

## Research Paper

# 2D Modeling of Wave Propagation in Shallow Water by the Method of Characteristics

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In this paper, a 2D numerical modeling of sound wave propagation in a shallow water medium that acts as a waveguide, are presented. This modeling is based on the method of characteristic which is not constrained by the Courant–Friedrichs–Lewy (CFL) condition. Using this method, the Euler time-dependent equations have been solved under adiabatic conditions inside of a shallow water waveguide which consists of one homogeneous environment of water over a rigid bed. In this work, the stability and precision of the method of characteristics (MOC) technique for sound wave propagation in a waveguide were illustrated when it was applied with the semi-Lagrange method. The results show a significant advantage of the method of characteristics over the finite difference time domain (FDTD) method.

**Keywords:** wave propagation; shallow water; MOC method; waveguide; transmission loss.



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## 1. Introduction

The propagation of sound in the sea has been studied to an extreme degree from the beginning of the Second World War, when it was realized that insight into this matter was indispensable to the successful performance of anti-submarine warfare working. These early estimations were fastly converted into constructive, albeit primeval, forecasting tools. Naval necessities motivate progress in all features of underwater acoustic modeling, especially the modeling of sound propagation. The investigation of sound wave propagation in seawater is essential for understanding and forecasting all underwater acoustic phenomena. The essentiality of wave propagation models is intrinsic in the ranking of acoustic models is illustrated in Fig. 1.

Sound propagation depends on the physical characteristics and the environment (HOSSEINI *et al.*, 2018). Many studies have been done on the physical characteristics of shallow waters such as the Persian Gulf. Such environments create an almost homogeneous layer of water due to the shallow depth and turbulence caused by wind and tides (KHALILABADI *et al.*, 2015; KHALILABADI 2016a; 2016b; 2016c; MAHPEYKAR, KHALILABADI, 2021; MOLLAESMAEILPOUR *et al.* 2019). In this

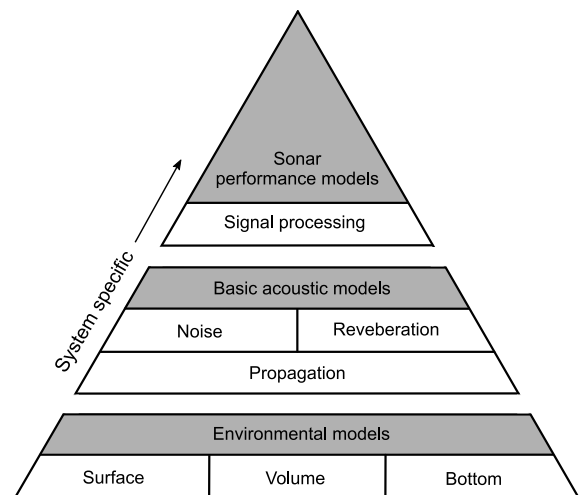


Fig. 1. Relationship between the environmental and acoustic models.

paper, a new method for simulating sound propagation within such environments is discussed. Simple intuitive developments have been given to present the physics of acoustic propagation in shallow water layer. The structure of a simple waveguide has been illustrated in Fig. 2.

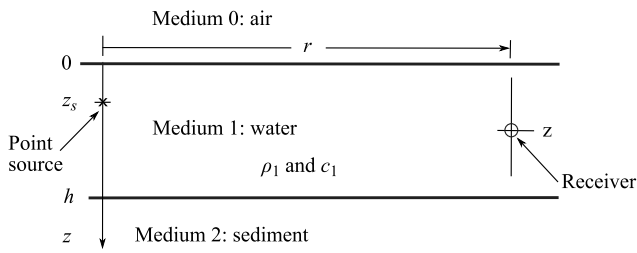


Fig. 2. Fundamental of a general waveguide.

Resulting from the computer programming evolution, numerical methods of wave propagation have been studied widely (MATSUMURA *et al.*, 2017). For high-performance soundwave field prediction, the progression of the precise numerical method is an essential subject (ARA *et al.*, 2011; MATSUMURA *et al.*, 2015; OSHIMA *et al.*, 2014).

Regarding the evolutionary process of studying and modeling waves in waveguides and shallow water, especially in recent years, we can mention the latest works. KIRBY and DUAN (2018) used modeled the sound wave propagation in the seawater using a normal mode approach and by finite elements method. Then they used a semi-analytical method for simulating the wave propagation in a waveguide (DUAN, KIRBY, 2019).

JENA *et al.* (2019) proposed a new solution of wave equations arising in shallow water wave propagation. LI *et al.* (2019) presented one method based on multi-layer boundary element for direct numerical modeling of acoustic wave propagation in shallow water areas.

DUAN and KIRBY (2020) calculated the characteristics of edge waves in 3D Plates using another numerical approach. WANG *et al.* (2020) predicted sound intensity vector field in shallow water waveguide using a prediction method. LI *et al.* (2021) determined the characteristics of sound wave propagation in shallow water waveguides for very low-frequency waves.

## 2. Materials and methods

The aim of this investigation is to illustrate the stability and precision of the method of characteristics (MOC) technique (FIEVISOHN, YU, 2016; LIU, 2021; MAZUMDAR, GUPTA, 2018; SONG *et al.*, 2020; SUBBOTINA, KRUPENNIKOV, 2017) with semi-Lagrange method (JIANG *et al.*, 2020; CHO *et al.*, 2021; PIAO *et al.*, 2018; SAADAT *et al.*, 2020) applied for sound wave propagation in a waveguide. The waveguide is a homogeneous water layer overlying a rigid sea bottom (JIHUI *et al.*, 2020; LI *et al.*, 2021; VERLINDEN *et al.*, 2017).

In numerical method, the MOC is a method to solve the partial differential equations (CAO, LIU, 2020; JEWELL, 2019; KAUFFMANN *et al.*, 2018; TWYMAN, 2018). In most cases, this technique applies to the first-order equations, albeit generally the MOC is re-

liable for each hyperbolic partial differential equation. The technique is reducing a partial differential equation to a group of ordinary differential equations along which the solution process can be integrated from some initial data given on an appropriate hyper-surface (ALI *et al.*, 2020; AYAS *et al.*, 2019; GAO *et al.*, 2021; COSTA *et al.*, 2021).

The basic equations have been written in cylindrical coordinates. For surface and bottom boundary conditions we consider free pressure in the surface and a rigid sea bed. 1D Euler and continuity equations under the circumstances adiabatic environment can be written in these forms:

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x}, \quad (1)$$

$$\frac{\partial p}{\partial t} = -\rho c^2 \frac{\partial u}{\partial x}. \quad (2)$$

By multiplying Eq. (1) by  $\pm c^2$  and collect with Eq. (2), we can obtain:

$$\frac{\partial f^+}{\partial t} + c \frac{\partial f^+}{\partial x} = 0, \quad (3)$$

$$\frac{\partial f^-}{\partial t} - c \frac{\partial f^-}{\partial x} = 0, \quad (4)$$

where

$$f^+ = \rho c u + p, \quad (5)$$

$$f^- = \rho c u - p. \quad (6)$$

Equations (1) and (2) are advection equations with soundwave speeds of  $+c$  and  $-c$ , respectively. The parameters  $f^+$  and  $f^-$  are advection along its characteristics. Thus new amounts at the subsequent time can be calculated by finding the up-wind amounts along with characteristics as illustrated in Fig. 3.

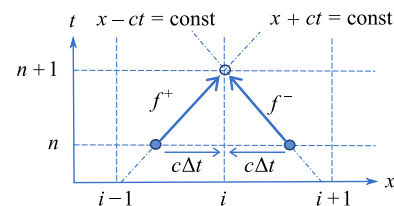


Fig. 3. Plan of advection.

The Courant–Friedrichs–Lewy (CFL) condition (ASCHER, VAN DEN DOEL, 2013; DOMINGUES *et al.*, 2013; HERSH, 2013; JELTSCH, KUMAR, 2013; LAX, 2013; LEFLOCH, 2013; RHEBERGEN, COCKBURN, 2013; SCHNEIDER *et al.*, 2013) can be written as:

$$c_0 \Delta t \sqrt{(1/\Delta r)^2 + (1/\Delta z)^2} \leq 1. \quad (7)$$

If  $CFL = 1$ ,  $f^+$  and  $f^-$  propagate the quantities from one specific cell to the next cell in time iterations.

If the CFL number is not a natural number, we can apply the constrained interpolation profile (CIP) technique (MATSUMURA *et al.*, 2017; YABE *et al.*, 2001). Therefore by addition and subtracting the Eqs (5) and (6), the pressure and particle velocity can be written as:

$$p = \frac{f^+ - f^-}{2}, \quad (8)$$

$$u = \frac{f^+ + f^-}{2\rho c}. \quad (9)$$

In 2D cases, we can solve these equations by a directional splitting technique (GENDRE *et al.*, 2017; NAKAMURA *et al.*, 2001). At the first, the equation of advection can be solved in the range direction, then this equation can be solved in depth direction.

The numerical model designed in this study, uses square grids. In this model, all of the physical quantities (the particle velocity and pressure) are collocated.

### 3. Results and discussion

The numerical model prepared in this research was implemented in a homogeneous seawater layer overlying a rigid seabed with a depth of 100 m. The projector and the receiver established at a same depth (50 m).

Figure 4 illustrates the sound wave propagation in this waveguide during running the model program. The model also calculate the transmission loss [dB] versus range [km].

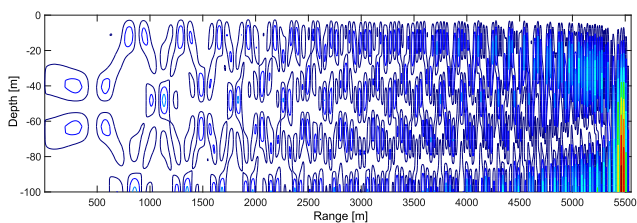


Fig. 4. Sound propagation in the waveguide during running the program.

The comparison of transmission loss between the model and theory is shown in Fig. 5. As seen in this figure, the range lowers than about 1 km where all modes have propagated, the model is well matches the

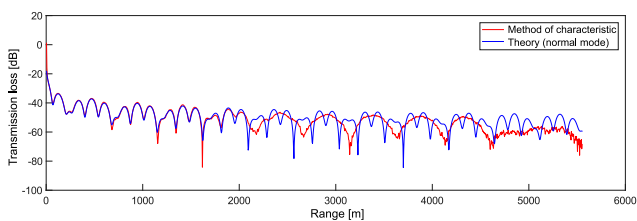


Fig. 5. Comparison between theory and model for transmission loss.

theory. As the range increases the difference between theory and model increases.

Then we changed the setup and put the source at the bottom. Figure 6 shows the sound propagation in the waveguide during running the program in this condition, and Fig. 7 shows the comparison of transmission loss between model and theory for this status. As discussed above, in the lower ranges where all modes have propagated, the model well matches the theory, and as the range increases the difference between theory and model increases.

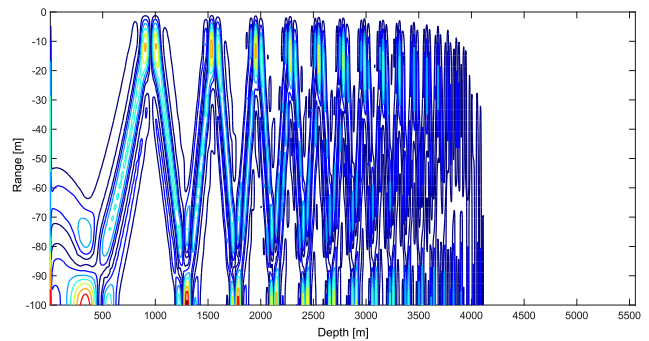


Fig. 6. Sound propagation in the waveguide during running the program.

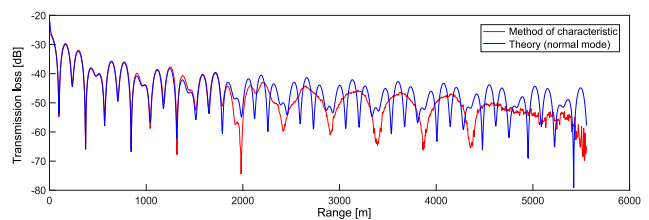


Fig. 7. Comparison of transmission loss between model and theory.

### 4. Conclusions

In this research, the propagation of acoustic waves within a waveguide is modeled using the MOC. The results compared with the finite difference time domain (FDTD) method. By examining the findings and modeling results, the following facts can be drawn:

- There is no difference in computational time between the MOC method and the FDTD per iteration, however, the FDTD is bound by more severe CFL conditions.
- When a rectangular mesh is used, the maximum amounts are so large that FDTD will take at least 1.4 times the calculation time required.
- In the method of characteristics, the phase properties of are more precise than the FDTD method. At low frequencies, this difference is not significant, but with increasing frequency, this difference becomes significant.

- It is determined by comparison of transmission losses that in the lower ranges where all modes have propagated, the model and the theory will be compatible. But as the range increases, the difference between model and theory will increase.

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