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GENERALIZED PERFORMANCE CHARACTERISTICS OF TURBINE STAGE GROUPS AN ATTEMPT TO SUPPLEMENT THE FLÜGEL'S - STODOLA'S LAW

The knowledge of performance characteristics of turbine stage groups is still insufficient, particularly in the general case of changes of operating conditions. This situation is caused mainly by the scarcity of experimental data available. In such case, the opportunity to obtain the required data, using mathematical modelling and numerical simulation of the operation of stage groups under off-design conditions instead of physical experiment, seems to be attractive.

The application of this idea for impulse type turbine stage groups was presented in [1], [2]. Here we discuss similar results but obtained for reaction type turbine stage groups, that is:

- mathematical model for computer simulation of operation of reaction type turbine stage group, under variable regime (based on Ainley's and Mathieson's method with some improvements);
- simulation results for a number of stage groups designed according to former BBC and traditional concepts;
- more general properties of these groups (in relation to flow capacity and efficiency) obtained from the analysis of simulation data;
- comparison of observed properties of impulse and reaction type turbine stage groups.

1. Introduction

The group of similar stages is, beside the single stages with special properties, the basic element of the flow part of steam turbines. However, in

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contrast to the single stages, the knowledge of stage group properties is still slight and far inadequate to their role and importance. In particular, this concerns characteristics of turbine stage groups needed for practical application in various analyses of turbine performance under off-design operating conditions. These problems belong to the most difficult and least explored and recognised ones. Therefore, the research of turbine stage group characteristics has cognitive significance - searching for more general features of stage groups, as well as practical significance - application of the results in analysis of turbine performance under off-design conditions.

In general, two basic approaches to determining turbine stage group characteristics under off-design operating conditions by calculations can be distinguished, i.e. the approach based on expanded mathematical models of the group, requiring very detailed data about the group and complex calculations, and the approach based on approximation models, using simpler dependencies of integral type. In spite of the considerable development of the methods belonging to the first type approach, the second one, based on approximation models, is still generally accepted and applied in practice for more general analyses in the cases, when e.g. detailed geometric data about group flow part are unknown or unobtainable. The law of stage group flow capacity is a classic example here.

The approximation approach, in which a general lack of proper data and information occurs, is considered in the paper presented.

The revision of prevailing works [2], [3], [4] proved that in the general case of variable operating conditions of a stage group, i.e. with simultaneous changes in π and \bar{n} (see further on for explanation), there is a deficit of proper data. Often even a qualitative assessment, in particular, of the group efficiency is difficult to be defined.

The fundamental reason for such state of affairs is the scarcity of appropriate experimental data in this field, caused, as it seems, mainly by the large costs of construction and operation of research stands needed. It is hard to predict further real progress in solving the considered problem without a considerable number of new adequate examples of stage group characteristics.

In this situation, the opportunity to obtain the needed data in other way than physical experiment has become very attractive. The application of mathematical modelling and computer simulation of stage groups' operation under off-design conditions creates such opportunities. This idea was accepted in the Power Engineering Division of the Institute of Heat Engineering of the Warsaw University of Technology, developing models and codes and running series of calculations, modelling stage groups' operation under off-design conditions. The model and the simulation code are replacing here the research stand, and can supply practically any data we wish to request, without limitations of types of changes of the operating conditions. It should be expected that systematically stored and analysed results of such research would allow us to reveal new, more general properties of turbine stage groups.

Encouraging results obtained here in the case of impulse type turbine stage groups have stimulated application of the similar approach to the reaction type turbine stage groups. Further on it is presented the obtained results (selected results have been already shown [11]) as well as some comparison of observed properties of impulse and reaction type turbine stage groups.

2. Off - design operation of turbine stage group

Practical analyses of turbine performance under off-design condition of operation need the knowledge of two essential characteristics of turbine stage groups: flow capacity and efficiency changes at variable conditions.

The characteristics of turbine stage group, with values of Reynolds number large enough to omit its influence and under assumption of working medium with properties sufficiently close to those of a perfect gas or ideal steam in the Eichelberg's understanding, accept generally the following form:

- flow capacity

$$\bar{m} = \frac{\dot{m}}{\dot{m}_0} = \frac{p_\alpha}{p_{\alpha 0}} \sqrt{\frac{j_{\alpha 0}}{j_\alpha}} E(\pi, \bar{n}) \quad (1)$$

$$\pi \equiv \frac{p_\omega}{p_\alpha}; \quad j \equiv \frac{k}{k-1} p_\alpha v_\alpha; \quad \bar{n} \equiv \frac{n}{n_0} \sqrt{\frac{j_{\alpha 0}}{j_\alpha}}$$

- internal efficiency

$$\eta = \eta(\pi, \bar{n}) \quad (2)$$

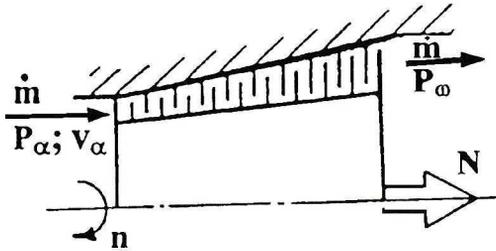


Fig. 1 Scheme of turbine stage group

where \dot{m} means mass flow rate (Fig. 1); p_α, v_α - overall working medium parameters at the stage group inlet; p_ω - static working medium pressure at the stage group outlet; j - normal enthalpy; n - rotational speed; k - isentropic exponent; the α index concerns conditions at the stage group inlet; the ω index - at the outlet; the 0 index - relative conditions (of reference). The usual assumption is that the relative conditions are very close to those, which correspond to the maximum efficiency of the group η_{\max} . The form of the

functions E and η (in general unknown) depends on the properties of the definite stage group.

The determination of the form of these functions is the research subject of the approach based on approximation models.

There are only two well investigated specific cases of varying operating conditions of a group, in which, particularly as far as flow capacity is concerned, some generalisation and some synthesis of the investigation results has taken place, i.e. the case of constant velocity ratio $\bar{n}=1$ and variable pressure ratio $\pi = \text{var}$, and the case of constant pressure ratio $\pi = \pi_0 = \text{const}$ and variable velocity ratio $\bar{n} = \text{var}$, e.g. [8].

Concerning the flow capacity with $\bar{n}=1$, the majority of authors agrees that, with sufficient accuracy for practical application, the following dependence could be used:

$$\frac{\bar{m}}{\dot{m}_o} = \frac{\dot{m}}{\dot{m}_o} = \frac{P_\alpha}{p_{\alpha 0}} \sqrt{\frac{j_{\alpha 0}}{j_\alpha}} \frac{E}{E_0} \quad (3)$$

where

$$E = 1 \quad \text{if} \quad \pi \leq \pi_{cr0}$$

$$\text{or} \quad E = \sqrt{1 - \left(\frac{\pi - \pi_{cr0}}{1 - \pi_{cr0}} \right)^2} \quad \text{for} \quad \pi > \pi_{cr0}$$

and similarly

$$E_0 = 1 \quad \text{if} \quad \pi_0 \leq \pi_{cr0}$$

$$\text{or} \quad E_0 = \sqrt{1 - \left(\frac{\pi_0 - \pi_{cr0}}{1 - \pi_{cr0}} \right)^2} \quad \text{for} \quad \pi_0 > \pi_{cr0}$$

Symbol π_{cr} means critical pressure ratio of the group (Fig. 4).

In the general case of variable operating conditions of stage group, i.e. with simultaneous changes in π and \bar{n} , there is a deficit of proper data for determining the flow capacity as well as, in particular, the changes in the group efficiency. Often even a qualitative assessment is difficult to be accomplished here.

Dependencies (1) and (2) determine connections between operation parameters and stage group performances at off-design conditions. From their nature they have comprehensive, integral character - their form results from similarity theory. However, this synthetic result, described by characteristics (1) and (2), is in reality a consequence of overlapping of many particular influences of various nature.

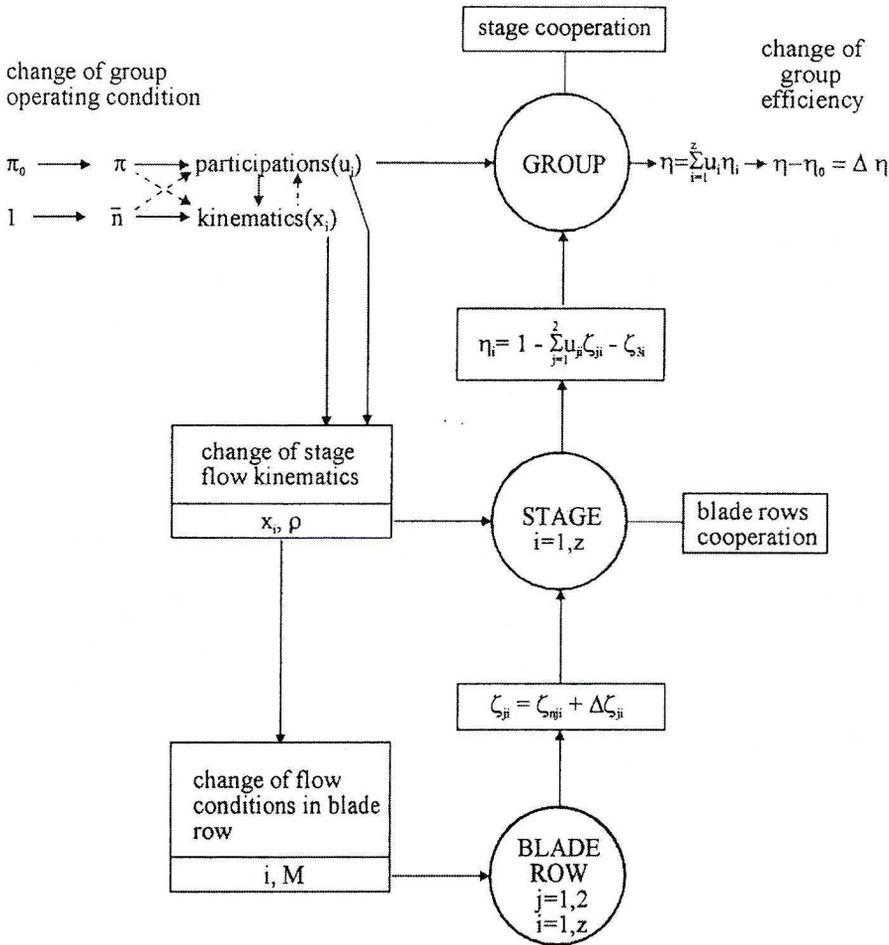


Fig. 2 Schem of „mechanism” of efficiency changes of turbine stage group caused by operating condition changes where: x_i - velocity ratio of the i -th stage; $i = 1 \div z$; z - number of stages in a group; u_i - participation of the i -th stage in isentropic enthalpy drop of group; η_i - internal efficiency of the i -th stage; ρ - degree of reaction; ζ_{ji} - losses of the j -th row of the i -th stage; ζ_{3i} - leaving velocity loss and additional losses of the i -th stage

For illustration, in the Fig. 2 it is shown an attempt of explanation of „mechanism” of efficiency changes of a turbine stage group at off-design operation. The change of a group efficiency $\Delta\eta = \eta_0 - \eta$, caused by operating condition changes (regarding to relative ones) is a result here of several mutually connected effects and influences. The three levels of cooperation of group elements: elementary level-blade row, turbine stage level - where blade rows cooperate and stage group level - where stages cooperate, are separated. If

it is taken into account only the main effects and influences then the change of flow conditions in a blade row (elementary level) determines incidence angle i and Mach number M ; next the changes of cooperation conditions of blade rows in a stage determine stage flow kinematics and influence of neighbouring stages; while the changes of cooperation conditions of stages at a group - participations of stages in constant or changed isentropic enthalpy drop of the group, specific speed and interaction between stages. Losses in separate blade row ζ_{ij} (elementary level), changed as a result of changes of flow conditions in blade rows, are one of main effects which determine the changes of efficiency of a separate stage, directly influencing also on its flow kinematics. Next, further on - efficiencies of separate stages together with their participations (u_i) in the isentropic enthalpy drop of the group determine its efficiency. At the same time, there exist here strong „feedbacks”, as changes of flow conditions of separate blade rows (elementary level) are a global result of changes of cooperation conditions on the group and separate stages levels, which is also indicated in Fig. 2. The complexity of the problem of off-design turbine stage group operation can be readily appreciated. This complexity makes it practically impossible to perform full analytical analysis of the problem. So it remains to use numerical approaches.

3. Mathematical model and computer code for simulation

The construction principles of mathematical models for simulation of turbine stage group operation under off - design conditions were formulated in the work [4]. In such mathematical model, the impulse type turbine stage group is divided into particular stages [1], [2] and the reaction type turbine stage group - into particular blade rows, later treated as repeated similar elements of common mathematical description. It was distinguished that the mathematical model of this definite element of the group should be based on two types of relations:

- dependencies determining the flow capacity, in a form selected adequately to the considered case; the flow capacity equations make it possible to divide the enthalpy drop at the group, i.e. to determine the participation of the particular stages (or blade rows) in the changed enthalpy drop of the group;
- dependencies determining the efficiency changes (or changes in losses) in new operating conditions, obtained mainly by generalisation of properly selected experimental data.

Because of the different properties, the development of mathematical models separately for the groups of impulse (chamber) and reaction (drum) type is suitable.

The model for simulation of operation of reaction type turbine stage group can be based on the concept and the method of performance estimation for axial - flow turbines given by D.G. Ainley and G.C.R. Mathieson [6]. In the search

for a model, a group is divided into particular blade rows. The proper description of blade row properties, as in the case of losses in the range of flow capacity, is the advantage of the Ainley's and Mathieson's method. Results accuracy (with improvements) as well carefully elaborated algorithms, motivate for the choice of this approach as the basis for the mathematical model of reaction type turbine stage group.

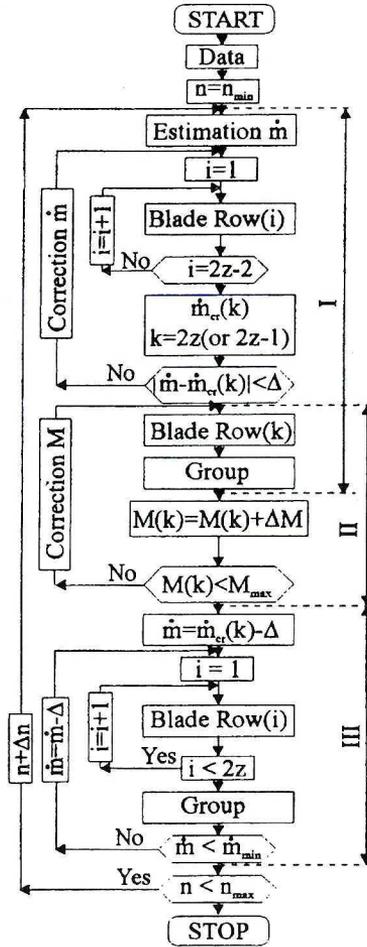


Fig. 3 Simplified structure of computer code

For „automatic” determination of a whole field of stage group characteristics with any given distribution of definite points, the appropriate computer code was elaborated. Simplified structure of this code is presented in Fig. 3. The computations are performed at constant values of the rotational speed chosen from the given range: n_{\min} to n_{\max} with step Δn .

The computations consist of three main parts:

I-the determination of the critical mass flow rate (\dot{m}_{cr}) and the critical pressure ratio of the group (π_{cr}). It is assumed here (for normal design of stage group) that critical state occurs firstly at the last stage (in nozzle $k = 2z - 1$ or in rotor $k = 2z$ row; z -number of stages).

II - the computations for a super - critical flow range ($\dot{m} = \dot{m}_{cr}$; $\pi < \pi_{cr}$) within a range from the critical Mach number $M(k)$ to the assumed maximum value M_{max} with step ΔM .

III - the computations for sub- critical flows ($\dot{m} < \dot{m}_{cr}$; $\pi > \pi_{cr}$) within a range of the mass flow rate up to the given value \dot{m}_{min} with step $\Delta \dot{m}$ starting from \dot{m}_{cr} . In the block diagram presented in Fig. 3. symbol \dot{m}_{cr} denotes the determination of the critical mass flow rate for the blade row under consideration; symbol Blade Row-algorithm in the model of row and Group denotes determination of overall performance of the given group of stages.

4. Stage groups design for simulation aims

The search for more general properties of stage groups using the considered simulations requires specification of the geometry and characteristics of the investigated blade rows, stages and groups.

The form of the needed characteristic (2) generally depends on the geometry of the stage group, and this relation can be presented as:

$$\eta \in F(\underline{G}) \quad (4)$$

where \mathbf{F} means set of functions for all types of groups, and \underline{G} is a vector of geometric parameters of stage group and can be distinguished as:

$$\underline{G} = \sum_{i=1}^z (\text{stage geometry})_i, \quad (5)$$

where z means number of stages in a group.

By the definition, stage geometry we mean here all the geometric parameters of the stage, including the shape of the profiles and the configuration of the blade rows. The fact that the problem of finding a form of the function practically can not be solved, because of the large number of components of the vector \underline{G} and elements of the set \mathbf{F} , is obvious.

However, the number of parameters describing group's properties can be reduced, limiting the considerations to the most commonly used in practice stage groups, constructed according to the principles of rational designing. Such groups contain similar stages with typical properties and characteristics. In the calculations, the isentropic enthalpy drops at the particular stages are equalised, and the values of these drops selected from the recommended ranges. The

nominal velocity ratios are close to the optimum ones, and the stage group has proper value of the Parsons number.

In the case of group of similar stages, in which a typical turbine stage is used as a repeatable element, it seems that the vector \underline{G} can be reduced to one element - the number z of stages in the group, i.e.

$$\eta \in f(z) \quad (6)$$

where f means set of functions for definite type of turbine stage. Further considerations deal with this form of the task set.

In the case of group flow capacity, it seems that the problem is less complicated, because of the generally accepted forms of relations (1). Only slight correction for taking the influence of the changes in \bar{n} into account should be provided.

For investigation of impulse type stage groups, the stage of TK2-TW2 type from the modernised stock of stages of the Zamech works in Elbląg [2], [8] was selected. Basing on its characteristics, nine stage groups were designed and tested [2].

For research on reaction type groups, the specially designed group of stages followed the example of flow part of the 1K12 steam turbine of the former company BBC (now ABB Alstom Power Systems) used for feed water pump drive was selected. The 1K12 turbine is exploited at variable rotational speed and pressure ratio and comes from leading in reaction type turbine building company, what motivates this choice. Diminishing subsequently three last stages of this group, starting from the twelfth one, three groups with stage number nine, six and three are received. The nominal data of these groups are set in Table 1.

For comparison aims, it was also designed the resembling group of 12 stages but according to traditional concept with fully similar blade rows based on the soviet TN-2 profile. The nominal data of these groups are set in Table 2.

Table 1-1K12

z	π_0	$\Pi_0=1/\pi_0$	η_0
3	0.617	1.63	0.8828
6	0.346	2.89	0.9005
9	0.161	6.21	0.9161
12	0.050	20.0	0.9330

Table 2 -TN-2

z	π_0	$\Pi_0=1/\pi_0$	η_0
3	0.867	1.15	0.8420
6	0.726	1.38	0.8561
9	0.606	1.65	0.8590
12	0.449	2.23	0.8640

5. Simulation results- flow capacity ratio

Using the described model, computer code and the mentioned eight projects of stage groups designed according to BBC and traditional concepts,

two-dimensional fields of characteristics of these eight groups were determined. This very large set of data and information was subjected to various tests and analyses, in attempt to find more general properties of stage groups.

A typical group flow capacity characteristics, taking the influence of the rotational speed into account, is presented in Fig. 4. The changes in the rotational speed go with changes in the critical mass flow \dot{m}_{cr} i.q. changes in the critical pressure ratio π_{cr} . The same propriety was corroborated in all the cases of investigated stage groups (also impulse type). Treating the curves $\dot{m}_{cr} = f(\pi)$ in the area of subcritical flows ($\pi > \pi_{cr}$) also as ellipses, analogous to the case of nominal rotational speed n_0 , the flow capacity equation (3) can be written in a more general form, taking the influence of the changes in the rotational speed into account. This influence is conceived in the coefficients $A(\bar{n})$ and $B(\bar{n})$, defined as follows:

$$A(\bar{n}) = \frac{\dot{m}_{cr} P_{\alpha 0}}{\dot{m}_{cr 0} P_{\alpha}} \sqrt{\frac{j_{\alpha}}{j_{\alpha 0}}} \quad \text{and} \quad \pi_{cr} = \pi_{cr 0} B(\bar{n}), \quad (7)$$

where the "0" index concerns the relative conditions (for $\bar{n} = 1$ obviously $A = 1$, $\pi_{cr} = \pi_{cr 0}$).

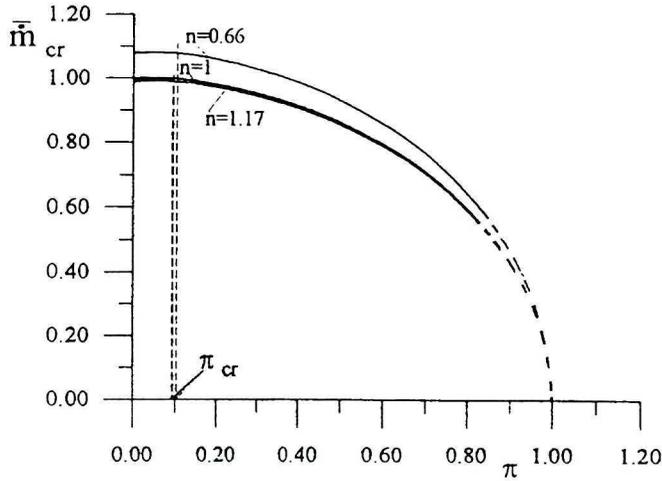


Fig. 4 Typical example of influence of the changes in the rotational speed on the group flow capacity

Therefore, the stage group flow capacity equation, with taking the influence of the changes in the rotational speed into account, accepts the form

$$\frac{\dot{m}}{\dot{m}_0} = A \frac{P_{\alpha}}{P_{\alpha 0}} \sqrt{\frac{j_{\alpha 0}}{j_{\alpha}} \frac{E}{E_0}}, \quad (8)$$

where

$$E_0 = 1 \quad \text{if} \quad \pi_0 \leq \pi_{cr0},$$

$$\text{or} \quad E_0 = \sqrt{1 - \left(\frac{\pi_0 - \pi_{cr0}}{1 - \pi_{cr0}} \right)^2} \quad \text{for} \quad \pi_0 > \pi_{cr0}$$

and similarly

$$E = 1 \quad \text{if} \quad \pi \leq \pi_{cr0} \cdot B$$

$$\text{or} \quad E = \sqrt{1 - \left(\frac{\pi - \pi_{cr0} \cdot B}{1 - \pi_{cr0} \cdot B} \right)^2} \quad \text{for} \quad \pi > \pi_{cr0} \cdot B$$

The functions $A(\bar{n})$ and $B(\bar{n})$, for the investigated stage groups, are presented in Fig. 5. The influence of the number of stages in a group on the functions $A(\bar{n})$ and $B(\bar{n})$ is obvious, so, according to the simulation results, the dependencies have the form $A(\bar{n}, z)$ and $B(\bar{n}, z)$, where z is the number of stages in the group.

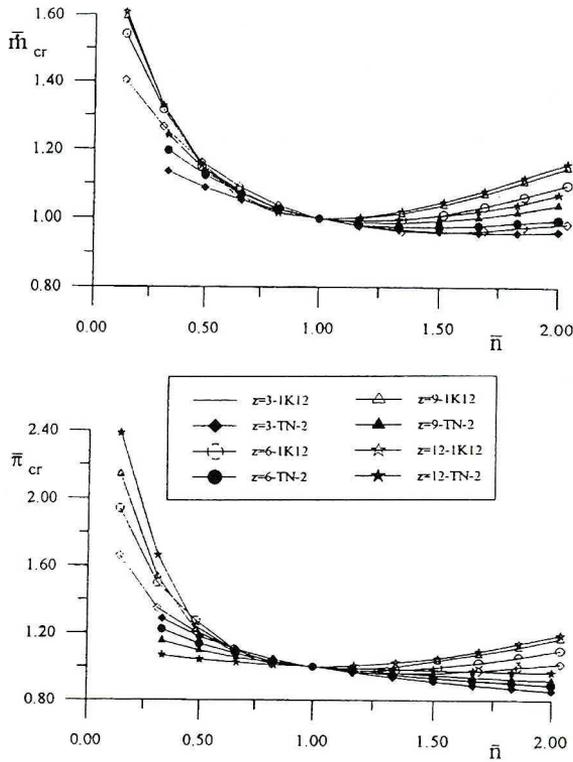


Fig. 5. Relative critical mass flow rate - coefficient $A(\bar{n})$ and relative critical pressure ratio - coefficient $B(\bar{n})$ as a function of the rotational speed and the number of stages in the group for the 1K12 and TN-2 groups

The analysis of the simulation results allowed us, as it seems, to obtain more general form of the flow capacity equation, such as (7) and (8), and also coefficients A and B (Fig. 5) for reaction type stage groups, used in the former BBC works and uniform ones. They can be applied to similar cases. This research should be repeated for other type of blading. It should be stressed that influence of changes in rotational speed on group flow capacity of reaction type is considerable greater than in the case of impulse type ones [1], [2].

6. Simulation results – efficiency

It is much more difficult to find similar regularities for the stage group efficiency (2) than in the case of the flow capacity, at the very least because of the lack of any positive examples so far (besides [1], [2]). For illustration, efficiency characteristics of selected groups resulting from simulations are presented in Fig. 6. Explicit irregularity in the course of the curves is visible here, observed repeatedly in various works e.g. [1], [2], [6], making any generalisation attempts impossible.

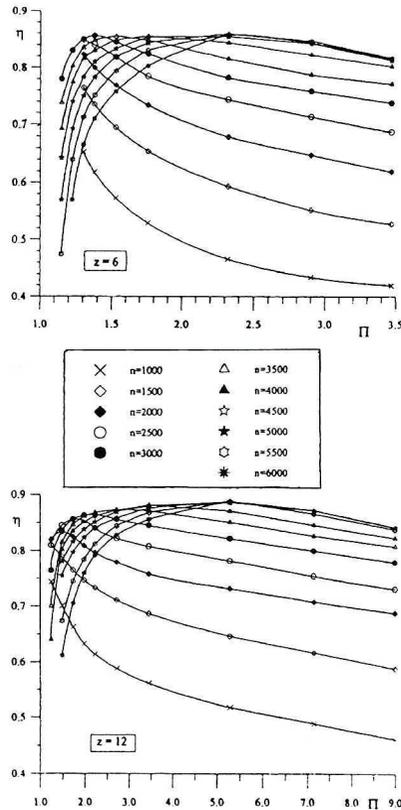


Fig. 6 Exemplary efficiency characteristics of the tested stage groups in the form resulting from the simulation where $\Pi = 1/\pi$ and $p_\alpha = p_{\alpha 0}$; $j_\alpha = j_{\alpha 0}$

Nevertheless, appropriately changing the variables, it seems to be possible to obtain the needed group characteristics in adequate regular form, making it possible to search for analytic description. The first step is here the transformation of the characteristics to the form proposed by Traupel [9], as in Fig. 7, using the variables:

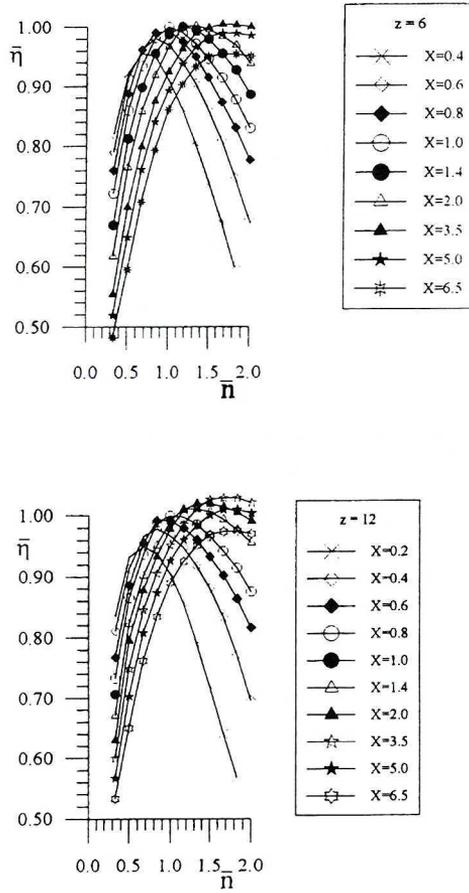


Fig. 7 Efficiency characteristics (the same as on Fig.6) in the form $\bar{\eta} = \bar{\eta}(\bar{\pi}, X)$

$$\bar{\eta} = \frac{\eta}{\eta_0} = \bar{\eta}(\bar{\pi}, X) \tag{9}$$

where $X = \frac{\Pi - 1}{\Pi_0 - 1}$; $\Pi = \frac{1}{\pi}$

can be named a rate of pressure ratio (pressurisation). Similar shape of all the curves obtained in this way, with diversified location of their peaks (maximums) and extension along the axis $\bar{\pi}$, can be observed.

That suggests the need of introducing new variables:

$$\bar{\eta} = \frac{\bar{\eta}}{\bar{\eta}_{\max}}; \quad \bar{n} = \frac{\bar{n}}{\bar{n}_{opt}} \quad (10)$$

connected with new relation (reduction) of the value of the relative efficiency $\bar{\eta}$ and reduced rotational speed \bar{n} , this time to the peak (maximum) co-ordinates $\bar{\eta}_{\max}$ and \bar{n}_{opt} , separately for each curve $X = const$ of the characteristics of the particular groups (Fig. 7). The values of $\bar{\eta}_{\max}$ and \bar{n}_{opt} are functions of X . In the so defined co-ordinate system, it is practically possible to obtain fitting of the particular curves $X = const$ of the group characteristics together, as in Fig. 8. This propriety was confirmed also in the case of other stage groups, including impulse type ones [1], [2].

Indicated regularity of efficiency characteristics of turbine stage groups seems to be the major result of this paper.

Similarly as in the case of impulse type stage groups [1], [2], the following approximation for efficiency characteristics (Fig. 8) was proposed:

$$\bar{\eta} = 1 - \left(1 - \bar{n}\right)^{a_1} \quad \text{for} \quad \bar{n} < 1 \quad (\text{left branch}) \quad (11)$$

and

$$\bar{\eta} = 1 - a_3 \left(\bar{n} - 1\right)^{a_2} \quad \text{for} \quad \bar{n} > 1 \quad (\text{right branch}) \quad (12)$$

Values of coefficients a_1, a_2, a_3 were found using least square method, and appropriate curves are also shown in Fig. 8. The statistical tests of the proposed approximations proved very high values of the curvilinear correlation coefficient, on the level of 0.99, and values of the test function. F considerably exceeding the value of F_{kr} on the level of confidence equal to 0.05.

Values of $\bar{\eta}_{\max}$ and \bar{n}_{opt} (Fig. 9) mainly depend on the rate of pressure ratio X but influences of the number of stages in the group and of the type of stage also are significant. The scope of obtained data till now is too small for generalizations.

Contrary to the case of impulse type turbine stage groups [1], [2], the coefficients a_1, a_2, a_3 depend here both on the number of stages in the group and on the type of the stage. This can be observed in Fig. 10. In the investigated scope both these influences are of the similar range. It can be also noticed that the value of a_2 coefficient 2.5÷2.7, proposed by Kreuter [8] is too small for the case analysed (average value of 3.5 up to 4.0 for individual groups).

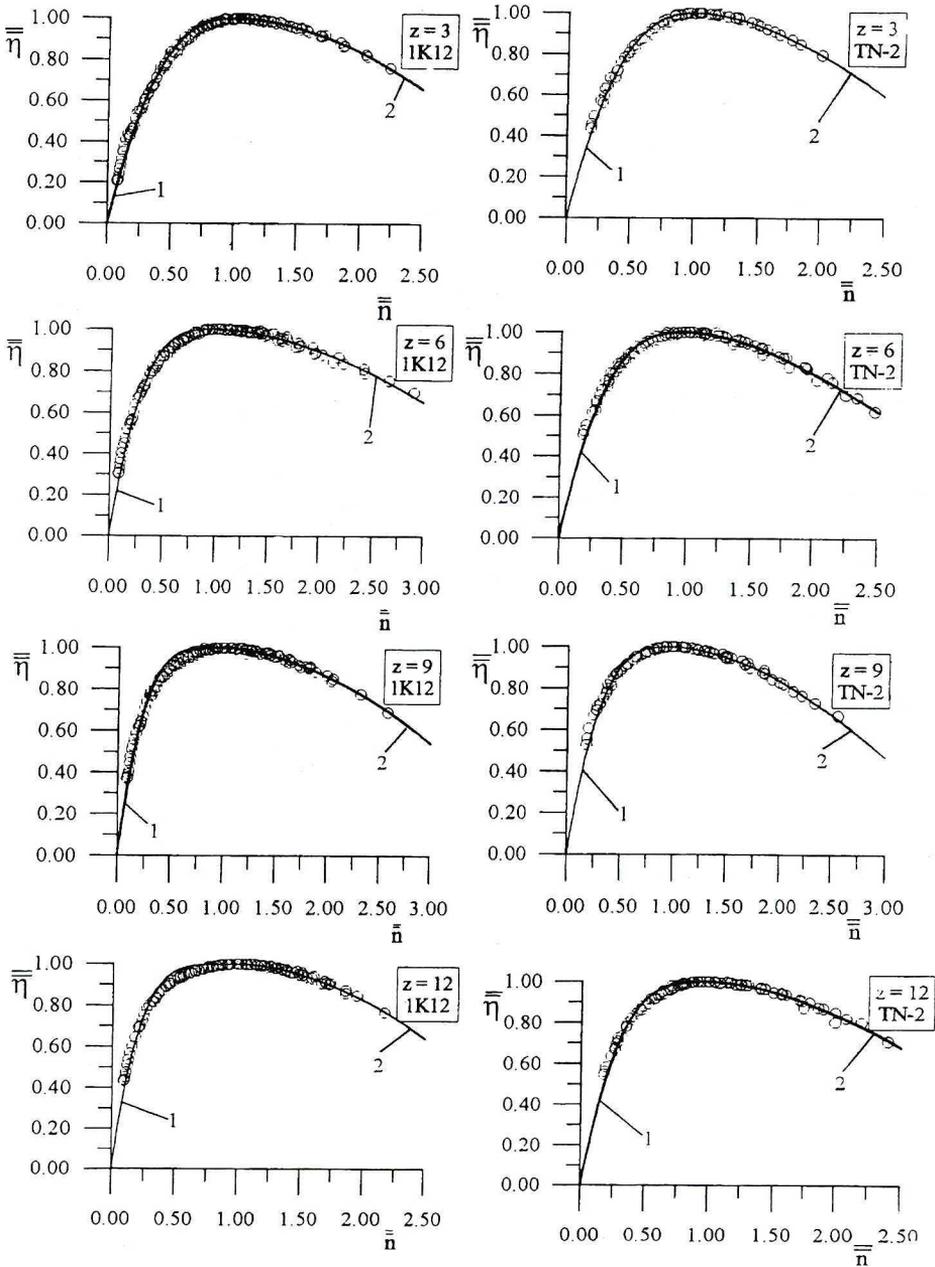


Fig. 8 Efficiency characteristics of the tested stage groups in the form $\eta = \eta(n)$;
 1- from eq.(11); 2 - from eq. (12)

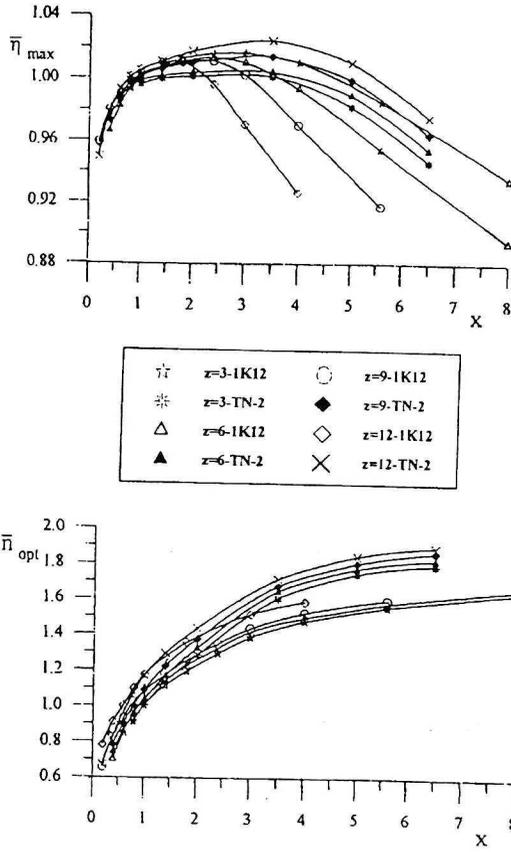


Fig. 9 Maximum efficiencies and optimum relative rotational speed as functions of the pressure ratio indicator X and the number of stages in the group for the 1K12 and TN-2 groups

Obtained values of coefficients a_1, a_2, a_3 are presented in Table 3 and Table 4.

Table 3-1K12

z	a_1	a_2	a_3
3	2.7	1.8	0.165
6	3.5	1.7	0.147
9	4.3	1.7	0.140
12	5.0	1.8	0.173

Table 4 -TN-2

z	a_1	a_2	a_3
3	2.6	1.7	0.202
6	3.0	1.6	0.179
9	3.4	1.6	0.163
12	3.5	1.7	0.159

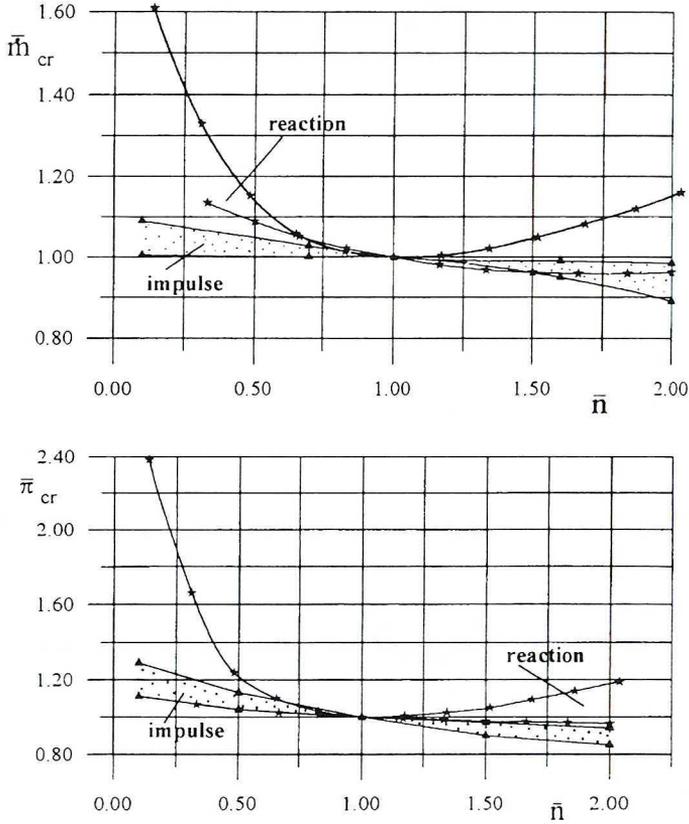


Fig.10 Comparison of courses of the relative critical mass flow rates and the relative critical pressure ratio for impulse and reaction type groups tested

7. Comparison of impulse and reaction type groups

The obtained results make it possible to perform some comparisons of performance characteristics of impulse and reaction type groups of stages.

In the scope of group flow capacity, the equation (8) can be treated, as it seems, as a more general form of equation (3) for both-impulse and reaction type stage groups. However, coefficients $A(\bar{n})$ and $B(\bar{n})$ are different here. For illustration, in Fig. 10 the relative critical mass flow rate and the relative critical pressure ratio as a function of the rotational speed for all investigated cases are set together. It can be seen that influence of changes in rotational speed on group flow capacity of reaction type is much greater then in the case of impulse ones. The considerable influence of number and specific features of stages in reaction type groups is also apparent here.

In the case of group efficiency, the possibility to present characteristics (2) in form (11) and (12) for both impulse and reaction type investigated stage groups was indicated. As it seems, this observed regularity of efficiency characteristics

can have more general sense. When seen in greater detail, groups of impulse and reaction type have different properties here. It can be observed that in the case of impulse type stage groups investigated coefficients a_1, a_2, a_3 in relations (11) and (12) are practically independent of the stage number in group. However for reaction type stage group especially coefficient a_1 strongly depends on stage number.

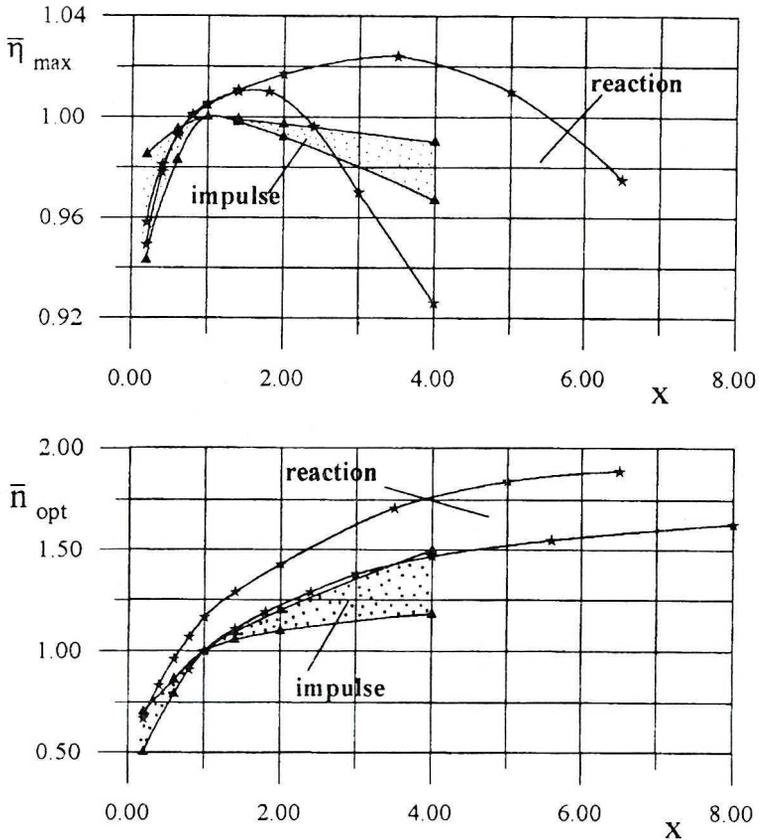


Fig.11 Comparison of courses of the maximum efficiencies and the optimum relative rotational speed for impulse and reaction type groups tested

Comparison of courses in Fig. 11 shows much greater and difficult to predict influence of number and specific features of stages on the value of $\bar{\eta}_{max}$ for reaction type turbine stage groups. It can be noticed here that efficiency of reaction type group at some off - design conditions can be greater than its nominal value.

8. Conclusions

The application of mathematical modelling and computer simulation of turbine stage group operation in off-design conditions makes it possible to obtain results, which can be used to reveal new, more general proprieties of groups of both impulse [1], [2] and reaction type [3], [11].

The proposals of corrections (7), obtained in this way, allow us, as it seems, to determine the flow capacity equation for turbine stage groups of both types in the form (8), taking the influence of the changes in the rotational speed into account. This proposal seems to be better than those published so far.

The influence of the changes in the rotational speed on capacity of reaction type group is considerable, significantly greater than in the case of impulse type groups.

It is much more difficult to find similar regularities for the stage group efficiency. Nevertheless, by indicated twice done change of variables, in all investigated cases, high regularity of its efficiency characteristics was obtained. It seems to prove the existence of regularity in this range, e.g. the possibility to reduce efficiency characteristics of turbine stage groups to the form $\bar{\eta} = \eta(\bar{n})$.

The efficiency characteristics of turbine stage group are more dependant on the particular features of the group than it is its flow capacity. Because of the lack of possibilities to perform general analysis here, further research is necessary for particular cases of groups using indicated in the paper approach.

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REFERENCES

- [1] Miller A., Lewandowski J., Kamal A.B.: A Contribution to the Flügel's - Stodola's Law for Flow Capacity of Turbine Stages Group. VDI BERICHTE Nr.1185, 1995, pp. 391+405.
- [2] Kamal A.B., Miller A., Lewandowski J.: Performance Characteristics of Impulse Type Turbine Stage Groups. The Archive of Mechanical Engiennring, 3/4, 1995 pp.211+231.
- [3] Trzeńska Z.: Praca grupy reakcyjnych stopni turbinowych w warunkach odmiennych od obliczeniowych. Ph.D. Thesis, Warsaw University of Technology, Warsaw, 1996.
- [4] Miller A.: Modelowanie pracy turbin parowych i sprężarek wirnikowych dla celów sterowania procesami technologicznymi. DSc.Thesis, Warsaw University of Technology, Warsaw 1975 (in Polish).
- [5] Flügel: Die Dampfturbinen. J.A. Barth, Leipzig 1931.
- [6] Ainley D.G., Mathieson G.C.R.: A Method of Performance Estimation for Axial-Flow Turbines. British Aeronautical Research Council R & M 2974, 1951.
- [7] Craig H.R.M., Cox H.J.A.: Performance Estimation of Axial-Flow Turbines. Proc. Inst. Mech. Eng. 1970-71, vol. 185, 32/71.

- [8] Cordes G.: - Stömungstechnik der Gasbeuschlagten Axial – Turbine. Springer Verlag, Berlin, 1963, pp. 266÷274.
- [9] Traupel W.: - Thermische Turbomaschinen. Springer Verlag, Berlin, 1982, pp.12÷24.
- [10] Szczegłajew A.W. - Parowyje Turbiny. Energia, Moskwa,1967, pp. 214÷217.
- [11] Trzcińska Z., Miller A., Lewandowski J.: Performance characteristics of Reaction Type Turbine Stage Groups. Proceedings of the 2nd European Conference on Turbomachinery - Fluid Dynamics and Thermodynamics, Technological Institute Antwerpen, 1997.
- [12] Atlas stopni turbinowych. Instytut Techniki Ciepłej, Łódź, 1988.

Uogólnione charakterystyki grup stopni turbinowych

Próba uzupełnienia prawa Flügla – Stodoli

Streszczenie

Znajomość charakterystyk grup stopni turbinowych jest stale jeszcze niewystarczająca, zwłaszcza w ogólnym przypadku zmian warunków pracy. Stan ten jest spowodowany szczupłością dostępnych danych doświadczalnych. W tej sytuacji atrakcyjne staje się wykorzystanie możliwości uzyskania potrzebnych danych przez modelowanie matematyczne i symulację cyfrową pracy grupy stopni w zmienionych warunkach, zamiast z eksperymentu fizycznego.

Zastosowanie tej koncepcji w przypadku grupy stopni turbinowych budowy komorowej („akcyjnych”) przedstawiono w pracach [1], [2]. Tutaj dyskutowane są podobne wyniki, otrzymane dla grup reakcyjnych stopni turbinowych (budowy bębnowej) tj.:

- model matematyczny dla symulacji pracy grup reakcyjnych stopni turbinowych w zmienionych warunkach (oparty na metodzie Ainley’a i Mathieson’a z pewnymi ulepszeniami);
- wyniki symulacji dla szeregu grup stopni zaprojektowanych wg. koncepcji byłej wytwórni BBC oraz „tradycyjnych” zasad;
- ogólniejsze właściwości tych grup (odnośnie przelotności i sprawności) otrzymane w wyniku analizy danych z symulacji;
- porównanie zaobserwowanych właściwości grup stopni typu „akcyjnego” i „reakcyjnego”.