

Key words: *active damping, journal bearings, rotor, self-excited vibration*

ZBIGNIEW STARCZEWSKI^{*)}

ACTIVE VIBRATION DAMPING OF AN ASYMMETRIC RIGID ROTOR SUPPORTED ON 5-LOBE JOURNAL BEARINGS

The paper concerns efficiency of active magnetic stabilization in damping of self-excited vibration of an asymmetric rotor supported on 5-lobe journal bearings with 5 oil gaps. The dependencies describing the pressure distribution in the oil film are presented. The components of the hydrodynamic uplift forces in the bearings are described. Equations of motion are derived using a numerical simulation method. It was found that active magnetic stabilization was effective for symmetric and non-symmetric systems. Exemplary trajectories of the journal bearing motion as well as the time histories are presented.

1. Introduction

The hydrodynamic theory dealing with lubrication problems focuses on the so-called wedge effect in the oil film and related interactions observed in the journal-oil film-bearing shell. High rotation speeds of turbines and machine shafts require stable behavior of the journal. In the general case, for a given set of structural and operating parameters, the journal should always be positioned in the so-called static equilibrium journal center position curve. The fundamental error made by designers in routine calculations of slide bearings consists in neglecting dynamic criteria, such as critical states, journal vibration amplitudes, loss of stability. Examination of these dynamic phenomena reveals many unexpected effects, like increased energy consumption and lowered quality of the operating. Especially dangerous can be self-excited vibrations, whose amplitudes can be much bigger than the resonant ones. The problem of reducing self-excited vibration generated by the oil film is a very common subject of numerous investigations. There are few methods to reduce

^{*)} Warsaw University of Technology, Faculty of Automobiles and Heavy Machinery Engineering; ul. Narbutta 84, 02-524 Warszawa, Poland; E-mail: dzk@simr.pw.edu.pl

self-excited vibration. One of them is to use slide bearings with non-circular contours of bearing bushes. Predominantly, this non-circular contour is a pericycloidal curve with different multiplication factors. This shape causes that the critical speed at which the self-excited vibration appears is greater than that of cylindrical bearings. Some examples related to this problem can be found in [1], [2], [3] and [4].

The latest trends in damping or reducing self-excited vibration are based on the methods with active damping systems. A special system of sensors follows the journal, and sends on-line voltage signal to actuators bringing the journal back to stable work. In [5] a method of active vibration damping of a symmetric rigid rotor supported on n-lobe journal bearings was described. Two pairs of electro-magnetic actuators were used as the stabilization system.

The main intention of the author of the present work is to examine the efficiency of active magnetic stabilization in absorbing self-excited vibration of an asymmetric rotor supported on n-lobe journal bearings.

2. Model of the journal bearing system

In the paper, one assumes the model of a rigid rotor supported on n-lobe journal bearings with the multiplication factor 5 and with five oil gaps, as shown in Fig. 1. Two kinds of asymmetry are taken into account:

1. Geometric asymmetry related with the position of the gravity center of the rotor between the bushes.
2. Physical asymmetry related with the asymmetry of bushes (different clearances, oil viscosities, etc.).

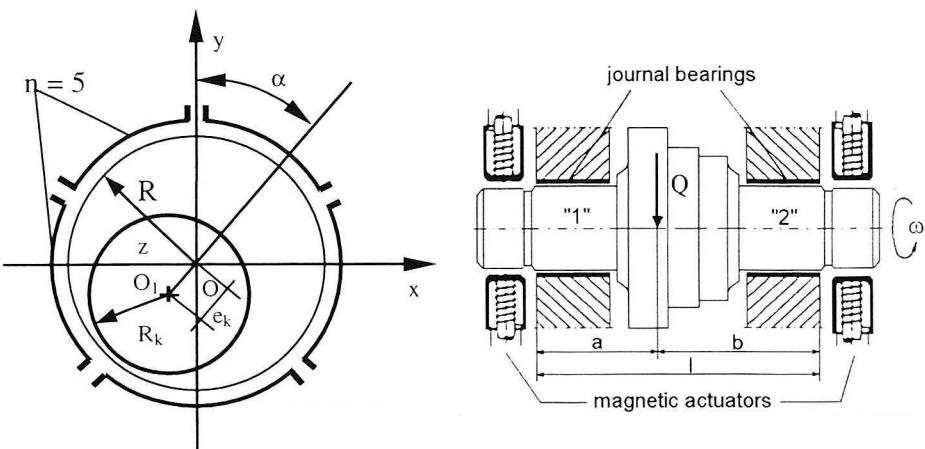


Fig. 1 Model of the journal bearing system

3. Pressure distribution in the oil film

For the so-called short bearing model, the pressure distribution takes the following form [6]:

$$\begin{aligned}
 p_{ik}(\varphi_{ik}, z_{ik}) = & \frac{3\mu_k}{\varepsilon_k^2} (\omega - 2\dot{\alpha}_k) \cdot \frac{\beta_k \cdot \sin(\varphi_{ik} - \alpha_k)}{\left[1 + \beta_k \cdot \cos(\varphi_{ik} - \alpha_k) + \lambda^* \cdot K_n \left[1 - \cos \left(\frac{\pi}{n} \left(2i - \frac{1}{2} (\cos n\pi + 3) \right) - \varphi_{ik} \right) \right] \right]^3} \\
 & \cdot \left[\frac{L_k^2 - 4z_k^2}{4} \right] + \frac{3\mu_k \omega}{\varepsilon_k^2} \cdot \frac{\lambda^* \cdot K_n \cdot \sin \left[\frac{\pi}{n} \left(2i - \frac{1}{2} (\cos n\pi + 3) \right) - \varphi_{ik} \right]}{\left[1 + \beta_k \cdot \cos(\varphi_{ik} - \alpha_k) + \lambda^* \cdot K_n \left[1 - \cos \left(\frac{\pi}{n} \left(2i - \frac{1}{2} (\cos n\pi + 3) \right) - \varphi_{ik} \right) \right] \right]^3} \\
 & \cdot \left[\frac{L_k^2 - 4z_k^2}{4} \right] + \frac{3\mu_k}{\varepsilon_k^2} \dot{\beta}_k \cdot \frac{\cos(\varphi_{ik} - \alpha_k)}{\left[1 + \beta_k \cdot \cos(\varphi_{ik} - \alpha_k) + \lambda^* \cdot K_n \left[1 - \cos \left(\frac{\pi}{n} \left(2i - \frac{1}{2} (\cos n\pi + 3) \right) - \varphi_{ik} \right) \right] \right]^3} \\
 & \cdot \left[\frac{4z_k^2 - L_k^2}{4} \right]
 \end{aligned} \tag{1}$$

where:

- i – number of the bushing segment, $i=1,2,3,4,5$, $k=1,2$,
- p_{ik} – pressure in the i -th segment of the k -th bearing,
- ε_k – absolute clearance in the k -th bearing,
- ω – rotation speed,
- $\dot{\alpha}_k$ – circumferential component of the journal velocity in the k -th bearing,
- $\dot{\beta}_k$ – radial component of the journal velocity in the k -th bearing,
- $\beta_k = \frac{e_k}{\varepsilon_k}$ – relative eccentricity in the k -th bearing,
- n – number of the bearing lobe,
- λ^* – relative eccentricity of the pericycloid,
- K_n – approximation coefficient of the pericycloid countour,
- L_k – length of the k -th bearing,
- z_{ik} – coordinate measured along the bearing axis,
- φ_{ik} – angle measured along the circumference in the i -th section of the k -th bearing,
- α_k – angle of the line between the journal center and the center of the bushing of the k -th bearing,
- R, R_k – journal radius and the radius of the circle inscribed in the pericycloid,
- e_k – distance between the journal and the bushing centers.

4. Hydrodynamic uplift forces and equation of motion

The natural consequence of determination of the oil film pressure distribution in individual bearings is derivation of components of hydrodynamic uplift forces. To do this, one needs to integrate function (1) describing the pressure within the limits of positive pressure. In the Cartesian coordinate system x,y,z , the hydrodynamic uplift force components can be expressed as:

$$\begin{aligned}
 P_{x_i} = & -R_i \int_{\varphi_{p_i}}^{\varphi_b} \left[\frac{\mu_i L_i^3}{2\varepsilon_i^2} (\omega - 2\dot{a}_i) \cdot \frac{\beta_i \cdot \sin(\varphi - \alpha_i)}{\left[1 + \beta_i \cdot \cos(\varphi - \alpha_i) + \lambda^* \cdot K_n \left[1 - \cos \left(\frac{\pi}{n} \left(2i - \frac{1}{2}(\cos n\pi + 3) \right) - \varphi \right) \right] \right]^3} \right. \\
 & + \frac{\mu_i L_i^3}{2\varepsilon_i^2} \omega \cdot \left. \frac{\lambda^* \cdot K_n \cdot \sin \left[\frac{\pi}{n} \left(2i - \frac{1}{2}(\cos n\pi + 3) \right) - \varphi \right]}{\left[1 + \beta_i \cdot \cos(\varphi - \alpha_i) + \lambda^* \cdot K_n \left[1 - \cos \left(\frac{\pi}{n} \left(2i - \frac{1}{2}(\cos n\pi + 3) \right) - \varphi \right) \right] \right]^3} \right] \cdot \sin \varphi d\varphi \\
 & + R_i \int_{\varphi_{p_i}}^{\varphi_b} \left[\frac{\mu_i L_i^3}{\varepsilon_i^2} \dot{\beta}_i \cdot \frac{\cos(\varphi - \alpha_i)}{\left[1 + \beta_i \cdot \cos(\varphi - \alpha_i) + \lambda^* \cdot K_n \left[1 - \cos \left(\frac{\pi}{n} \left(2i - \frac{1}{2}(\cos n\pi + 3) \right) - \varphi \right) \right] \right]^3} \right] \cdot \sin \varphi d\varphi \\
 (2)
 \end{aligned}$$

$$\begin{aligned}
 P_{y_i} = & -R_i \int_{\varphi_{p_i}}^{\varphi_b} \left[\frac{\mu_i L_i^3}{2\varepsilon_i^2} (\omega - 2\dot{a}_i) \cdot \frac{\beta_i \cdot \sin(\varphi - \alpha_i)}{\left[1 + \beta_i \cdot \cos(\varphi - \alpha_i) + \lambda^* \cdot K_n \left[1 - \cos \left(\frac{\pi}{n} \left(2i - \frac{1}{2}(\cos n\pi + 3) \right) - \varphi \right) \right] \right]^3} \right. \\
 & + \frac{\mu_i L_i^3}{2\varepsilon_i^2} \omega \cdot \left. \frac{\lambda^* \cdot K_n \cdot \sin \left[\frac{\pi}{n} \left(2i - \frac{1}{2}(\cos n\pi + 3) \right) - \varphi \right]}{\left[1 + \beta_i \cdot \cos(\varphi - \alpha_i) + \lambda^* \cdot K_n \left[1 - \cos \left(\frac{\pi}{n} \left(2i - \frac{1}{2}(\cos n\pi + 3) \right) - \varphi \right) \right] \right]^3} \right] \cdot \cos \varphi d\varphi \\
 & + R_i \int_{\varphi_{p_i}}^{\varphi_b} \left[\frac{\mu_i L_i^3}{\varepsilon_i^2} \dot{\beta}_i \cdot \frac{\cos(\varphi - \alpha_i)}{\left[1 + \beta_i \cdot \cos(\varphi - \alpha_i) + \lambda^* \cdot K_n \left[1 - \cos \left(\frac{\pi}{n} \left(2i - \frac{1}{2}(\cos n\pi + 3) \right) - \varphi \right) \right] \right]^3} \right] \cdot \cos \varphi d\varphi \\
 (3)
 \end{aligned}$$

$$\begin{aligned}
P_{x_{2i}} = & -R_2 \int_{\varphi_{pi}}^{\varphi_{ki}} \left[\frac{\mu_2 L_2^3}{2\epsilon_2^2} (\omega - 2\dot{a}_2) \cdot \frac{\beta_2 \cdot \sin(\varphi - \alpha_2)}{\left[1 + \beta_2 \cdot \cos(\varphi - \alpha_2) + \lambda^* \cdot K_n \left[1 - \cos \left(\frac{\pi}{n} \left(2i - \frac{1}{2}(\cos n\pi + 3) \right) - \varphi \right] \right]^3} \right] \\
& + \frac{\mu_2 L_2^3}{2\epsilon_2^2} \omega \cdot \left[\frac{\lambda^* \cdot K_n \cdot \sin \left[\frac{\pi}{n} \left(2i - \frac{1}{2}(\cos n\pi + 3) \right) - \varphi \right]}{\left[1 + \beta_2 \cdot \cos(\varphi - \alpha_2) + \lambda^* \cdot K_n \left[1 - \cos \left(\frac{\pi}{n} \left(2i - \frac{1}{2}(\cos n\pi + 3) \right) - \varphi \right] \right]^3} \right] \cdot \sin \varphi d\varphi \\
& + R_2 \int_{\varphi_{pi}}^{\varphi_{kli}} \left[\frac{\mu_2 L_2^3}{\epsilon_2^2} \dot{\beta}_1 \cdot \frac{\cos(\varphi - \alpha_1)}{\left[1 + \beta_2 \cdot \cos(\varphi - \alpha_2) + \lambda^* \cdot K_n \left[1 - \cos \left(\frac{\pi}{n} \left(2i - \frac{1}{2}(\cos n\pi + 3) \right) - \varphi \right] \right]^3} \right] \cdot \sin \varphi d\varphi
\end{aligned} \tag{4}$$

$$\begin{aligned}
P_{y_{2i}} = & -R_2 \int_{\varphi_{pi}}^{\varphi_{ki}} \left[\frac{\mu_2 L_2^3}{2\epsilon_2^2} (\omega - 2\dot{a}_2) \cdot \frac{\beta_2 \cdot \sin(\varphi - \alpha_2)}{\left[1 + \beta_2 \cdot \cos(\varphi - \alpha_2) + \lambda^* \cdot K_n \left[1 - \cos \left(\frac{\pi}{n} \left(2i - \frac{1}{2}(\cos n\pi + 3) \right) - \varphi \right] \right]^3} \right] \\
& + \frac{\mu_2 L_2^3}{2\epsilon_2^2} \omega \cdot \left[\frac{\lambda^* \cdot K_n \cdot \sin \left[\frac{\pi}{n} \left(2i - \frac{1}{2}(\cos n\pi + 3) \right) - \varphi \right]}{\left[1 + \beta_2 \cdot \cos(\varphi - \alpha_2) + \lambda^* \cdot K_n \left[1 - \cos \left(\frac{\pi}{n} \left(2i - \frac{1}{2}(\cos n\pi + 3) \right) - \varphi \right] \right]^3} \right] \cdot \cos \varphi d\varphi \\
& + R_2 \int_{\varphi_{pi}}^{\varphi_{kli}} \left[\frac{\mu_2 L_2^3}{\epsilon_2^2} \dot{\beta}_1 \cdot \frac{\cos(\varphi - \alpha_1)}{\left[1 + \beta_2 \cdot \cos(\varphi - \alpha_2) + \lambda^* \cdot K_n \left[1 - \cos \left(\frac{\pi}{n} \left(2i - \frac{1}{2}(\cos n\pi + 3) \right) - \varphi \right] \right]^3} \right] \cdot \cos \varphi d\varphi
\end{aligned} \tag{5}$$

where:

φ_{pi} – beginning of the oil film in the i-th section of the first bearing, for the wedge effect;

φ_{ki} – end of the oil film in the i-th section of the first bearing, for the wedge effect;

φ_{pli} – beginning of the oil film in the i-th section of the first bearing, for the squeeze effect;

φ_{kli} – end of the oil film in the i-th section of the first bearing, for the squeeze effect;

$i = 1, 2, 3, 4, 5$;

R_1, R_2 – radii of the first and second journal, respectively.

In the Cartesian coordinate system, the equation of motion, on the basis of [7], can be expressed as:

$$\begin{aligned}\ddot{x}_1 &= \frac{I_2 a}{I_1 l} \omega (\dot{y}_2 - \dot{y}_1) + \left(\frac{1}{m} + \frac{a^2}{I_1} \right) \left(P_{x_1} + \gamma \frac{V_1}{\delta^2} \dot{x}_1 + \left(\frac{1}{m} - \frac{ab}{I_1} \right) \right) P_{x_2} + \gamma \frac{V_1}{\delta^2} \dot{x}_2, \\ \ddot{y}_1 &= \frac{I_2 a}{I_1 l} \omega (\dot{x}_2 - \dot{x}_1) + \left(\frac{1}{m} + \frac{a^2}{I_1} \right) \left(P_{y_1} + \gamma \frac{V_1}{\delta^2} \dot{y}_1 + \left(\frac{1}{m} - \frac{ab}{I_1} \right) \right) P_{y_2} + \gamma \frac{V_1}{\delta^2} \dot{y}_2 - \frac{Q}{m}, \\ \ddot{x}_2 &= \frac{I_2 b}{I_1 l} \omega (\dot{y}_2 - \dot{y}_1) + \left(\frac{1}{m} + \frac{ab}{I_1} \right) \left(P_{x_1} + \gamma \frac{V_2}{\delta^2} \dot{x}_1 + \left(\frac{1}{m} - \frac{ab}{I_1} \right) \right) P_{x_2} + \gamma \frac{V_2}{\delta^2} \dot{x}_2, \\ \ddot{y}_2 &= \frac{I_2 b}{I_1 l} \omega (\dot{x}_2 - \dot{x}_1) + \left(\frac{1}{m} + \frac{ab}{I_1} \right) \left(P_{y_1} + \gamma \frac{V_2}{\delta^2} \dot{y}_1 + \left(\frac{1}{m} - \frac{b^2}{I_1} \right) \right) P_{y_2} + \gamma \frac{V_2}{\delta^2} \dot{y}_2 - \frac{Q}{m}\end{aligned}\quad (6)$$

where:

a, b – coordinates defining the point of application of the external loading Q ;
 l – bearing length;

$$\gamma = \gamma_D \frac{\mu_0 \cdot A \cdot N^2}{2r};$$

γ_D – gain factor in the state-based control system;

μ_0 – magnetic permeability of the air;

A – cross-section area of the core;

N – number of windings;

r – coil resistance;

V_1, V_2 – voltage;

$$\delta = \delta_1 + \frac{l_1}{2\mu_1};$$

δ_1 – magnetic clearance;

μ_1 – relative magnetic permeability;

l – core length;

$\dot{x}_1, \dot{y}_1, \dot{x}_2, \dot{y}_2$ velocities of the journal in the x, y directions;

I_1, I_2 – principal geometrical moments of inertia of the rotor;

$$P_{x_1} = \sum_{i=1}^5 P_{x_{1i}}; \quad P_{y_1} = \sum_{i=1}^5 P_{y_{1i}}; \quad P_{x_2} = \sum_{i=1}^5 P_{x_{2i}}; \quad P_{y_2} = \sum_{i=1}^5 P_{y_{2i}} \quad (7)$$

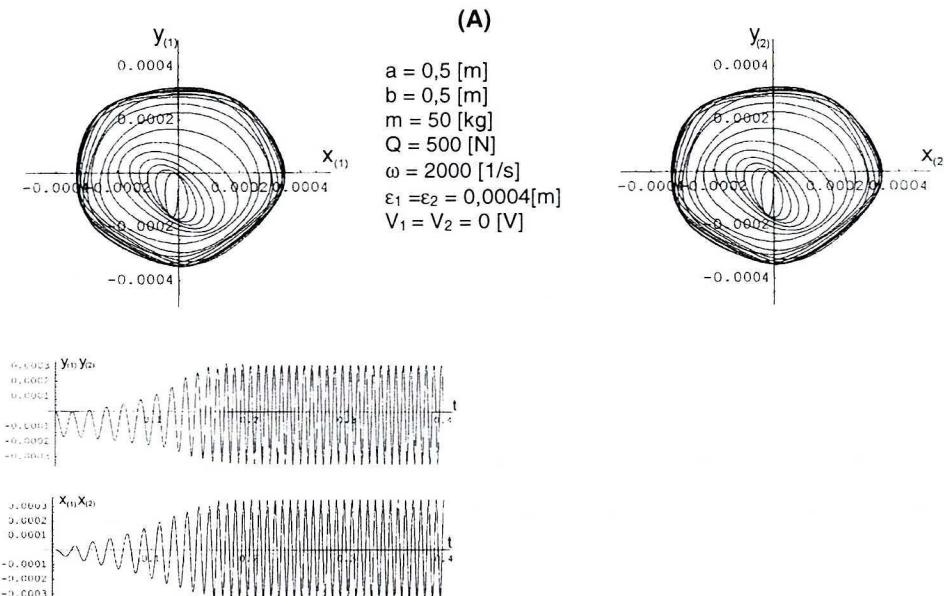
In order to investigate motion of the journal bearing centers in the assumed Cartesian coordinate system x, y, z on the basis of equations (6), we must express $\beta_1, \beta_2, \dot{\alpha}_1, \dot{\alpha}_2, \dot{\beta}_1, \dot{\beta}_2, \sin\alpha_1, \cos\alpha_1, \sin\alpha_2$, and $\cos\alpha_2$ in the following form:

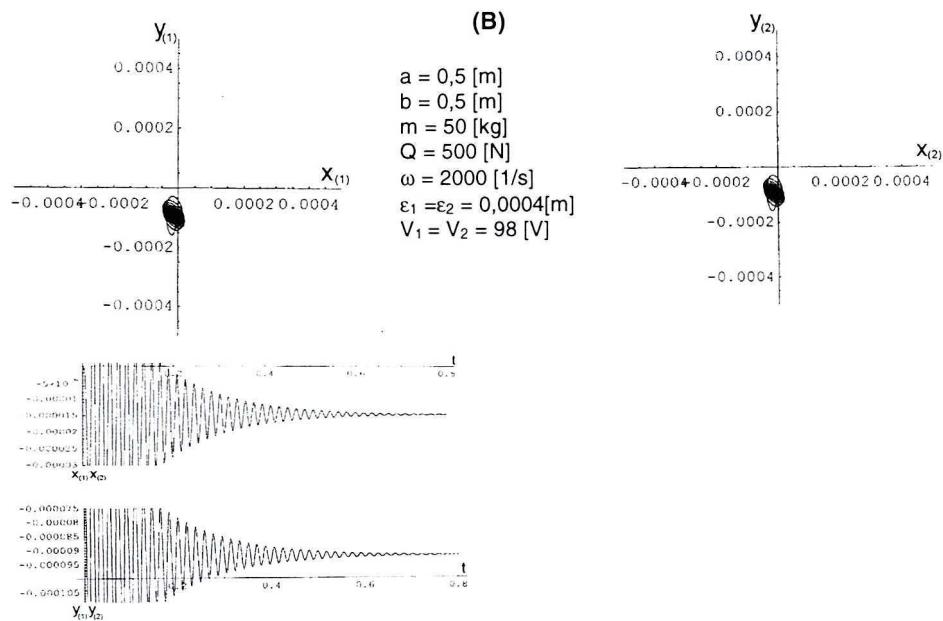
$$\begin{aligned}
 \beta_1 &= \frac{\sqrt{x_1^2 + y_1^2}}{\varepsilon_1}; & \dot{\beta}_1 &= \frac{x_1 \cdot \dot{x}_1 + y_1 \cdot \dot{y}_1}{\beta_1 \cdot \varepsilon_1^2}; & \dot{\alpha}_1 &= \frac{x_1 \cdot \dot{y}_1 - y_1 \cdot \dot{x}_1}{\beta_1^2 \cdot \varepsilon_1^2} \\
 \cos \alpha_1 &= -\frac{x_1}{\beta_1 \cdot \varepsilon_1}; & \sin \alpha_1 &= -\frac{y_1}{\beta_1 \cdot \varepsilon_1}; \\
 \beta_2 &= \frac{\sqrt{x_2^2 + y_2^2}}{\varepsilon_2}; & \dot{\beta}_2 &= \frac{x_2 \cdot \dot{x}_2 + y_2 \cdot \dot{y}_2}{\beta_2 \cdot \varepsilon_2^2}; & \dot{\alpha}_2 &= \frac{x_2 \cdot \dot{y}_2 - y_2 \cdot \dot{x}_2}{\beta_2^2 \cdot \varepsilon_2^2} \\
 \cos \alpha_2 &= -\frac{x_2}{\beta_2 \cdot \varepsilon_2}; & \sin \alpha_2 &= -\frac{y_2}{\beta_2 \cdot \varepsilon_2};
 \end{aligned} \tag{8}$$

and then substitute them into equations (2), (3), (4) and (5).

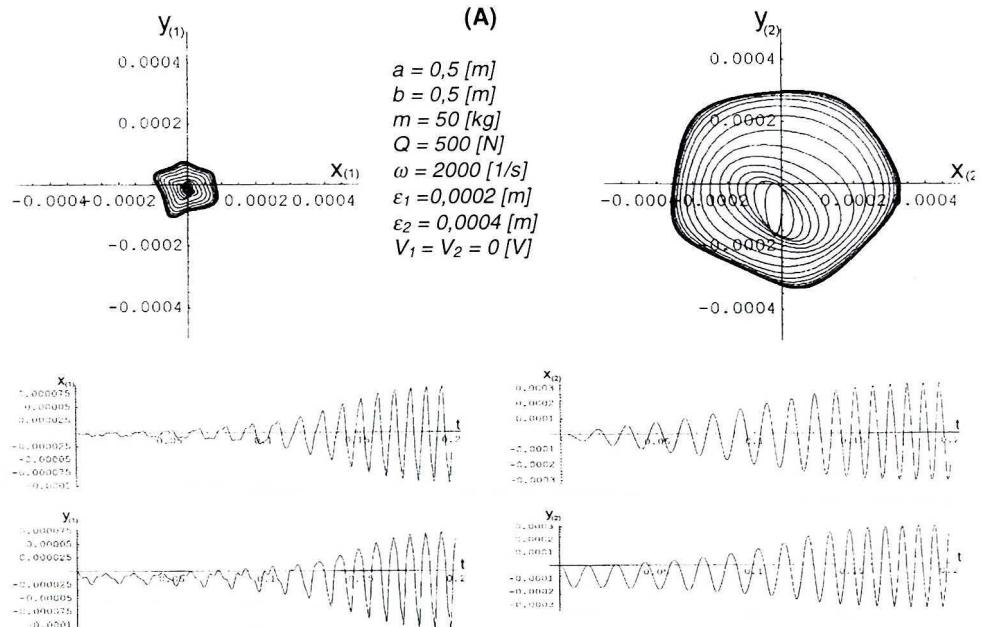
5. Numerical simulation

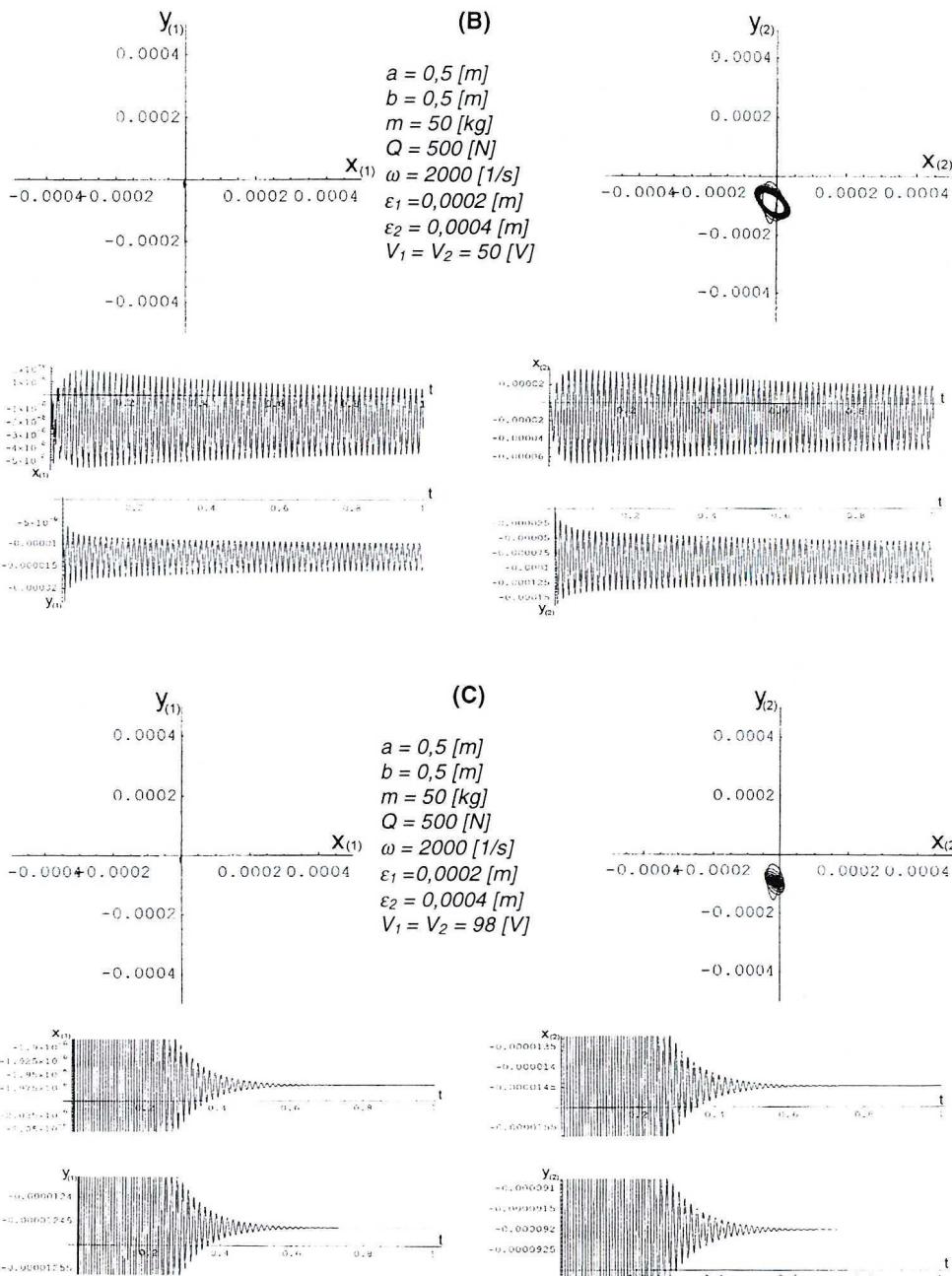
Equations (6) are strongly nonlinear and coupled. They cannot be solved analytically. In order to investigate motion of the journal bearing centers, and the effect of the active magnetic stabilization in reducing the self-excited vibration, a numerical simulation method was used. Exemplary results for the two kinds of asymmetry and the effect of the voltage are presented in Figs 2, 3, 4 and 5.



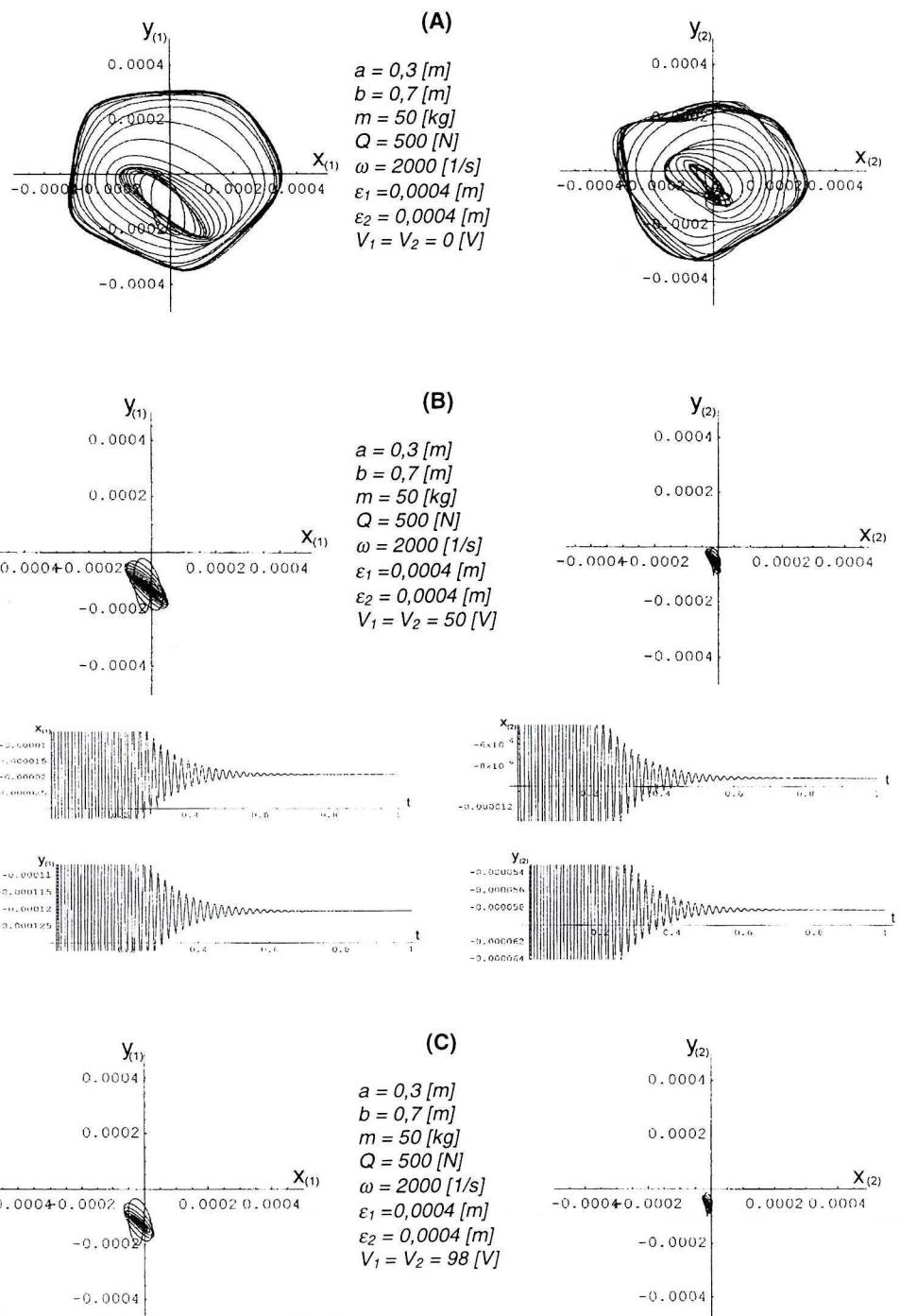


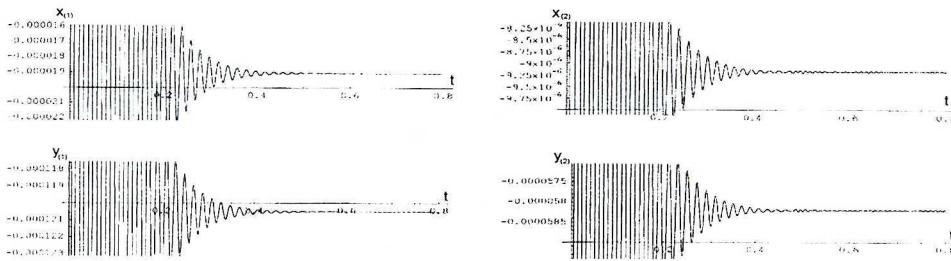
Rys. 2. Trajectories of motion of journal bearings „1”, „2” and the corresponding time histories.
Different voltages in the stabilization system, ($A=0[V]$, $B=98[V]$).
Symmetric journal bearing system



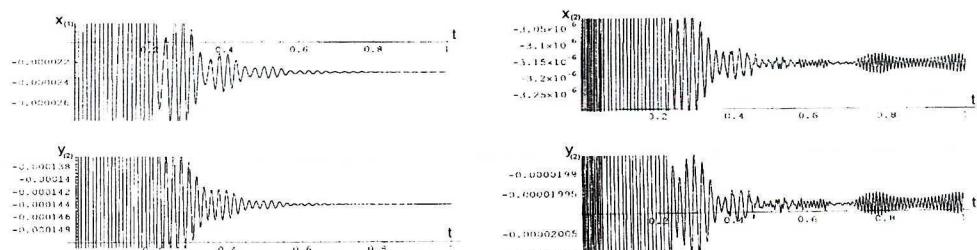
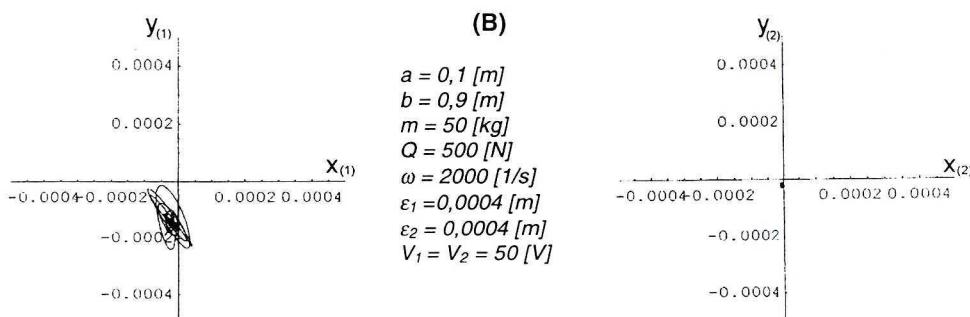
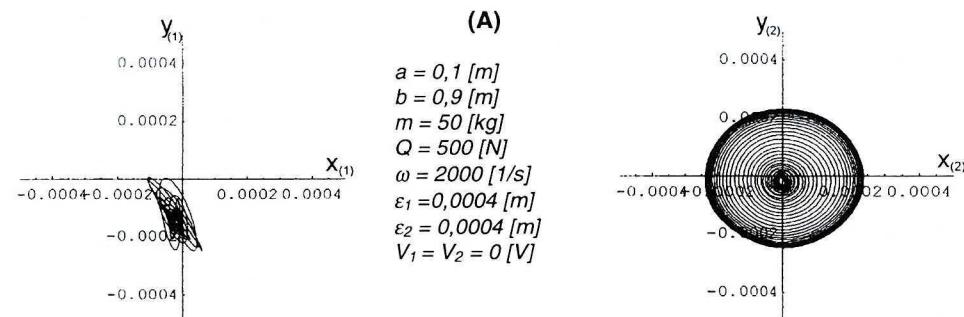


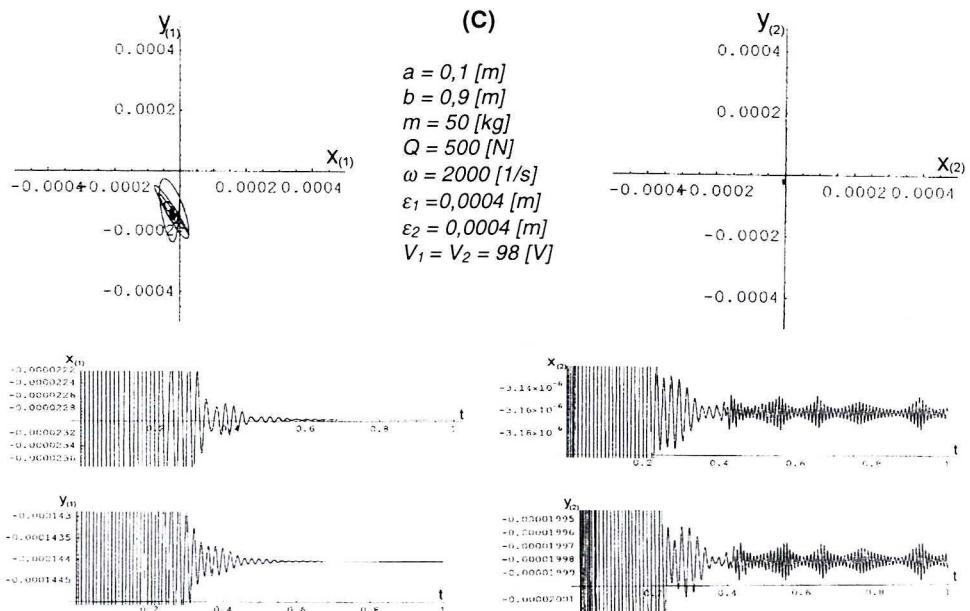
Rys. 3. Trajectories of motion of journal bearings „1”, „2” and the corresponding time histories.
Different voltages in the stabilization system, ($A=0[V]$, $B=50[V]$, $C=98[V]$),
Non-symmetric journal bearing system, physical asymmetry





Rys. 4. Trajectories of motion of journal bearings „1”, „2” and the corresponding time histories.
Different voltages in the stabilization system ($A=0[V]$, $B=50[V]$, $C=98[V]$).
Non-symmetric journal bearing system, geometrical asymmetry





Rys. 5. Trajectories of motion of journal bearings „1”, „2” and the corresponding time histories.
 Different voltages in the stabilization system ($A=0[V]$, $B=50[V]$, $C=98[V]$).
 Non-symmetric journal bearing system, geometrical asymmetry.

6. Final remarks

On the grounds of the obtained numerical results, we can ascertain the following facts:

- active magnetic stabilization of the arisen self-excited vibration of the journal bearings is effective for both symmetric and non-symmetric systems (considerable reduction or even disappearance of vibration is observed),
- stabilization proves applicable for two kinds of asymmetry,
- voltage growth increases the efficiency of the applied stabilization system.

Manuscript received by Editorial Board, May 23, 2001;
 final version, September 11, 2001.

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Aktywne tłumienie drgań samowzbudnych niesymetrycznego sztywnego wirnika podpartego w łożyskach ślizgowych o zarysie niekołowym

Streszczenie

W pracy rozważono możliwości tłumienia drgań samowzbudnych sztywnego wirnika podpartego w łożyskach ślizgowych o zarysie niekołowym pięciopowierzchniowym z pięcioma szczelinami smarnymi. Zamieszczono zależności opisujące rozkłady ciśnienia w filmie olejowym. Opisano składowe hydrodynamicznej siły wyporu smaru w łożyskach. Otrzymane równania ruchu zbadano metodą symulacji cyfrowej. Zastosowana aktywna magnetyczna stabilizacja okazała się być skuteczna w obu rodzajach występującej w układzie asymetrii. Zamieszczono przykładowe trajektorie ruchu i przebiegi czasowe.