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Research paper

Randomized Earned Value Method for the rolling assessment of construction projects advancement

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Abstract: The Randomized Earned Value Method enable to control the time and cost of works during the implementation of a construction project. The method allows to assess the compliance of the current advancement in time and actually incurred costs with the adopted plan. It also allows to predict the date and amount of the project completion costs. Individual assessment indicators (BCWS, BCWP, ACWP) are calculated after the ongoing control of the progression of works. In the case of randomly changing of implementation conditions, the calculated in this way values of the indicators may be unacceptable because of overlarge differences in comparison to actual values. Therefore, it is proposed an EVM enhancement and additional risk conditions analysis. In this approach data from the quantity survey of works are randomized based on analysis of variations between actually measured and planned values of duration and cost of implemented works. It is estimated the randomized values of individual indicators after successive controls of the progress of works. After each project advancement control the duration and cost of the works that remain to be performed are estimated. Moreover, new verified overall time and total cost of the project implementation are also estimated. After the last inspection, randomized values of the final date and total cost of completion of the project are calculated, as well as randomized values of time extension and total cost overrun. Of course, for randomized values, standard deviations of individual quantities are calculated. Therefore, the risk of time and the risk of cost of the project implementation are presented in the risk charts. The proposed approach provides a better assessment of the progress of works under risk conditions. It is worth to add that the method does not require significant changes to the typical construction management process, however, it ensures realistic consideration of the influence of random factors on the course and results of individual works and the entire project.

Keywords: construction project, duration, cost, advancement, rolling assessment

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1. Introduction

Construction project means here arrangements and implementation of construction works with the aim of erecting, repair or overhaul of construction objects such as tunnels, bridges, roads, highways, building houses, high-rise buildings, office buildings, pipelines, and the like. Such projects are developed in several stages. The stage of works execution at the construction site is particularly important. Because of random disruptive threats a course of the implementation can be unsettled. Thus, the actual works advancement of each performance stage should be periodically controlled. This means, rolling construction site inspections and assessments of the project advancement. Based on the inspection data the advancement of works implementation should be assessed and, as it is required, rightly adjust the schedule and cost estimate to the actual and projected randomly evolved conditions of works implementation. The truthfulness of the advancement assessment, apart from correct data, also requires the use of specialized assessment methods. Well known methods that can be applied for projects advancement state assessment cannot be widely used when projects are implemented in random conditions. The basic problem that is not solved concerns a risk analysis.

It may be a source of errors and wrong assessment of the durability and cost of works. As a consequence, the schedule and the cost estimate can be incorrectly adjusted to the actual and projected works implementation conditions. It is especially important when random disruptive events can strongly affect the course and results of works execution. Consequently, the facility in accordance with design documentation and in compliance with technical specification of performance and acceptance of the work, on schedule, within budget and safety requirements may not be completed.

In practice, three kinds of conditions of the project implementation can be considered, that is: *the conditions of certainty, the conditions of risk and the conditions of uncertainty*.

The conditions of certainty means circumstances in which the type and condition of the activity implementation are known with the reasonable certainty. Such situations are most often considered in practice. It is often a main cause of mistakes in the project plan. In fact, the classic EVM method is also used with tacit assuming deterministic conditions of the works execution.

The conditions of risk means circumstances in which individual capabilities and benefits associated with each possible action are known with some probability – the probability of possible states of the outside world is known or can be estimated. Such situation is considered in developed the randomized EVM method.

The conditions of uncertainty means circumstances in which individual capabilities and benefits of each possible action are unknown or poorly known. In some situations even the type of the acceptable activities cannot be known. Moreover, the risk and possible consequences connected with each activity option also may not be known – the probability of possible states of the outside word is unknown and cannot be estimated.

In the article the conditions of risk have been defined and for them the Randomized Earned Value Method has been worked out. It is assumed that conditions of risk will be identified periodically, from determined time to time, during the project in-situ inspection.

After each inspection an assessment of construction project advancement are elaborated. Such process here has been called the rolling assessment of construction project advancement. This method can be used by investors for the rolling assessment of construction projects advancement during its performance on the construction site. It is predicted that the method deployment will be simple and will not require important changes of the standard project management. As concern scientific side of the method one should mention it is developed on the basis of theory of probability. Especially on the basis of randomization theory. The issues of the theory of probability and theory of randomization used in the method have been explained in details in the process of Randomized Earned Value Method description.

2. Basic assumption and characteristics of the Earned Value Method

Earned Value Management (EVM) is a technique that is used to track the progress and status of a project and forecast its likely future performance. It allows to control delays and acceleration of construction works as well as to estimate their cost and completion date [3, 6, 7, 21, 22].

The Earned Value Management (EVM) method plays an important role in the investment control process, in particular construction [1, 13, 27, 28]. One of the first versions of EVM method was developed by the Defence Department (DoD) – The American National Standards Institute [2]. Since 2005, EVM has been a part of general federal project risk management. Today EVM is a mandatory requirement of the US government. EVM is also used in the private sector by companies in a variety of industries [11, 24], also construction industry [4, 5, 8, 18]. It should be emphasized that, compared to the traditional approach to the control of construction investments, EVM allows not only to combine the assessment of the progress of construction works with the current financial result of the investment, but also by using indices, gives the possibility to forecast future trends in the pace of construction works on individual investment tasks and costs incurred [16].

Based on the analysis of the current state of knowledge and applications of the Earned Value Management [9, 10, 19, 20, 26], it can be determined that it has many advantages, including:

1. This is one of the investment control methods that is particularly useful during the execution phase of works on the construction site.
2. It allows to determine the degree of advancement of works in time and control the investment budget in accordance with it.
3. It also makes it possible to determine the delay or acceleration of works in time.
4. It allows to combine a temporary assessment of the progress of works with their financial advancement against the planned values.
5. It allows to estimate the final cost and completion date of the investment based on the trends that have emerged in the implementation of the facility so far.

6. Some indicators of the EVM method allow to assess the situation at a given moment of the investment, while others allow to forecast the situation at the time of completion of the investment based on the situation during the works.

Unfortunately, like most methods, it also has its drawbacks:

1. The analysis concerns only deterministic data.
2. The analysis does not use control data that describe random times and costs of works obtained under real disturbance conditions.
3. Lack of analysis of the work time, as the time analysis is actually estimated on the basis of cost data.
4. The main problem that may arise is the detailed investment data necessary to obtain.
5. The analysis of investments in deterministic conditions does not allow taking into account probabilities, random events and the most frequently related risks with a delay in time and an increase in the cost of works.

Accordingly, there was a need for a probabilistic analysis for the detailed control of construction projects. The conducted research, discussed in this article, shows that the randomized EVM method and the precise determination of individual risk factors for a specific investment in combination with the probabilistic method enable a more detailed and precise control of the construction investment and the final values ??of works forecast with greater efficiency.

3. Comprehensive analysis of the construction project advancement

3.1. Analysis of the construction project advancement

Construction works should be executed according to the schedule and cost estimate, and according to the contractual quality requirements. Because of influence of random events, the course of the works may be disturbed. For this reason the accomplishment of works may randomly delay, costs may coincidentally increase and completion of works quality may deteriorate. In order to prevent such unfavorable changings, the advancement and actual state of works should be periodically identified during consecutive inspections that is, during the works rolling assessments at the construction site. Based on quantity survey the actual scope and costs of works should be estimated. Usually, as a result of random threats impact, durations and costs of individual works as well as the final deadline and the total cost of the project may require adjustments. This means that the schedule and cost estimate of project may also require verification.

3.2. Randomized Earned Value Method

Randomized Earned Value Method (REVM) can be directly used in the random project implementation conditions. Considering mentioned above explanations and assumptions the method provide comprehensive analysis of the advancement of construction works execution. The method can be used for assessment of different construction projects advancement. Here, the method is generally described and additionally explained with the example of small bridge erection. It can be done in the following steps:

Step 1: Identification of the original project implementation data:

1. Modelling of the construction object structure technology, that is the construction technology of the object erected within the project:

$$S = \langle G, L \rangle \tag{3.1}$$

$G = \langle Y, U, P \rangle$ – coherent and a-cyclic unigraph with a single initial node and a single final node that describes interdependence and permissible sequence of the works execution,
$Y = \{y_1, \ldots, y_i, \ldots, y_k, \ldots, y_m\}$ – set of the nodes of the graph representing events where works begin or end that is, indicates beginning or completing individual works,
$U = \{u_1, \ldots, u_j, \ldots, u_l, \ldots, u_n\}$ – set of the arcs (arrows) of the graph representing relatively independent works (activities) that are constrained by initial $y_i \in Y$ and final $y_k \in Y$ nodes,
$P \subset Y \times U \times Y$, $\langle y_i, u_j, y_k \rangle \in P$ – three-term relation that assigns to each arc $u_j \in U$ the initial node $y_i \in Y$ and final node $y_k \in Y$,
$L : U \to R^+$– function defined on the set U of arcs of the graph G which describes bill of quantities of works $u_j \in U$, $l = \{l_1, \cdots, l_j, \cdots, l_n\}$.

2. Modelling of the construction object performance technology that is, technology of works that are performed within the project:

$$\mathcal{L}\{\langle H, K, T \rangle, S\}, \tag{3.2}$$

$H = \{H^1, \ldots, H^r, \ldots, H^S\}, H^r = \{1, \ldots, h, \ldots h^r\}$ – set of rational or optimal task teams H^r for works $u_j \in U^r$ execution, h – rudimentary resources (staff, laborers, tools, machines and etc.),
T: $(H \times U) \to R^+$ – function defined on the set $\boldsymbol{H}$ of teams H^r and the set U of works u_j which determines durations t_j of works $u_j \in U$, $t = \{t_1, \cdots, t_j, \cdots, t_n\}$,
K: $(H \times U) \to R^+$ – function defined on the set H of teams H^r and the set U of works u_j which determines costs k_j of works $u_j \in U$, $k = \{k_1, \cdots, k_j, \cdots, k_n\}$;

3. Duration at completion (DAC) i.e. the total performance time of the project which is equal the earliest time v_m of completion of all works $u_j \in U$ executed within the project:

$$t = \sum_{i=1}^{m} v_i \to \min, \tag{3.3}$$

under the constraints: $v_k - v_i \geq t_j for u_j \in U j = 1, 2, \ldots, n; \langle y_i, u_j, y_k \rangle \in P; v_{i,} v_k \geq 0$

4. Budget at completion (BAC) i.e. the total budget allocated to the project:

$$K = \sum_{j=1}^{n} k_j \tag{3.4}$$

These data and solution for the small bridge erection are presented at Fig. 1, Fig. 2 and Table 1.

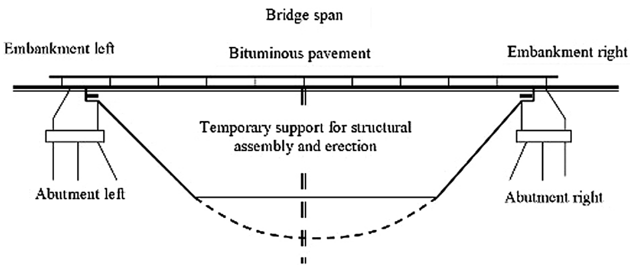

Fig. 1. Bridge model and construction tasks list

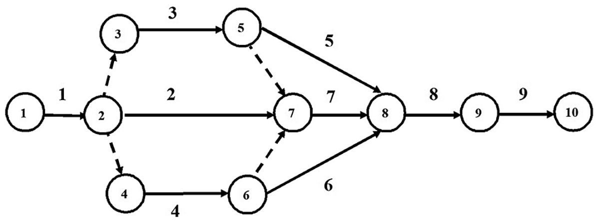

Fig. 2. Graph of the bridge structure technology

Table 1. The design characteristics model of the bridge structure *S* and the bridge construction technology $\mathcal{L}$

	No	Title	y_i	u_j	y_k	H^r	Planned duration (days) T_j	Planned costs (PLN) K_j	Earliest start $ES(Vi)$	Latest start $ES(Vk)$
	1	2	3	4	5	6	7	8	9	10
	1	Land development and preconstruction works	1	1	2	1	44	58560	0	0
	2	Temporary support for structural assembly and erection	2	2	7	2	46	39396	44	44
	3	Abutment left	3	3	5	3	167	1683600	44	44
FI 120	4	Abutment right	4	4	6	4	140	1491111	44	70
	5	Embankment left	5	5	8	5	56	575000	210	210
	6	Embankment right	6	6	8	6	74	528001	184	210
SI 270	7	Bridge span	7	7	8	7	224	2501000	210	210
	8	Bituminous pavement	8	8	9	8	13	20333	434	434
	9	Finishing works	9	9	10	1	49	68625	447	447
		Budgeted Cost (BCC) and Duration (BDC) at Completion						6965627	496	496

Step 1. Planning and organization the inspection of the project implementation:

– organizing an inspection team and defining the rules of control,
– determination the dates of rolling inspections: $t^I = \{t^{FI}, t^{SI}, t^{TI}, \cdots\}$, e.g. *Date of the First Inspection (FI), Second Inspection (SI), Third Inspection (TI)* etc.

Step 2. First step of rolling inspection:

1. Quantity survey of works performed:

– the set U^{FI} of works u_j^{FI} that have been started and partly completed by the date t^{FI}:

$$U^{FI} = \left\{\cdots, u_f^{FI}, \cdots, u_j^{FI}, \cdots\right\} \tag{3.5}$$

– the set L^{AC} of current quantity survey of works $u_j^{FI} \in U^{FI}$ that is the total scope of works u_j^{FI} performed by the reporting date t^{FI}:

$$L^{AC} = \left\{\cdots, l_f^{AC}, \cdots, l_j^{AC}, \cdots\right\} \tag{3.6}$$

l_j^{AC} – actual quantity survey of part of work u_j^{FI} that has been started and performed by the date t^{FI} (e.g. m^3);

– the set T^{AC} of actual duration of works performed (ADWP) that is the total time taken to complete the work u_j^{FI} as of a reporting date t^{FI}:

$$T^{AC} = \left\{\cdots, t_f^{AC}, \cdots, t_j^{AC}, \cdots\right\} \tag{3.7}$$

$$t_j^{AC} = l_j^{AC}\pi_j; \vee t_j^{AC} = \frac{l_j^{AC}}{\lambda_j} \tag{3.8}$$

t_j^{AC} – total time spent on part of work u_j^{FI} that has been started and performed by the date t^{FI} (e.g. hours),
π_j – labor consumption (e.g. h/m3),
λ_j – work productivity (e.g. m3/h);

– the set K^{AC} of actual costs of works performed (ACWP) that is the total cost taken to complete the work u_j^{FI} as of a reporting date t^{FI}:

$$K^{AC} = \left\{\cdots, k_f^{AC}, \cdots, k_j^{AC}, \cdots\right\} \tag{3.9}$$

$$k_j^{AC} = \kappa_j l_j^{AC} \quad \text{or} \quad k_j^{AC} = \kappa_j t_j^{AC} \tag{3.10}$$

k_j^{AC} – total cost spent on part of work u_j^{FI} that has been started and performed by the date t^{FI} (e.g. PLN),
κ_j – piece work or hourly rate paid for performed work u_j^{FI} (e.g. PLN/m3 or PLN/h).

2. Bill of quantities and cost estimate of works performed:

– the set T^{PV} of budgeted durations of works scheduled (BDWS) that is the total time taken to complete the work u_j^{FI} as of a reporting date t^{FI}:

$$T^{PV} = \left\{\cdots, t_f^{PV}, \cdots, t_j^{PV}, \cdots\right\}, \tag{3.11}$$

$$t_j^{PV} = \pi_j l_j^{PV}; \vee t_j^{PV} = \frac{l_j^{PV}}{\lambda_j}, \tag{3.12}$$

t_j^{PV} – total time scheduled on part of the work u_j^{FI} that, according to the schedule, should be started and performed by the date t^{FI} (e.g. hours),
l_j^{PV} – bill of quantities of part of the work u_j^{FI} that, according to the schedule, should be performed by the date t^{FI} (e.g. m^3),
π_j – labour consumption (e.g. h/m3),
λ_j – work productivity (e.g. m3/h).

– the set K^{PV} of budgeted costs of works scheduled (BCWS) that is the total cost of the work u_j^{FI} scheduled as of a reporting date t^{FI}:

$$K^{PV} = \left\{\cdots, k_f^{PV}, \cdots, k_j^{PV}, \cdots\right\} \tag{3.13}$$

$$k_j^{PV} = \kappa_j t_j^{PV} \vee k_j^{PV} = \kappa_j l_j^{PV} \tag{3.14}$$

k_j^{PV} – total cost of part of the work u_j^{FI} that, according to the construction works estimate, has been scheduled by the date t^{FI} (e.g. PLN),
κ_j – piece work or hourly rate for performed work u_j^{PV} (e.g. PLN/m^3 or PLN/h),

– the set of budgeted duration of works performed (BDWP) that is the total duration of the work u_j^{FI} completed/performed as of a reporting date t^{FI}:

$$T^{EV} = \left\{\cdots, t_f^{EV}, \cdots, t_j^{EV}, \cdots\right\} \tag{3.15}$$

$$t_j^{EV} = \begin{cases} t_j^{PV} & \text{when } t_j^{AC} \geq t_j^{PV} \\ t_j^{AC} & \text{when } t_j^{AC} < t_j^{PV} \end{cases} \tag{3.16}$$

t_j^{EV} – portion of total time spent on part of work u_j^{FI} that has been started and actually completed by the date t^{FI} (e.g. hours),

– the set of budgeted cost of works performed (BCWP) that is the total cos t of the work u_j^{FI} completed/performed as of a reporting date t^{FI}:

$$K^{EV} = \left\{\cdots, k_f^{EV}, \cdots, k_j^{EV}, \cdots\right\}, \tag{3.17}$$

$$k_j^{EV} = \begin{cases} k_j^{PV} & \text{when } k_j^{AC} \geq k_j^{PV} \\ k_j^{AC} & \text{when } k_j^{AC} < k_j^{PV} \end{cases} \tag{3.18}$$

k_j^{EV} – portion of total cost of part of the work u_j^{FI} that started and actually performed by the date t^{FI} (e.g. PLN).

3. Analysis of works performed before the date t^{FI}:

a) data of time:

– variance – BDWS – ADWP:

$$\Delta t_j^{SV} = t_j^{AC} - t_j^{PV} \quad \text{for } u_j^{FI} \in U^{FI} \tag{3.19}$$

– absolute value of the time variances mass:

$$t^{SV} = \sum_{u_j \in U^{FI}} \left| \Delta t_j^{SV} \right| \tag{3.20}$$

– absolute value of the negative and positive time variances mass:

$$t_n^{SV} = \sum_{u_j \in U^{FI}} \left| -\Delta t_j^{SV} \right| t_p^{SV} = \sum\nolimits_{u_j \in U^{FI}} \Delta t_j^{SV} \tag{3.21}$$

– standard deviation of absolute value of the time variances mass:

$$\Delta t^{SV} = \sqrt{\frac{\sum\limits_{u_j \in U^{FI}} \left(\Delta t_j^{SV} - \Delta \bar{t}^{FI} \right)^2}{\text{card} \left| U^{FI} \right|}} \tag{3.22}$$

card $\left| U^{FI} \right|$ – cardinality of the set U^{FI},

– standard deviation of absolute value of the time variances mass %:

$$\Delta t_{\%}^{SV} = \frac{\Delta t^{SV}}{t^{SV}} 100\%, \quad \Delta t_{\%n}^{SV} = \frac{t_n^{SV}}{t^{SV}} 100\%, \quad \Delta t_{\%p}^{SV} = \frac{t_p^{SV}}{t^{SV}} 100\% \tag{3.23}$$

– coefficients of time optimism and time pessimism:

$$\underline{p}^{FI} = \frac{\Delta t^{SV}}{t^{SV}} \frac{t_n^{SV}}{t^{SV}} \overline{p}^{FI} = \frac{\Delta t^{SV}}{t^{SV}} \frac{t_p^{SV}}{t^{SV}} \tag{3.24}$$

b) data of cost:

– cost variance – BCWS–ACWP:

$$\Delta k_j^{SV} = k_j^{AC} - k_j^{PV} \quad \text{for } u_j^{FI} \in U^{FI} \tag{3.25}$$

– absolute value of the cost variances mass:

$$k^{SV} = \sum_{u_j \in U^{FI}} \left| \Delta k_j^{SV} \right|, \tag{3.26}$$

– absolute value of the negative and positive cost variances mass:

$$k_n^{SV} = \sum_{u_j \in U^{FI}} \left| -\Delta k_j^{SV} \right|, \quad k_p^{SV} = \sum_{u_j \in U^{FI}} \Delta k_j^{SV} \tag{3.27}$$

– standard deviation of absolute value of the cost variances mass:

$$\Delta k^{SV} = \sqrt{\frac{\sum\limits_{u_j \in U^{FI}} \left(\Delta k_j^{SV} - \Delta \overline{k}^{FI} \right)^2}{\text{card} \left| U^{FI} \right|}} \tag{3.28}$$

card $\left|U^{FI}\right|$ – cardinality of the set U^{FI},

– standard deviation of absolute value of the cost variances mass %:

$$\Delta k_{\%}^{SV} = \frac{\Delta k^{SV}}{k^{SV}} 100\%, \quad \Delta k_{\% n}^{SV} = \frac{k_n^{SV}}{k^{SV}} 100\%, \quad \Delta k_{\% p}^{SV} = \frac{k_p^{SV}}{k^{SV}} 100\% \tag{3.29}$$

– coefficients of cost optimism $\underline{p}^{FI}$ and cost pessimism $\overline{p}^{FI}$:

$$\underline{p}^{FI} = \frac{\Delta k^{SV}}{k^{SV}} \frac{k_n^{SV}}{k^{SV}}, \quad \overline{p}^{FI} = \frac{\Delta k^{SV}}{k^{SV}} \frac{k_p^{SV}}{k^{SV}} \tag{3.30}$$

c) data of time and cost of small bridge construction at the date t^{FI}.

4. Data randomization of works to be performed after FI:

a) the set U^{SI} of works to be performed after the date t^{FI}:

$$U^{SI} = U - U^{FI} = \left\{ u_0^{SI}, \cdots, u_f, \cdots, u_j, \cdots, u_n \right\} \tag{3.31}$$

b) normative performance time and cost of works $u_j \in U^{SI}$:

$$T^{SI} = \left\{ t_j : u_j \in U^{SI} \right\}, \qquad K^{SI} = \left\{ k_j : u_j \in U^{SI} \right\} \tag{3.32}$$

t_j – the most probable normative performance time,

k_j – the most probable normative performance cost,

c) PERT-beta distribution parameters of duration and cost of works $u_j \in U^{SI}$:

– time characteristics:

$$t_j^E = \frac{t_j^o + 4t_j^m + t^p}{6} \quad \text{for } u_j \in U^{SI} \tag{3.33}$$

t_j^E – expected duration of works $u_j \in U^{SI}$,

$t_j^o = \left(1 - \underline{p}^{FI}\right) t_j$ – optimistic duration of works $u_j \in U^{SI}$,

$t^p = (1 + \overline{p}\,, | \, FI)\, t_j$ – pessimistic duration of works $u_j \in U^{SI}$,

– cost characteristics:

$$k_j^E = \frac{k_j^o + 4k_j^m + k^p}{6} \quad \text{for } u_j \in U^{SI} \tag{3.34}$$

k_j^E – expected cost of works $u_j \in U^{SI}$,

$k_j^o = \left(1 - \underline{p}^{FI}\right) k_j$ – optimistic cost of works $u_j \in U^{SI}$,

$k^p = (1 + \overline{p} \mid FI)\, k_j$ – pessimistic cost of works $u_j \in U^{SI}$.

5. Projected randomized duration and cost of budgeted works to be completed after the FI:
   - modelling the construction object structure technology S^{SI} and construction object performance technology $\mathcal{L}^{SI}$ after the FI:
     $S^{SI} = \langle G^{SI}, L^{SI} \rangle$, $\mathcal{L}^{SI} = \{\langle H^{SI}, K^{SI}, T^{SI} \rangle, S^{SI}\}$,
     $G^{SI} = \langle Y^{SI}, U^{SI}, P^{SI} \rangle$, $U^{SI} = \{u_0^{SI}, \cdots, u_f, \cdots, u_j, \cdots, u_n\}$,
     $Y^{SI} = \{y_0^{SI}, \cdots, y_i^{SI}, \cdots, y_k^{SI}, \cdots, y_m\}$, H^{SI}, K^{SI}, T^{SI} – similarly as before, but for the sets adequate to SI,
   - randomized budgeted duration to complete (RDTC) i.e. the estimated expected total time v_m^E required to complete the remainder of the project after the FI that is, find the expected earliest date v_m^E of the project completion and the expected earliest starting terms v_i of works $u_j \in U^{SI}$ that remain to be performed after the FI:
     $v = \sum_{i=0}^{i=m} v_i \longrightarrow \min$, under the constraints: $v_k - v_i \geq t_j^E$ for $u_j \in U^{SI}$, $j = 0, \ldots, n$; $\langle y_i, u_j, y_k \rangle \in P^{SI}$; $v_i, v_k \geq 0$,
   - randomized budgeted duration at completion (RDAC) i.e. the estimated expected time T^E of works $u_j \in U$ at the end of the project from works start to finish: $T^E = t^{FI} + v_m^E$,
   - randomized budgeted estimate to complete (RETC) k^E i.e. the estimated expected cost required to complete the remainder of the project after the FI: $k^E = \sum_{u_j \in U^{SI}} k_j^E$,
   - randomized budgeted estimate at completion (REAC) i.e. the estimated expected performance cost K^E of the project at the end of the project from works start to finish: $K^E = k^E + k^{FI}$, where k^{FI} – cost of works performed to the date t^{FI}.
6. Data of time and cost of small bridge construction at the date t^{FI} have been presented at Fig. 3, Table 2 and results of the small bridge construction randomized analysis have been presented at Fig. 4 and Table 3.

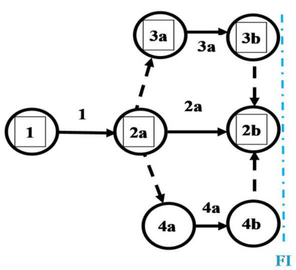

Fig. 3. Graph of the bridge structure technology – part of works performed before the date t^{FI}

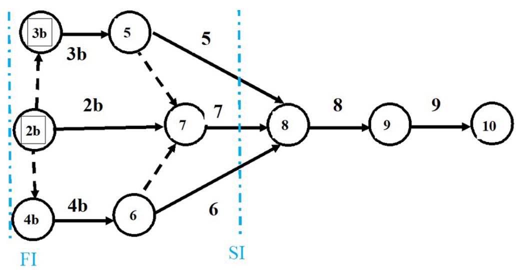

Fig. 4. Graph of the bridge structure technology – part of works performed before the date t^{FI}

Table 2. Randomized data of time and cost of small bridge construction at the date t^{FI} (Fig. 3)

No	Works started but not completed	y_i	u_j	y_k	H^r	Planned duration (days)	Planned cost (PLN)	Earliest start	Latest start	Percent of work performed	BDWS	BDWP	ADWP	Time variance ADWP – BDWS	BCWS	BCWP	ACWP	Cost variance ACWP – BCWS
						T_j	K_j	$ES\ (V_i)$	$LS\ (V_i)$	%								
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	Land development and preconstruction works	1	1	2a	1	44	58 560	0	0	100%	44	40	40	–4	58 560	56 950	56 950	–1 610
2a	Temporary support for structural assembly and erection	2a	2a	2b	2	46	39 396	44	44	100%	46	46	52	6	39 396	39 396	44 778	5 382
3a	Abutment left	3a	3a	3b	3	76	770 276	44	44	46%	76	76	87	11	770 276	770 276	808 488	38 212
4a	Abutment right	4a	4a	4b	4	76	810 741	44	44	54%	76	70	70	–6	810 741	783 387	783 387	–27 354
	Budgeted Duration and Cost of Works Performed to FI	1		2b		120	1 678 972	Mass of time variances		27	10	17	Mass of cost variances			72 558	28 964	43 595
Standard deviation										8.06	Standard deviation					26 997		
Standard deviation %										30%	37%	63%	Standard deviation %			37%	40%	60%
Coefficient of time optimism										0.11	Coefficient of cost optimism					0.15		
Coefficient of time pesymism										0.19	Coefficient of cost pesymism					0.22		

Table 3. Randomized data of works that remain to be performed after FI (Fig. 4)

							Planned values		Earliest start	Randomized duration				Randomized cost			
	No	Title	y_i	u_j	y_k	H^r	Duration	Cost	ES(Vi)	EV	OV	MP	PV	EV	OV	MP	PV
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
FI	FI	Budgeted duration and cost of works performed to FI	1		2b		120	1 678 972	0	120				1 678 972			
	2b	Temporary support for structural assembly and erection	2b	2b	7	2	0	0	0	0	0	0	0	0	0	0	0
	3b	Abutment left	3b	3b	5	3	90	913 324	0	92	80	90	107	924 745	777 675	913 324	1 117 494
	4b	Abutment right	4b	4b	6	4	64	680 371	0	65	57	64	76	688 878	579 320	680 371	832 465
	5	Embankment left	5	5	7	5	56	575 000	92	57	50	56	66	582 190	489 600	575 000	703 540
	6	Embankment right	6	6	8	6	74	528 001	65	75	66	74	88	534 603	449 581	528 001	646 033
SI	7	Bridge span	7	7	8	7	224	2 501 000	92	227	199	224	266	2 532 272	2 129 545	2 501 000	3 060 089
	8	Bituminous pavement	8	8	9	8	13	20 333	318	13	12	13	16	20 588	17 313	20 333	24 879
	9	Finishing works	9	9	10	1	49	68 625	332	49	43	49	58	69 483	58 433	68 625	83 966
Randomized Budgeted Duration to Completion									381	Randomized Budgeted Cost at Completion				5 352 758			
Randomized Budgeted Duration to Completion									501	Randomized Budgeted Cost at Completion				7 031 731			
Randomized Cost of works partly performed before FI														1 613 622			

Step 3 and the next steps are concerned the analysis the rolling assessment of construction projects advancement after the consecutive inspections. The analysis can be done according to the step 3 using data received during the subsequent inspections. Results of such analysis of the small bridge construction have been presented at Fig. 5 and 6, and in Tables 4 and 5.

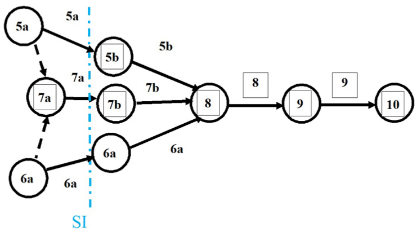

Fig. 5. Graph of the bridge structure technology – part of works started and not performed before SI

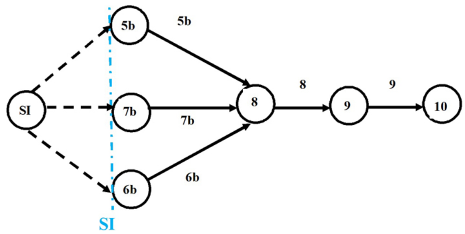

Fig. 6. Graph of the bridge structure technology – part of works to be performed after the date t^{SI}

Finally, the risk of time and the risk of cost can be calculated by using the formulas:

$$p(t) = P\left[E(T) \geq t\right] = 1 - P\left[E(T) \leqslant t\right] = 1 - \Phi\left[\frac{t - E(T)}{\sqrt{D^2(T)}}\right] \tag{3.35}$$

$$p(k) = P\left[E(K) \geq k\right] = 1 - P\left[E(K) \leqslant k\right] = 1 - \Phi\left[\frac{K - E(K)}{\sqrt{D^2(K)}}\right] \tag{3.36}$$

Charts of the risk of time and the risk of cost have been presented at the Fig. 7, 8, 9 and 10.

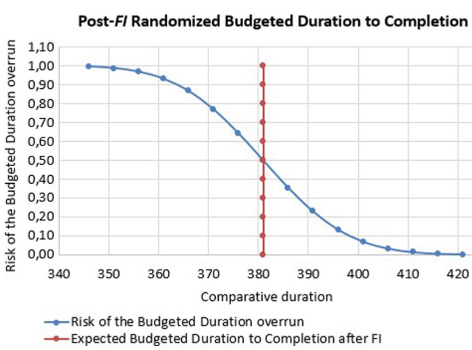

Fig. 7. Chart risk of time after the FI

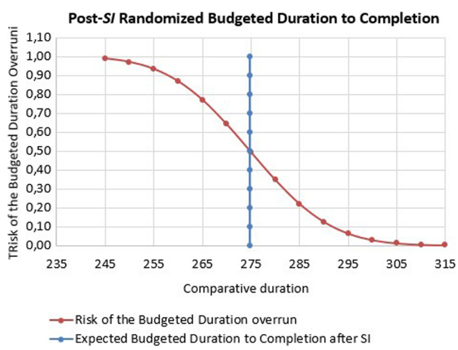

Fig. 8. Chart risk of time after the SI

The small bridge construction advancement assessment after the first and the second inspections has been presented in Table 6.

Table 4. Randomized data of works started but not finished before SI (Fig. 5)

No	Works started but not completed	y_i	u_j	y_k	H^r	Planned duration (days)	Planned cost (PLN)	Earliest start	Latest start	Percent of work performed	BDWS	BDWP	ADWP	Time variance ADWP – BDWS	BCWS	BCWP	ACWP	Cost variance ACWP – BCWS
						T_j	K_j	$ES\ (V_i)$	$LS\ (V_i)$	%								
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
5a	Embankment left	5a	5a	5b	5	28	291 892	92	92	51%	28	28	60	32	90 439	90 439	191 165	100 727
6a	Embankment right	6a	6a	6b	6	55	392 419	65	92	74%	55	55	70	15	15 112	15 112	423 642	408 530
7a	Bridge span	7a	7a	7b	7	28	317 401	92	92	13%	28	28	192	164	8 709	8 709	665 189	656 479
SI	Budgeted works that started before and are being finished after SI	7a		7b		55	1 001 712	Mass of time variances		210	0	210	Mass of cost variances			1 165 736	0	1 165 736
									Standard deviation	81.49					Standard deviation	278 413		
									Standard deviation %	39%	0%	100%			Standard deviation %	24%	0%	100%
									Coefficient of time optimism	0.00					Coefficient of cost optimism	0.00		
									Coefficient of time pesymism	0.39					Coefficient of cost pesymism	0.24		

Table 5. Randomized data of works that remain to be performed after SI (Fig. 6)

						Planned values		Earliest start	Randomized duration				Randomized cost			
No	Title	y_i	u_j	y_k	H^r	Duration	Cost	ES(Vi)	EV	OV	MP	PV	EV	OV	MP	PV
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
FI	Budgeted duration and costs of works performed before FI	1	net	29		120	1 678 972	0					1 678 972			
	Budgeted duration and cost of works that started before and finished after FI	2b	net	7a			1 613 622	120					1 613 622			
	Budgeted works that started before and are being finished after SI	7a	net	7b			1 001 712						1 001 712			
	Second inspection SI					270		270								
5b	Embankment left	5b	5b	8	5	28	283 108	0	29	28	28	38	294 377	283 108	283 108	350 723
6b	Embankment right	6b	6b	8	6	19	135 581	0	20	19	19	26	140 978	135 581	135 581	167 962
7b	Bridge span	7b	7b	8	7	195	2 183 599	0	208	195	195	271	2 270 517	2 183 599	2 183 599	2 705 108
8	Bituminous pavement	8	8	9	8	13	20 588	208	14	13	13	19	21 407	20 588	20 588	25 505
9	Finishing works	9	9	10	1	49	69 483	222	53	49	49	69	72 249	69 483	69 483	86 078
Randomized Budgeted Duration to Completion								275	Randomized Budgeted Cost to Completion				2 799 529			
Randomized Budgeted Duration at Completion								545	Randomized Budgeted Cost at Completion				7 093 836			

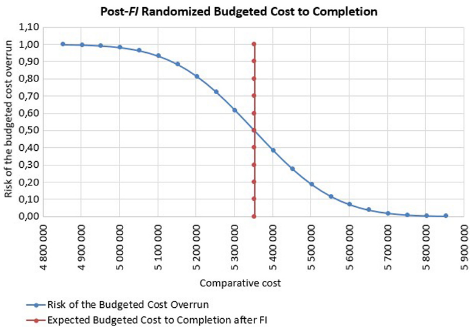

Fig. 9. Chart risk of cost after the FI

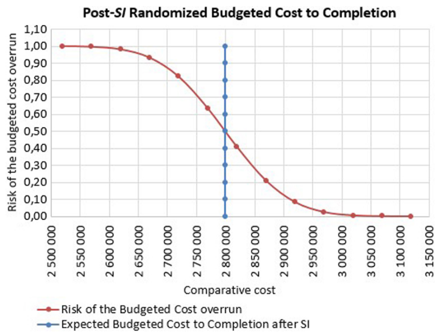

Fig. 10. Chart risk of cost after the SI

Table 6. Final results of the small bridge construction advancement assessment after FI and SI

Duration at Completion	**496**	
Estimate at Completion	**6 965 627**	
Randomized Budgeted Duration to Completion projected after FI	381	
Randomized Budgeted Cost to Completion projected after FI	5 352 758	
Randomized Budgeted Cost at Completion projected after FI	7 031 731	
Randomized Budgeted Duration at Completion projected after FI	501	
Overrun the Budgeted Cost at Completion projected after *FI*	**66 104**	**1.00%**
Overrun the Budgeted Duration at Completion projected after *FI*	**5**	**1.00%**
Randomized Budgeted Duration to Completion projected after SI	275	
Randomized Budgeted Cost to Completion projected after SI	2 799 529	
Randomized Budgeted Cost at Completion projected after SI	7 093 836	
Randomized Budgeted Duration at Completion projected after SI	545	
Overrun the Budgeted Cost at Completion projected after *SI*	**128 209**	**1.84%**
Overrun the Budgeted Duration at Completion projected after *SI*	**49**	**9.81%**

Based on the final results of the small bridge construction efficiency assessment one can confirm the correctness of reasoning but definitive proof of the method correctness should be verified yet. This has been done in the next point of the article.

4. Final conclusion

Randomized Earned Value Method for the rolling assessment of construction projects advancement allow to track a progress of the project performance under the influences of random disturbances. Such threats can be cause an extending of final term and overrun of total cost of the project performance. The results of the advancement of project assessment, received after each inspection, determine the randomized schedule of the works remaining to performance as well as randomized duration and randomized cost of the works completion, and randomized final deadline and total cost of the project. It means that using the method one can projected probable randomized extend of time and randomized increase in price of the project completion. Based on such information, in the early stage of performance it is possible even to cease the project. In the subsequent stages of performance, when project stoppage can be too expensive, it is possible to adjust the project to an actual situation and according to the assessment results.

The method can be improved by using computer-analysed program and big data that describe historically realized construction projects under random conditions of implementation. Such works have been undertaken by the authors.

Summarizing the above:

- Deterministic analysis does not provide exact control of the project implementation in turbulent circumstances.
- There is a need for the probabilistic method of analysis for detailed inspection of the construction project.
- Randomized EVM method and precise determination of individual risk factors for a specific project in combination with the probabilistic assessment method enable a more detailed and accurate control of the construction project and more exact forecast the final values of works.
- The presented research allows to confirm that the identification of the advancement of works by the randomized EVM method together with a probabilistic analysis of the implementation process allows for a better estimate of the actual completion date and total costs of works and the correction of the construction plan.
- The method improves control of the project implementation, including efficiency of analysis and more real knowledge of the works performed and the future, adjusted costs and deadlines for the execution of individual works and the entire project.
- The risk assessment of construction works consists in the analysis of a random execution situation and random characteristics of works, defining threats and implementation opportunities, calculating the randomized time and randomized costs of works, and estimating the probability of exceeding or not exceeding various contractual values ??of time and costs of works in the anticipated conditions of implementation.
- The risk assessment of construction works is the last stage of the analysis before making the final decision on the correction of the works and possible assumptions regarding the permissible values of shortening or extending the time and reducing or increasing the costs of works. The risk assessment is the basis for a realistic estimate of the likely benefits or losses of the investor and the contractor in connection with the performance of the construction works contract.

All mentioned above problems are still considered by Authors. The analysis carried out so far show that the method can be significantly improved, mainly due to the efficiency of its use. Such improvement is the basis for the method application in the practice of operational management of construction projects.

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Randomizowana metoda wartości wypracowanej dla kroczącej oceny zaawansowania przedsięwzięć budowlanych

Słowa kluczowe: przedsięwzięcie budowlane, czas trwania, koszt, zaawansowanie, ocena krocząca

Streszczenie:

Randomizowana metoda wartości wypracowanej pozwala kontrolować czas i koszty robót podczas realizacji przedsięwzięcia budowlanego. Metoda pozwala ocenić zgodność aktualnego tempa i rzeczywiście poniesionych kosztów z przyjętym planem. Pozwala prognozować datę i wielkość kosztów zakończenia przedsięwzięcia. Poszczególne wskaźniki (BCWS, BCWP, ACWP) obliczane są po bieżącej kontroli postępu robót. W przypadku losowo zmiennych warunków realizacji różnice obliczonych i rzeczywistych wartości stosowanych wskaźników ocenowych mogą być nie do zaakceptowania, dlatego zaproponowano rozszerzenie EVM o analizę warunków ryzyka. Proponuje się po kolejnych kontrolach postępu robót randomizację danych z kontroli oraz szacowanie randomizowanych wartości poszczególnych wskaźników. Po każdej kontroli postępu robót obliczane są randomizowane wartości czasu trwania i kosztów przedsięwzięcia, które pozostały do zakończenia przedsięwzięcia oraz randomizowane wartości czasu i całkowitych kosztów wykonania całego przedsięwzięcia. Po ostatniej kontroli obliczane są randomizowane wartości końcowego terminu i całkowity koszt zakończenia przedsięwzięcia oraz randomizowane wartości wydłużenia czasu i przekroczenia całkowitych kosztów przedsięwzięcia. Ryzyko czasu i kosztów realizacji przedsięwzięcia przedstawiane są na wykresach ryzyka, odpowiednio czasu i kosztów. Oczywiście, dla wartości randomizowanych obliczane są odchylenia standardowe poszczególnych wielkości. Proponowane podejście zapewnia lepszą ocenę postępu robót w warunkach ryzyka. Metoda nie wymaga istotnych zmian procesu kierowania budową, jednak zapewnia realistyczne uwzględnienie wpływu czynników losowych na przebieg i wyniki poszczególnych robót i całości przedsięwzięcia.

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