Vol. XLVIII 2001 Number 3

Key words: *fine-pitch worm gears, counter gears*

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WORM GEAR WITH A WORM HAVING CIRCULAR SECTION OF THE THREAD

This paper presents requirements for meshing of simplified type of worm gears applied in fine mechanisms. There are given some geometrical dependencies referring to meshing of the worm with the thread made of wire (having circular section). The paper describes also a computer program aiding design of such kind of worm gears.

1. Initial remarks

In precision machines and devices, one usually applies constant ratio worm gears [4], [6], [7], and at the same time in majority of cases worm is the driving wheel (gear ratio $i = z_2 / z_1 > 1$). On the contrary, in fine mechanisms (FM), one also applies other types of worm gears, provided the constant ratio is not required. They are of different design solutions than machine gears. At the same time, worm can be the driving element, e.g. in various kinds of counter gears (in clock-work with quartz-crystal resonator, electricity meters, anemometers, etc.) as well as the driven element, e.g. in multiplying gears driving friction-centrifugal or fly governors (in striking mechanisms of mechanical clocks, in musical boxes).

Besides worm gears with constant ratio (Fig. 1a), in many kinds of FM there can be applied worm gears with simplified structure, whose instantaneous ratio can vary within one pitch length (Fig. 1b) [1], [5].

While realising the former works [2], [3] concerning worm gears applied in FM, one specific meshing turned out to be very interesting. It has been studied

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only in a superficial way up to this time. This meshing consist in a wormwheel, which is usually a typical spur gear or a helical gear with involute profile, made of metal sheet or polymer, and a worm having the thread created by winding a wire around a smooth steel bar (Fig. 2). An advantage of this solution is simplicity of manufacturing of the worm while keeping small dimensions of the gear accordingly.

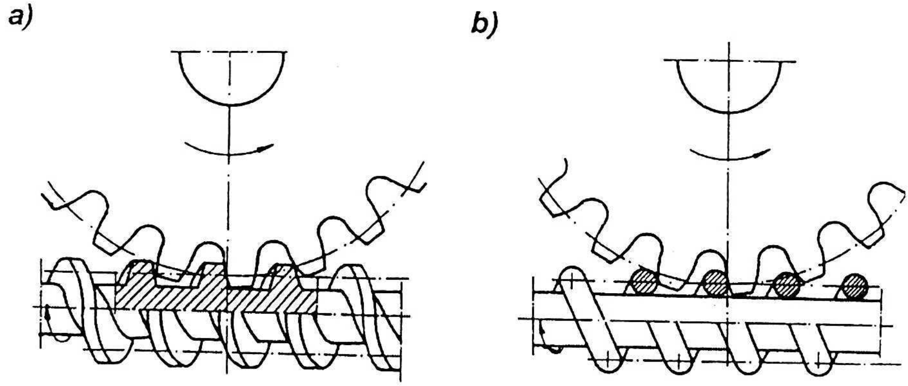

Fig. 1. Constructive scheme of fine pitch worm gear a) with worm made of full bar, b) with worm whose thread is made of wire

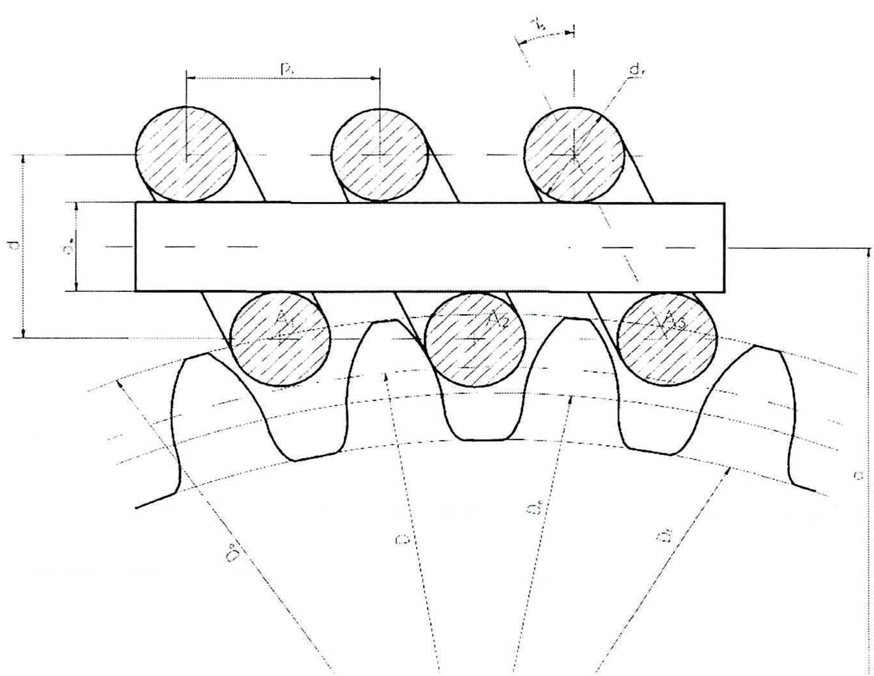

Fig. 2. Simplified type of worm gear whose worm has the thread made of wire

This kind of simplified worm gear can be commonly applied in small and very small counter mechanisms, being cheap at the same time. Condition of their correct operation is, among others, the correct selection of the pitch of worm thread with respect to the pitch of the applied toothed wheel playing the role of the wormwheel.
The gears described in this paper are the subjects of a Grant of the State Committee for Scientific Research, which has been realised at present.

2. Requirements on meshing for simplified types of worm gears applied in fine mechanisms

In works [2], [3] it was pointed out that operating conditions and requirements, which should be met by the worm gears applied in fine mechanisms (FM), are different from those of worm gears applied in machines and precision mechanisms and instruments. Main requirements concerning worm gears applied in FM are as follows:

* simplicity of manufacturing (low accuracy of manufacturing) – especially in the case of mass production,
* in some application, possibly low variability of instantaneous torque ratio,
* high average efficiency,
* design that allows working with backlash and radial clearance,
* high durability (wear resistance).

In majority of applications - because of small angular velocities of worm and wormwheel – *meshes applied in these gears do not have to meet the requirement of constancy of the instantaneous gear ratio*, which can be periodically variable within one pitch length.

So, in these gears there can be applied meshing with both involute tooth profile and much simplified tooth profile (deformed) with respect to the involute tooth profile.

The basic advantage of gearing with an instantaneous ratio periodically variable within one pitch length is the fact that it is possible to obtain the most advantageous location of the active path of contact, i.e. a location, for which the variability of the instantaneous torque ratio is possibly low [1]. This is obtained thanks to a proper selection of the geometrical parameters of meshing of the collaborating wheels. A problem of variability of the instantaneous torque ratio is of a significant importance in the case of multiplying worm gears.

3. Geometrical dependencies of simplified type of meshing applied in reduction worm gears with a worm having the thread made of wire (of circular section) and wormwheel of involute profile

There is a worm gear presented in Fig. 3, where the driving element is a one-thread worm of a specific design, and the driven element is a helical tooth wheel. The worm is made of a bar having diameter d_w, onto which there is tightly winded a wire having diameter d_r. The basic parameters determining dimensions of the worm are the following:

- axial pitch (lead) p_t of the worm,
- diameter of the wire threads d_r,
- helix angle of the threads γ (or bar diameter d_w).

Dimensions of the wormwheel collaborating with the worm are resulted by defining the following quantities

- module of gearing m,
- number of teeth z_2,
- basic rack of the tooth profile,
- helix angle γ_k.

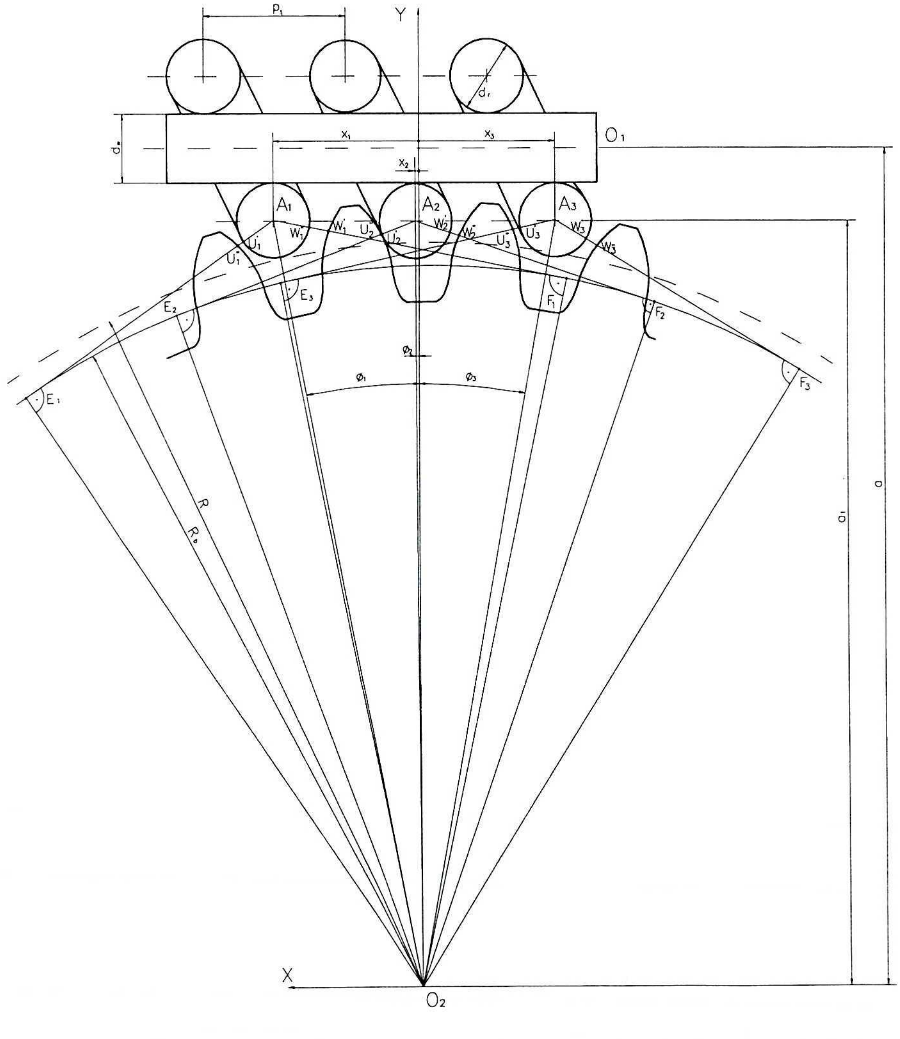

Fig. 3. Geometry of meshing for the worm gear whose worm has the thread made of wire

The problem of designing a worm gear with worm having the thread made of wire boils to matching the worm parameters and determining such a distance a between the axes, so that the gearing can work properly. It is obvious that the worm parameters must be matched to the dimensions of the wormwheel, not the other way round, what is resulted by standardisation of the module of the wheel m.

Collaboration of the worm threads and the wormwheel teeth is presented in Fig. 2 and Fig. 3. Worm rotates as indicated by the arrow, and its motion causes rotation of the toothed wheel to the left. The collaboration can start both before and behind the line O_1O_2 being orthogonal to both axes. For determined parameters of the toothed wheel, location of the initial point of collaboration A_1 is dependent on the diameter of the wire thread d_r, axial pitch (lead) of the worm p_t and the distance a.

The end of collaboration should not exceed the point where the circle of cross-section of the thread and the tooth involute come into contact at the tip circle diameter, and still have a common normal (point A_2). Keeping contact of both teeth in succeeding positions will result in an edge-type of collaboration. Worm lead p_t, distance between the axes a and the wire diameter d_r should be matched in such a way that the recess takes place a bit earlier.

The instantaneous velocity ratio of such gearing is obviously variable within one pitch length (this ratio would be constant only in the case when the worm collaborating with the involute wormwheel had a trapezoid thread profile).

Because of the fact that the basic rack as well as the module pitch of the driven wheel (wormwheel) are standardised, the parameters of the worm are matched to this wheel.

Therefore, the following are given: module m, number of teeth of the driven wheel z_2, helix angle γ_k and the other parameters of the basic rack of the driven wheel. If the gear is to be self-locking, then the angle γ_k, must meet the following condition:

$$\gamma_k < \rho \tag{3.1}$$

where ρ is the friction angle for the materials the worm and the wormwheel are made of.

In order to obtain the longest meshing angle, it is recommended to use a diameter of the wire threads d_r calculated from the following formula:

$$d_r = m\left[z_2\left(1-\frac{\cos\alpha_0}{\cos\alpha_{p1}}\right)+2y\right] \tag{3.2}$$

where the angle α_{p1} is determined from the equation:

$$inv\alpha_{p1} = \Phi + inv\alpha_0 + \frac{d_r}{mz_2\cos\alpha_0} - \frac{\pi}{z_2} \tag{3.3}$$

The bar diameter d_w of the worm should be matched in such a way as to ensure meeting two criteria: self-locking of the gear and correct design of the spring constituting the worm threads. The diameter d_w of the worm bar ensuring self-locking of the gear can be determined from the following condition:

$$\mathrm{tg}\gamma < \mu$$

but

$$\mathrm{tg}\gamma = \frac{p_t}{\pi \cdot d} = \frac{D_s}{z \cdot (d_w + d_r)} \cong \frac{D_1}{z_2 \cdot (d_w + d_r)} = \frac{m}{d_w + d_r}$$

After transformation we receive a simplified dependence:

$$d_w > \frac{m}{\mu} - d_r \tag{3.4}$$

In the case of the considered gear, the worm threads are manufactured as a cylindrical helical spring clasping the bar of diameter d_w. The ratio C_s of the average diameter of wire winding, i.e. approximately pitch diameter of the worm d to the wire diameter d_r, should be the same as in the case of typical cylindrical springs, i.e. 6÷12. Therefore, the diameter d_w accepted in design should meet the conditions (3.3) and (3.4):

$$C_s = \frac{d}{d_r} = \frac{d_w + d_r}{d_r} = (6 \div 12) \tag{3.5}$$

The normal backlash j_n should be large enough, as to reduce a possibility of seizure of the gear. Its value should be no lower than the one applied in fine-pitch meshing designed for work with backlash, i.e.:

$$j_n = (0.05 \div 0.1)\pi m$$

Distance between the axes is dependent on the accepted value of the normal backlash j_n, according to the following formulas:

$$a = O_2P_{10} + 0.5(d_r + d_w) \tag{3.6}$$

Point P_{10} determines location of the centre of the worm thread symmetrically placed in the tooth space of the wormwheel. In this position, normal backlash is from the both sides of the thread of the same value, and is equal to half of the assumed backlash value j_n.

The distance O_2P_{10} is described by the formulas containing the normal backlash j_n:

$$O_2P_{10} = 0.5\, m z_2 \frac{\cos\alpha_0}{\cos\alpha_{p10}} \tag{3.7}$$

where the angle α_{p10} can be determined from the formula:

$$inv\alpha_{p10} = \Phi + inv\alpha_0 + \frac{d_r + j_n}{m z_2 \cos\alpha_0} - \frac{\pi}{z_2} \tag{3.8}$$

The axial pitch of the worm can be accepted as the nominal one, according to the formula:

$$p_{t0} = \frac{\pi D_s}{z_2} \tag{3.9}$$

where: D_s – diameter of a circle tangent to the pitch cylinder of the worm d.

This pitch ensures almost symmetrical segmentation of the path of contact into approach and recess. If it is necessary to shift the path of contact towards recess, the pitch should be accepted as: $p_t' > p_{t0}$, whereas $p_t'' < p_{t0}$ in the case when it is to be shifted towards approach. In both cases, one should determine limiting values of the pitches that still do not cause edge-type collaboration, and check out the conditions for avoiding interference.

More detailed considerations on matching the diameter of wire playing the role of the worm thread, as well as geometrical conditions which must be met in order to ensure collaboration of such gearing without any teeth interference, are provided in work [3].

4. Computer program aiding calculations of the simplified type of worm gear with a worm having the thread made of wire

On the basis of the formulas presented in section 3, it is possible to design a simplified type of worm gear whose worm is made of wire. Taking into consideration complicity of the derived formulas, it seemed necessary to elaborate for this purpose a computer software. The use of computer will not only cut the time of calculations but will additionally make it possible to check out many variants of the worm parameters and, on this basis, to select the most optimal variant.

It is possible to formulate few procedures of the calculating process of simplified type of worm gear. They differ as far as the degree of intervention of designer into the calculation process is concerned. According to some of them, values of the majority of parameters are accepted on the basis of the knowledge and experience of the designer, or on the basis of strictly defined criteria, in the case of the other. The role of designer in the second case consists in selection of the most suitable set of gearing parameters out of many variants presented in the calculation process.

Calculating process of the simplified type of worm gear with a desired value of the backlash can be realised according to the following algorithm (Fig. 4):

- accepting parameters of the toothed wheel with involute helical teeth as wormwheel in the designed gear, and calculating its dimensions;
- calculating diameter of the wire d_r' while assuming that the circle of diameter d_r' located within tooth space is tangent to adjoining tooth flanks and to the tip circle of the toothed wheel (theoretically, it is the maximal diameter of the wire $d_{r\,\max}$ determined for the distance of the axes $a = a_{gr}$);
- accepting the diameter of the wire as the closest standardised value;
- determining diameter of the worm shaft d_w according to one of the accepted criteria:

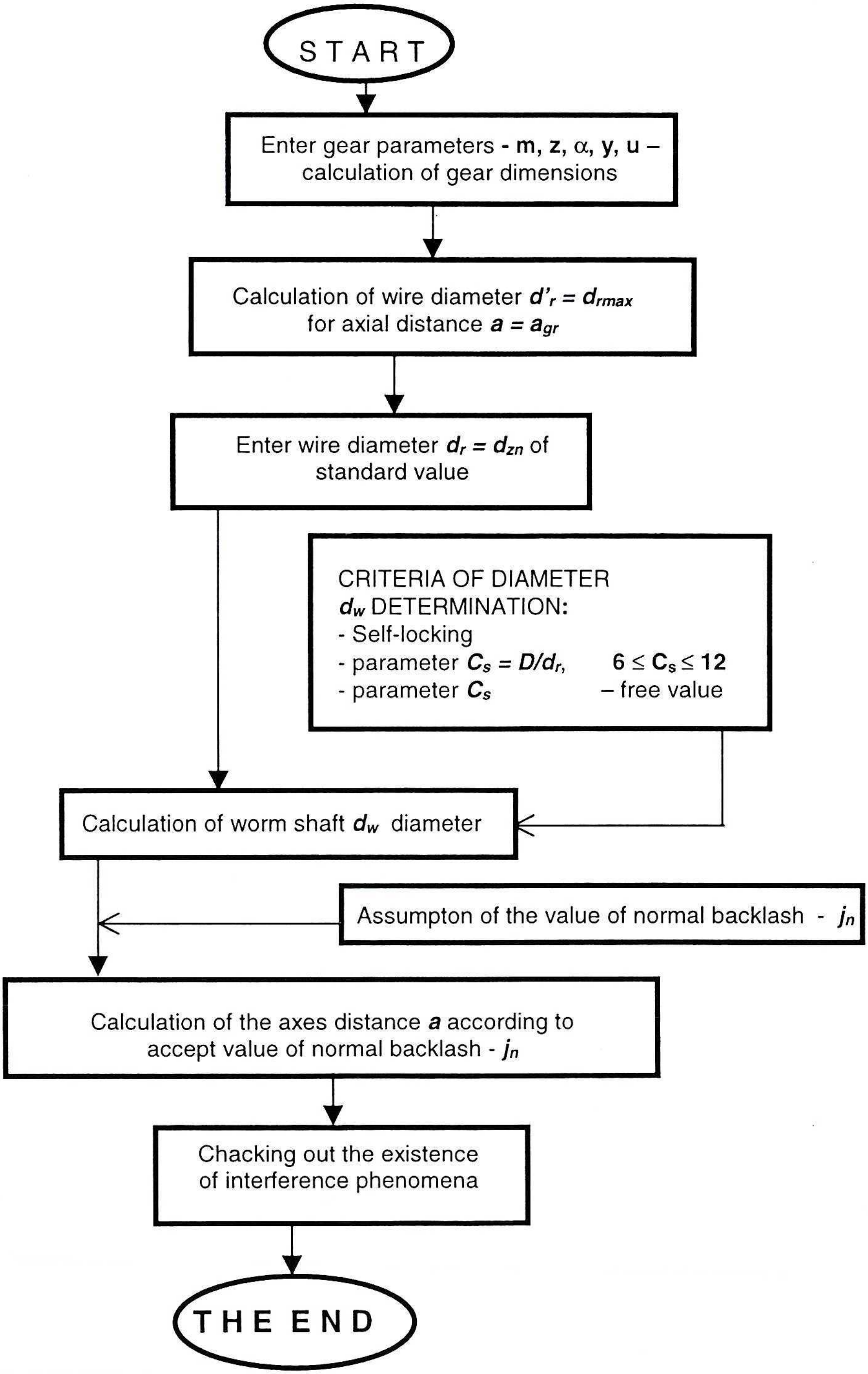

Fig. 4. Algorithm for calculating process of worm gear with a desired value of the backlash

- self-locking of the worm gear,
- ratio of the average diameter of threads D to the wire diameter d_r is found within the accepted interval $C_s = D/d_r = 6 \div 12$,
- ratio of the average diameter of threads D to the wire diameter d_r has a value assumed by the designer, e.g. $C_s = D/d_r = 15$;

– assuming value of the normal backlash j_n in the gearing;
– calculating the distance between the axes $a > a_{gr}$, matched in such a way as to ensure the desired normal backlash j_n between teeth of the wheel and threads of the worm;
– checking out the conditions for existence of interference of the tooth profiles.

This algorithm allows for a quite accurate matching of the parameters of the simplified type of worm gear. Designer determines criteria of matching or assumes value of an indicator, i.e. a parameter used for determining values of the gearing parameters.

A gearing calculated according to the above algorithm should have the maximal length of the active path of contact, possible for a given gearing. It results from the accepted wire diameter. Of course, the length of the path of contact will vary with respect to the accepted normal backlash j_n, along with variations of the distance between the axes a.

Some users of the program might not be familiar with the problems concerning worm gears and/or computer usage. Therefore, the structure of the program should be simple, clear and its interface user-friendly.

The language applied in the program was Pascal.

5. Conclusions

In the case of miniature worm gears, where the transmitted torques are of small values, and it is not required to ensure the constancy of the velocity ratio, it is purposeful to apply meshing with periodically variable velocity ratio within one pitch length. An advantage of this type of meshing is the fact that it is possible to determine a position of the path of contact in the way that makes it possible to obtain the most suitable collaborating conditions. This is realised owing to appropriate matching of the geometrical parameters of the profile of worm thread and wormwheel teeth.

1) Designing a reduction worm gear whose worm has the thread made of wire (having circular section) and whose wormwheel has involute teeth can be performed according to the following pattern:
   – assuming parameters of teeth of the toothed wheel (wormwheel): number of teeth z_2, module m, and, if needed, helix angle γ (if the wheel has a considerable thickness),
   – determining the wire diameter (worm thread) d_r and diameter of the bar d_w,
   – matching the axial pitch of worm p_t,

- defining (if needed) other parameters of gearing, e.g. instantaneous velocity ratio, efficiency, etc.

  One can make use of the formulas presented in section 3 and in [3] to determine these quantities.

2) In order to simplify the process of matching parameters of reduction worm gears whose worm has the thread made of wire, one elaborated a computer program. It makes use of the formulas presented in the paper and checks out the conditions for interference (section 4),
3) In order to ensure possibly high efficiency and durability of fine-pitch worm gear, one should apply manufacturing techniques ensuring small roughness of surfaces being in contact, especially on the worm.

This gear is the subject of a Grant of the State Committee for Scientific Research No. 7 T07C 011 17 entitled „Miniaturowe przekładnie ślimakowe o specjalnej konstrukcji - analiza oraz optymalizacja cech geometrycznych i kinematycznych" (manager: Z. Mrugalski).

Manuscript received by Editorial Board, January 15, 2001;
final version, July 24, 2001.

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Przekładnia ślimakowa ze ślimakiem o kołowym zarysie zwoju

Streszczenie

W artykule omówiono warunki poprawnego zazębienia uproszczonej przekładni ślimakowej stosowanej w drobnych mechanizmach. Przedstawiono najważniejsze zależności geometryczne dotyczące zazębienia ślimaka ze zwojami wykonanymi z drutu o przekroju kołowym z walcowym kołem zębatym. Omówiono program komputerowy wspomagający projektowanie tego typu przekładni.