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THE PRESSURE DISTRIBUTION IN TURBULENT LIQUID FLOW WITHIN A CONICAL AXIAL CLEARANCE

Deflection of balancing disc in a multistage centrifugal pump, being the consequence of pressure forces, causes the change of shape of the axial clearance between the rotating balancing disc and the stationary counterdisc. It results in reduction of axial force acting on the rotating wall of the clearance, and the total balancing force. The final effect is a significant decrease (e.g. of 20%) of disc to counterdisc distance. The approximate formulae for radial pressure distribution in turbulent flow through hydraulically smooth conical axial clearance are derived in the paper and a numerical example is given. The method of integral relations was applied.

NOMENCLATURE

a, b	– diameter ratios; $a = d_{ou}/d_{in}$, $b = 1/a$,
c	– mean velocity of liquid; $c = Q_l/2\pi rs$,
d_{in}, d_{ou}	– diameters, as shown in Fig. 1,
F_{ax}	– axial thrust,
F_{bal}	– balancing force,
F_g	– force acting on the rotating surface of the gap,
k, $\bar{k}$	– local (eq.12) and average value of liquid swirl between r_{in} and r_{ou},
N	– velocity ratio; $N = u/c$,
p	– static pressure (time averaged),
Δp, Δp_g, Δp_{in}	– differential pressure, as shown in Fig.1,
Q_l	– leakage through the gap,

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Re	– Reynolds number; $\mathrm{Re} = c \cdot 2s/\nu$,
u	– peripheral velocity; $u = \omega r$,
υ_r, υ_φ, υ_z	– radial, peripheral and axial fluid velocity (time averaged),
w	– mean relative velocity (relating to the gap surface),
α	– coefficient in Blasius formula for fluid stress on the wall,
β	– coefficient for calculation of relative velocity,
γ	– angle of the gap conicity,
τ	– shear stress in liquid,
Φ	– coefficient of the pressure drop increase due to rotation of the gap surface,
ω	– angular speed of rotating disc.

LOWER INDICES:

con	– refers to conical gap,
diff	– refers to the pressure increase because of the gap diffuserness,
in	– refers to inner diameter of the gap,
ou	– refers to outer diameter of the gap,
s	– refers to stationary wall,
t	– total (i.e. resultant),
ω	– refers to rotating wall,
r, φ, z	– refers to radial, angular (circumferential) or axial coordinate.

1. Introduction

When designing a system with balancing disc in a multistage pump, we usually assume that the surfaces on both sides of the radial annular gap (i.e. axial clearance) s are parallel (Fig. 1a). However, due to the forces resulting from high difference of pressures acting on both surfaces, the disc deforms, and the gap takes a conical shape (Fig. 1b).

In order to carry the same axial force F_{ax}, the rotating disc must move closer to the counterdisc, in result of which the width s_{in} at the inlet of the clearance may become too low. A threat then arises that the two discs may touch, and a severe damage of the pump may occur.

When designing the balancing device, one must first estimate the deflection angle γ of the disc (Fig. 1b), and then calculate the parameters of fluid flow in the clearance, as well as the actual force F_g acting on the rotating surface of the gap and the diminished width s_{in}. Only then can one prevent negative effects of disc deflection by

- making a disc with a "counter-cone", $\gamma < 0$ for $\omega = 0$, or applying a conical counterdisc,
- adequately increasing outer diameter of the disc d_{ou}.

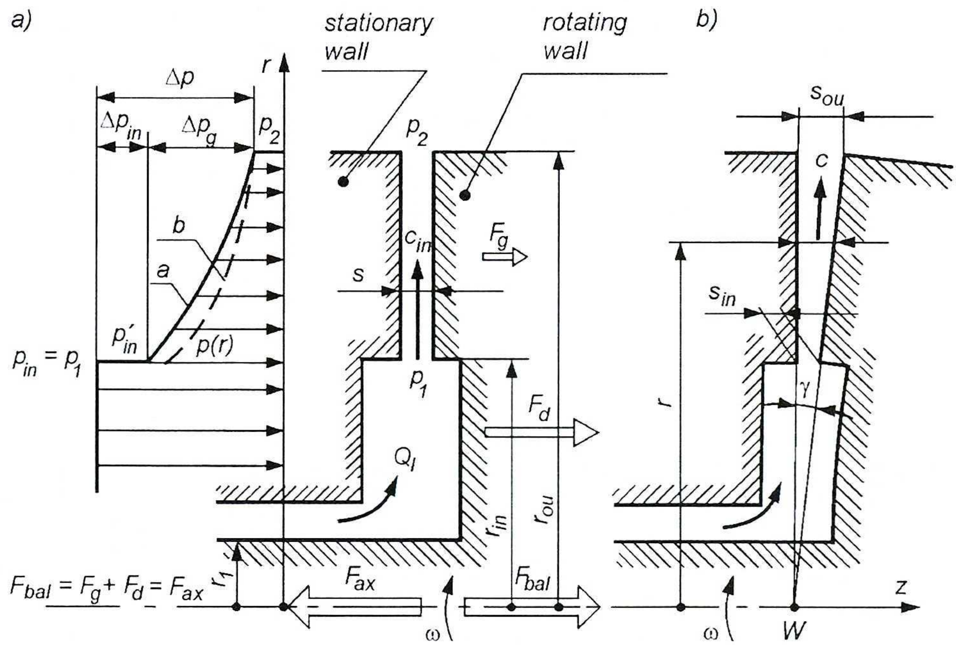

Fig. 1. Radial annular gap (axial clearance) with:
a) two parallel surfaces, b) rotating surface of conical shape

The solution to a turbulent flow in a clearance of parallel walls was obtained in [1] in a form of a set of algebraic relationships. The objective of this study is to present the method of calculations, of an adequate accuracy, which would allow finding the distribution $p(r)$ and pressure drop $\Delta p(Q_l, \omega)$ in a turbulent flow through a conical clearance. A suitable form of conicity is assumed to facilitate the calculations (the apex W of the cone lies at the point where the axis of rotation passes through the surface of the stationary disc; if we assumed a different apex location, the calculations would be significantly more complicated, and obtaining a convenient form of final formulae would become practically impossible).

To find the solution, which would also have the form of a set of algebraic relationships, one applied the method of integral relations. When deriving these relations, one used the following formulae, which explicitly result from Fig. 1, the equation of continuity of flow with velocity c averaged over the width s, the definition of circumferential velocity u, and the Reynolds number Re:

$$\operatorname{tg}\gamma = \frac{s}{r} = \frac{s_{\text{in}}}{r_{\text{in}}}, \quad \text{thus} \quad s = r\frac{s_{\text{in}}}{r_{\text{in}}} \tag{1}$$

$$c = \frac{Q_l}{2\pi rs}, \quad \text{thus} \quad c_{\text{in}}\, r_{\text{in}}\, s_{\text{in}} = c \ \ r \ \ s \tag{2}$$

and

$$c = c_{\text{in}}\left(\frac{r_{\text{in}}}{r}\right)^2 \tag{3}$$

$$u = \omega r = u_{\text{in}}\frac{r}{r_{\text{in}}} \tag{4}$$

$$\text{Re} = \frac{c \cdot 2s}{\nu} = \text{Re}_{\text{in}}\frac{r_{\text{in}}}{r} \tag{5}$$

$$N = \frac{u}{c} = N_{\text{in}}\left(\frac{r}{r_{\text{in}}}\right)^3 \tag{6}$$

2. Initial equation system

It is assumed that there is a steady state, axially symmetric turbulent flow of fluid of constant viscosity in the clearance. The initial segment close to the inlet (the entrance region) has been neglected, as its length is very small (below 0.1 of the length of a typical axial clearance). The Reynolds equation in radial direction (after making typical simplification [1], [2]), and the equation of continuity have in this case the forms

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\rho\left(\upsilon_r\frac{\partial\upsilon_r}{\partial r} + \upsilon_z\frac{\partial\upsilon_r}{\partial z} - \frac{\upsilon_\varphi^2}{r}\right) + \frac{\partial\tau_{zr}}{\partial z} \tag{7}$$

$$\frac{\upsilon_r}{r} + \frac{\partial\upsilon_r}{\partial r} + \frac{\partial\upsilon_z}{\partial z} = 0 \tag{8}$$

Although the walls of the clearance are almost parallel, one cannot neglect the velocity component υ_z. It follows from Fig. 2 that $\operatorname{tg}\vartheta = \frac{\upsilon_z}{\upsilon_r} = \frac{z}{r}$, thus $\upsilon_z = \upsilon_r\frac{z}{r}$, and $\frac{\partial\upsilon_z}{\partial z} = \frac{\upsilon_r}{r} + \frac{z}{r}\frac{\partial\upsilon_r}{\partial z}$;

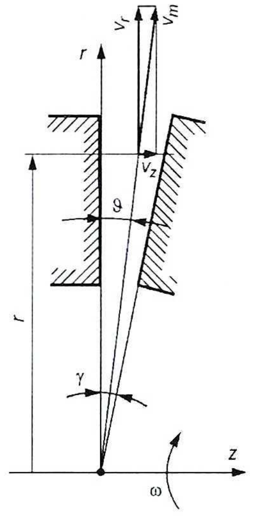

Fig. 2. Components v_r, v_z of meridional velocity v_m ($v_m + v_\varphi = v$)

Substituting this relation into we obtain

$$\frac{\partial v_r}{\partial r} = -\left(2\frac{v_r}{r} + \frac{z}{r}\frac{\partial v_r}{\partial z}\right) \quad \text{thus} \quad v_r\frac{\partial v_r}{\partial r} + v_z\frac{\partial v_r}{\partial z} = -2\frac{v_r^2}{r}$$

and then equation (7) takes the form

$$\frac{\mathrm{d}p}{\mathrm{d}r} = 2\rho\frac{v_r^2}{r} + \rho\frac{v_\varphi^2}{r} + \frac{\partial \tau_{zr}}{\partial z} \tag{9}$$

Considering very small value of angle γ[1], one can assume that the velocity distributions $v_r(z)$ and $v_\varphi(z)$ are practically the same as for a clearance of parallel walls. Similarly as in [1], the velocity distributions assumed here are power functions with exponent $1/n$ (Fig. 3), given by the equations

$$\left.\begin{aligned} v_r &= v_{r\max}\left(\frac{2z}{s}\right)^{\frac{1}{n}}, && \text{for} \quad 0 \le z \le \frac{s}{2} \\ v_r &= v_{r\max}\left[2\left(1 - \frac{z}{2}\right)\right]^{\frac{1}{n}}, && \text{for} \quad \frac{s}{2} \le z \le s \end{aligned}\right\} \tag{10}$$

[1] The disc deflection shown in Fig. 2 is greatly exaggerated; actually $\gamma \cong 0.02° \dots 0.04°$, and the deflection of rotating wall is practically invisible, as it is in Fig. 3a.

where
$$v_{r\max} = \frac{n+1}{n} c,$$

and

$$\left.\begin{aligned} v_\varphi &= k\omega r\left(\frac{2z}{s}\right)^{\frac{1}{n}}, && \text{for } 0 \le z \le \frac{s}{2} \\ v_\varphi &= \omega r\left\{1 - (1-k)\left[2\left(1-\frac{z}{2}\right)\right]^{\frac{1}{n}}\right\}, && \text{for } \frac{s}{2} \le z \le s \end{aligned}\right\} \tag{11}$$

where

$$k = \frac{(v_\varphi)_{z=s/2}}{\omega r} \le 0.5 \tag{12}$$

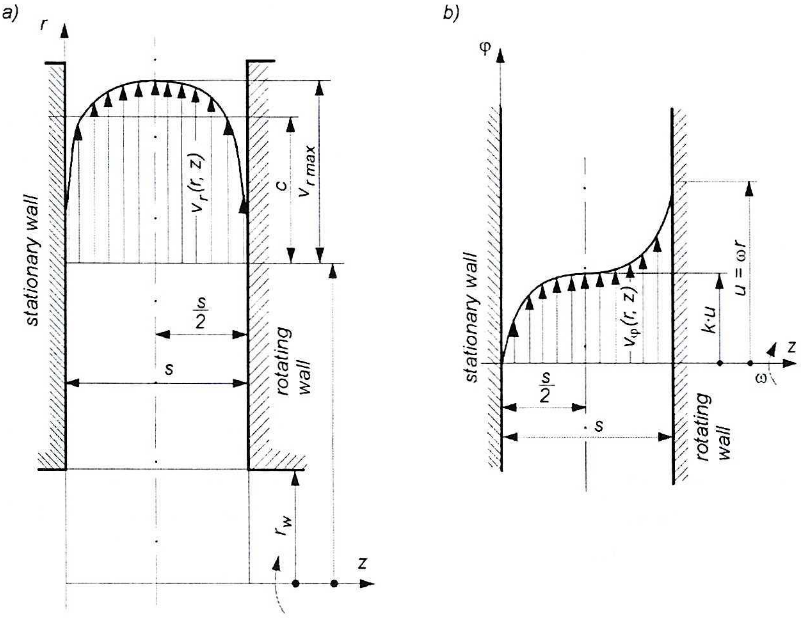

Fig. 3. Distributions of velocities averaged over time, $v_r(z)$ and $v_\varphi(z)$

3. Pressure distribution in conical clearance

After substituting (10) and (11), both sides of equation (9) are multiplied by dz, integrated first over the limits 0-s/2, s/2-s, and then added side-to-side and integrated from r_{in} to r. Assuming – similarly as in [1] – that $\bar{k} = k_{\text{av}} = \text{const}$, and taking into account formulae (1) ... (6) we obtain

$$p(r) = p_{in}' + \delta p_{\text{diff}} + \delta p_{\omega} - \delta_{pf} \tag{13}$$

where δp_{diff} is the pressure increase resulting from the drop of velocity c (the clearance is a diffuser)

$$\delta p_{\text{diff}} = \rho \frac{c_{\text{in}}^2}{2} \frac{(n + 1)^2}{n(n + 2)} \left[1 - \left(\frac{r_{\text{in}}}{r} \right)^4 \right] \tag{14}$$

δp_{ω} is the increase of pressure due to rotation of the movable wall

$$\delta p_{\omega} = \frac{\rho \omega^2}{2} (r^2 - r_{\text{in}}^2) \frac{1 + \bar{k}n + \bar{k}^2 n + \bar{k}^2 n^2}{(n + 1)(n + 2)} \tag{15}$$

and δp_f is the pressure drop due to friction on both walls of the clearance

$$\delta_{pf} = \int_{r_{\text{in}}}^{r_{\text{ou}}} \frac{\tau_{sr} + \tau_{\omega r}}{s} \mathrm{d}r \tag{16}$$

while $p_{\text{in}}' = p_{\text{in}} - \Delta p_{\text{in}}$, where Δp_{in} is the pressure drop at the clearance inlet (Fig. 1a), and $\bar{k}$ – average value of liquid swirl coefficient between r_{in} and r_{ou}.

Denotations of radial components of stresses on the walls are as follows:
- stationary wall (s) – $(\tau_{zr})_{z=0} = \tau_{sr}$
- rotating wall (ω) – $(\tau_{zr})_{z=s} = \tau_{\omega r}$.

The values of stresses τ_s on the stationary wall and on the rotating one, t_{ω} (Fig. 4) were calculated, similarly as in [1], from the following formula

$$\tau = \varrho \frac{v_t^2}{2} \frac{\alpha}{\mathrm{Re}_t^{\frac{2}{n+1}}} \tag{17}$$

that was obtained by generalising the known relationship describing the flow in a pipe of circular section, as well as other cases of flows, such as flows in plane gaps and in axial annuli.

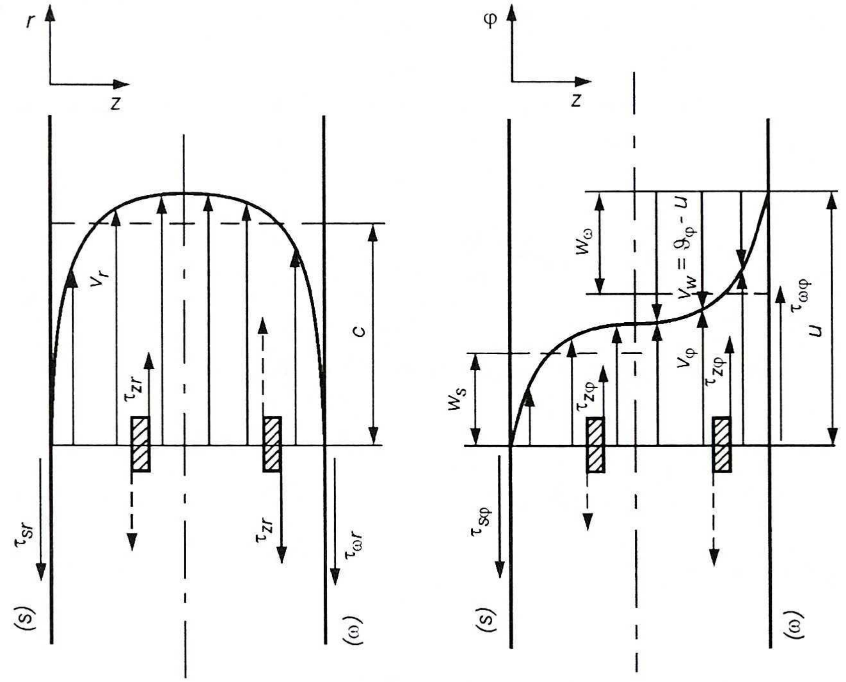

Fig. 4. Radial (r) and circumferential (φ) stresses in the fluid and on the clearance walls, the rotating one (ω) and the stationary one (s)

The resulting fluid flow velocity v_t with respect to the walls was calculated as a mean value over the clearance width, separately for both halves of the clearance,

$$v_t = \sqrt{c^2 + w^2} = \sqrt{c^2 + (\beta u)^2} \tag{18}$$

where
$$w = |\mathbf{w}| = \beta u \tag{19}$$

$$\left.\begin{aligned} \beta_s &= \bar{k}\frac{n}{n+1} \\ \beta_\omega &= (1-\bar{k})\frac{n}{n+1} \end{aligned}\right\} \tag{20}$$

(Fig. 3a)
$$c = \frac{n}{n+1} v_{r\max} \tag{21}$$

A conventional Reynold's number that pertains to this value equals

$$\mathrm{Re}_t = \frac{v_t \cdot 2s}{\nu} = \mathrm{Re}\frac{v_t}{c} \tag{22}$$

It follows on the assumption expressed by formula (17) that the stresses τ on the walls, and the velocities v_t, have the same directions but opposite senses (Fig. 5). From this property, there follows the equality of respective angles ϑ and the relationship valid for both walls

$$\tau_r = \tau \cos \vartheta = \tau \frac{c}{v_c} = \left[1 + (\beta N)^2\right]^{-1/2} \tag{23}$$

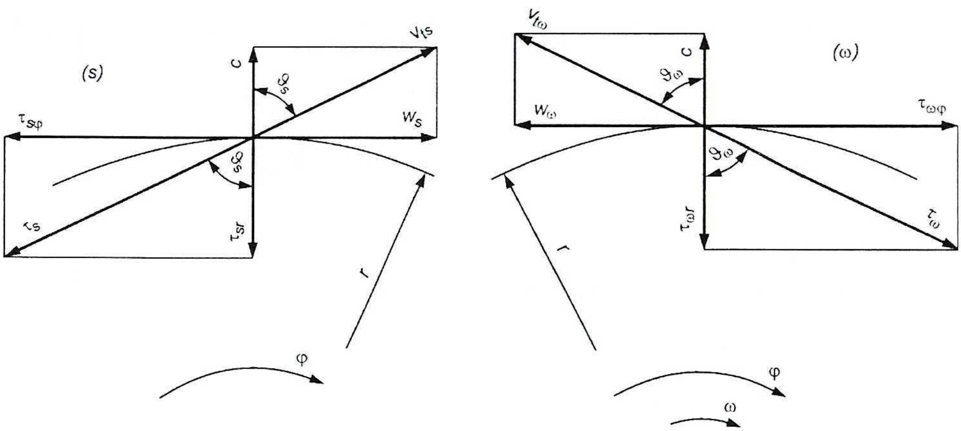

Fig. 5. Velocities, total stresses and their components on the stationary (s) and the rotating wall (ω)

On the basis of formulae (17) ... (23) one finally obtains the values of stresses on the stationary (s) and the rotating wall (ω)

$$\left.\begin{aligned}\tau_{nr} &= \rho \frac{c^2}{2} \alpha \mathrm{Re}^{-\frac{2}{n+1}} \left[1 + (\beta_s N)^2\right]^{\frac{n-1}{2(n+1)}} \\ \tau_{\omega r} &= \rho \frac{c^2}{2} \alpha \mathrm{Re}^{-\frac{2}{n+1}} \left[1 + (\beta_\omega N)^2\right]^{\frac{n-1}{2(n+1)}}\end{aligned}\right\} \tag{24}$$

In order to avoid solving elliptic integrals when calculating (16), one must expand the expressions in the square brackets into Taylor series. However, because there could be $\beta N \le 1$ as well as $\beta N > 1$, one should apply two different expansions, which would be inconvenient in practice. Then, similarly as it was done in [1], in this work the expressions in brackets were approximated by one approximate polynomial (with an error < 4.5% for $\beta N \le 3$):

$$\left[1 + (\beta N)^2\right]^{\frac{n-1}{2(n+1)}} \cong 1 + \frac{n - 1}{2(n + 1)} (\beta N)^2 - \frac{(n - 1)(n + 3)}{12(n + 1)^2} (\beta N)^3$$

Carrying out integration (16) with the use of the above relations we obtain

$$\delta_{pf} = \delta_{pf0}\,\varphi_{\text{con}} \tag{25}$$

where

$$\delta_{pf0} = \rho\frac{c_{\text{in}}^2}{2}\frac{\alpha}{\text{Re}_{\text{in}}^{\frac{2}{n+1}}}\frac{d_{\text{in}}}{2s_{\text{in}}}\frac{n+1}{2n+1}\left[1-\left(\frac{r_{\text{in}}}{r}\right)^{\frac{4n+2}{n+1}}\right] \tag{26}$$

is the pressure drop between r_{in} and r for a conical clearance of immobile walls, and the increase of the pressure resulting from rotating motion of the disc is expressed by the coefficient

$$\begin{aligned}\varphi_{\text{con}} = 1 &+ \frac{(n-1)(2n+1)n^2}{2(n+1)^3(2n+4)}N_{\text{in}}^2(1-2\bar{k}+2\bar{k}^2)\frac{\left(\frac{r}{r_{\text{in}}}\right)^{\frac{2n+4}{n+1}}-1}{1-\left(\frac{r_{\text{in}}}{r}\right)^{\frac{4n+2}{n+1}}} + \\ &- \frac{(n-1)(2n+1)(n+3)n^3}{12(n+1)^5(5n+7)}N_{\text{in}}^3\cdot(1-3\bar{k}+3\bar{k}^2)\frac{\left(\frac{r}{r_{\text{in}}}\right)^{\frac{5n+7}{n+1}}-1}{1-\left(\frac{r_{\text{in}}}{r}\right)^{\frac{4n+2}{n+1}}}\end{aligned} \tag{27}$$

On the basis of equations (13) ... (15) and (25) ... (27), one can calculate the force F_g acting on the rotating surface of conical gap. One can also calculate the total pressure drop in a clearance limited by the radii r_{in} and r_{ou}:

$$\Delta p = \Delta p_{\text{in}} + \Delta p_f - \Delta p_{\text{diff}} - \Delta p_{\omega} \tag{28}$$

where

$$\Delta p_{\text{in}} = \rho\frac{c_{\text{in}}^2}{2}\zeta_{\text{in}} \cong \rho\frac{c_{\text{in}}^2}{2}\cdot 1.05 \tag{29}$$

and the remaining pressure differences can be calculated by substituting $r = r_{\text{ou}}$ into formulae (14), (15), (26), (27).

For a wide range of turbulent flows, one usually assumes $n = 7$ and $\alpha = 0.0971$, the validity of which has been experimentally verified for axial annuli. In the here-discussed case, we introduce the symbols that simplify the notation

$$\frac{d_{ou}}{d_{in}} = a, \qquad \frac{d_{in}}{d_{ou}} = \frac{1}{a} = b, \tag{30}$$

and then we obtain

$$\Delta p_{diff} = \frac{64}{63}\rho\frac{c_{in}^2}{2}(1 - b^4) \tag{31}$$

$$\Delta p_{\omega} = \rho\frac{\omega^2}{2}d_{in}^2(a^2 - 1)\frac{1 + 7\bar{k} + 56\bar{k}^2}{288} \tag{32}$$

$$\Delta p_{f0} = 0.02109\,\rho c_{in}^2\frac{d_{in}}{2s_{in}}\mathrm{Re}_{in}^{-1/4}\left(1 - b^{\frac{15}{4}}\right) \tag{33}$$

$$\begin{aligned}\Phi_{con} = 1 &+ \frac{5\cdot 49}{1024}N_{in}^2(1 - 2\bar{k} + 2\bar{k}^2)\frac{a^{\frac{9}{4}} - 1}{1 - b^{\frac{15}{4}}} + \\ &- \frac{25\cdot 49}{128\cdot 512}N_{in}^3(1 - 3\bar{k} + 3\bar{k}^2)\frac{a^{\frac{21}{4}} - 1}{1 - b^{\frac{15}{4}}}\end{aligned} \tag{34}$$

$$\Delta p_f = \Delta p_{f0}\,\Phi_{con} \tag{35}$$

The averaged value $\bar{k} = k_{av}$ of the fluid swirl coefficient in axial clearance depends on the value k_{in} at the clearance inlet, and the latter depends on the course of function $k_1(r)$ between the outlet of axial annulus (before the counterdisc) and the inlet to the radial gap. Based on [2] and making some additional estimations, one can assess that this value should lie within the limits $k = 0.3 \ldots 0.38$. A more precise value of $\bar{k}$ can be found by calculating first $\bar{k}(r)$ in the region between r_1 and r_{in}, and then $k(r)$ in the axial clearance, using the methodology described in [2].

The above formulae have been derived on the assumption that the surfaces of the clearances are hydraulically smooth, which is confirmed by the results of measurements done for parallel-walled clearances, as well as for axial annuli. It seems that the formulae are applicable up to $\mathrm{Re}_{in} \cong 5 \cdot 10^4$, or, according to [1]:

$$\mathrm{Re}_{in} \leq \mathrm{Re}_{in,\,lim} \cong 126\left(\frac{s_{in}}{h_{av}}\right)^{8/7} \tag{36}$$

where h_{av} is the average height of wall surface roughness.

4. Force acting on conical disc surface

The force F_g acting on the rotating, conical surface of radial gap (Fig. 1) constitutes 25–30% of the total loading force F_{bal}. It is lower by 8–15% (depending on geometry of the system with balancing disc) than the force acting on the surface of a clearance of parallel walls. The difference is due to a greater velocity c_{in} and an increased pressure drop Δp_{in} at the clearance inlet. The force can be calculated from the relationship that is obtained after substituting the relationships (13)÷(15) and (25)÷(27) into the definition formula

$$F_g = 2\pi \int_{r_{in}}^{r_{ou}} p(r) r dr. \tag{37}$$

Assuming – as in the previous case – $n = 7$, one obtains

$$\begin{aligned} F_g = \frac{\pi}{4} d_{in}^2 + \Bigg\{ &(a^2 - 1)\Delta p_g + \frac{32}{63}\rho c_{in}^2 (a^2 - 2 + b^2) + \\ &+ \rho\omega^2 \frac{d_{in}^2}{4}(a^2 - 1)^2 \frac{1 + 7\bar{k} + 56\bar{k}^2}{288} - 0.02109 \rho c_{in}^2 \frac{d_{in}}{2 s_{in}} \mathrm{Re}_{in}^{-1/4} \Bigg[a^2 + b^{7/4} + \\ &- \frac{15}{7} + \frac{5 \cdot 49}{2 \cdot 512} N_{in}^2 (1 - 2\bar{k} + 2\bar{k}^2)\left(\frac{8}{17} a^{\frac{17}{4}} - a^2 + \frac{9}{17}\right) + \\ &- \frac{25 \cdot 49}{128 \cdot 512} N_{in}^3 (1 - 3\bar{k} + 3\bar{k}^2)\left(\frac{8}{29} a^{\frac{29}{4}} - a^2 + \frac{21}{29}\right) \Bigg] \Bigg\}. \end{aligned} \tag{38}$$

If we have assumed, as one usually does, a rectilinear distribution of pressure $p(r)$ in the clearance, the calculated value of force F_g would be greater by approximately 10...15%. This fact confirms the purposefulness of using formula (38).

5. Example of calculations

The below-presented numerical example shows how, with a constant force F_{ax} carried by the balancing disc, conical shape of axial clearance influences the decrease of pressure drop in it, and consequently leads to the decrease of s_{in} at the inlet. The boiler feed pump having rate of flow $Q = 275$ m^3/h and head $H = 1820$ m is selected for the example.

The dimensions of parallel-walled clearance are: $d_{ou} = 300$ mm, $d_{in} = 200$ mm, $s = 0.088$ mm. The remaining data: $\omega = 488$ rad/s ($n = 4660$ rpm); $\rho = 910$ kg/m^3, $\nu = 0.2 \cdot 10^{-6}$ m^2/s (hot water $t = 158$°C); $c_{in} = 63.1$ m/s. With the above data we have: $u_{in} = 48.8$ m/s; $N_{in} = 0.7734$; $\mathrm{Re}_{in} = 5.553 \cdot 10^4$.

Using the formulae derived in [1] one calculates: $\Delta p_{diff} = 1.0224$ MPa; $\Delta p_{\omega} = 0.1531$ MPa (for $\bar{k} = 0.3$); $\Delta p_{f0} = 7.4196$ MPa; $\Phi = 1.1018 \rightarrow \Delta p_f = 8.175$ MPa; $\Delta p_{in} = 1.19022$ MPa (assuming $\zeta_{in} = 1.05$), and then $\Delta p = \Delta p_{in} + \Phi \Delta p_{f0} - \Delta p_{diff} - \Delta p_{\omega} = 8.9017$ MPa.

Applying the formulae (28) ... (35) for a conical clearance of inlet width $s_{in} = 0.088$ mm, the same as that mentioned before, one obtains the pressure drop as low as $\Delta p = 4.84$ MPa. In order to satisfy again the condition $F_{bal} = F_{ax}$ and to maintain the same through-flow Q_l, and – at the same time – keep the same pressure drop in both the axial annulus and the radial gap, the balancing disc must be shifted closer to the counterdisc, which means that the width must be set to $s_{in} < 0.088$ mm. Using the above formulae and applying the method of successive approximations, one calculates $s_{in} = 0.0729$ mm ($\gamma = 0.042°$), and $c_{in} = 76.59$ m/s; $N_{in} = 0.6372$; $\Delta p_{in} = 2.803$ MPa; $\Delta p_{diff} = 2.176$ MPa $\rightarrow \Delta p = 9.025$ MPa. In the calculations, one assumes $\bar{k} = 0.35$ – a value greater than that for a parallel-walled clearance – taking into account that the ratio $N = u/c$ strongly increases with the radius r, and in consequence the swirl of fluid is greater for $r > r_{in}$. It can be easily proven that the influence of the $\bar{k}$ value on the final result is small (for $\bar{k} = 0.16$ one would obtain $\Delta p = 9.223$ MPa, so that it would be only 2.2% greater than with $\bar{k} = 0.35$).

It must be emphasised that the above calculation is approximate, because – despite the same value of total pressure drop Δp in both clearances – their pressure distributions are different (Fig. 1), and thus the forces F_g must be different, too. The smaller force for conical clearance will result in a decrease of force F_{bal} so that re-establishing the force balance $F_{bal} = F_{odc}$ would be possible only after an additional decrease of the clearance width at the inlet, down to the value of $s_{in} \cong 0.07$ mm.

A more accurate value s_{in} can be determined by performing a complete calculation for the whole balancing disc, with respect to the abovementioned condition of equality between the balancing force and the axial thrust.

5. Conclusions

1. The result of deflection of the balancing disc due to the pressure forces acting on it is that the clearance between the disc and the counterdisc becomes conical. It leads to a decrease of the clearance width s_{in} at its inlet, and causes a simultaneous increase of the speed c_{in}. These effects deteriorate, in an obvious way, functional properties of a pump with balancing disc, and increase probability of pump's failure.
2. A more precise numerical examination of the influence of disc deflection needs calculating this deflection (e.g. using the Finite Element Method) and determining flows and pressure distributions in the whole balancing device, taking into account the distribution $p(r)$ of pressure in the region between the radial annular gap and the axial annulus, i.e. for $r_1 \leq r \leq r_{in}$ (Fig. 1).
3. In order to prevent the negative effects of conical axial clearance, one should give the surface of the balancing disc a "counter-conical" shape, so that, after deflection of the disc, the walls would become parallel, or one can increase the disc diameter d_{ou}. Additionally, in order to compensate also the influence of shaft deflection, one can apply a counterdisc with elastic support [4].

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Rozkład ciśnień w turbulentnym przepływie cieczy przez stożkową szczelinę poprzeczną

S t r e s z c z e n i e

Projektując układ z tarczą odciążającą w pompie wielostopniowej zakłada się, że powierzchnie tworzące szczelinę poprzeczną (promieniową) są do siebie równoległe (rys. 1a). Jednak tarcza odkształca się, wskutek sił spowodowanych dużą różnicą ciśnień działających na jej powierzchnie, wskutek czego szczelina staje się stożkowa (rys. 1b). Powoduje to zmianę rozkładu ciśnień w szczelinie i zmniejszenie siły osiowej F_g działającej na wirującą powierzchnię szczeliny, w wyniku tego zaś – zmniejszenie siły odciążającej F_{bal}. Aby przenieść nie zmieniony napór osiowy F_{ax} musi wzrosnąć siła $F_{bal} = F_{ax}$, co wymaga zbliżenia wirującej się tarczy do nieruchomej przeciwtarczy i zmniejszenia szerokości szczeliny. Szerokość s_{in} na wlocie szczeliny może stać się wówczas zbyt mała, co grozi zetknięciem się obu tarcz i poważną awarią pompy.

Aby zapobiec negatywnym skutkom ugięcia tarczy należy móc uprzednio dostatecznie dokładnie obliczyć parametry przepływu cieczy w szczelinie. W pracy wyprowadzono zależności na rozkład ciśnień w hydraulicznie gładkiej poprzecznej szczelinie stożkowej dla przypadku przepływu turbulentnego. Do rozwiązania zastosowano metodę związków całkowych zakładając przepływ osiowo-symetryczny i potęgowe rozkłady prędkości promieniowych i obwodowych. Wykorzystano znane zależności półempiryczne na naprężenia na obu powierzchniach tworzących szczelinę. Wyprowadzono także wzór na siłę F_g dla szczeliny stożkowej.

Zamieszczono przykład liczbowy pokazujący, jak stożkowość szczeliny wpływa na zmniejszenie się spadku ciśnienia Δp w szczelinie oraz siły F_g, a w konsekwencji – na zmniejszenie się szerokości s_{in} (o ok. 20%) w układzie odciążającym pompy zasilającej kocioł, o wydajności $Q = 275$ m^3/h i wysokości podnoszenia $H = 1820$ m.