ríos-Moreno: Expert control systems implemented in a pitch control of wind turbine: A review. *IEEE Access*, **7** (2019), 13241–13259. DOI: 10.1109/ACCESS.2019.2892728.

[34] Z. Lendek, T. Guerra, R. Babuşka and B. De-Shutter: *Stability Analysis and Nonlinear Observer Design Using Takagi-Sugeno Fuzzy Models*. Studies in Fuzziness and Soft Computing, **262** Springer, India, 2010.

[35] M. Chadli, H. Karimi and P. Shi: On stability and stabilization of singular uncertain Takagi-Sugeno fuzzy systems. *Journal of the Franklin Institute*, **351** (2014), 1453–1463. DOI: 10.1016/j.jfranklin.2013.11.008.

[36] Purtojo and Wahyudi: Integral anti-windup scheme of full-state feedback control for point-to-point (PTP) positioning system. In: *International Conf. on Electronic Design*, Ed. IEEE, Penang, Malaysia, (2008), 1–6. DOI: 10.1109/ICED.2008.4786777.

[37] W. Liu, Y. Zheng, Q. Chen and D. Geng: An adaptive CGPC based anti-windup PI controller with stability constraints for the intermittent power penetrated system. *International Journal of Electrical Power & Energy Systems*, **130** (2021), 106922. DOI: 10.1016/j.ijepes.2021.106922.

[38] P. Bui, S. You, H. Kim and S. Lee: Dynamics modelling and motion control for high-speed underwater vehicles using H-infinity synthesis with anti-windup compensator. *Journal of Ocean Engineering and Science*, 2021, Preprint. DOI: 10.1016/j.joes.2021.07.002.

[39] K. Ogata: *Modern Control Engineering*. 5th edition, Pearson Education, Inc., Upper Saddle River, New Jersey, 2010.

[40] M. Grant and S. Boyd: CVX: Matlab software for disciplined convex programming. Version 2.0 beta, 2013, Available from: http://cvxr.com/cvx/download/.

[41] R. Sree and M. Chidambaram: Simple method of tuning PI controllers for stable inverse response systems. *Journal of the Indian Institute of Science*, **83** (2003), 73–85.

[42] J. Jeng and S. Lin: Robust proportional-integral-derivative controller design for stable/integrating processes with inverse response and time delay. *Industrial & Engineering Chemistry Research*, **51** (2012), 2652–2665. DOI: 10.1021/ie201449m.

[43] A. Ma'Arif, A. Cahyadi, S. Herdjunanto and O.Wahyunggoro: Tracking control of high order input reference using integrals state feedback and coefficient diagram method tuning. *IEEE Access*, **8** (2020), 182731–182741. DOI: 10.1109/ACCESS.2020.3029115.