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## Prediction of mineral product price based on mean reversion model

### Introduction

The price change of mineral products directly determines the value behavior of mining planning. Accurately predicting the price change of mineral products can better provide a strong decision-making basis for the initial planning of mine-project investment and construction, and improve the anti-risk ability of enterprises (Savolainen et al. 2022). Traditional time-series models (Shrestha and Bhatta 2018) are methods that use historical data to reveal the development, direction and trend patterns of phenomena over time in order to make predictions about long-term conditions. However, time series are in reality non-stationary, so the long-term mineral-product price prediction rarely uses time series as well as supply and demand prediction methods. Additionally, the deterministic prediction results obtained

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by traditional time-series prediction methods make it difficult to quantitatively evaluate the uncertainty and risk level of mineral-product price through statistical analysis.

Currently, time-series methods, regression models, fuzzy analysis, neural networks, machine learning, and various types of hybrid forecasting models (Liu and Long 2020; Garcia et al. 2018; Livieris et al. 2020) have been applied to the prediction of mineral prices, but there are few studies of stochastic fluctuations and uncertainties in the prediction of mineral-product prices.

Wadi et al. (Wadi et al. 2018) utilized the autoregressive integrated moving average (ARIMA) model in predicting the closed time-series data which shows that the ARIMA model has significant results for short-term prediction. Mohamed (Mohamed 2020) presented an ARIMA model and regression with an ARIMA errors models to forecast the monthly CPI in Somaliland for the May 2020–April 2021 period and selected the best-fitting model based on how well the model captures the stochastic variance in the data. It was observed that the ARIMA (0, 1, 3) model is a reasonable and acceptable model for forecasting Somaliland's CPI. Yao and Wang (Yao and Wang 2021) propose a hybrid forecasting model combining LSTM with GM (1,1) models to analyze monthly, weekly and daily data of international crude-oil price series. It shows that the GM (1,1) model based on the rolling prediction method is more sensitive to new information and has better performance. Rathnayaka and Seneviratna (Rathnayaka and Seneviratna 2019) propose a Taylor series approximation and unbiased GM (1,1)-based new hybrid statistical approach (HTS\_UGM (1,1)) for forecasting gold-price demands. With regard to the traditional time-series approaches, it suggested that HTS\_UGM (1,1) is more suitable and appropriate for handling incomplete, noisy and uncertain data in multidisciplinary systems. Liu et al. (Liu et al. 2022) developed a hybrid neural network with Bayesian optimization and wavelet transform to forecast the copper price. The results indicated that the hybrid methods, either LSTM or GRU, can appropriately predict the copper price in both the short- and long-term with the mean squared error both below 3%.

However, the above research shows that due to the influence of national policies, the supply and demand relationship, production technology and other factors on the price of mineral products, there are irregular stochastic changes in each time increment, and the prediction results are usually not unsatisfied (Ramos et al. 2019).

The research on the uncertainty analysis of mineral product prices started late. Postali and Picchetti (Postali and Picchetti 2006) present a quantitative analyses model of the oil-price path based on GBM. Savolainen et al. (Savolainen et al. 2022) propose a complementary approach by combining block sequencing software and the GBM modelling of metal prices to build a managerial decision-making system for mine-plan selection. Rubaszek et al. (Rubaszek et al. 2020) present an analysis of the dynamics of real prices for the main industrial metals: aluminum, copper, nickel and zinc. The results provide ample evidence that mean-reverting models deliver significantly better forecasts than the naive random walk.

However, the current literature incorporating the uncertainty analysis into mineral-price prediction models is relatively limited and mostly focuses on the qualitative analysis stage.

In order to further study the prediction and uncertainty analysis of mineral product prices, this paper proposed a mineral product-price-prediction model based on the mean reversion process, and uses the Monte Carlo simulation method to perform the simulation of the stochastic process of prices. The effectiveness of the proposed model applied to mineral product-price forecasting is demonstrated by comparing the results of the prediction model with actual data. Meanwhile, the role of the mean reversion model in data support and quantitative risk decision-making in price uncertainty analysis is illustrated through probabilistic methods.

## 1. Methodology

### 1.1. Mineral product-price-fluctuation characteristics

As special resource commodities, mineral products have the properties of ordinary commodities and follow conventional market value laws, while also having certain characteristics of their own. Under the cross influence of various micro and macro factors such as the scarcity of mineral resources, substitutability, market supply and demand, production technology, costs and national economic policies, the prices of mineral products are bound to show complex changes and fluctuations (Gleich et al. 2013; Labys et al. 1999; Chen et al. 2019).

#### (1) Complex long-/short-term price fluctuation trends

In a long-time dimension, the market price of a mineral product is a function of the mineral product cost, which is reduced to some extent with the technological development and the expansion of economy. At the same time, the demand for a mineral product is constantly increasing according to the long-term development needs of society. Therefore, driven by a combination of both of these factors, the mineral product prices are the result of a balance between the two in the long-term dimension.

Due to short-term irregular changes in the supply and demand of mineral products, while influenced by national mining policies, the international political situation and the global economic situation, it often has a sudden short-term uncontrollability, thus having an impact on the price of a mineral product with significant fluctuations in the short term. Short-term fluctuations tend to exhibit mean-reversibility, trend inelasticity and high-rate changes at some time points.

#### (2) Mineral product price changes show stochasticity

In the complex system that affects the fluctuation and change of mineral product prices, the correlation relationship of various factors cannot be fully explained by the existing economic theory, in other words, the relationship between some influencing factors and the mineral product price cannot be determined. For example, the relationship between mineral-

-product price and costs, quality can be determined, while the relationship with economic, political, policy, military and other nonquantitative influencing factors cannot be determined, which leads to considerable uncertainty and stochasticity in mineral-product price changes.

## 1.2. Mean reversion model

### 1.2.1. Mean reversion theory

In the financial options theory, it is usually assumed that the pattern of stock-price behavior obeys the geometric Brownian motion (GBM) (Fama and French 1988). Geometric Brownian motion is a special type of Markov process (Dynkin 2012), which indicates that the future predicted value is only related to the current value of the variable. Because real options have their own special characteristics, the differences in market mechanisms dictate that sometimes, the pattern of asset-price behavior for real options cannot be described by simply applying geometric Brownian motion. Therefore, many researchers believe that the movement of real asset prices is gradual, and its future prices are to some extent predictable (Obthong et al. 2020). Furthermore, their market prices will exhibit mean-reversion characteristics with a high probability in the long term (Poterba and Summers 1987).

The mean-reversion model considers that the research object follows the mean-reversion process – the price changes are fluctuating up and down around its long-term equilibrium price (Schwartz 1997). The mean-reversion process is characterized by inevitability, periodic uncertainty, and asymmetry. Inevitability means that over the long-term, prices cannot always deviate from the value pivot and will definitely revert to their intrinsic value, which is mean-reverting in nature. Periodic uncertainty means that the period of each mean reversion is uncertain, showing a random walk nature. Asymmetry refers to the fact that in the mean-reversion process, positive and negative returns return at different rates, with the rate and magnitude of negative returns significantly greater than that of positive returns. Domestic and foreign scholars have performed research on energy-product-price prediction based on the mean-reversion model, which confirms this result to some extent. Related studies also pointed out that when mineral product prices are close to the marginal cost in the long term, they are bound to reach a constant level even if there is a short-term deviation, which is in line with the characteristics of mean reversion.

Assuming that  $t$  is the time and  $S$  is the price of mineral product, then the A common specification for the mean-reversion process can be expressed as Equation (1):

$$dS = \alpha(p - S)dt + \sigma S^\gamma dz \quad (1)$$

Where  $\alpha$  is the mean reversion rate, which represents the speed of reversion to the long run equilibrium level;  $p$  is the mean level of  $S$ , which can represent the price's historical

trend;  $dz$  is a Wiener process,  $dz = \varepsilon\sqrt{dt}$ , and  $\varepsilon$  is a random value selected from the standard normal distribution. When  $\gamma = 0$ , that is the most conventional mean reversion model, also known as Ornstein–Uhlenbeck process, shown as Equation (2):

$$dS = \alpha(p - S)dt + \sigma dz \quad (2)$$

However, Equation (2) is not suitable to represent the evolution of a commodity price since it allows negative prices. Therefore, a variation of this process when  $\gamma = 1$  is developed as Equation (3):

$$dS = \alpha(p - S)dt + \sigma Sdz \quad (3)$$

The discrete formula of Equation (3) can be written as Equation (4):

$$\Delta S = \alpha(p - S)\Delta t + \sigma S\varepsilon\sqrt{\Delta t} \quad (4)$$

The mean-reversion model presents very useful properties – both supply and demand tend to show a slow speed of adjustment in response to disequilibria of prices. Meanwhile, the mean-reverting process does not have a constant expected growing rate. This process is more realistic in the extent that the growing rate responds to deviations of spot prices from their average levels.

### 1.2.2. Parameter determination

The parameters in the mean regression equation are calculated as follows:

- ◆  $p$  is the mean level of  $S$ , and therefore, the average price of mineral product  $\bar{S}$  can be taken as  $p$ .
- ◆  $\alpha$  is the mean reversion rate. The higher  $\alpha$ , the higher the speed of reversion. Its formula is as Equation (5):

$$\alpha = \frac{\sum_{i=1}^n \frac{S_{i+1} - S_i}{\bar{S} - S_i}}{n} \quad (5)$$

Where  $S_i$  is the historical price of mineral product;  $\bar{S}$  is the average price of mineral product during a period;  $n$  is the number of data.

- ◆  $\sigma$  is the volatility, meaning an increasing expected price variability as the time horizon increases. Its estimated value can be obtained from historical data. The formula is as Equation (6):

$$\sigma = \frac{S}{\sqrt{\tau}} \quad (6)$$

where  $\tau$  is the unit interval length.

Based on the above analysis, the key parameters of the mean-reversion model can be determined based on the historical data of mineral-product prices. For Equation (4),  $p$ ,  $\alpha$  and  $\sigma$  are known,  $\Delta t$  is the time interval of price change. Therefore, stochastic simulation of mineral product price can be performed based on the mean-reversion model using the Monte Carlo method. It can obtain multiple sets of predictions and probability distributions of mineral-product price at different moments in time.

## 2. Case study

### 2.1. Data description

To study the prediction model and risk analysis of mineral-product price, this paper takes the prediction and uncertainty analysis of gold price as an example. The principles and methods are also applicable to other mineral products.

The trend of daily gold spot trading prices from January 2018 to December 2021 is shown in Figure 1. From the historical data, the change of gold price over time has complex randomness and uncertainty, showing irregular changes, which is more similar to stock price fluctuations, and its future movement trend is more difficult to predict.

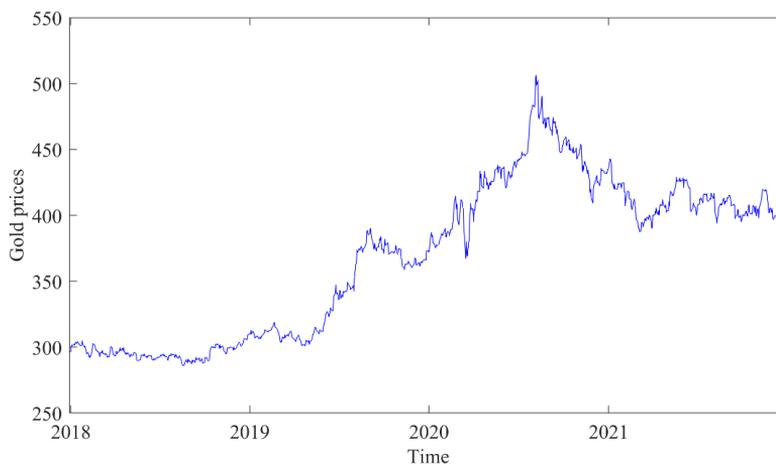


Fig. 1. Original gold price trend from January 2018 to December 2021 (RMB/gram)

Rys. 1. Pierwotny trend cen złota od stycznia 2018 do grudnia 2021 r. (RMB/gram)

## 2.2. Gold-price prediction based on MR

According to this original gold price, the data from January 2018 to December 2021 is used to predict the gold price in 2021. Under the given conditions, the gold price is predicted based on the mean-reversion (MR) model, and the Monte Carlo simulation process is implemented by MATLAB (MATLAB 2016).

### (1) Parameters: gold price mean level $p$ , mean reversion rate $\alpha$ , volatility $\sigma$

- ◆  $p = 355.39$ ,
- ◆  $\alpha = 0.012$ ,
- ◆  $\sigma = 0.035$ .

### (2) Prediction results based on the MR model

Based on the parameter estimation of the mean-reversion model, 1000 stochastic simulations of the gold price trend in 2021 were conducted using the Monte Carlo method and twenty-five stochastic simulations were taken to display the results, as shown in Figure 2.

The distribution of the stochastic simulation results of gold prices for each month is analyzed, as shown in Figure 3. Meanwhile, the prediction results of the gold price in 2021 are tabulated by monthly mean values, as shown in Table 1.

From Figure 2, Figure 3 and Table 1, traditional time-series models can only obtain a prediction result based on the estimation. The Monte Carlo simulation method based on MR models can obtain a set of prediction results with the same probability based on the probability distribution of input variables in the case of uncertain future market conditions.

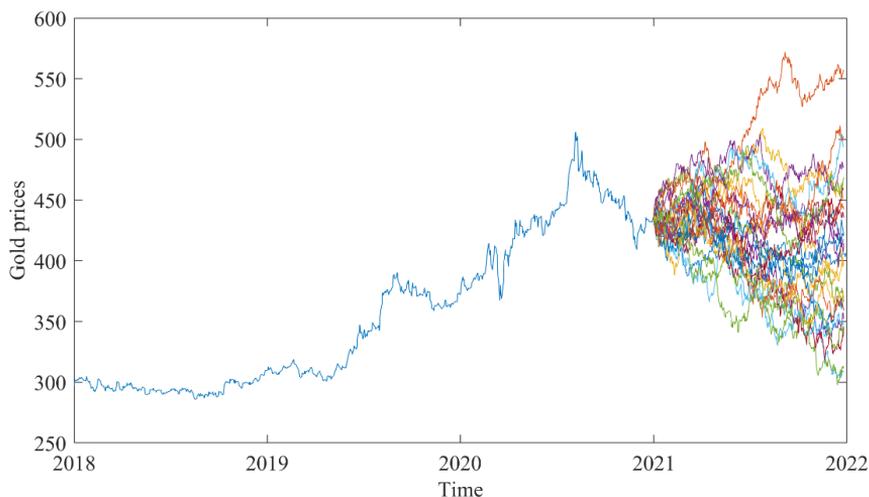


Fig. 2. Prediction results of the gold price in 2021 (RMB/gram)

Rys. 2. Wyniki prognozy ceny złota w 2021 r. (RMB/gram)

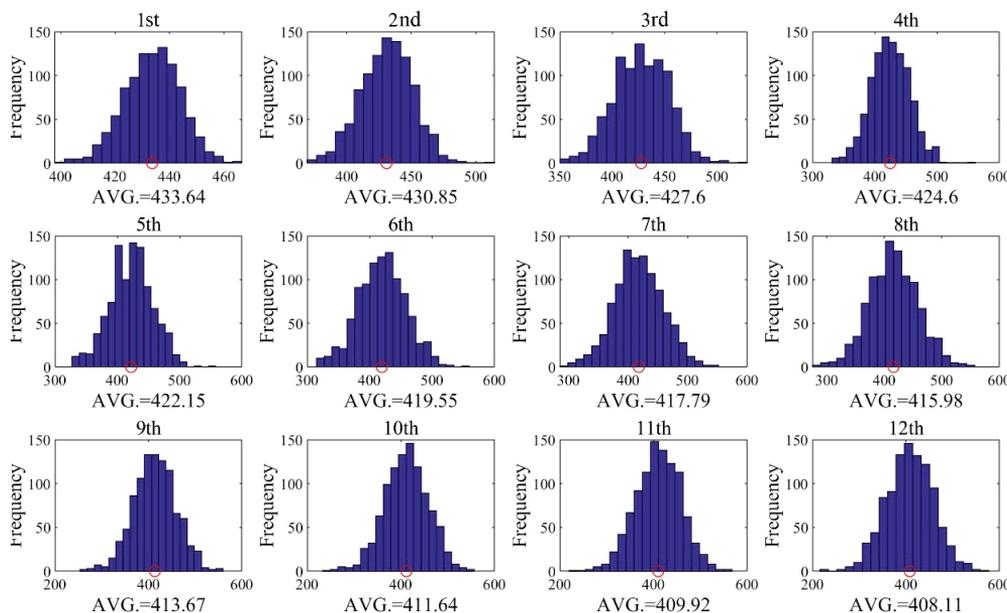


Fig. 3. Monthly statistics of gold price prediction results (RMB/gram)

Rys. 3. Miesięczne statystyki wyników prognoz cen złota (RMB/gram)

Table 1. Gold price prediction results based on MR

Tabela 1. Wyniki prognozowania ceny złota na podstawie MR

Time	Observation	Mean	Error	Maximum	Minimum	Variance	Standard deviation	Skewness	Kurtosis
M1	426.12	433.64	1.76%	467	398	106	10	-0.157	0.057
M2	411.62	430.85	4.67%	514	369	408	20	-0.091	0.085
M3	394.49	427.60	8.39%	529	350	690	26	-0.016	-0.019
M4	404.97	424.60	4.85%	561	331	971	31	0.024	0.139
M5	420.39	422.15	0.42%	558	327	1,199	35	-0.036	0.022
M6	415.62	419.55	0.95%	560	315	1,454	38	-0.009	-0.023
M7	412.67	417.79	1.24%	552	285	1,674	41	0.018	0.046
M8	407.58	415.98	2.06%	557	275	1,845	43	0.006	0.188
M9	404.74	413.67	2.21%	560	254	2,045	45	-0.096	0.163
M10	402.69	411.64	2.22%	557	233	2,184	47	-0.168	0.233
M11	410.22	409.92	-0.07%	568	219	2,339	48	-0.165	0.185
M12	401.32	408.11	1.69%	577	216	2,500	50	-0.098	0.162

Statistical analysis shows that the overall price forecasts are better and the relative errors remained low (11 items with relative errors below 5%). Meanwhile, the results of the stochastic simulation of gold prices within each period are normally distributed, which proves the validity of the MR parameter estimation.

## 2.3. Results and discussion

### 2.3.1. Comparison and analysis of gold-price prediction results

To further study the mineral-product price prediction from the perspective of uncertainty, the gold price fluctuation process is assumed to follow the geometric Brownian motion (GBM) and time-series (TS) models. The unknown parameters are estimated by the maximum likelihood estimation method, and the stochastic process is simulated by using Monte Carlo simulation.

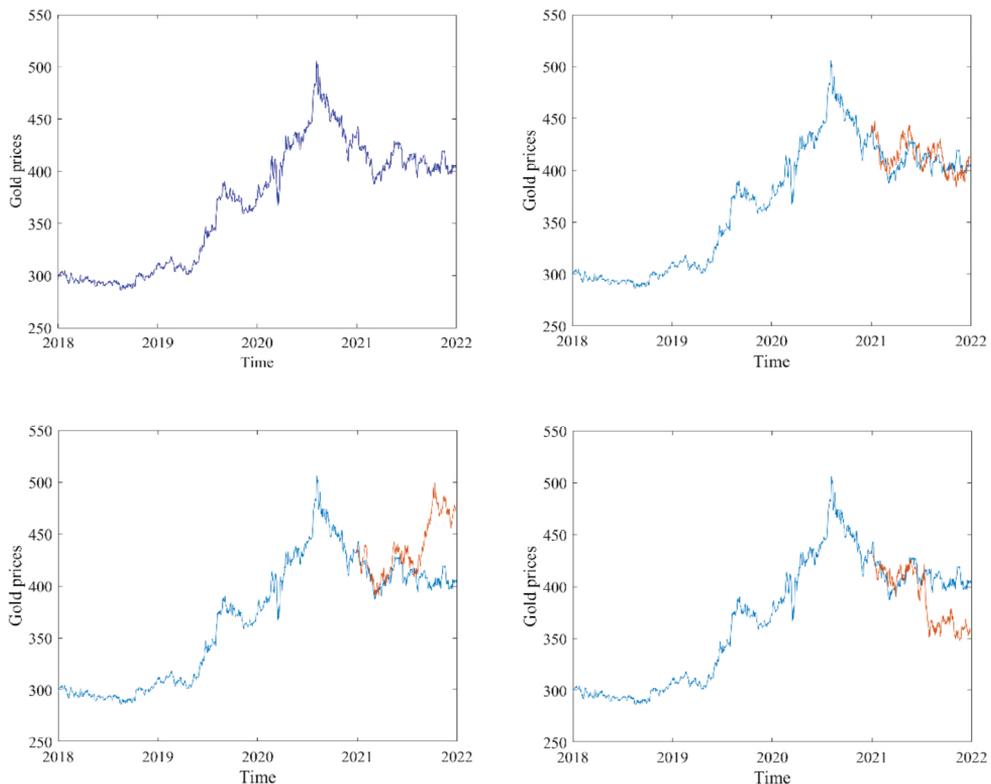


Fig. 4. Comparison of three prediction results

(1) observed values; (2) average trend of MR results; (3) average trend of GBM results; (4) the trend of TS results

Rys. 4. Porównanie trzech wyników predykcji

(1) obserwowanych wartości; (2) średni trend wyników MR; (3) średni trend wyników GBM; (4) trend wyników TS

Based on the results of the previous section, the trend comparison between the actual observed values and the three-price prediction models is shown in Figure 4. From left to right and from top to bottom, the four trajectory diagrams are observed values (OV), mean reversion (MR) result, geometric Brownian motion (GBM) result and the time series (TS) result, respectively. It can be intuitively seen that the simulated numerical trajectories in the mean-reversion process are the closest to the observed values.

The statistical indicators comparison of three prediction results are as shown in Table 2. There are eleven items with relative errors less than  $\pm 5\%$  in the MR results, eight items with relative errors less than  $\pm 5\%$  in the GBM results, and seven items with relative errors less than  $\pm 5\%$  in TS model results, further reflecting the accuracy of the MR model in gold-price prediction.

Tabela 2. Porównanie wskaźników statystycznych trzech wyników predykcji

Table 2. Statistical indicators comparison of three prediction results

Time	OV	MR	Error	GBM	Error	TS	Error
M1	426.12	433.64	1.76%	426.18	0.01%	419.94	-1.45%
M2	411.62	430.85	4.67%	418.11	1.58%	416.72	1.24%
M3	394.49	427.60	8.39%	398.81	1.10%	404.28	2.48%
M4	404.97	424.60	4.85%	409.19	1.04%	412.01	1.74%
M5	420.39	422.15	0.42%	429.70	2.21%	418.18	-0.53%
M6	415.62	419.55	0.95%	429.79	3.41%	406.17	-2.27%
M7	412.67	417.79	1.24%	425.37	3.08%	392.48	-4.89%
M8	407.58	415.98	2.06%	421.44	3.40%	360.16	-11.63%
M9	404.74	413.67	2.21%	452.10	11.70%	364.51	-9.94%
M10	402.69	411.64	2.22%	482.46	19.81%	369.52	-8.24%
M11	410.22	409.92	-0.07%	476.25	16.10%	353.47	-13.83%
M12	401.32	408.11	1.69%	468.54	16.75%	359.44	-10.43%

Prediction accuracy is the description of the conformity degree between the prediction results and the actual values. It is the benchmark to measure whether the prediction model is suitable for the prediction object. The commonly used prediction accuracy indicators include relative error (RE), mean square error (MSE), root mean square error (RMSE), mean absolute error (MAE) and mean absolute percentage error (MAPE).

The prediction accuracy of MR, GBM and TS results is statistically analyzed, and the comparison results are shown in Table 3. Among them, RMSE and MAPE are very sensitive

to the reflection of large or extra errors in a set of observations, which can well reflect the fitting degree of the stochastic process. According to the statistics in Table 3, each accuracy index of MR results is the smallest. Therefore, the MR model is a stochastic process with the best fitting degree for the price prediction of mineral products under uncertainty.

Tabela 3. Porównanie dokładności wyników trzech modeli predykcyjnych

Table 3. Accuracy comparison of three prediction models results

	MR	GBM	TS
MSE	185.80	1517.09	887.70
RMSE	13.63	38.95	29.79
MAE	10.31	27.13	23.28
MAPE	2.54%	6.68%	5.72%

### 2.3.2. Risk analysis of gold-price prediction results

The frequency histogram and cumulative probability distribution of gold-price prediction results are shown in Figure 5 and 6. From the cumulative probability distribution, the blue solid line is the experimentally calculated cumulative probability distribution, the red solid line is the theoretically fitted cumulative probability distribution, and the blue and red dashed lines are the upper and lower boundaries of the 95% confidence interval.

From Figure 5 (1), the predicted gold price in the first month fluctuates between RMB 400/gram and RMB 460/gram, with most of the results concentrated between RMB 410/gram and RMB 450/gram. The probability that the gold price is less than RMB 420/gram is about 10%, and the probability that it is less than RMB 446/gram is 90%, which means that the average grade has nearly 80% probability of being between RMB 420/gram and RMB 446/gram.

From Figure 5 (2), (3) and (4), the probability of the predicted gold price in the fourth, eighth and twelfth at RMB 400/gram is nearly the highest, which keeps a good consistency with the actual value of each month. As shown in Figure 6, it is the frequency histogram and cumulative probability distribution of gold-price prediction results for the whole year, which keeps a good consistency with the average value of the actual gold price in 2021.

To sum up, it is clear that traditional price prediction models, such as Holt model, ARIMA model and BP neural network, can only obtain a prediction result based on the estimation of relevant parameters, and the uncertainty and risk accuracy of their prediction results is difficult to evaluate quantitatively by statistical methods. The gold-price prediction method based on MR can quantitatively evaluate the uncertainty of the predicted result. Meanwhile,

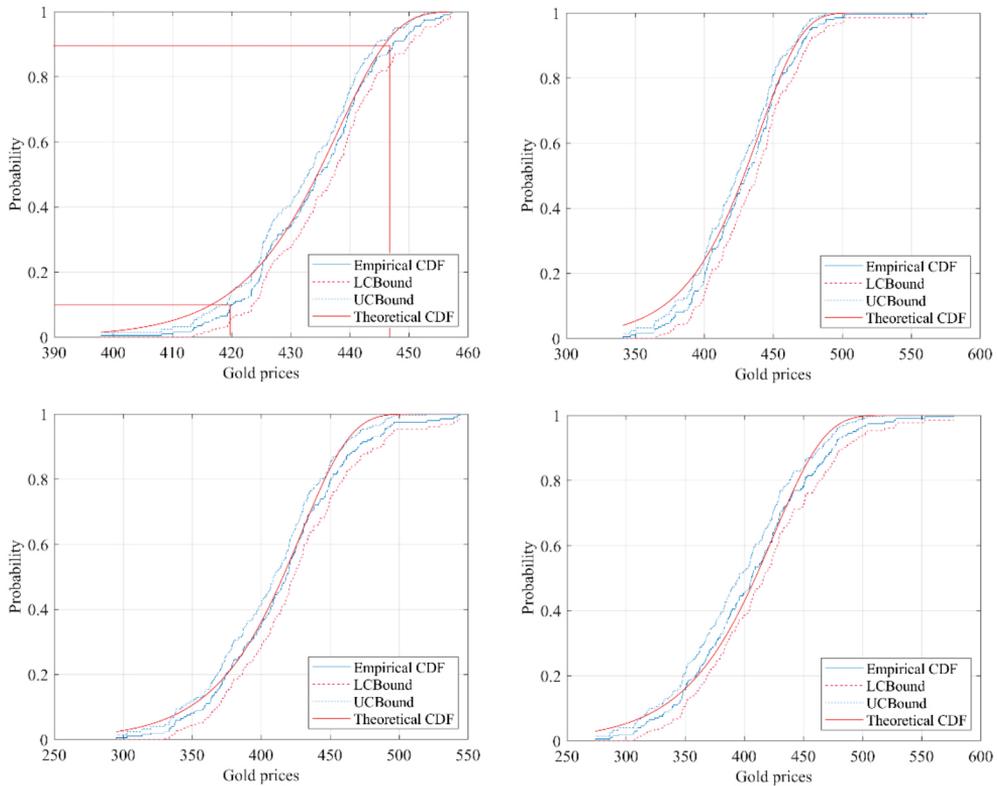


Fig. 5. Cumulative probability distribution of gold price prediction results for the 1<sup>st</sup>, 4<sup>th</sup>, 8<sup>th</sup> and 12<sup>th</sup> months (RMB/gram)

Rys. 5. Skumulowany rozkład prawdopodobieństwa wyników prognoz cen złota dla 1, 4, 8 i 12 miesiąca (RMB/gram)

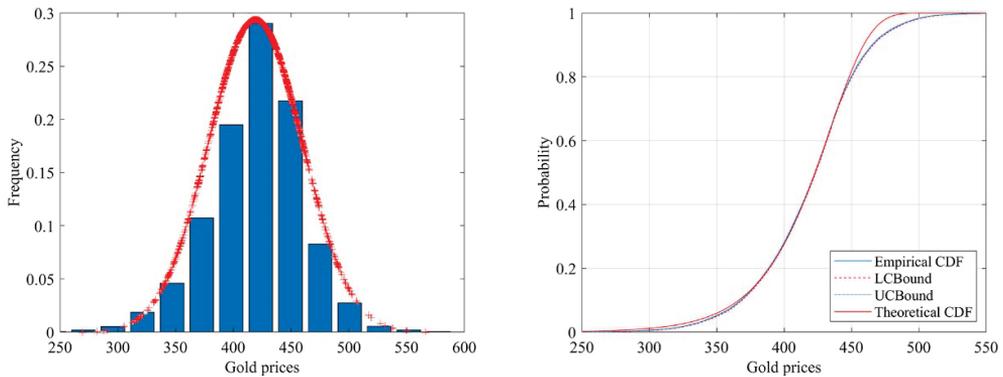


Fig. 6. Frequency histogram and cumulative probability distribution of gold price prediction results in 2021 (RMB/gram)

Rys. 6. Histogram częstotliwości i skumulowany rozkład prawdopodobieństwa wyników prognozy ceny złota w 2021 r. (RMB/gram)

probability and risk-analysis methods can be used to precisely describe the floating range, reliability and expected realization probability of each index, which can visually assist in evaluating the uncertainty of the price-prediction results.

## Conclusion

1. An analysis of mineral product price fluctuations has been conducted, on the basis of which, the stochastic process of mineral product price is studied based on the MR model. The maximum likelihood estimation method was used to estimate the parameters and the Monte Carlo simulation method was used to simulate the price trend of mineral products.
2. By comparing the prediction results of observed data, MR, GBM and TS results, it can be seen that the MR prediction result has better simulation superiority for the price trend of mineral products, which proves the effectiveness of the mean-reversion process in describing the mineral-product-price fluctuation.
3. The uncertainty and risk accuracy of the prediction results of the traditional time-series model are difficult to evaluate quantitatively by statistical methods. The stochastic simulation method based on the MR model can perform mathematical statistics and risk analysis on the expectation, standard deviation, and probability distribution of the prediction results. This provides powerful data support and the decision-making basis for the risk analysis of mineral-product prices under economic uncertainty.

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**PREDICTION OF MINERAL PRODUCT PRICE BASED ON MEAN REVERSION MODEL****Key words**

mineral-product price, mean reversion model,  
Monte Carlo simulation, uncertainty analysis

**Abstract**

The mean-reversion model is introduced into the study of mineral product price prediction. The gold price data from January 2018 to December 2021 are selected, and a mean-reverting stochastic process simulation of the gold price was carried out using Monte Carlo simulation (MCS) method. By comparing the statistical results and trend curves of the mean-reversion (MR) model, geometric Brownian motion (GBM) model, time series model and actual price, it is proved that the mean-reversion process is valid in describing the price fluctuation of mineral product. At the same time, by comparing with the traditional prediction methods, the mean-reversion model can quantitatively assess the uncertainty of the predicted price through a set of equal probability stochastic simulation results, so as to provide data support and decision-making basis for the risk analysis of future economy.

**PROGNOZOWANIE CENY PRODUKTU MINERALNEGO W OPARCIU O MODEL ŚREDNIEJ REWERSJI****Słowa kluczowe**

cena produktu mineralnego, model średniej rewersji,  
symulacja Monte Carlo, analiza niepewności

**Streszczenie**

W badaniach predykcji cen produktów mineralnych wprowadzono model średniej rewersji. Wybrano dane dotyczące cen złota od stycznia 2018 do grudnia 2021 r., a symulację ceny złota w procesie odwracania średniej przeprowadzono metodą symulacji Monte Carlo (MCS). Porównując wyniki statystyczne i krzywe trendu modelu średniej rewersji (MR), modelu geometrycznego ruchu Browna (GBM), modelu szeregów czasowych i rzeczywistej ceny, udowodniono, że proces średniej rewersji jest prawidłowy w opisie fluktuacji cen na produkt mineralny. Jednocześnie, porównując z tradycyjnymi metodami predykcji, model średniej rewersji może ilościowo oszacować niepewność przewidywanej ceny za pomocą zestawu wyników symulacji stochastycznej równego prawdopodobieństwa, w celu zapewnienia wsparcia danych i podstawy decyzyjnej do analizy ryzyka przyszłej gospodarki.

