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ANALYSIS OF PUMP ENERGY CONSUMPTION IN THE UPPER SLOPE REGION OF EFFICIENCY CHARACTERISTIC CURVE

The article presents methods of evaluating energy consumption of water pumps. Pump characteristics during work at variable rotational speed and constant head are discussed. Characteristics of the upper slope of pump efficiency curve were determined for the cases of constant and variable growth of rotational speed The left-sided coefficient of pump efficiency correction was defined.

ESSENTIAL TERMS

a	– pipe friction loss coefficient,
e	– unit energy consumption of pumping unit,
g	– gravitational constant [m/s²],
H	– medium head [m],
H_n	– default medium head,
H_{st}	– static head of pumping plant,
H_t	– head of pressure in pressure collector,
H_{uk}	– head of pumping plant,
H_{zn}	– geometric head of inflow,
$h^* = \frac{H}{H_n}$	– relative medium head,
m	– number of working pump sets,
n	– rotational speed [rpm],
n_n	– default rotational speed,
$n^* = \frac{n}{m_n}$	– relative rotational speed,

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$n_q = \frac{n}{60} \frac{\sqrt{Q}}{H^{3/4}}$ – specific speed,
P_{el} – electric power consumed by pumping unit [kW],
p_s – pressure at pump suction branch,
p_t – pressure at pump discharge branch,
Q – capacity of pump set [m^3/h],
Q_n – default capacity of pump set,
$q^* = \frac{Q}{Q_n}$ – relative capacity of pump set,
η – pump efficiency,
η_f – efficiency of frequency converter,
η_g – efficiency of pumping unit,
η_{max} – maximum pump efficiency at default rotational speed,
η_n – characteristic curve approximating pump efficiency,
η_{opt} – optimum pump efficiency,
η_s – efficiency of induction motor,
η_z – efficiency of pump set,
ρ – density of medium [kg/m^3].

1. Introduction

In 1993, 62.500 GWh electricity, or 56% of national output of electricity, was used in Poland to power a variety of devices [2]. Most of the energy was powering pumps. The share of energy consumption by pumps is estimated at (25–30)% of the national electricity consumed to drive machinery. A number of methods are applied to save electricity. Manufacturers implement solutions to increase efficiency of pumps and motors. Pump control, featuring regulated rotational speed, limits energy-consumption of pumping water that is typical for throttling or discharge methods. In some automatic solutions, expert systems such as neural networks, fuzzy logic or genetic algorithms are applied beside the classic regulators.

The article analyses efficiency characteristics of small and medium default power pumps. These pumps are widely used in pump plants that satisfy demand for drinking water in building complexes, housing estates, small towns, etc. Pumps are also commonly used to supply technological water, for instance, to food or chemical industry.

For purposes of the article, it was assumed that the classic regulator PI is used to adjust the pump's rotational speed. Additional savings of electricity are possible after implementing a method of control that would ensure pump operation in the vicinity of maximum efficiency point. The article is intended to present a method of analysing efficiency characteristics of the pump as an object of control at variable rotational speed, which will allow for pump operation in changing conditions at best possible efficiency, thus reducing energy consumption of pumping.

2. Energy consumption of pumping units

Application of modern electronic power components in pumping unit powering systems enables development of measures to rationalise energy consumption. The amount of consumed electricity depends on parameters of pumping set including a pump and its electric powering equipment. Power drive of a pump unit consists of a motor and its powering device in the form of frequency converter or one of other electric devices that power the motor directly from three-phase system.

Pump sets in series or parallel connection between suction and discharge branches make up one pumping unit. A pump unit including one pump set is a special case. It is assumed that pumps within a pump unit have identical rated capacities Q_n, and heads H_n. Appropriate operation of a pumping unit requires its fitting with:

- Hydraulic fixtures including check and shut-off valves,
- Control and measurement components including pressure converters, water-meters, manometers,
- Controlling and safety components.

Pumping unit components including controlling systems should reduce electricity consumption, which is equivalent to minimising momentary values of electric power P_{el} consumed by a pumping unit, defined as follows:

$$P_{el} = \sum_{i=1}^{m} \frac{\rho g Q_i H_i}{\eta_{zi}}, \tag{1}$$

where:

$$\eta_z = \eta \eta_s \eta_f. \tag{2}$$

Dividing the expression (1) by momentary total capacity of pump unit $Q = \sum_{i=1}^{m} Q_i$, adding water density $\rho = 1000$ kg/m^3, and gravitation constant $g = 9{,}81$m/s^2, and assuming identical head H of each pump set, the unit energy-consumption e of pumping unit in the process of pumping 1 m^3 water is obtained as follows:

$$e = \frac{1}{367}\frac{H}{Q}\sum_{i=1}^{m}\frac{Q_i}{\eta_{zi}}\left[\frac{kWh}{m^3}\right]. \tag{3}$$

Assuming identical efficiencies η_z of pumping sets, the following expression is obtained to define the unit energy consumption e_1:

$$e_1 = \frac{1}{367}\frac{H}{\eta_z}\left[\frac{kWh}{m^3}\right]. \tag{4}$$

Total efficiency of a pumping unit (a group of pumps) η_g is described in the following relation:

$$\eta_g = \frac{Q}{\sum_{i=1}^{m}\frac{Q_i}{\eta_{zi}}}. \tag{5}$$

Depending on:

- Application of high-efficiency water pumps,
- Selection of pumps appropriate to parameters of water supply system,
- Use of energy-saving motors whose design allows for powering with frequency converters,
- Application of frequency converters plotting a sinusoid or rectangular course of motor feed voltage,
- Method of controlling of pumping unit work.

One of the factors that rapidly increases efficiency of a pumping unit η_g while reducing the unit energy consumption e at a minimum expenditure is a good choice of the control method.

The unit energy consumption e reaches minimum values at maximum efficiency η_{zi} of each pumping set (3). Efficiency of a pumping set η_z depends on the pump's operating point, determined by capacity Q, head H, and rotational speed n. Maximum value of efficiency η_z occurs for one point of flow characteristic curve. In few pump applications is it reasonable for the pump to operate at one rotational speed. Rotational speed n is adjusted through frequency variations of the voltage powering the induction motor. When frequency variations of the voltage powering the induction motor are within the range (35÷55) Hz, motor efficiency η_s and efficiency of frequency converter η_f vary up to 5% in relation to their maximum values [11]. Such variations have slight impact on electricity consumption of a unit. Therefore, efficiency changes η^s and η_f are assumed to exert negligible influence on efficiency of a pump set η_z as defined in (2). Assuming the above, the relation (3) indicates that, within the range (35÷55)Hz of frequency of voltage powering the pump motor, the value of unit energy consumption e is significantly dependent upon the pump efficiency η.

3. Representation of performance chart with a two-element function set

A performance chart contains information about variations of head H depending on capacity Q, rotational speed n, and pump efficiency η. It can be generally stated:

$$H = f(Q, n, \eta). \tag{6}$$

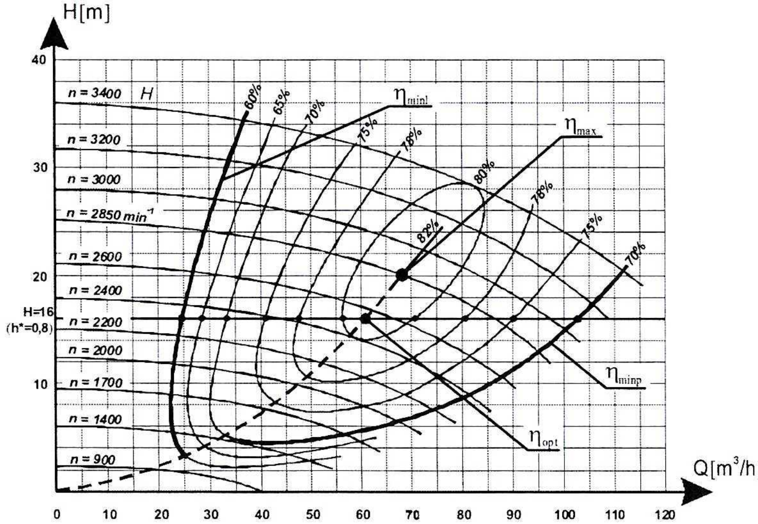
Design differences between pumps influence distribution of performance chart characteristics on the plane (H,Q). One of fundamental figures describing pump characteristics is the specific speed n_q. Distribution of electric power P_{el} in function of capacity Q depends on the specific speed n_q [5]. The following analysis concerns single-flow, two-flow, single-stage, and multistage pumps with a specific speed n_q which ensures reduced electricity consumption at decreasing capacity Q, i.e. $n_q \cong 20...70$.

Analysis of pump energy consumption was based on a sample performance chart [7] shown in Fig. 1a. The energy consumption criterion assumed minimizing of unit energy consumption e (3). Low pumping energy consumption occurs when a pump operates in the vicinity of its maximum efficiency values η. To know pump properties with regard to its energy consumption in detail, it is necessary to analyse planar distribution of pump efficiency η (Q, n). Distribution of pump efficiency depends on the method of controlling operation of a pumping set. In most industrial applications, a control method is chosen that guarantees constant head H (H = const) throughout the range of pump capacities Q. Such a control method is included in analysis of pump efficiency characteristics.

Information about pump efficiency η in function of rotational speed n, of capacity Q, with H = const, is included in the performance chart. The presentation of pump efficiency η as a performance chart is not very clear and useful for evaluation of pumping energy consumption. A new presentation of performance chart requires that the function $H = f(Q, n, \eta)$ be divided into the following elementary functions, constituting six elements of a set **M**:

$$\mathbf{M} = \left| \begin{array}{lll} \{1\}: & \eta = f_1(q^*) & \text{where: } n^* = \text{const} \\ \{2\}: & \eta = f_2(q^*) & \text{where: } h^* = \text{const} \\ \{3\}: & \eta = f_3(n^*) & \text{where: } h^* = \text{const} \\ \{4\}: & \eta = f_4(n^*) & \text{where: } q^* = \text{const} \\ \{5\}: & \eta = f_5(h^*) & \text{where: } n^* = \text{const} \\ \{6\}: & \eta = f_6(h^*) & \text{where: } q^* = \text{const} \end{array} \right. \qquad (7)$$

a)



b)

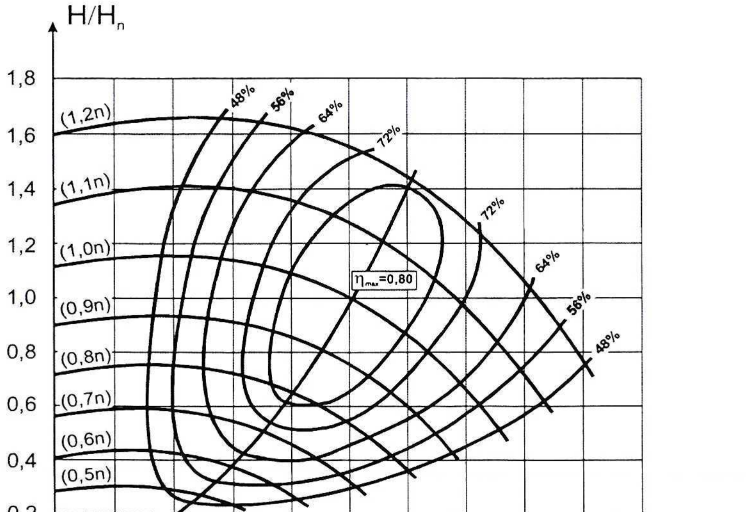


Fig. 1. Performance chart of a pump a) on the basis of [7] with specific speed $n_q \cong 42$, and b) based on [6] with specific speed $n_q \cong 20 \div 30$

Full information included in the performance chart as the function (6) is interpreted by any two elements of set **M** within the range {1}÷{6}. Assuming number k = 6 of all elements in set **M**, and introducing arbitrary subsets of the set, containing any m = 2 elements of **M** each, the following total number of subset combinations is obtained:

$$C_k^m = \frac{k!}{m!\,(k - m)!} \tag{8}$$

The total number of two-element subsets is 15. It means that pump characteristics can be analysed by evaluating one of the fifteen two-element subsets. In view of the diverse relations among elements {i} of set **M**, four subset groups are produced:

1. subset group where arguments of functions f_i and f_j constitute the same variable:

$$\begin{aligned} &\{1\} + \{2\} \text{ common variable } q^* \\ &\{3\} + \{4\} \text{ common variable } n^* \\ &\{5\} + \{6\} \text{ common variable } h^* \end{aligned} \tag{9}$$

2. subset group where constant-value parameters for functions f_i and f_j constitute the same variable:

$$\begin{aligned} &\{2\} + \{3\}\ h^* = \text{const} \\ &\{1\} + \{5\}\ n^* = \text{const} \\ &\{4\} + \{6\}\ q^* = \text{const} \end{aligned} \tag{10}$$

3. subset group where a parameter with constant value of function in f_i is replaced with argument of the function f_j, and vice versa:

$$\begin{aligned} &\{2\} + \{6\}\ h^* \leftrightarrow q^* \\ &\{3\} + \{5\}\ h^* \leftrightarrow n^* \\ &\{1\} + \{4\}\ q^* \leftrightarrow n^* \end{aligned} \tag{11}$$

4. subset group where there is no logical interdependence between parameters and arguments of functions f_i and f_j:

$$\begin{aligned} &\{1\} + \{3\};\ \{1\} + \{6\};\ \{3\} + \{6\} \\ &\{2\} + \{4\};\ \{2\} + \{5\};\ \{4\} + \{5\} \end{aligned} \tag{12}$$

where:
i, j – are natural numbers within the range <1,6>, and $i \neq j$.

The resulting two-element subsets must be appraised in respect of their applicability to the analysis of pump characteristic curve η. Results of such analysis would make it possible to find a method of controlling a unit operation that would allow for low energy consumption.

The pumping process was assumed to occur at variable rotational speed n of the pump. These variations result in varied capacity q^* and efficiency η of the pump. Pump capacity q^* ought to conform with current water consumption. The conformity of capacity q^* with water consumption is reflected in the relation of geometric head of pressure in suction branch H_t, to the default head H_{zad}. If the head H_t is larger than H_{zad} the pumps rotational speed n^* needs to be lowered in order to reduce capacity q^*. In case $H_t < H_{zad}$ one should try to increase the pump's rotational speed. This rule of controlling the pump operation is the basis for defining the criterion of utility of the subset of set **M** for evaluation of pumping energy consumption in the process of capacity q^* regulation. It is assumed that a subset of **M**, chosen for purposes of analysing energy consumption, should include the functions f_i, f_j, with arguments capacity q^* or rotational speed n^*. It is also assumed that, in the process of regulation, head h^* in the suction branch is constant ($h^* = \text{const}$). The above assumptions, which result from the method of regulating the pumps operation, limit the analysis of pump efficiency on the basis of performance chart to one subset: $\{2\} + \{3\}$.

4. Pump efficiency at constant head

Regulation of pump rotational speed n^* while maintaining a constant head H influences the pump efficiency η as well. Knowledge of pump efficiency η characteristic curve in function of rotational speed n^*, and capacity q^* at $H = \text{const}$, becomes a major element in the analysis of pump energy consumption. Pump efficiency η characteristic curves $\eta = f(n^*)$, and $\eta = f(q^*)$ at $H = \text{const}$ are to a certain extent dependent upon the pump type, e.g. one-, two-flow, single- or multi-stage. One can obtain a proper course of pump efficiency η only on the basis of reliable performance charts.

The process of pumping water adjusts capacity Q of a pumping unit to water consumption. Varying consumptions produce diverse speeds of water flows, which causes pressure losses in the line. In effect, if one wishes to obtain constant pressure at the least favourable point of the water supply system, pressure in the suction branch has to correspond to the line (plant) characteristic curve, as defined in the equation:

$$H_{uk} = H_{st} + aQ^2. \quad (13)$$

In specific conditions, the following equation has to obtain:

$$H = H_{uk}. \quad (14)$$

Assuming identical water speeds c_s in the suction branch s and speed c_t in the discharge branch t, and negligible difference of altitude of suction branch and discharge branch axes Δz, the following relationship defining the pump head H obtains [5]:

$$H = \frac{p_t - p_s}{\rho g}. \quad (15)$$

The expression (15) can be replaced with the following dependence:

$$H = H_t - H_{zn}. \quad (16)$$

Considering the equations (13), (14), (16), the relationship defining head of pressure in the suction branch H_t in relation to momentary capacity Q of pump unit is obtained:

$$H_t = H_{st} + H_{zn} + aQ^2. \quad (17)$$

The relationship (17) determines the head in the suction branch H_t necessary to recompense pressure loss in the pipeline due to water flows. Variations of head H_t are mainly due to the changes of capacity Q. There are applications where head of inflow H_{zn} exerts a substantial influence upon head H_t at the same time enforcing a reduced pump head H (16).

Most applications work based on the condition of maintaining constant head of pressure H_t in suction branch. In the case of constant inflow head H_{zn}, constant temporary head H of the pump is obtained. The assumption of constant temporary head concerns the variable of the pumping process which remains constant for even a few minutes. Constant head H is obtained through a change of capacity Q of one or several pumps in a unit. Capacity Q is most often altered by changing the pump's rotational speed n^*.

Sample characteristic curves of pump efficiency in relation to non-dimensional constants of head h^*, flow q^*, and rotational speed n^*, are shown in Fig. 2. Pump efficiency characteristics η were determined on the basis of performance chart characteristic curve described in [6]. For purposes of the analysis, pump efficiency η was assumed to vary within the range 0.48÷0.8. Pump efficiency η exhibits slight differences for diverse heads h^*, and constant flow q^* less than the rated flow of one (Fig. 2a). In the case of capacity q^* below the rated value, variations of efficiency do not exceed 5%. In the case of relative flow $q^* > 1$, values of efficiency η vary to a significant degree. Differences among pump efficiencies, with identical capacity q^*, over the rated flow reach as much as 15% within the range of head $h^* \in (0,6; 1,2)$.

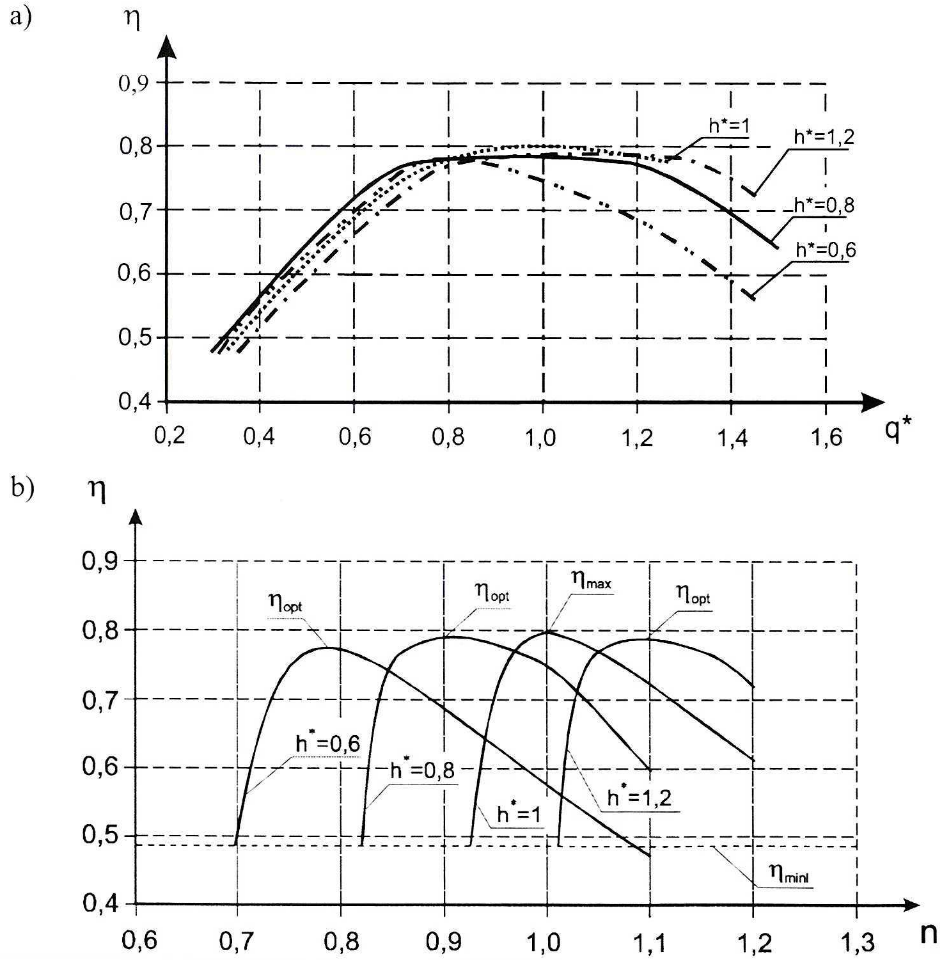


Fig. 2. Pump efficiency η for constant head h^*, obtained on the basis of [6]

Pump efficiency η is strongly dependent on variations of rotational speed n^* (Fig. 2b). Reduction of rotational speed n^* o 0,1 while keeping constant head h^* lowers the value of pump efficiency by 0.3. A large drop of pump efficiency is ineffective in respect of operating costs [3].

4.1. The upper slope of the pump efficiency characteristic curve

The courses of pump efficiency in function of rotational speed n^* were analysed on the basis of available performance charts of various type pumps. Pump efficiency η was determined while maintaining constant head h^*. On the basis of available source material, the variation range of relative pump head was assumed to be within the range $h^* \in (0{,}6 \div 1)$.

In order to fully analyse the nature of variation of pump efficiency (, efficiency characteristic curve was divided into two slopes: upper and lower. Both parts of the efficiency characteristic curve have their domains, delimited by boundary values of rotational speed n^* and corresponding boundary pump efficiencies η. Boundary values of efficiency for the upper slope are defined as follows:

- Left side efficiency η_{minl} corresponding to the curve for minimum efficiency η_{minl} in the region of capacity Q lower than rated capacity (Fig. 1).
- right side efficiency η_{opt}, corresponding to optimum pump efficiencies η_{opt} situated on the common similarity parabola.

Optimum efficiency η_{opt}, determined for the rated speed, is defined as maximum efficiency. Maximum pump efficiencies η_{max}, usually have values in the range 20%–90% depending on the pump's rated power. Significant differences of maximum pump efficiencies mean that comparative analysis of various types pumps is not sufficiently general. Analysis of pump efficiency characteristic curve is carried out by introducing the notion of relative pump efficiency η^*, or quotient of pump efficiency η and maximum efficiency η_{max}:

$$\eta^* = \frac{\eta}{\eta_{max}}. \tag{18}$$

By analogy to the general definition of relative pump efficiency, the notion of minimum relative efficiency $\eta^*_{min\,l}$ for the upper slope of the pump efficiency characteristic curve, and optimum relative efficiency η^*_{opt} are introduced and defined as follows (Fig. 2.b):

$$\eta^*_{min\,l} = \frac{\eta_{min\,l}}{\eta_{max}}, \tag{19}$$

$$\eta^*_{opt} = \frac{\eta_{opt}}{\eta_{max}}. \quad (20)$$

Assuming arbitrary value of head, for instance $h^* = 0.8$, one characteristic curve of pump efficiency η^* in function of rotational speed n^* is created. Plotting of efficiency characteristic curve consists in coupling points (η^*, n^*) situated on the line corresponding to pump head $h^* = 0.8$ (H = 16 m), intersecting with curves of constant efficiency on the performance chart (Fig. 1). Intersection points of the line $h^* = 0.8$, and a curve corresponding to any constant value of pump efficiency η^*_i make up a matrix of points $(\eta^*_i; n^*_i)$ on the pump efficiency characteristic curve η^*. Matrix of points $(\eta^*_i; n^*_i)$ is defined in Table 1. Boundary values of pump efficiency correspond to the following rotational speeds:

a) Minimum rotational speed n^*_{min1} corresponds to the pump's operation at minimum efficiency η^*_{min1},
b) Optimum rotational speed n^*_{opt} corresponds to the pump's operation at optimum efficiency η^*_{opt}.

Table 1

Pump efficiency η^*, and its approximating curve η^*_n defined for various frames of reference, assuming h^* = const – upper slope of pump efficiency characteristic curve

Frame of reference	η^*, η^*_n	η^*_{min1}	η^*_2	...	η^*_{opt}
$(N; \eta^*)$, $(N; \eta^*_n)$	n^*	n^*_{min1}	n^*_2	...	n^*_{opt}
$(X; \eta^*)$, $(X; \eta^*_n)$	x	1	2	...	x_{max}
$(Z; \eta^*)$, $(Z; \eta^*_n)$	z	z_1	z_2	...	z_{max}

The number of point couples (η^*, n^*) varies with each pump type depending on available source material. The set of point couple (η^*, n^*) plots a non-continuous course of efficiency characteristic curve η^*. Points of efficiency characteristic curve η^* determine the approximating function $\eta^*_n = f_1(x) = f_2(z) = f_3(n^*)$ which interprets the efficiency course in three independent frames of reference (Fig. 4):

1. The first domain, X, belongs among natural numbers. Element x_i of the domain X determines the consecutive point number (η^*, n^*) defined on the basis of performance chart (Fig. 1) for rotational speed variations by a constant value δn^* (Table 1). Thus, element $x_1 = 1$ corresponds to the first point of pump efficiency characteristic curve (Fig. 4). Consecutive points x_i are determined using the following relationship:

$$x_i = x_1 + \frac{n_i^* - n_{min1}^*}{\delta n^*} \quad \text{for } i = 1,2,3,...,max \tag{21}$$

2. The second domain, Z, belongs among natural numbers. Element z_i of the domain Z determines the consecutive point number (η^*, n^*) calculated in relation to rotational speed $n^* = 0$. For rotational speed $n^* = 0$, element $z_i = 0$. Consecutive point numbers (η^*, n^*) are multiples of speed variation δn^*. The set of elements $z \in (z_1; z_{max})$ determines the domain of pump efficiency characteristic curve (Fig. 4) within which the pump should operate owing to low energy consumption. Elements of the domain Z fit the following dependence:

$$z_i = \frac{n_i^*}{\delta n^*} \quad \text{for } i \in C \tag{22}$$

where: C – set of integers,

z_i – subsequent values of calculation step in the domain Z.

3. Domain N of rotational speed assumes values within the set of integers R. The origin of 0N and 0Z axes is common. The set of rotational speed elements $n^* \in (n_{min1}^*; n_{opt}^*)$ defines the domain of pump efficiency characteristic curves within which the pump should operate owing to low energy consumption.

The domains X and Z of the function η_n^* are closely related to domain N of rotational speed. Initial values x_1 and z_1 are related to the boundary value of rotational speed n_{min1}^*. The consecutive points x_i and z_i are related to the value of rotational speed ni which meets the following condition:

$$n_i^* = n_{min1}^* + (i - 1)\,\delta n^* \quad \text{for } i = 1,2,3,...,max \tag{23}$$

where: δn^* – growth of rotational speed relative value.

A sample analysis of the upper slope of the pump efficiency η^* characteristic curve was based on the performance chart of a single stage, single-flow pump of the type 10A25A (Warsaw Pump and Fittings Unit, $P_n = 55$ kW, $n_n = 2950$ rpm). The non-dimensional head $h^* = 0.8$ was assumed. Pump efficiency characteristic curve is defined by the distribution of the set of pump efficiency η^* point couples and their corresponding rotational speeds n^*. The matrix of elements (η^*, n^*) forms the basis for determining variation of relative pump efficiency η^* in function of rotational speed n^* and in the domain X (Fig. 3). Pump efficiency η^* is approximated with the curve η_n^*. The absolute error of the deviation of approximation curve η_n^* from the actual pump efficiency η^* characteristic curve is not in excess of (2÷3)%. Comparable results were obtained for all available variations of pump efficiency on the basis of performance charts. The approximation shown in Figure 3 results in the approximation function η_n^* that is defined as follows:

$$\eta_n^* = 0.088 \ln(x) + 0.7368 \tag{24}$$

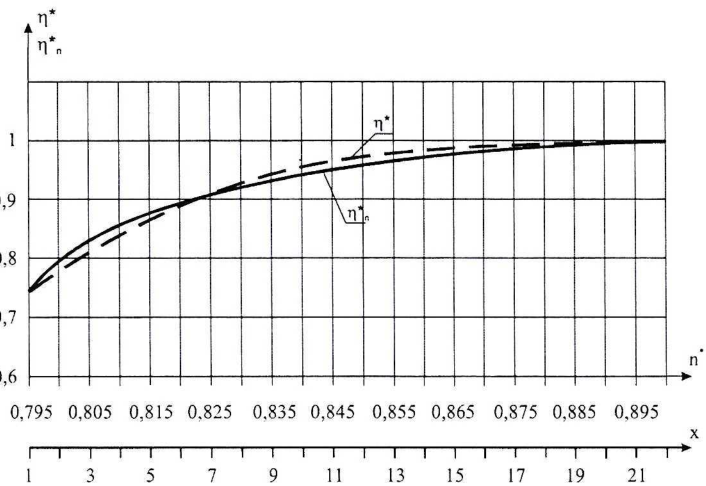


Fig. 3. The upper slope of the pump efficiency η^* characteristic curve 10A25A ($n_q \cong 38$), and approximation curve η_n^* in function of rotational speed n^* and of argument of the domain X, where $h^* = 0.8$

The result of approximation of pump efficiency η_n^* becomes a logarithm function regardless of what source material including performance charts for various types pumps was used. The generalised dependence of approximation curve η_n^* of the upper slope of pump efficiency, determined on the basis of known performance charts, is defined as follows:

$$\eta_n^* = a \ln(x) + b \tag{25}$$

The characteristic curve of approximated efficiency η_n^* is defined in the domain X. The domain X determines consecutive numbers of pair couples (η_i^*; η_i^*), i.e., a sequence of numbers $x_i = i = 1, 2, 3, \ldots, x_{max}$, within the range $x \in (x_1, x_{max})$. The minimum value $x_1 = 1$ relates to a point on the pump efficiency characteristic curve and defined by the minimum value of rotational speed n^*_{min1}. Rotational speed n^*_{min1} for the minimum relative pump efficiency η^*_{min1} is variable and dependent on head h^* of water. As a result, the origin of the reference system $(X; \eta^*_n)$ is shifted by the value d (Fig. 4) in relation to the reference system $(N; \eta^*_n)$ and $(Z; \eta^*_n)$. The length of section d is variable, dependent on the rotational speed n^*_{min1}, and defined as follows:

$$d = \frac{n^*_{min1}}{\delta n^*} - 1 = z_1 - 1 \tag{26}$$

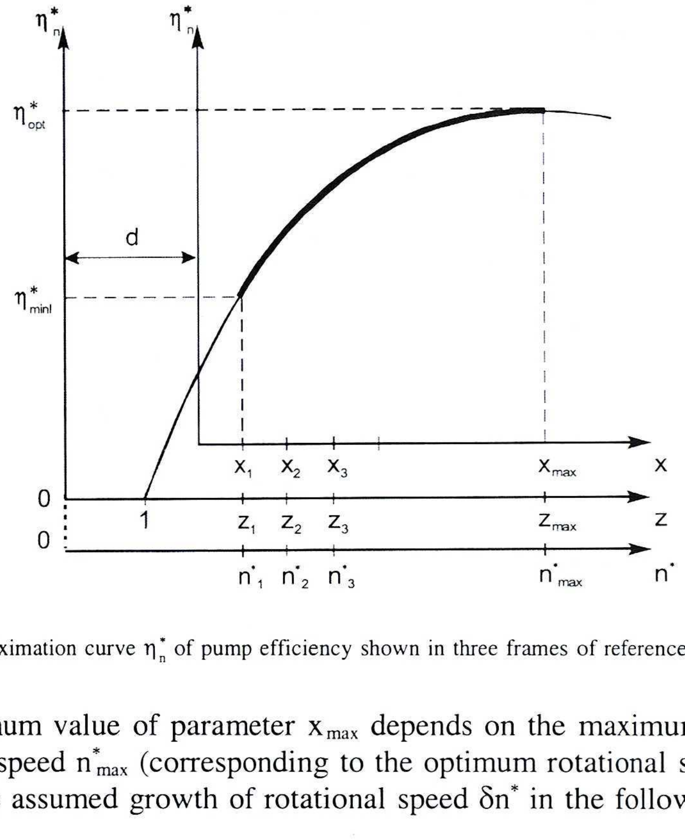


Fig. 4. Approximation curve η^*_n of pump efficiency shown in three frames of reference: X, Z, and N

Maximum value of parameter x_{max} depends on the maximum value of rotational speed n^*_{max} (corresponding to the optimum rotational speed n^*_{opt}), and on the assumed growth of rotational speed δn^* in the following way:

$$x_{max} = \frac{n^*_{opt} - n^*_{min1}}{\delta n^*} + 1 \quad \text{for } x_{max} \in N. \tag{27}$$

For the courses to be unique, it is essential to define parameters a and b in the equation (25). Generalised determination of parameters a and b is possible by reducing the efficiency approximation curve η^*_n from the reference system $(X; \eta^*_n)$ to the reference system $(Z; \eta^*_n)$, and then $(N; \eta^*_n)$.

Shifting the frame of reference $(X; \eta^*_n)$ by the value d results in each pump efficiency characteristic curve in the reference system $(X; \eta^*_n)$ having identical initial value $\eta^*_n = b$ (25). For clarity of the analysis of efficiency characteristic curve, one needs to convert the domain X of function η^*_n to the new frame of reference (Z; η^*_n). Shifting of argument X in relation to the reference system (Z; η^*_n) by the value d determines the following dependence among arguments of the function η^*_n in the domains X and Z:

$$x = z - z_1 + 1. \tag{28}$$

The frame of reference (Z; η^*_n) does not shift in relation to the reference system (N; η^*_n), and their origins are common. The shift of domain X→Z (Fig. 4) changes the argument of logarithm function (25) into what follows:

$$\eta^*_n = a \ln(z - z_1 + 1) + \eta^*_{min1} \text{ for } z \geq z_1. \tag{29}$$

Elements of the domain Z of the approximating function η^*_n belong to the set of natural numbers in the range $z \in (z_1; z_{max})$. Elements z assume diverse values depending on arbitrary rotational speed n^*, and on speed growth δn^*.

Making the domain Z dependent on the domain N via the equation (22) makes the efficiency η^*_n, approximating in the domain X, dependent on the relative rotational speed n^*. The range of variations of rotational speeds n^* depends on head h^* and fits within the range $n^* \in (n^*_{min1}; n^*_{opt})$.

It was assumed that the initial value of relative efficiency η^* defined by the relationship (29) and determined for $z = z_1$ corresponds to the minimum efficiency η^*_{min1}. This assumption means that parameter b in the equation (29) fits the following dependence:

$$b = \eta^*_{min1}. \tag{30}$$

When determining parameter a, it is assumed that the maximum value of efficiency η^*_n defined in the dependences (22) and (29), equals η^*_{opt}. Assuming $z = z_{max}$, and $\eta^*_n = \eta^*_{opt}$, the value of parameter is defined by the following dependence:

$$a = \frac{\eta^*_{opt} - \eta^*_{min1}}{\ln(z_{max} - z_1 + 1)} \tag{31}$$

where:

$z_1 = \dfrac{n^*_{min1}}{\delta n^*}$ – number of calculation steps within the range of rotational speeds $n \in (0; n^*_{min1})$,

$z_{max} = \frac{n^*_{opt}}{\delta n^*}$ – number of calculation steps within the range of rotational speeds

$n \in (0;\ n^*_{opt})$.

The value of parameter at head $h^* = const$ is constant and determines the nature of variations of pump efficiency throughout the range $\Delta\eta^* = \eta^*_{opt} - \eta^*_{min1}$.

4.2. Unified form of the upper slope of efficiency characteristic curve

In the range of pump rotational speed $(n^*_{min1};\ n^*_{opt})$ the number of points x depends on growth of the rotational speed δn^* which assumes the value x_{max}. For various heads h^*, variable ranges of rotational speed are obtained $(n^*_{min1};\ n^*_{opt})$. In effect, each head h^* corresponds to a variable number of elements of the domain set $X \in (x_1;\ x_{max})$.

It is assumed that the maximum number of consecutive points x_{max} is identical for different ranges of rotational speed $(n^*_{min1};\ n^*_{opt})$ of the pump under examination. There may be any number of points x_{max} within the range $(10 \div 300)$. The assumption is right in relation to variations of pump efficiency $(\Delta\eta^*)_{max}$ which is defined as follows:

$$(\Delta\eta^*)_{max} = \eta^*_{opt} - \eta^*_{min1} \quad \text{for } h^* = const \tag{32}$$

Boundary values of efficiency η^* correspond to the boundary values of rotational speed in the range $n^* \in (n^*_{min1};\ n^*_{opt})$. For the family k of pump efficiency characteristic curves η^* at diverse heads h^*, the following expression is obtained which defines the minimum rotational speed n^*_{min1}:

$$n^*_{min1}(h^*) = n^*_{opt}(h^*) - (x_{max} - 1)\,\delta n^*_h \quad \text{for } h^* \in (0.6;\ 1) \tag{33}$$

where:

δn^*_n – growth of pump rotational speed, assuming constant value for one head h^* (Fig. 7).

At each h^*, the pump operates within the assumed maximum range of efficiency variations $(\Delta\eta^*)_{max}$. Maximum efficiency variations $(\Delta\eta^*)_{max}$ correspond to the range of maximum changes of rotational speed Δn^*_{max1} which is defined as follows:

$$\Delta n^*_{max1}(h^*) = n^*_{opt}(h^*) - n^*_{min1}(h^*) \quad \text{for } h^* \in (0.6;\ 1) \tag{34}$$

Considering identical value of parameter x_{max} determined for variable heads h^*, variable speed growths are defined δn_h^* according to the dependence:

$$\delta n_h^* = \frac{\Delta n_{max1}^*(h)}{x_{max} - 1} \tag{35}$$

Maintaining the constant value of parameter x_{max} as a condition, pump efficiency η^* characteristic curve was analysed at four heads $h^* = 1.2$, $h^* = 1$, $h^* = 0.8$, $h^* = 0.6$. Pump efficiency characteristic curves were plotted on the basis of performance chart shown in [6]. Pump efficiency η^* characteristic curves are approximated by the function η_n^* in the domain X. Individual relationships are described by the following equations:

$$\left.\begin{aligned} \eta_n^* &= 0.1407\ln(x) + 0.5865 \text{ for } h^* = 1.2 \\ \eta_n^* &= 0.1344\ln(x) + 0.6099 \text{ for } h^* = 1 \\ \eta_n^* &= 0.1331\ln(x) + 0.629 \text{ for } h^* = 0.8 \\ \eta_n^* &= 0.128\ln(x) + 0.5743 \text{ for } h^* = 0.6 \end{aligned}\right\} \tag{36}$$

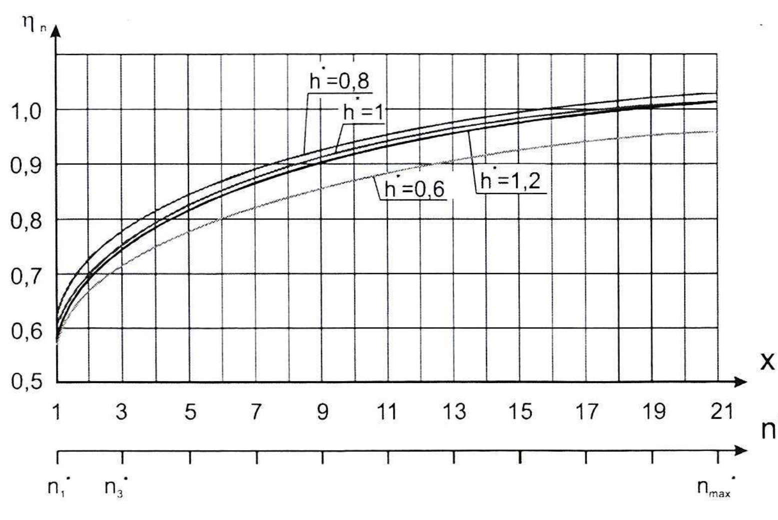


Fig. 5. Relative pump efficiencies η_n^* at $h^* = 1.2$; $h^* = 1$; $h^* = 0.8$; $h^* = 0.6$ on the basis of performance chart in Figure 1b [6]. Rotational speeds n_1^* and n_{max}^* for different heads are defined in the dependence (36)

The dependences (36) are determined for diverse domains of rotational speed $n^* \in (n_1^*, n_{max}^*)$. Boundary values of rotational speeds n_1^* and n_{max}^* at selected heads h^* are defined by the following relationships:

$$\begin{array}{lll} n_l^* = 1 & n^*_{max} = 1.1 & \text{for } h^* = 1.2 \\ n_l^* = 0.925 & n^*_{max} = 1 & \text{for } h^* = 1 \\ n_l^* = 0.82 & n^*_{max} = 0.91 & \text{for } h^* = 0.8 \\ n_l^* = 0.7 & n^*_{max} = 0.78 & \text{for } h^* = 0.6 \end{array} \tag{37}$$

Parameters of functions η_n^* defined in the relationship (36) display slight variation. The courses of pump efficiency characteristic curves as the function η_n^* at different heads are shown in Figure 5. Variations among values of individual functions η_n^* (Fig. 5) for identical argument are up to 8% of the relative value of pump efficiency. These results suggest that it is possible to plot the upper slope of pump efficiency η^* characteristic curve for any head h^* on the basis of the unified form of the function, defined as follows:

$$\eta^* = a \ln(x) + b \tag{38}$$

Parameters a, b in the dependence (38) are defined by the equations (30) and (31) under conditions corresponding to head $h^* = 1$. Once the same parameters a, b are applied, the unified nature of the function (38) allows for determination of the distribution of the upper slope of pump efficiency η^* characteristic curve for the remaining heads h^* in the range (0.6; 1).

Shifting the argument of the function defined by the relationship (38) from the domain $X \rightarrow Z$, and considering the equation (26), the following expression is obtained that defines the unified form of pump efficiency η_n^*:

$$\eta_n^*(z) = a \ln(z - z_l + 1) + b \tag{39}$$

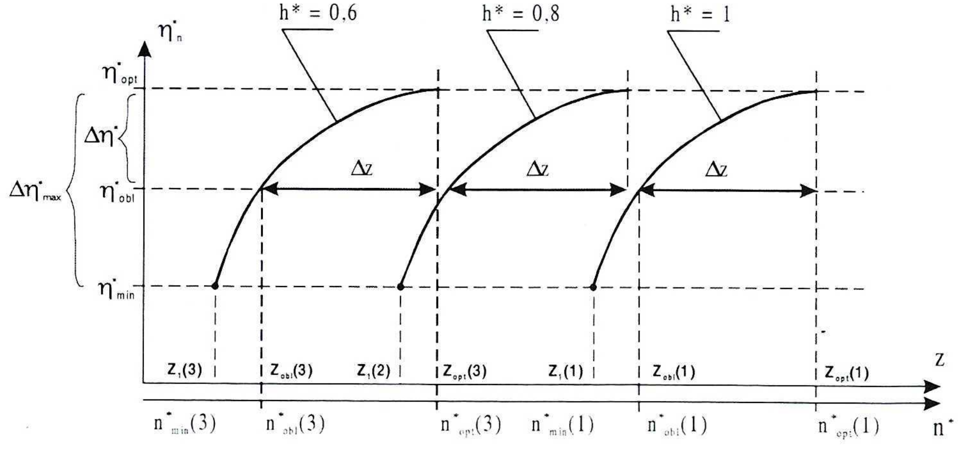


Fig. 6. Relative pump efficiency η_n^* in the domains Z and N for any three heads h^*

Pump efficiency η_n^* characteristic curves in the domain Z are shown in Figure 6 with heads $h^* = 0.6$, $h^* = 0.8$, $h^* = 1$. Each efficiency characteristic curve was plotted while keeping the condition of constant value of the argument x_{max}. Value of the argument x_{max} corresponds to the Z-domain difference of arguments z_{max} and z_1 defined as: $\Delta z_{max} - z_1$. The values of arguments x_{max} and Δz_{max} do not change throughout the range of head h variations.

One can assume any growth of pump efficiency $\Delta\eta^*$ that would meet the condition $\Delta\eta^* < (\Delta\eta^*)_{max}$. The assumed variations of pump efficiency $\Delta\eta^*$ correspond to the range of variations of argument Δz in the domain Z and the corresponding change of relative rotational speed Δn_h^* in the domain N, which is expressed as follows:

$$\Delta n_h^* = n_{opt}^*(h^*) - n_{obl}^*(h^*) \tag{40}$$

where:
$n_{obl}^*(h^*)$ – calculation rotational speed where the pump operates at minimum calculation efficiency, η_{obl}^*, which meets the condition $\eta_{obl}^* > \eta_{min1}^*$.

Fig. 7 shows the field of pump operation within the range of head (0.6; 1). The curve corresponding to the minimum pump efficiency η_{min1}^* is related to the first argument $z_1(h^*)$ belonging to the domain Z. The curve plotting the similarity parabola of optimum efficiency η_{opt}^* is related to the maximum value of argument $z_{opt}(h^*)$. Both the efficiency curves, η_{min1}^* and η_{opt}^* have a common point of intersection Wsp.

Selecting the unified form of pump efficiency function η_n^* (39), an identical range of variation of argument Δz is obtained for diverse heads h^* within the field of variations of pump efficiency $\Delta\eta^*$ (Figures 6 and 7). Fig. 7 shows the distribution of argument Δz of identical value at diverse heads h^*. Each head h^* and argument Δz corresponds to a change of rotational speed Δn_h^* defined in the following relationship:

$$\Delta n_h^* = \Delta z \cdot \delta n_h^* \tag{41}$$

Fig. 7 implies that constant growth of efficiency $\Delta\eta^*$, corresponding to the constant value of argument Δz, variations of rotational speed Δn_h^* and growth of rotational speed δn_h^* increase with growing head h^*. After estimating the value of variable Δz on the basis of dependence (39), one can determine the permissible variation of pump rotational speed Δn_h^* (41) in comparison to optimum rotational speed n_{opt}^*. Minimum calculation pump rotational speed n_{obl}^* is defined according to the equation (40).

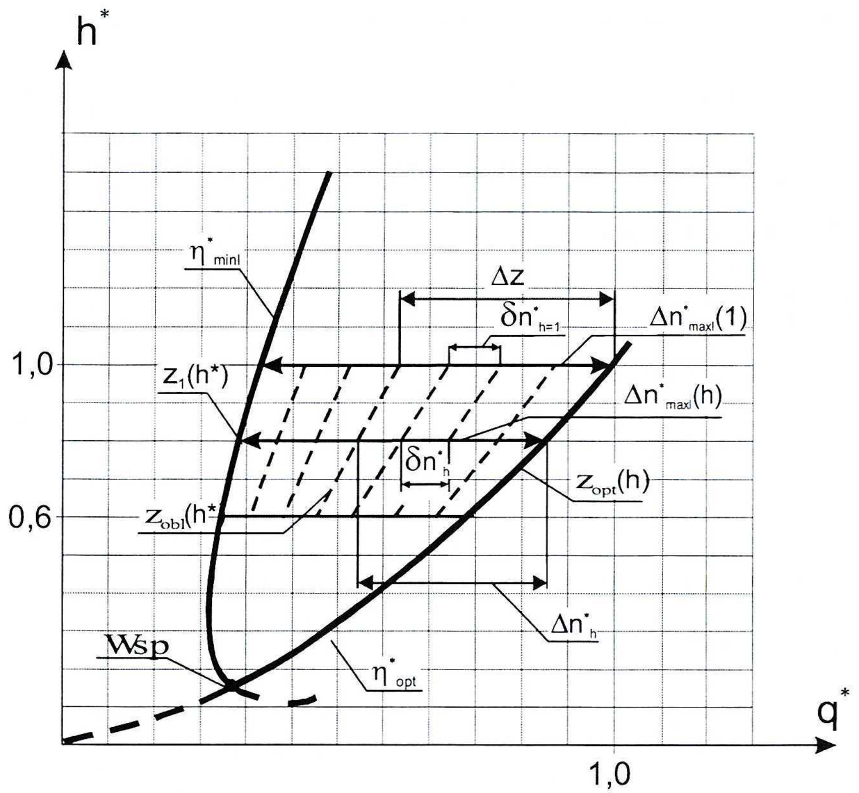


Fig. 7. Distribution of rotational speed variations $\Delta n^*_{max\,l}(h^*)$, parameter Δz, growth of rotational speed δn^*_h within the field of head variations $h \in (0.6;\ 1)$ concerning the pump shown in the performance chart, Fig. 1a

Calculation speed n^*_{obl} is assumed to correspond to the calculation value of argument z_{obl}. The growth of argument Δz is defined by the following expression:

$$\Delta z = \Delta z_{opt} - \Delta z_{obl} \tag{42}$$

Where the dependences Δz_{opt} and Δz_{obl} are defined as follows:

$$\Delta z_{opt} = z_{opt} - z_1 \tag{43}$$

$$\Delta z_{obl} = z_{obl} - z_1 \tag{44}$$

The dependence (39) in the case of pump operation at the point of maximum efficiency η^*_{opt}, and calculation efficiency η^*_{obl} looks as follows:

$$\eta^*_{obl} = a \ln(\Delta z_{obl} + 1) + b \tag{45}$$

$$\eta^*_{opt} = a \ln(\Delta z_{opt} + 1) + b \tag{46}$$

On the basis of equations (45), (46), the relationships determining the growths Δz_{obl}, and Δz_{opt} are defined as follows:

$$\Delta z_{obl} = e^{\lambda_{obl}} - 1 \tag{47}$$

$$\Delta z_{opt} = e^{\lambda_{opt}} - 1 \tag{48}$$

where parameters λ_{obl} and λ_{opt} are defined in the expressions:

$$\lambda_{obl} = \frac{\eta^*_{obl} - b}{a}$$

$$\lambda_{opt} = \frac{\eta^*_{opt} - b}{a}$$

The ultimate relationship defining growth of parameter Δz for pump efficiency within the range $(\eta^*_{obl}; \eta^*_{opt})$ is based on the dependences (42), (47), (48) and forms the following equation:

$$\Delta z = e^{\lambda_{opt}} - e^{\lambda_{obl}} \tag{49}$$

Considering the relationships (40), (41), and (49), the calculation rotational speed n^*_{obl} is defined as follows:

$$n^*_{obl} = n^*_{opt} - \Delta n^*_h = n^*_{opt} - (e^{\lambda_{opt}} - e^{\lambda_{obl}})\, \delta n^*_h \tag{50}$$

The dependence (50) defined the calculation pump rotational speed n^*_{obl} during operation assuming the minimum calculation efficiency of η^*_{obl}. The resulting calculation rotational speed n^*_{obl} and the optimum rotational speed n^*_{opt} determine the permissible range of pump rotational speed $(n^*_{obl}; n^*_{opt})$ for the upper slope of pump efficiency η^* characteristic curve while maintaining the assumed variations of relative pump efficiency $\Delta\eta^*$ at $h^* = \text{const}$.

Unification of pump efficiency η^* function is important for purposes of determining the criterion of controlling pump operation. It becomes feasible to implement an energy-saving method of controlling pump rotational speed n^* within the assumed range of pump efficiency $\eta_n^* \in (\eta^*_{min\,l}; \eta^*_{opt})$ at diverse heads $h^* \in (0.6; 1)$.

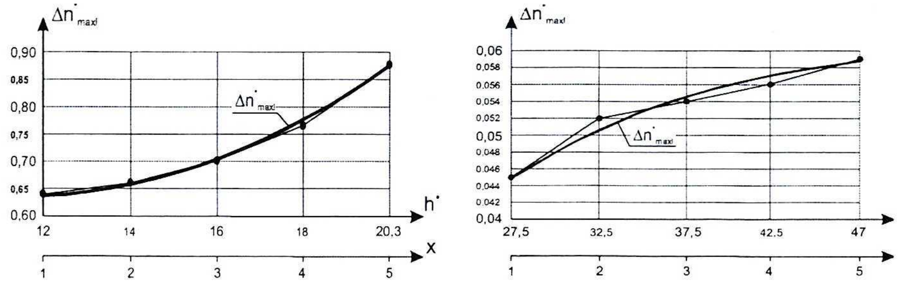
4.3. Left-side coefficient of pump efficiency correction

Determination of calculation rotational speed n^*_{obl} following the dependence (50) implies a change of speed growth δn_h^* every time h^* changes. A change of rotational speed growth δn_h^* is determined in consideration of correction coefficient k_{hl} according to the following dependence:

$$\delta n_h^* = \delta n^*_{h=1} \cdot k_{hl} \tag{51}$$

where:
$\delta n^*_{h=1}$ – growth of rotational speed determined for head $h^* = 1$.

Changes of rotational speed growth δn_h^* are related to variations of rotational speed $\Delta n^*_{max\,l}$ according to the dependence (35). The knowledge of distribution of rotational speed $\Delta n^*_{max\,l}$ characteristic speed is the basis for determining the coefficient of efficiency correction k_{hl}. Sample characteristic curves of maximum speed variations $\Delta n^*_{max\,l} = f(h^*)$ in relation to head h^* are shown in Fig. 8 as curves featuring dots. Fig. 8a shows a characteristic curve of speed variations $\Delta n^*_{max\,l}$ on the basis of performance chart in Fig. 1. Fig. 8b shows a sample characteristic curve of speed variations $\Delta n^*_{max\,l} = g(h^*)$ for single-stage, single-flow pump type 20A40 (Warsaw Pump and Fittings Unit, $P_n = 90$ kW, $n_n = 1480$ rpm).



a) b)

Fig. 8. Maximum variations of rotational speed $\Delta n^*_{max\,l}$ in function of head h for a) pump defined in [7] with specific speed $n_q \cong 42$, b) pump 20A40 with specific speed $n_q \cong 30$

Approximating functions of characteristic curves of rotational speed $\Delta n^*_{max\,l}$ variations, shown as curves without dots and including a negligible error, display the same distribution as the original characteristic curve. The characteristic curve of rotational speed variations and its approximating curve are marked with the same symbol: $\Delta n^*_{max\,1}$. Approximating functions of characteristic curves of rotational speed $\Delta n^*_{max\,1}$ variations are defined as follows:

$$\Delta n^*_{max\,1} = 0.0015x^2 - 0.0031x + 0.0658 \text{ for Fig. 8a} \tag{52}$$

$$\Delta n^*_{max\,1} = 0.0006x^2 + 0.0066x + 0.0396 \text{ for Fig. 8b} \tag{53}$$

Characteristic curves of rotational speed $\Delta n^*_{max\,1}$ variations based on generally available performance charts become quadratic functions which are defined as follows:

$$a_{hl}h^{*2} + b_{hl}h^* + c_{hl} = \Delta n^*_{max\,1}(h^*) \tag{54}$$

Assuming any three heads h^*_1, h^*_2, h^*_3, one obtains the following point couples $(\Delta n^*_{max\,1}, h^*_1)$, $(\Delta n^*_{max\,2}, h^*_2)$, $(\Delta n^*_{max\,3}, h^*_3)$ covered by the equation (54). The resulting points determine the following system of equations:

$$\left.\begin{aligned} \Delta n^*_{max\,1} &= a_{hl}h^{*2}_1 + b_{hl}h^*_1 + c_{hl} \\ \Delta n^*_{max\,1} &= a_{hl}h^{*2}_1 + b_{hl}h^*_1 + c_{hl} \\ \Delta n^*_{max\,1} &= a_{hl}h^{*2}_1 + b_{hl}h^*_1 + c_{hl} \end{aligned}\right\} \tag{55}$$

The system of equations (55) is resolved into the following relationships that define parameters a_{hl}, b_{hl}, c_{hl}:

$$\left.\begin{aligned} a_{hl} &= \frac{(\Delta n^*_{max\,1} - \Delta n^*_{max\,3})(h^*_1 - h^*_3) - (\Delta n^*_{max\,1} - \Delta n^*_{max\,2})(h^*_1 - h^*_3)}{(h^*_2 - h^*_1)(h^*_3 - h^*_2)(h^*_3 - h^*_1)} \\ b_{hl} &= \frac{\Delta n^*_{max\,1} - \Delta n^*_{max\,3}}{h^*_1 - h^*_3} - a_{hl}(h^*_1 + h^*_3) \\ c_{hl} &= \Delta n^*_{max\,1} - a_{hl}\,h^{*2}_1 - b_{hl}h^*_1 \end{aligned}\right\} \tag{56}$$

Based on the performance chart, the maximum value of rotational speed $\Delta n^*_{max\,1}(h^* = 1)$ is calculated at head $h^* = 1$. Assuming the known nature of speed variations $\Delta n^*_{max\,1}(h)$ (54), the value of correction coefficient khl is defined as follows:

$$k_{h1} = \frac{\Delta n^{*}_{max\,1}(h^{*} \neq 1)}{\Delta n^{*}_{max\,1}(h^{*} = 1)} \tag{57}$$

Coefficient k_{h1} becomes the equivalent of function defining $\Delta n^{*}_{max\,1}$ depending on the head h^{*}.

5. Conclusion

In view of a substantial number of electricity-powered pump drives, it must be regarded as reasonable to conduct a wide range of research that could reduce the energy consumption of the water pumping process. In the process of controlling a pumping unit operation, the overall unit efficiency η_g, is heavily influenced by the pump efficiency η^{*}. Literature does not contain any analytic evaluations of pump efficiency characteristic curves. The paper divides the performance chart into elementary functions that make up two-element subsets. Maximum number of combinations of elementary function subsets was presented. On the basis of a sample performance chart, efficiency characteristic curves were plotted at constant pump head. The extreme of efficiency characteristic curve was determined, and the nature of efficiency variation in function of rotational speed was described. A generalized upper slope of efficiency characteristic curve was defined as a logarithm function. A method of calculating the logarithm function parameters on the basis of performance chart for any head h^{*} was presented. Based on the logarithm function, and assuming the permissible efficiency deviation from the maximum value, one can estimate rotational speed variations in function of optimum speed.

This method of estimating the best values of rotational speed in the regulation process should result in possible reductions of unit energy-consumption e of the water pumping process. Simple results of the method, which can be easily implemented in widely-used PLC controllers, are a major advantage. This control method can involve application of well-known PI regulator. A full efficiency analysis will require determination of variations of the lower slope of pump efficiency characteristic curve.

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Analiza energochłonności pompy w obszarze narastającej części charakterystyki sprawności

Streszczenie

W artykule przedstawiono sposób oceny energochłonności pompowania wody. Omówiono własności pomp podczas ich pracy przy zmiennej prędkości obrotowej i stałej wysokości podnoszenia. Wyznaczono charakterystykę narastającej części sprawności pompy dla przypadków: stałej wartości przyrostu prędkości obrotowej, oraz zmiennej wartości przyrostu prędkości obrotowej. Zdefiniowano lewostronny współczynnik korekcji sprawności pompy.