THE ARCHIVES OF MECHANICAL ENGINEERING

VOL. LIII 2006 Number 4

Key words: random loading, mean stress, energy models

*ALEKSANDER KAROLCZUK *, KRZYSZTOF KLUGER *, TADEUSZ ŁAGODA**

ESTIMATION OF FATIGUE LIFE UNDER UNIAXIAL RANDOM LOADING WITH ZERO AND NON-ZERO MEAN VALUE BY MEANS OF THE STRAIN ENERGY DENSITY PARAMETER

The paper contains a comparison of the results of calculation and experiment for the 10HNAP alloy steel. Specimens made of this steel were subjected to uniaxial constant-amplitude and random loading with both zero and non-zero mean values of loading. For determination of the steel fatigue life, the energy parameter including the mean value of loading was proposed. Under random loading, cycles were counted with the rain flow algorithm, and fatigue damage was accumulated with the Palmgren-Miner hypothesis. For the registered stress histories, elastic-plastic strains were calculated with the kinematic hardening model proposed by Mróz.

1. Introduction

There are stress, strain and energy models which can be used for the analysis of fatigue test results including the influence of the mean loading value. At present, the energy models [1] are often used to describe multiaxial fatigue. However, in these models the influence of the mean value on fatigue is not examined in detail. The aim of this paper is to develop the energy model including the influence of the mean stress and its verification in fatigue tests of 10HNAP steel. The influence of the mean value for 10HNAP steel was analysed previously according to the stress models formulated by Goodman, Gerber [4] and Dang-Van [4], [5].

2. The tested material and specimen geometry

Plane specimens (Fig. 1) of 10HNAP steel [6] were tested on a fatigue test stand SHM 250. This stand enables tests to be performed under controlled

* *Department of Mechanics and Machine Design, Opole University of Technology, ul. Mikołajczyka 5, 45-271 Opole, Poland; E-mail: t.lagoda@po.opole.pl*

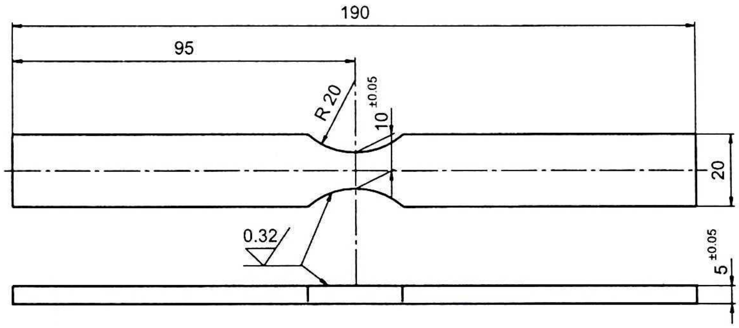

Fig. 1. Specimen geometry

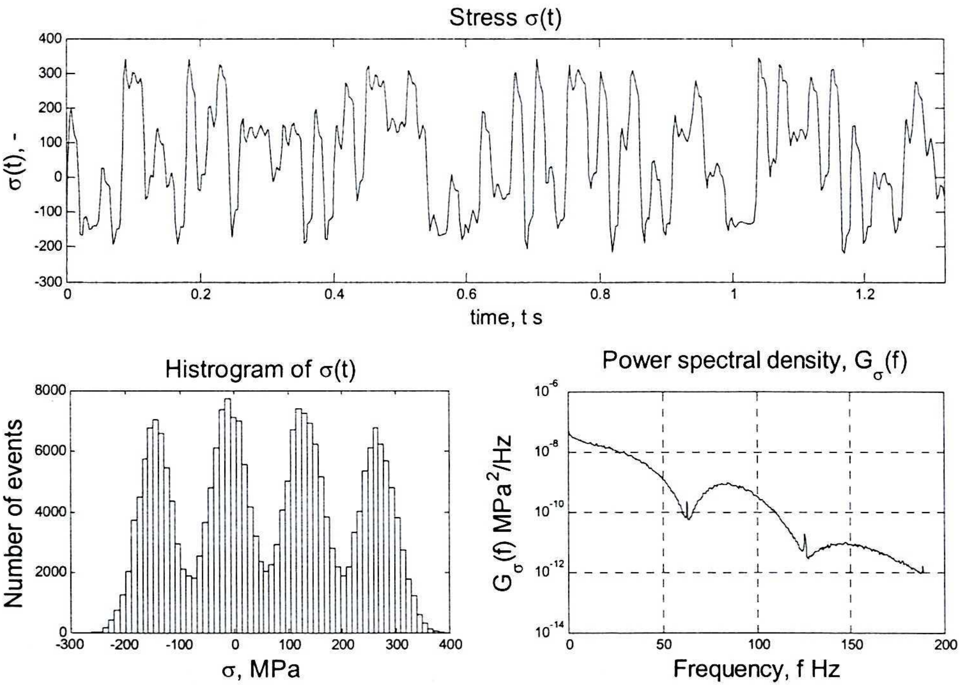

Fig. 2. Stress characteristics

force, displacement or strain of cyclic or random loading. Chemical composition of the tested alloy steel was as follows: C = 0.115%, Mn = 0.71%, Si = 0.41%, P = 0.082%, S = 0.028%, Cr = 0.81%, Cu = 0.30%, Ni = 0.50% in wt. and Fe = balance. Static properties were the following: E = 215 GPa, σ_{YS} = 414 MPa, σ_{TS} = 556 MPa, ν = 0.29, El_{10} = 31%, RA = 35% and cyclic properties were the following: σ'_f = 1136 MPa, b = −0.105, ε'_f = 0.114, c = −0.420, n' = 0.156, K' = 853 MPa. Under cyclic loading, the tests were performed for five different stress amplitudes and three levels of the mean loading, σ_m = 75 MPa, 150 MPa and 225 MPa. Under random loading, the tests were done for seven different values of root mean square of stress, σ_{RMS} and mean values, σ_m (zero, compressive and tensile). Observation time for random loading was T_0 = 649 s, and sampling time was $\Delta t = 2.641 \cdot 10^{-3}$s, i.e. 245760 instantaneous samples. As an example, a short part of stress history $\sigma(t)$, the histogram and the power spectral density function of stress are presented in Fig. 2.

3. The energy model

The strain energy density parameter in time domain can be calculated from

$$W(t) = 0.5\sigma(t)(\varepsilon(t) - \varepsilon_m)\,\mathrm{sgn}[\sigma(t), (\varepsilon(t) - \varepsilon_m)], \tag{1}$$

similarly to the model presented in [1], [2], [3], where ε_m is strain mean value calculated from the following equation

$$\varepsilon_m = \frac{1}{T_0}\int_0^{T_0} \varepsilon(t)dt \tag{2}$$

and

$$\mathrm{sgn}[\sigma(t), (\varepsilon(t) - \varepsilon_m)] = \frac{\mathrm{sgn}[\sigma(t)] + \mathrm{sgn}[\varepsilon(t) - \varepsilon_m]}{2}, \tag{3}$$

$$\mathrm{sgn[i,j]} = \frac{\mathrm{sgn(i)} + \mathrm{sgn(j)}}{2} = \begin{cases} 1 & \text{for} \quad \mathrm{sgn(i)} = \mathrm{sgn(j)} = 1 \\ 0.5 & \text{for} \quad (\mathrm{i} = 0, \mathrm{sgn(j)} = 1) \quad \text{or} \quad (\mathrm{j} = 0, \mathrm{sgn(i)} = 1) \\ 0 & \text{for} \quad \mathrm{sgn(i)} = -\mathrm{sgn(j)} \\ -0.5 & \text{for} \quad (\mathrm{i} = 0, \mathrm{sgn(j)} = -1) \quad \text{or} \quad (\mathrm{j} = 0, \mathrm{sgn(i)} = -1) \\ -1 & \text{for} \quad \mathrm{sgn(i)} = \mathrm{sgn(j)} = -1 \end{cases} \tag{4}$$

In the case when $\sigma(t) = 0$ or $\varepsilon(t) - \varepsilon_m = 0$, the total value of energy density parameter according to (1) is equal to zero. Thus, the expression (4) can be reduced to

$$\mathrm{sgn[i,j]} = \frac{\mathrm{sgn(i)} + \mathrm{sgn(j)}}{2} = \begin{cases} 1 & \text{for} \quad \mathrm{sgn(i)} = \mathrm{sgn(j)} = 1 \\ 0 & \text{for} \quad \mathrm{sgn(i)} = -\mathrm{sgn(j)} \\ -1 & \text{for} \quad \mathrm{sgn(i)} = \mathrm{sgn(j)} = -1 \end{cases}. \tag{5}$$

Function $\text{sgn}[\sigma(t), \varepsilon(t) - \varepsilon_m]$ defines the tensile and compressive phases of loading. It means that the parameter (1) assumes that material is under compression when sgn function (3) is negative and reverse, when sgn function is positive it is assumed that the material is under tension. Application of the function sgn in calculations causes that the history of the strain energy density parameter changes in time in a symmetric way, while cyclic stresses and strains change in relation to the mean values. Figs. 3 and 4 show the constant-amplitude and random stress histories $\sigma(t)$ with the mean value $\sigma_m = 75$ MPa and the corresponding history of strain $\varepsilon(t)$ as well as histories of the strain energy density parameter with $W(t)$ and without the sgn function $W^*(t)$. From the graphs it appears that application of the function sgn reduces the mean value of W_m.

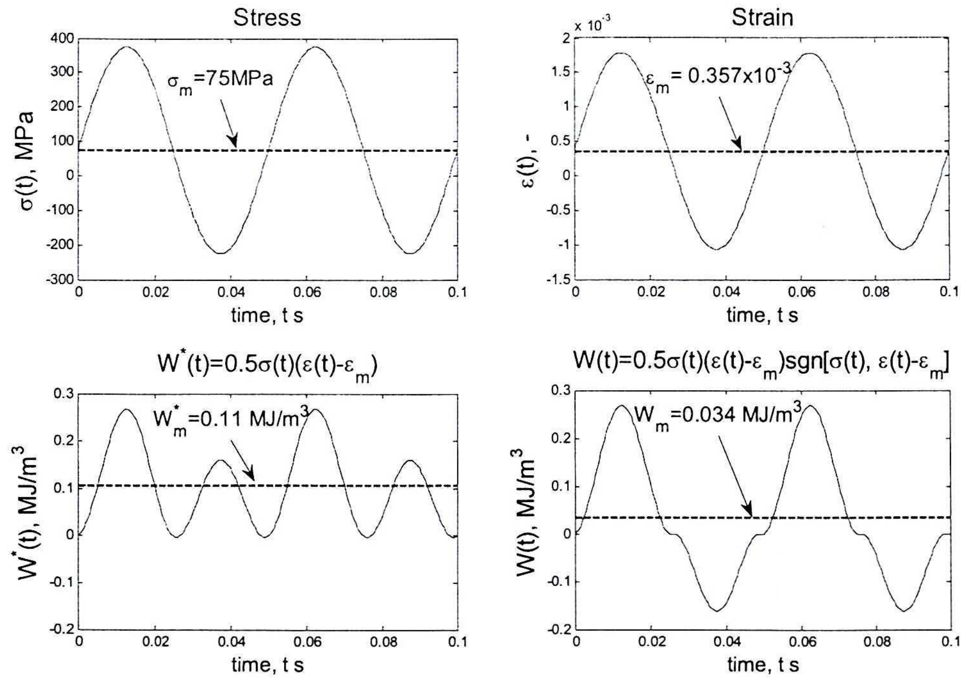

Fig. 3. Example histories of stress, strain and strain energy density parameters $W^*(t)$, $W(t)$ for constant-amplitude loading

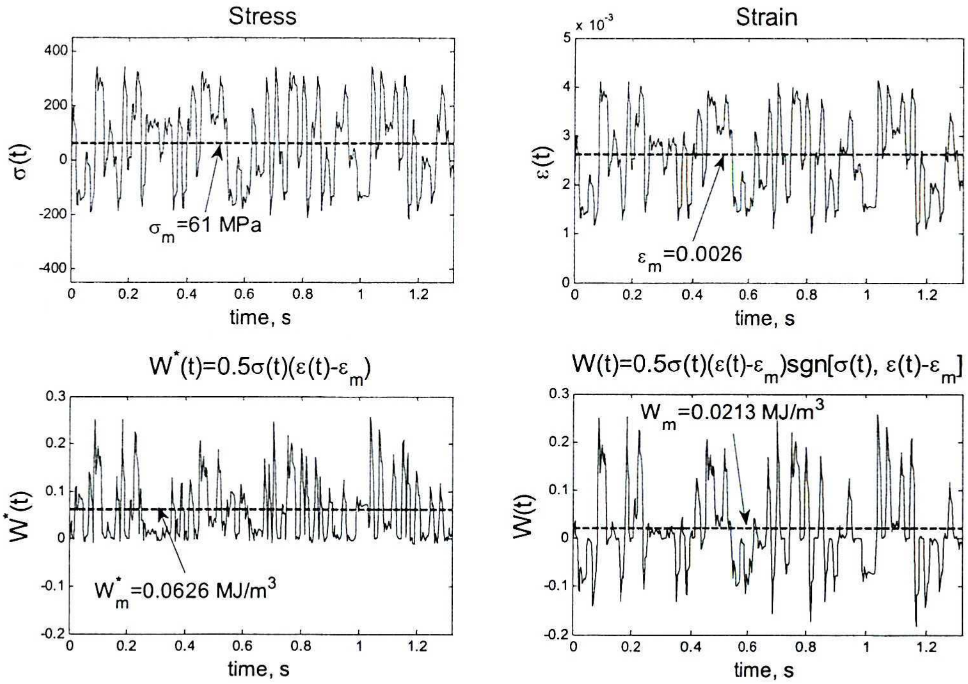

Fig. 4. Example histories of stress, strain and strain energy density parameter $W^*(t)$, $W(t)$ for random loading

The fatigue life N_f of the tested material for the low- and high-cycle regime can be calculated from combined fatigue characteristics: $N_f - \sigma_a$ (Basquin) and $N_f - \varepsilon_a$ (Manson-Coffin), [1]

$$W_a = \frac{\sigma'^2_f}{2E}(2N_f)^{2b} + 0.5\varepsilon'_f\sigma'_f(2N_f)^{b+c}, \tag{6}$$

where W_a is the amplitude of the strain energy density parameter and N_f is the number of cycles to failure.

4. Application of the Mróz model

The incremental kinematic model of material hardening formulated by Mróz [7] was used to calculate the strain history from the stress history. Mróz introduced the concept of fields of plastic moduli. According to this idea for the one-dimensional case, the non-linear curve of cyclic strain ($\sigma - \varepsilon$) is replaced by a sequence of linear segments. Each linear segment has its own modulus of plasticity (C_0, C_1, C_2, ..., C_{m-1}). The points on the new linearized curve of cyclic strain, where the moduli of plasticity change, determine fields in the nine-dimensional space of stresses with constant moduli of plasticity (fields of moduli of plasticity). The surfaces $f_1, f_2, \ldots, f_m$. with

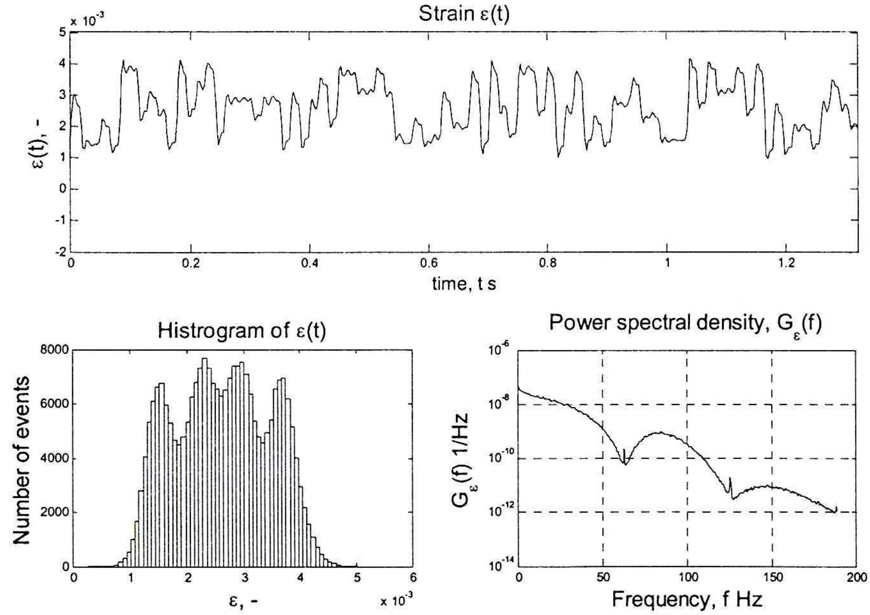

Fig. 5. Strain characteristics

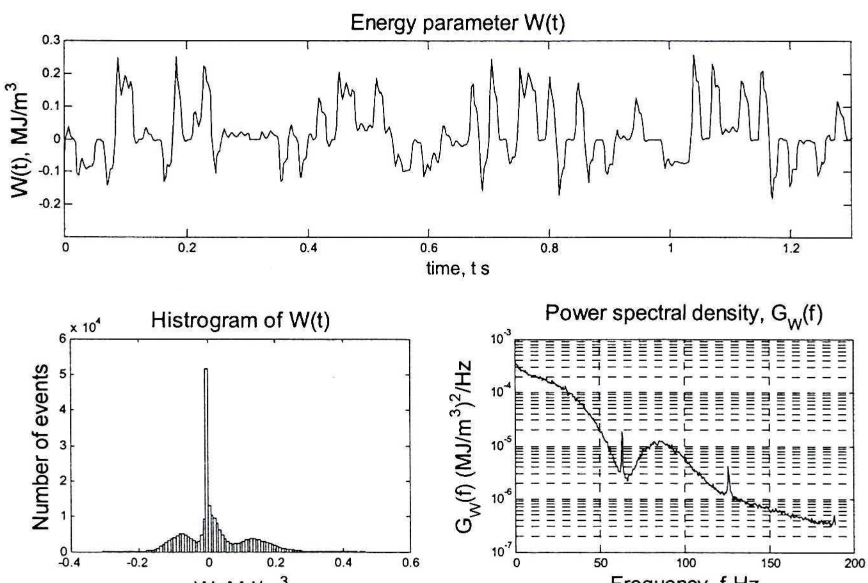

Fig. 6. Strain energy density parameter characteristics

constant moduli of plasticity are reduced to spherical surfaces in the case of selection of a proper scale and application of the Huber-Mises-Hencky condition of plasticity (H-M-H). The Mróz model assumes that the material is homogeneous, isotropic, and the influence of the loading rate can be neglected. Moreover, the model does not include thermal phenomena, uniaxial ratcheting and assumes constancy of the Young's and Poisson's modules. As an example, a short part of strain history $\varepsilon(t)$, the histogram and the power spectral density function of the total strain history $\varepsilon(t)$ are presented in Fig. 5. Fig. 6 presents the corresponding energy parameter characteristics.

5. Verification of the model

5.1. Constant-amplitude loading

The transform amplitudes W_{aT} of the strain energy density parameter were calculated from the following formula

$$W_{aT} = \frac{(\sigma_a + k\sigma_m)\,\varepsilon_a}{2} \quad \text{for} \quad \begin{matrix} \sigma_m \geqslant 0 & k = 1 \\ \sigma_m < 0 & k = 0 \end{matrix} \quad = \begin{cases} \dfrac{(\sigma_a + \sigma_m)\,\varepsilon_a}{2} & \text{for} \quad \sigma_m \geqslant 0 \\ \dfrac{\sigma_a^2 \varepsilon_a}{2} & \text{for} \quad \sigma_m < 0 \end{cases}. \tag{7}$$

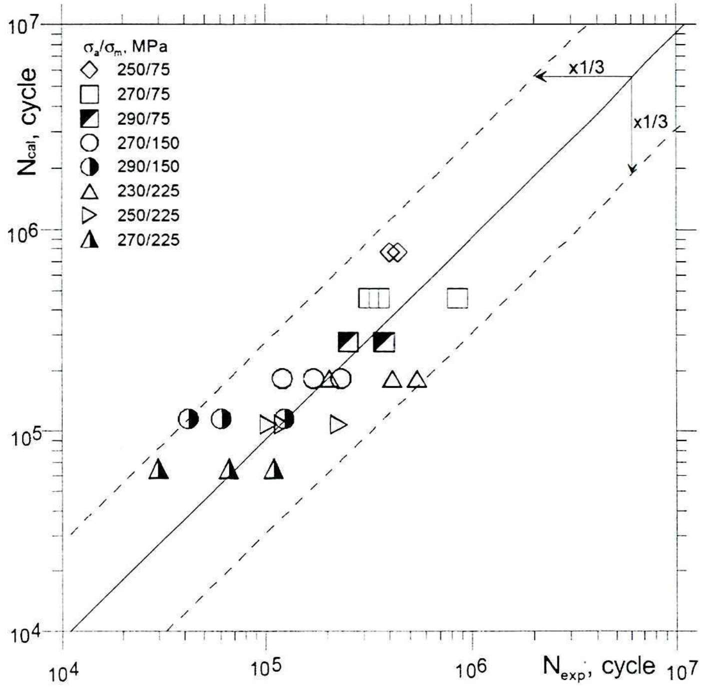

Fig. 7. A comparison of the calculated and experimental lives for 10HNAP steel under constant-amplitude tension-compression loading

Under constant-amplitude loading for $\sigma_m \geqslant 0$, Eq. (7) is connected with the Smith-Watson-Topper parameter (SWT) [8] according to the following equation

$$W_{aT} = 0.5P_{SWT} = 0.5\sigma_{\max}\varepsilon_a = 0.5(\sigma_a + \sigma_m)\varepsilon_a. \tag{8}$$

The number of cycles to failure was calculated according to Eq. (3), ($N_f = N_{cal}W_a = W_{aT}$). A graphical comparison of experimental and calculated lives is shown in Fig. 7. The solid line represents a perfect conformity of results, the dashed lines represent a scatter band with coefficient of 3, i.e. $N_{exp}/N_{cal} = 3$ (1/3), because constant-amplitude tests give such scatter [6].

5.1. Random loading

The algorithm for determination of the fatigue life of 10HNAP steel according to the presented model can be shown as:

- calculation of stresses $\sigma(t)$ from measurement of forces $F(t)$,
- numerical determination of strains $\varepsilon(t)$ corresponding to the given stresses according to the incremental kinematic model of material hardening formulated by Mróz,
- determination of the strain energy density parameter history according to Eq. (1),
- determination of amplitudes, $W_a^{(i)}$ and mean values, $W_m^{(i)}$ of cycles and half-cycles with the rain flow algorithm [9], ($i = 1 \ldots k$, i – subsequent cycle, $k = 32$ – number of classes to which cycles where assigned), (Fig. 8),
- determination of the transform amplitude of the strain energy density parameter from the previously determined amplitudes and mean values according to

$$W_{aT}^{(i)} = \begin{cases} W_a^{(i)} + W_m^{(i)} & \text{for} \quad W_m^{(i)} \geqslant 0 \\ W_a^{(i)} & \text{for} \quad W_m^{(i)} < 0 \end{cases}, \tag{9}$$

- determination of a damage degree according to the Palmgren-Miner hypothesis [10],

$$S(T_0) = \sum_{i=1}^{k} \frac{1}{N_f^{(i)}}, \tag{10}$$

where $N_f^{(i)}$ is determined from Eq. (6) for the transformed amplitudes $W_{aT}^{(i)}$

$$W_{aT}^{(i)} = \frac{\sigma'^2_f}{2E}(2N_f^{(i)})^{2b} + 0.5\varepsilon'_f\sigma'_f(2N_f^{(i)})^{b+c}, \tag{11}$$

- fatigue life determination according to the following relationship

$$T_{cal} = \frac{T_0}{S(T_0)}, \tag{12}$$

where T_0 is the observation time.

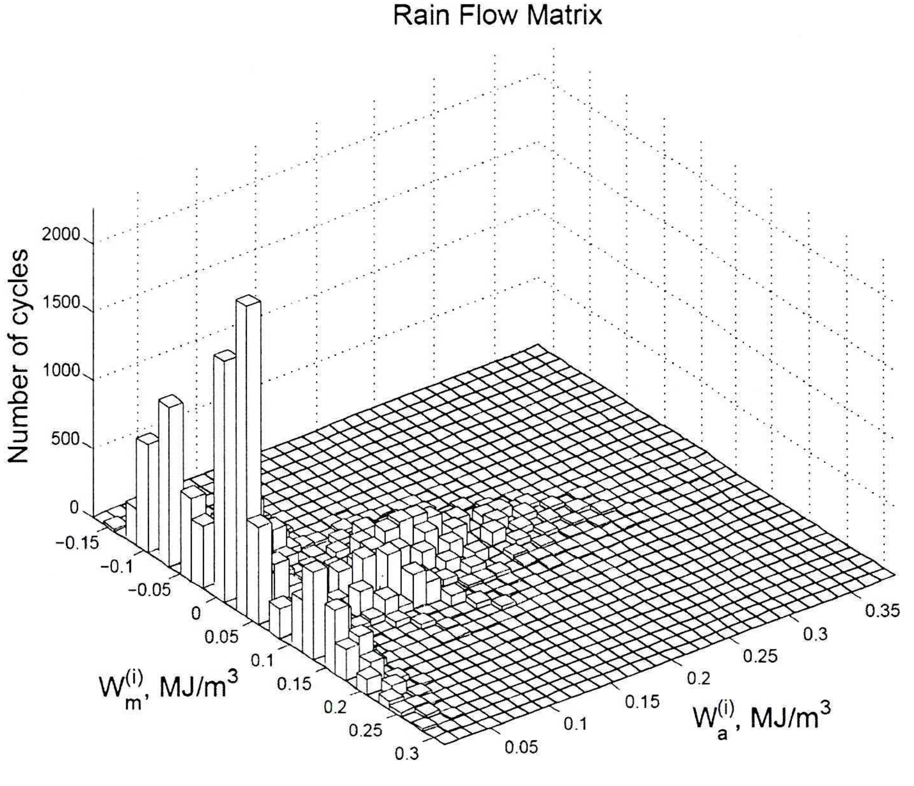

Fig. 8. Example of rain flow matrix of strain energy density parameter

Fig. 9 shows the comparison of the calculated and experimental lives for random loading with the zero and non-zero mean value of loading [12]. The solid line represents a perfect conformity of results, and the dashed lines represents a scatter band with coefficient of 3, i.e. $T_{exp}/T_{cal} = 3(1/3)$.

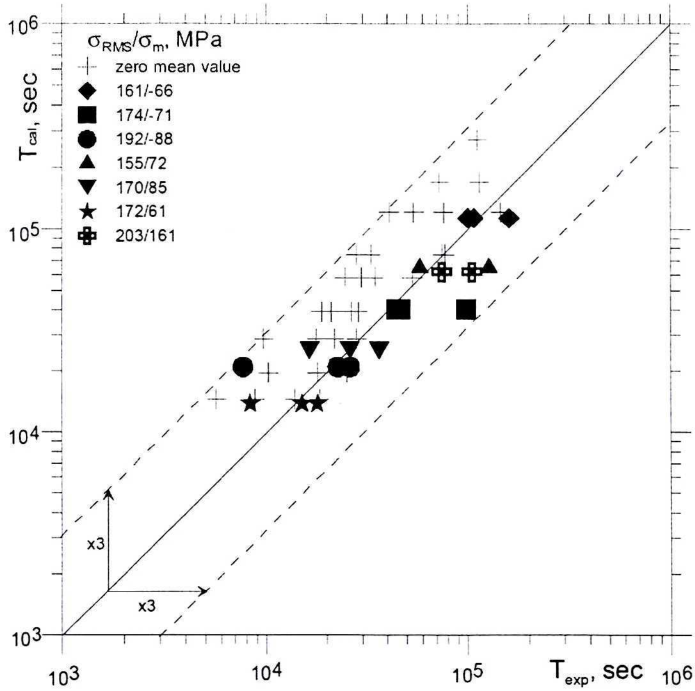

Fig. 9. A comparison of the calculated and experimental results for 10HNAP steel under random tension-compression loading

6. Conclusions

From the verification of the energy model for specimens made of 10HNAP steel we can draw the following conclusions.

1. Satisfactory correlation of results between calculated (N_{cal}, T_{cal}) and experimental fatigue lives (N_{exp}, T_{exp}) was obtained under constant-amplitude and random tension-compression with zero and non-zero mean values.
2. All the results for the considered loadings are within the scatter band with the coefficient of the factor 3.
3. For random loading from calculation using the rain flow algorithm, acceptable fatigue life results were obtained. Negative mean values of cycles were neglected.
4. The presented the strain energy density parameter includes the influence of the mean loading and for constant-amplitude loading it reduces to the known Smith-Watson-Topper model P_{SWT}.

With the support of the Commission of the European Communities under the FP5 contract No. GIMA-CT-2002-04058 (CESTI).

The work financed from the means for science In 2005/2006, as a research project.
The work financed by the European Social Fund of the European Community and the budget of Poland.

Manuscript received by Editorial Board, May 06, 2005;
final version, May 10, 2006.

REFERENCES

[1] Łagoda T.: Macha E., Będkowski W.: A critical plane approach based on energy concepts: Application to biaxial random tension-compression high-cycle fatigue regime. Int. J. Fatigue, 1999, Vol. 21, pp. 431÷443.
[2] Łagoda T.: Energy models for fatigue life estimation under random loading – Part I – The model elaboration. Int. J. Fatigue, 2001, Vol. 23, pp. 467÷480.
[3] Łagoda T.: Energy models for fatigue life estimation under random loading – Part II – Verification of the model. Int. J. Fatigue, 2001, Vol. 23, pp. 481÷489.
[4] Łagoda T., Macha E., Pawliczek R.: Fatigue life of 10HNAP steel under random loading with mean value. Int. J. Fatigue, 2001, Vol. 23, pp. 283÷291.
[5] Kluger K., Łagoda T.: Application of the Dang-Van criterion for life determination under uniaxial random tension-compression with different mean values. Fatigue and Fracture of Engineering Materials and Structures, 2004, Vol. 27, pp. 505÷512.
[6] Lachowicz C., Łagoda T., Macha E., Dragon A., Petit J.: Selections of algorithms for fatigue life calculation of elements made of 10HNAP steel under uniaxial random loadings. Studia Geotechnika et Mechanica, 1996, Vol. XVIII, pp. 19÷43.
[7] Mróz Z.: On the description of anisotropic work hardening. J. Appl. Phys. Solids, 1967, Vol. 15, pp. 163÷175.
[8] Smith K. N., Watson P., Topper T. H.: A stress strain function for the fatigue of metals. J. Materials, 1970, Vol. 5, pp. 767÷776.
[9] Downing S. D., Socie D. F.: Simple rain flow counting algorithms. Int. J. Fatigue, 1982, Vol. 4, pp. 31÷40.
[10] Miner M. A.: Cumulative damage in fatigue. Journal of Applied Mechanics, 1945, Vol. 12, pp. 159÷164.
[11] Palmgren A.: Die Lebensdauer von Kugelagern. VDI-Z, 1924, Vol. 68, pp. 339÷341.
[12] Karolczuk A., Łagoda T., Macha E., Determination of fatigue life of 10HNAP and 1208.3 steels with the parameter of strain energy density. Proc. of the Eight Int. Fatigue Congress, Fatigue 2002, Stockholm, Sweden, Ed. A.F. Blom, EMAS, Vol. 1, pp. 515÷522.

Wyznaczanie trwałości zmęczeniowej przy jednoosiowym, losowym obciążeniu z wartością średnią przy użyciu parametru gęstości ehergii odkształcenia

S t r e s z c z e n i e

Praca zawiera porównanie trwałości zmęczeniowej eksperymentalnej z obliczeniową. Próbki wykonane ze stali 10HNAP zostały poddane jednoosiowym obciążeniom cyklicznym oraz losowym z zerową oraz różną od zera wartością średnią. Trwałość zmęczeniowa została obliczona przy użyciu

parametru gęstości energii odkształcenia uwzględniającego wpływ wartości średniej obciążenia. Przy obciążeniach losowych cykle obciążeń zostały wyznaczone na podstawie przebiegu parametru gęstości energii odkształceń za pomocą metody zliczania cykli *płynącego deszczu* (rain flow). Kumulacja uszkodzeń zmęczeniowych z każdego cyklu i półcyklu została przeprowadzona na podstawie hipotezy kumulacji uszkodzeń zmęczeniowych Palmgrena-Minera.