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Research paper

Influence of the method of numerical modelling of the connections of the roof truss and vaults with the walls of historic masonry structures on local stress distribution

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Abstract: The work concerns the influence of the method of numerical modelling of the connections of the roof truss and vaults with the walls of historic masonry objects structures on the local stress distribution in the walls. At the outset, the need to search for rational modelling was justified due to the large size of the calculation models and the erroneous results obtained with oversimplification of the model. Four methods of modelling the connections between the walls and roof truss and vaults were analysed. The first method was to describe the elements of walls and foundations as solid elements, the ribs of the vaults and the roof truss as beam elements, and the vaulting webs as shell elements. The remaining methods 2–4 describe the walls as shell elements. In places where the walls join with the roof truss and vaults, fictitious/fictional elements in the form of rigid horizontally-oriented shells were used in model No. 2. In model No. 3, fictitious rigid horizontally-oriented shell elements in addition to local rigid vertically-oriented shells were used, while in model No. 4, only fictitious rigid vertically-oriented shell elements with stepwise decreasing protrusions were introduced. The best solution in terms of local stress distribution turned out to be the description of connections with fictitious shell elements in the case of model No. 4. This approach slightly increases the number of unknowns, and makes the results of stresses in the connection areas realistic in relation to full modelling with solid finite elements.

Keywords: connections, displacement, normal stresses, numerical modelling, roof truss, vault

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1. Introduction

Historic masonry structures are particularly sensitive to horizontal loads [1, 2]. Their increased susceptibility to this type of load results mainly from the weakening of the external walls with large openings [3, 4], significant distances between the bracing walls or pilasters and the lack of reinforced concrete ring beams. Ensuring the spatial rigidity of the structure requires that all loads are transferred to the stiffening elements in a uniform manner –which not only contributes to a more homogeneous stress distribution, but also significantly reduces the local values of extreme displacements. In residential buildings and in the predominant part of currently erected public buildings, a way of evenly transferring loads is provided by flat ceilings treated as rigid horizontal shields [1]. In historic masonry structures, ceilings of this type are usually absent, and their counterparts are coverings – vaults and roof truss structures [5, 6]. Unfortunately, compared to rigid ceilings, they have a much lower stiffness in the horizontal plane, which makes their impact on the increase in the vulnerability of the entire structure even greater. The spatial stiffness of historic masonry structures is also influenced by the type of the covering structure, as well as the geometry and type of joints used [7].

Internal forces in historic masonry structures can be determined using analytical methods based on static one or two-dimensional models.

In the case of methods based on two-dimensional models, the following methods are used: the Equivalent Frame Method [8, 9], the Wide-column Frame Method [10], Strip Methods [11], Limit Analysis Method [12, 13], the Line Thrust Method [13, 14] and the Uniqueness Theorem Method [12]. The last two methods are modifications of the Limit Analysis Method.

To determine internal forces in masonry structures, photo elastic methods based on two-dimensional models [15, 16] and numerical methods can also be used.

Nowadays, numerical methods are commonly used in modelling masonry structures – mainly the Finite Element Method (FEM) [17, 18], In the discretization process, it is possible to divide the separated areas of the modelled structure into different types of finite elements (1D – beam elements, 2D – plate, shield or shell elements, 3D – solid elements) with different shape functions and the required nodal mesh density. The Finite Element Method is especially useful in numerical analysis of structures characterized by high complexity, both in terms of geometric (curved elements such as arches, vaults, domes, etc.) and construction. For this reason, FEM is also eagerly used in modelling and numerical analysis of historic masonry structures [13, 19–21]. The key issue in the case of this type of numerical analysis, however, is the reliability of the obtained results and the large size of the computational models expressed by a very large number of unknowns. Hence, beam and shell finite elements (or shield elements [22]) are used instead of solid elements in the description. However, improper modelling of connections between elements, too large simplifications of the numerical model, or significant stiffening of the structure at the stage of its modelling, may lead to significantly different results and misinterpretation of the current state of stress in the structure, and thus the formulation of incorrect conclusions regarding recommendations related to possible repair and/or actions to strengthen the structure [21].

The work will review the ways in which the connections with the walls of the roof truss and vaults of historic masonry objects are modelled numerically (using software based on the Finite Element Method algorithm). Proprietary solutions for modelling connections of roofs with masonry walls will be proposed (taking into account the eccentric way of load transfer) and comparative numerical analyses will be carried out to determine the degree of influence of the adopted modelling method on the static work of the structure and reduction of local disturbances in the distribution of internal forces when describing masonry walls with finite shell elements.

2. Overview of the methods of numerical modelling in FEM of a roof truss and vaults in static analyses of historic masonry structures

2.1. Modelling of a roof truss

The method of modelling a roof truss in historic masonry structures depends on the size of the object to be analysed and the type of truss.

In the case when the conducted static analysis covers the complete structure of the roof truss, spatial (3D) schemes are often used, in which all load-bearing elements have real mutual stiffness relations in the nodes. Shaping the computational model in 3D space increases the level of accuracy of the obtained results, but at the same time increases the size of the computational task. Usually, insufficient computing power of computer equipment means that even at the stage of modelling spatial schemes of structures (especially large-size ones), it is necessary to significantly reduce the number of unknowns in the adopted numerical model [2, 23].

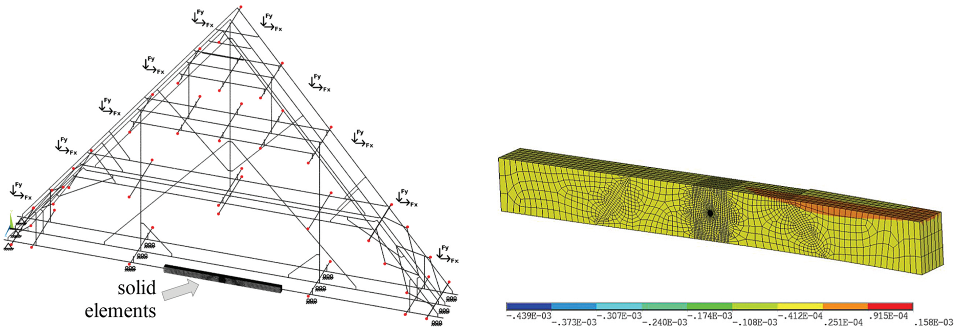
Virtually every roof truss of a historic masonry building contains repetitive sections of the structure. Numerical analyses can then be carried out on flat (2D) schemes [24].

In static analyses of roof trusses, beam elements are most often used, which allow for a significant reduction of the numerical model of the structure. Shell and solid elements in spatial (3D) analyses of roof trusses are usually not used.

The exception are numerical analyses aimed at determining the distribution of internal forces in connections between wooden elements and the support of the roof truss structure on masonry walls [25–27]. Analyses of this type can be carried out in two ways.

The first (and most frequently used method) assumes that the internal forces obtained as a result of the 3D static analysis of the complete structure (hereinafter referred to as the global spatial model) are used for the analysis of connections on separate, locally considered, detailed models [25–27]. The global spatial model is in this case described by beam elements. On the other hand, in detailed models, shell and solid elements are used, which more accurately model the connection geometry.

The second solution (described, inter alia, in the work of Tippner [24]), consists in shaping the calculation model in such a way that it immediately includes the appropriate density of the mesh nodes in places where the connection geometry should be described in detail. An example of this type of modelling is shown in Fig. 1.



(a) (b)

Fig. 1. An example of modelling a connection using beam and solid elements [24]:
a) location of solid elements in the structure model, b) discretization and deformation maps in solid elements

Modelling the roof truss with beam elements is also connected with the necessity to undertake additional activities. We often deal with the lack of cooperation (continuity) between these elements in the discrete model, as the axially guided beam finite elements will intersect at different heights. In such a situation, fictitious rigid connecting elements are introduced into the numerical model.

The issues of cooperation between beam elements are presented in more detail in [28, 29], in which a solution was proposed for the connection, inclined at a certain angle, of the load-bearing element of the roof truss with a horizontal or vertical element. The presented solution assumes the introduction of a fictitious rigid element as an extension of the load-bearing element of the inclined element or the introduction of a fictitious element oriented perpendicular to a horizontal or vertical beam (Fig. 2a).

In both cases, additional bending moments appear in the connection (Fig. 2b). Their influence on the results depends on the size of the cross-sections and the values of the combined internal forces.

Difficulties in modelling the roof truss may also be caused by the appropriate shaping of the support of the roof truss on masonry walls. In the case when the masonry walls in the three-dimensional model are described with solid elements, modelling the connection of the roof truss is not a major problem due to the possibility of supporting the load-bearing elements at any point in the cross-section of the masonry wall, but it is associated with a significant increase in the number of unknowns. For this reason, wall modelling often uses shell elements that enable a significant reduction of the size of the computational task [19, 30]. Unfortunately, in such a case, shaping the support of the roof truss, which does not always run along the axis of massive walls of large thickness, requires eccentric support of the roof truss elements on the shell elements located in the wall axis. In such a situation, additional rigid shell elements oriented vertically as well as horizontally and

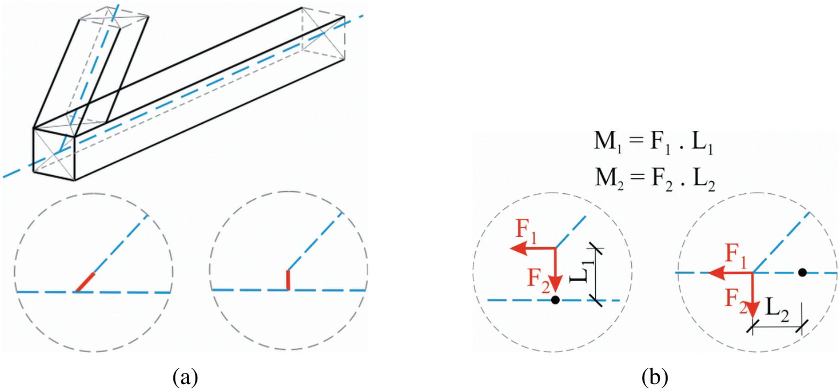


Fig. 2. Modelling of the connection of inclined and horizontal elements of the roof truss [28]: a) inclined and vertical fictitious rigid elements, b) diagram of determining additional bending moments (where: L_1, L_2 – values of eccentricities)

perpendicularly to the wall are introduced to the model, which ensure the continuity of the discrete model [31]. Such a solution causes significant local disturbances in the distribution of internal forces in the shell wall and may contribute to the misinterpretation of the obtained results.

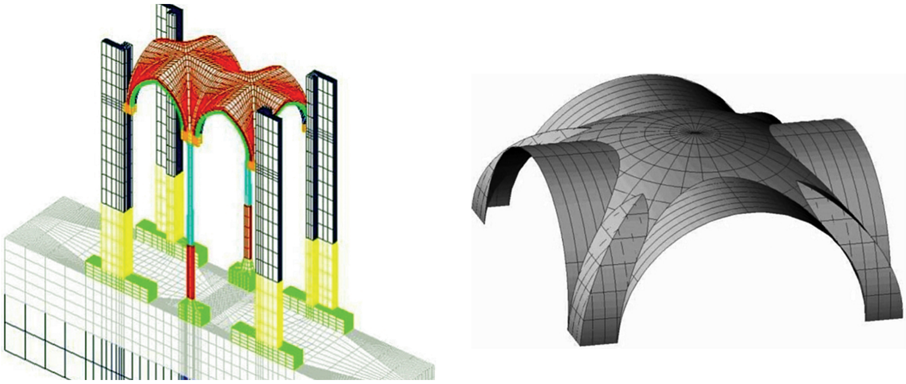
2.2. Modelling of vaults

The size of the computational task results from the scope of the conducted analyses and the level of spatial static work of the structure taken into account in the calculations. Brick vaults of historic masonry structures in static analyses can be modelled with the use of schemes separated from the structure as single spans, separate fragments of the structure covering several spans or complete three-dimensional vault structures [32, 33]. There are also solutions in which the vaults are modelled with masonry walls [34]. Exemplary models are shown in Fig. 3.

The structure of the vaults, as in the case of the roof truss, can be described with finite elements differing in terms of dimensions. Shell elements are most often used for this purpose. Modelling vaults with solid elements significantly increases the size of the computational task. In this case, particular attention should be paid to the appropriate modelling of the ribs.

In the case of using solid elements, which, with an appropriately dense discretization mesh, allow for a detailed description of their geometry, allowing for both the thickness of the ribs and the appropriate shape of the area of their support on the wall [35].

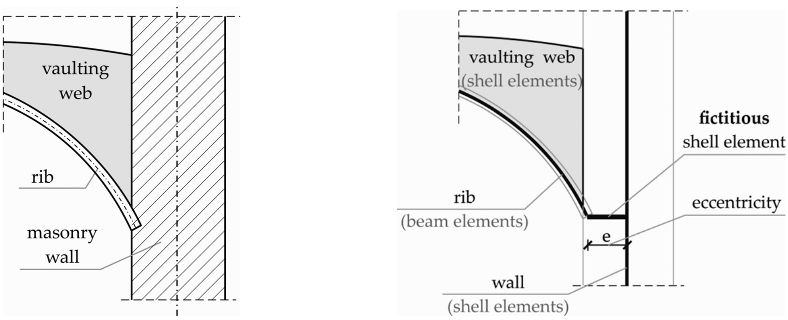
In the case of modelling with shell elements not only of vaults, but also of masonry walls, additional difficulties arise related to the description of the connection of the vaulting webs and main ribs with the walls. These difficulties are related to the real geometry of the structure and large wall thicknesses, which make it impossible to transfer the loads directly



(a) (b)

Fig. 3. Examples of modelling of vaults of historic masonry objects: a) a model with the cooperation of vaults with walls [34], b) model of the vault separated from the structure and considered independently [33]

in the discrete model. In such a case, it is necessary to take into account the eccentric way of transferring loads from the shell elements, with which the vaults are modelled – to the shell elements, with which the massive masonry walls are modelled. The eccentric loading is then usually realized by means of fictitious elements (beam or shell elements) [31] – as shown in Fig. 4.



(a) (b)

Fig. 4. Modelling of the connection of vaults with walls: a) a sketch illustrating how the loads are transferred from the vaults to the walls; b) sketch of the model of connection of eccentrically oriented shell elements [31]

According to the literature review, due to the recent development of computer techniques and methods, there has been a tendency to model structures (including historic objects) in natural spatial (three-dimensional) static schemes. Various types of simplifications are commonly used, such as the description of walls and vaults with beam or shell elements.

Then, artificial shifts of axes or middle planes of elements are introduced in the models. In turn, by introducing fictitious rigid elements (so-called "interfaces"), we generate additional eccentricities in the walls. The eccentricities cause additional moments that disturb the distribution of internal forces in this region and affect the statics of the entire walls.

3. Numerical analyses

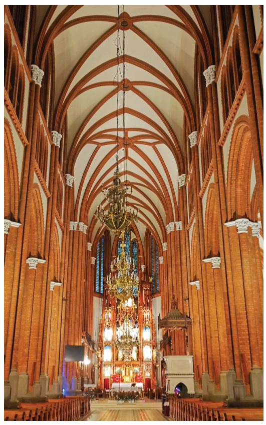
3.1. Numerical simulation software

Three-dimensional modelling of structures and numerical analyses in the field of linear statics were carried out using ANSYS 2021R1 computer simulation software based on the Finite Element Method (FEM).

3.2. Variant modelling of the structure

It was assumed that the subject of numerical analyses would be the construction of a historic building of a neo-Gothic church erected in Bialystok (Poland) in the years 1900–1905 (Fig. 5).

(a)

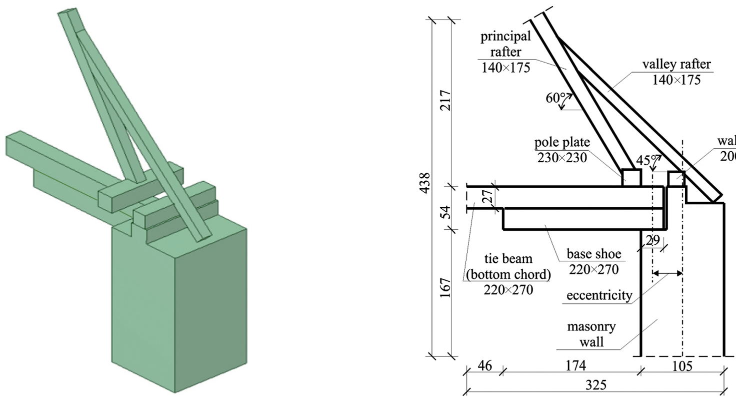


(b)

Fig. 5. Bialystok Cathedral: a) view of the Bialystok Cathedral with an indication of the location of the modelled connection, b) the interior of the cathedral with visible vaults and pillars of the nave

In the case of the analysed structure, we are dealing with the queen-post truss over the main church nave. A schematic of the geometry of the support of individual roof truss elements on the masonry walls is shown in Fig. 6.

In order to limit the size of the task, numerical analyses were carried out on strips separated from the nave part of the Cathedral, maintaining the spatiality of the model in the transverse direction.



(a) (b)

Fig. 6. Supporting a roof truss on masonry walls: a) three-dimensional geometric model of a separated structure strip, b) 2D sketch with marked structural elements of the connection

Four construction models were developed. In the reference model (No. 1; Fig. 7a), masonry walls and foundations were modelled with twenty-node solid finite elements with three translational degrees of freedom at each node. The structure of the roof truss and the main ribs of the vaults were modelled using three-node beam elements with six degrees of freedom in the node (three translational and three rotational). The vaults are described with eight-node shell elements – also with six degrees of freedom in the node. All finite elements used in model No. 1 had square shape functions.

In the remaining models (No. 2–4, Fig. 7b–7d), both masonry walls and vaults were described with eight-node shell finite elements. The planes of the walls modelled with the use of shell elements were located in the middle planes of the masonry walls – which is associated with the eccentric transfer of loads from the roof truss to the walls and the risk of local disturbances in the distribution of internal forces of the structure modelled in this way.

In models No. 2–4, in connection with the modelling of the walls with the use of shell elements located in their middle planes, a problem arises related to the eccentric transfer of loads from the roof truss to the walls. This problem was analysed in three variants.

In the first (model No. 2, Fig. 7b) the shell elements of the walls were connected with the beam elements of the roof trusses by means of fictitious shells oriented horizontally, perpendicular to the plane of the walls. The fictitious elements were modelled as rigid and weightless (with dimensions in the plan: projection 50 cm, width 25 cm), rigidly connected to the walls and jointed to the roof truss.

In model No. 3 the eccentric load transfer from the roof trusses to the walls was realized with the use of fictitious shell elements, oriented both horizontally (with a projection of 50 cm and a width of 25 cm) and vertically (with a projection of 50 cm and a height of 50 cm). However, in the model No. 3 vertical, rigid and weightless fictitious shells were

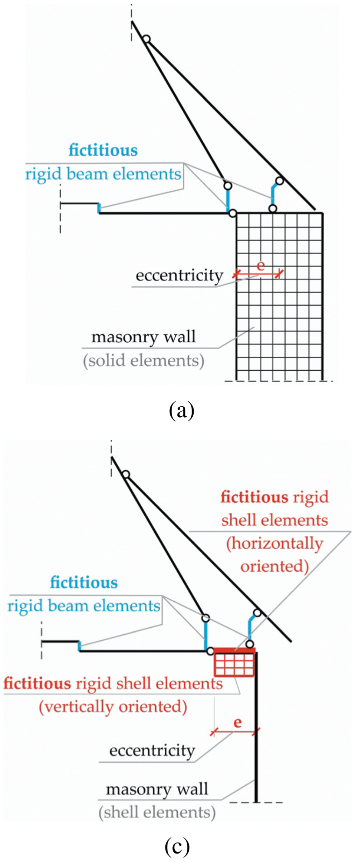


Fig. 7. Static schemes of the connection of the nave's roof truss with the walls and the rigid fictitious elements used in the models: a) reference solid model No. 1, b) model No. 2, c) model No. 3, d) model No. 4

applied perpendicular to the wall modelled with the shell elements. Whereas, in model No. 4 (Fig. 7d), three vertical rigid fictitious shells (each 16.7 cm high) were applied one above the other with stepwise decreasing protrusions (50.0 cm, 33.3 cm and 16.7 cm).

In the case of the No. 2–4 models, rigid and weightless fictitious elements had to be introduced in the places where the vaults (modelled with shell elements) and the ribs of the vaults (modelled with beam elements) connect with the wall shells located in the planes of the middle walls. They are schematically shown in Fig. 8b–8d.

In model No. 2, the possibility of transferring loads from beam elements (with which the main ribs of the vaults were modelled) and shell elements (with which the vaulting webs were modelled) to masonry walls modelled with shells elements in their middle planes, was ensured by introducing fictitious, rigid and weightless shell elements oriented horizontally (Fig. 8b).

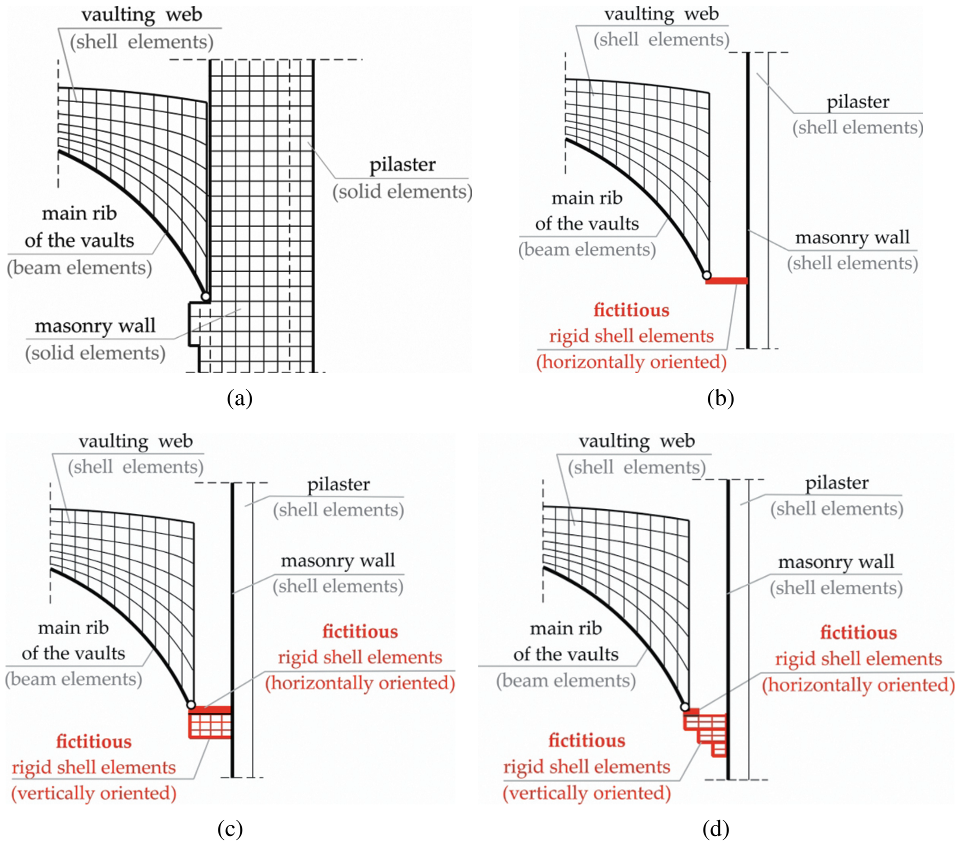


Fig. 8. Static schemes of the connection of the main ribs and vaults with the walls and the rigid fictitious elements used in the models: a) reference model No. 1, b) model No. 2, c) model No. 3, d) model No. 4

In model No. 3, there were introduced rigid and weightless horizontally oriented fictitious shell elements (similarly to the No. 2 model, jointed with ribs and vaulting webs and rigidly with the walls) and additional rigid fictitious shell elements oriented vertically.

In the case of model No. 4 (Fig. 8d) also provided horizontally oriented rigid fictitious shells (to allow the ribs and vaulting webs to articulate against the walls) but not connected to the walls. In model No. 4, similar to No. 3, vertically oriented rigid and weightless shells were introduced – analogous to modelling the connections of the roof truss.

In all models (No. 1–4), the material parameters of beam finite elements were adopted as for softwood of class C24 in accordance with BS EN 338 [36] mean modulus of elasticity parallel bending $E_{m,0,\text{mean}}$ = 11 GPa, mean characteristic value of modulus of elasticity in bending perpendicular to grain $E_{m,90,\text{mean}}$ = 0.37 GPa, mean shear modulus G_{mean} = 0.69 GPa).

In the case of masonry elements (shell and solid elements), the mechanical parameters were assumed as for the masonry made of solid ceramic brick on a lime mortar – according to the completed expert works (Young's modulus $E = 1.32$ GPa, Poisson's ratio $\nu = 0.2$, compressive strength $f_c = 2.2$ MPa).

Numerical analyses were carried out in the field of linear statics, assuming the work of the structure in the linear-elastic range. The influence of the subsoil (hard-plastic clay) was approximated by elastic support of the structure in the places of contact of the foundations with the ground. Each model takes into account the load of the self-weight of the structure and the weight of the roof covering.

Detailed information on the size of each of the computational models considered in the work is summarized in Table 1.

Table 1. The number of nodes and finite elements in models No. 1–4 (rigid fictitious shells)

Model No.	Nodes	Elements	Fictitious rigid shells
1	678 757	171 476	–
2	265 312	73 527	64
3	266 980	74 022	84
4	268 727	74 341	190

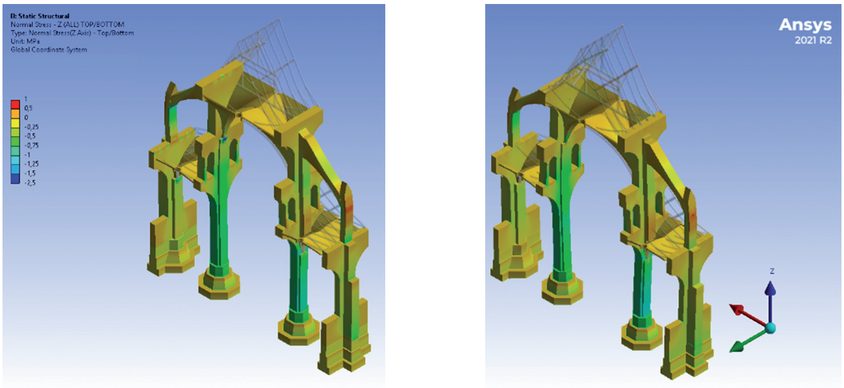
Table 1 shows that in the reference model with walls modelled using solid elements (No. 1), it was necessary to introduce more than two and a half times more nodes and slightly more than twice as many finite elements as compared to models No. 2–4, in which the walls were modelled shell elements.

3.3. Comparison and analysis of the results of the numerical simulations

As a result of the performed numerical analyses, detailed data on displacements, strains and stresses were obtained for each of the four models (No. 1–4). The selected results in the form of maps of normal stresses in the vertical direction (σ_z) and deformation of the structure are shown in Fig. 9.

Horizontal displacements (in the X axis direction) along the height of the walls and pillars of the nave at selected heights of the Cathedral (h_1 – support level of the side vaults, h_2 – the level of the lower support of the side truss, h_4 – support of the main vaults, h_5 – support of the roof truss on the walls) along the line running along the external face of the masonry wall, for each of the models analysed in the study are summarized in Table 2.

Table 2 shows that the separated part of the Cathedral adopted for numerical analysis deforms in the same way (identical deformation trend) – regardless of whether the walls were modelled with solid or shell elements. In each case, the largest horizontal displacements of the structure in the direction of the X axis is at a height slightly below the level of the bottom support of the side roof truss. The highest horizontal deformations are shown by



(a) (b)

Fig. 9. The stress-deformation behavior of the structure:a) reference model – No. 1 (walls modelled with solid elements), b) model No. 4

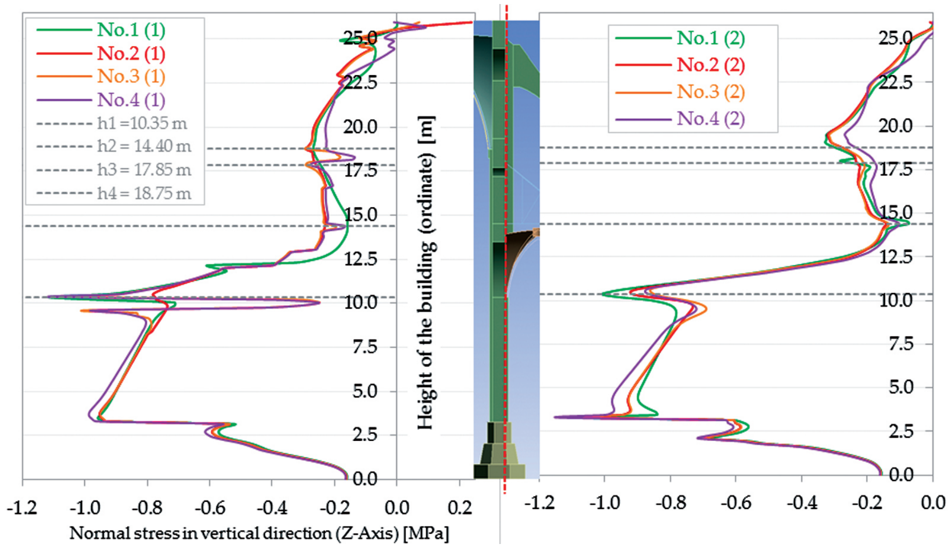
Table 2. List of horizontal displacements d_i at selected heights from the upper surface of the foundations (mm)

Model No.	h_1 = 10.35 m	h_2 = 14.40 m	h_4 = 18.75 m	h_5 = 25.95 m	d_{min}	d_{max}
1	–1.39	–2.28	–0.04	0.23	–2.34	0.57
2	–1.83	–2.19	–0.06	0.05	–2.34	0.69
3	–1.63	–2.01	–0.05	0.02	–2.14	0.68
4	–1.45	–1.86	–0.17	–0.01	–1.95	0.38

the reference model (No. 1) with walls modelled with solid elements ($d_{min} = -2.34$ mm). In the case of model No. 2, the extreme horizontal displacements are only 0.1% smaller than the displacements of the reference model. However, in the case of models No. 3 and No. 4, there are significantly larger differences in structure displacements compared to the reference model (respectively 8.7% and 16.9% lower than the maximum deformations of model No. 1).

There is much greater differentiation of results in the case of analysis of normal stresses in the vertical direction (σ_z). The resulting plots of normal stresses along the height of the walls and pillars of the nave in the vertical direction (in the direction of the Z axis) are shown in Fig. 10 (along the lines running along the external face of the masonry wall – indicated in the figure by a broken red line: a) in the axis of the cross-section through the pilaster and b) on the edge of the pillar at the bottom of the structure).

When analysing individual data series presented in Fig. 10, it should be stated that the highest values of normal stresses in the vertical direction (in the direction of the Z axis) and the most “homogeneous” results are obtained using the reference model (No. 1) with more than two and a half times the number of nodes compared to models in which the



(a) (b)

Fig. 10. Normal stresses in the vertical direction along the height of the wall and pillar along the lines running along the external face of the wall structure: a) in the axis of the cross-section through the pilaster; b) on the edge of the pillar

walls were modelled with shell elements (No. 2–4). In the case of less demanding models No. 2–4, where shell elements were used for modelling the walls and additional rigid and weightless fictitious elements to transfer forces from the roof truss and vaults, generally higher values of compressive stresses were obtained in the lower part of the walls/pillars. For all simplified models, much more distorted graphs of normal stress were also obtained at the height of the wall connections with the side vaults and the vault above the main nave, and at the height of the wall connections with roof trusses. This, of course, is related to the additional bending moments resulting from the eccentric transfer of loads to the walls.

The distribution of normal stresses at the height of the connection of the walls with the upper roof truss and at the height of the connection of the walls with the main vault and the side roof truss (in the axis of the cross-section through the pilaster) are shown in Fig. 11 and Fig. 12.

Selected values of normal stresses determined at the connection of the nave pillars with the upper surface of the foundations, the connection of the walls with the vaults and the roof truss in the axis of the cross-section through the pilaster are summarized in Table 3.

By analysing the values of normal stresses in the vertical direction (σ_z), generated along the height of the walls along the axis of the pilaster cross-section (along the red dotted line in Fig. 10) and summarized in table 3 and shown in Fig. 11, it should be stated that at the height of h_5 = 22.75 m (support of the main roof truss on the masonry walls), we deal with the best convergence of the results was obtained from model No. 4 (compressive residual stress of –0.010 MPa) compared with the results obtained for the reference model

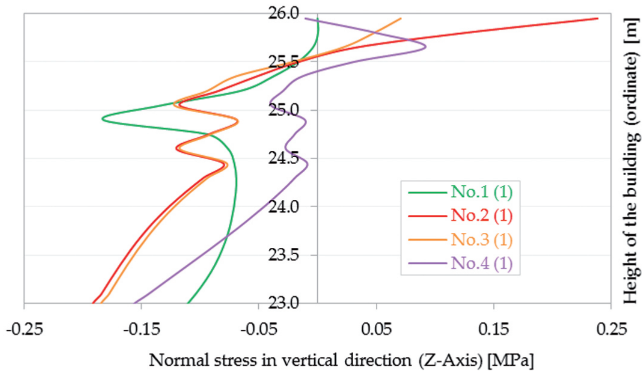


Fig. 11. Normal stresses in the wall in the vertical direction at the height of the connection with the roof truss (h_5 = 25.95 m) along the line running along the external face of the wall structure (in the axis of the cross-section through the pilaster)

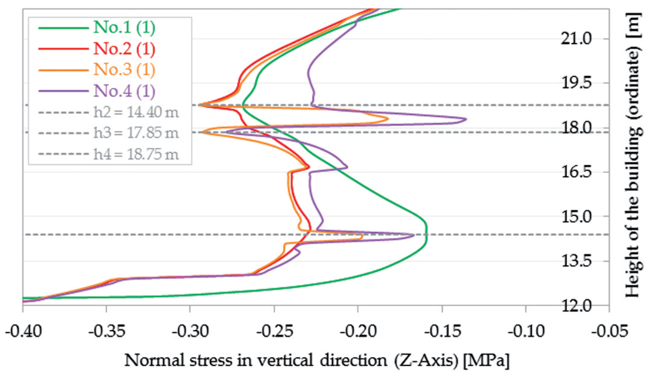


Fig. 12. Normal stresses in the wall in the vertical direction at the height of the connection with the main vault (h_4 = 18.75 m) and the upper support of the side truss (h_3 = 17.85 m) along the line running along the external face of the wall structure (in the axis of the cross-section through the pilaster)

No. 1 (residual compressive stress –5.5E–5 MPa). In the case of model No. 3, the tensile stress of 0.071 MPa appears in the same place, i.e. more than 7 times greater than in model No. 4. The greatest differences in relation to the reference model occurred in the case of model No. 2 (tensile stress at the level of 0.238 MPa). On the other hand, slightly below the support of the main roof truss on the walls (i.e. approximately 25.5 m horizontally), the best convergence of the results with the reference model is visible in the case of the model No. 3 (Fig. 11).

We deal with a slightly different tendency in the vertical stress distribution at the height of the support of the nave vaults on masonry walls (Fig. 12, level h_4 = 18.75 m). Compressive stresses at this point in model No. 2 (–0.293 MPa) are 8.9% larger than in the reference model. For model No. 3 the difference is 9.4% and 15.3% for model No. 4.

Table 3. List of normal stresses in the vertical direction (σ_z) at selected heights of walls/pillars (MPa) in the axis of the cross-section through the pilaster

Model No.	h_0 = 0 m	h_1 = 10.35 m	h_2 = 14.40 m	h_4 = 18.75 m	h_5 = 25.95 m	$\sigma_{z-\min}$	$\sigma_{z-\max}$
1	–0.162	–1.117	–0.159	–0.269	–5.5E–5	–1.117	3.5E–04
2	–0.164	–0.775	–0.231	–0.293	0.238	–0.952	0.238
3	–0.164	–1.014	–0.198	–0.294	0.071	–1.014	0.071
4	–0.166	–1.101	–0.168	–0.228	–0.010	–1.101	0.092

On the other hand, in the level of the lower support of the side truss (h_2 = 14.40 m), similarly to the height of the support of the main roof truss, the normal stresses in the vertical direction closest to the results obtained in the reference model were obtained for the model No. 4 (compressive stress only 5.2% greater than that of the No. 1 model). For models No. 3 and 2, the compressive stresses were respectively 24.2% and 44.8% higher than the reference results.

Also at the height of the support of the side vaults (h_1 = 10.35 m), the values of normal stresses closest to the reference results were obtained for the model No. 4 (difference 1.4%). Compressive stresses obtained from models No. 3 and No. 2 are lower than the reference model by 9.2% and 30.6%, respectively.

A comparative analysis of the extreme values of normal stresses in the direction of the Z axis ($\sigma_{z,\min}$) compiled along the path running along the face of the wall in the axis of the pilaster cross-section (Table 3 and Fig. 10a) also shows the best convergence of the model No. 4 with the reference model (difference at the level of 1.4%). In the case of other models, the obtained extreme normal stresses were by 9.2% and 14.8% lower than the values obtained for the reference model.

A similar, although not so much disturbed, distribution of normal stresses in the vertical direction occurs in the case of the values juxtaposed along the path running along the face of the wall near the outer edge of the pillar (Fig. 10b).

Analysing the above results, it should be stated that in the case of normal stresses in the vertical direction, the greatest convergence in relation to the reference model No. 1 was obtained in model No. 4 (differences do not exceed 15.3%), in which shell elements were used to describe the walls, and rigid weightless fictitious shell elements with stepwise decreasing protrusions were used to transfer the loads to the walls from the roof truss and vaults.

4. Summary and conclusions

Static calculations of historic masonry objects with the use of numerical methods are carried out with the use of computational models which vary in terms of the degree of the spatial static work of the structure and the type of finite elements used. Coverings –

the roof truss and vaults, which also ensure static cooperation between brick walls, play a significant role in ensuring the spatial rigidity of these specific structures. The way in which the structure and support of the roof truss and vaults on the masonry walls will be modelled, determines the credibility of the obtained calculation results and is the basis for the correct interpretation of the static work of the object and a proposal for possible strengthening of the structure.

The basic conclusion resulting from the analyses is that the currently used rigid fictitious elements (beam or shell) between the non-intersecting axes of structural elements cause large local disturbances in the distribution of internal forces in numerical simulations carried out with the use of the finite element method algorithm. The analyses show that a better solution is to describe these parts of the structure locally with solid or shell elements. Such an approach does not increase the number of unknowns much, and it makes the results of internal forces obtained from calculations in these areas of the structure realistic.

The article presents the results of numerical analyses related to the modelling of historic masonry structures with the use of spatial schemes that allow for the eccentricity of the action of loads and a partial reduction of local disturbances in the distribution of internal forces. Local disturbances in such systems result not only from the modelling method, but also from their complex geometry (buttresses, openings, pillars).

Moreover, it has been shown that the appropriate moulding of the connections in the three-dimensional beam-shell models allows to replace the computationally demanding models of this type (with a very high number of degrees of freedom) with much smaller computational models with an acceptable error of the obtained results.

The conducted numerical simulations, the fragmentary results of which are presented in Figs. 10–12, show that the best of the analysed solutions in terms of the normal stress distribution in the vertical direction (σ_z) is to use rigid, fictitious vertically oriented shell elements with stepwise decreasing protrusions (model No. 4) in the places of supporting the roof truss and vaults on the walls (modelled with shell elements). This method of modelling the structure reduces the size of computational tasks and is worth recommending to users of computational programs based on the FEM algorithm.

The largest dispersion and the weakest convergence of normal stresses in the vertical direction compared to the reference model were obtained in the case of model No. 2, with horizontally oriented rigid and weightless fictitious elements, which introduce locally concentrated moments and an unrealistic stress distribution – although the model is simpler.

On the other hand, a comparative analysis of the deformations of the reference model (No. 1) with the deformations of models in which the walls are described with shell elements (No. 2–4) indicates an increase in the stiffness of the structure, resulting in smaller deformations compared to the reference model. The deformations of the models described with shell elements are the smaller the greater the number of rigid fictitious elements used in the model. Therefore, if the designer who analyses this type of historic complex structures is more interested in obtaining reliable displacements than a reliable distribution of normal stresses, he should minimize the number of rigid and weightless fictitious elements.

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Wpływ sposobu numerycznego modelowania połączeń więźby dachowej i sklepień ze ścianami zabytkowych obiektów murowych na lokalny rozkład naprężeń

Słowa kluczowe: naprężenia normalne, numeryczne modelowanie, połączenia, sklepienia, więźba dachowa

Streszczenie:

Praca dotyczy wpływu sposobu numerycznego modelowania połączeń więźby dachowej i sklepień ze ścianami zabytkowych obiektów murowych na lokalny rozkład naprężeń w ścianach. Na wstępie uzasadniono potrzebę poszukiwań racjonalnego modelowania z uwagi na duże rozmiary modeli obliczeniowych lub błędnych wyników przy zbytnich uproszczeniach modelu. Przedstawiono aktualnie stosowane sposoby modelowania, w tym stosowanie sztywnych elementów skończonych między nieprzecinającymi się osiami konstrukcyjnymi elementów. Przeanalizowano cztery sposoby modelowania połączeń więźby dachowej i sklepień ze ścianami. Pierwszy sposób to opisanie elementów ścian i fundamentów elementami bryłowymi, żeber sklepień i więźby dachowej elementami belkowymi, a wysklepek sklepień elementami powłokowymi. Pozostałe sposoby 2–4 to opisanie ścian elementami powłokowymi. W miejscach połącznia ścian z więźbą dachową i ze sklepieniami w modelu No. 2 zastosowano fikcyjne elementy w postaci sztywnych powłok zorientowanych poziomo. W modelu No. 3 również wprowadzono fikcyjne sztywne powłoki poziome oraz lokalnie dodatkowo sztywne powłoki zorientowane pionowo, natomiast w modelu No. 4 wprowadzono tylko fikcyjne sztywne pionowe elementy powłokowe o skokowo zmiennym wysięgu. Najlepszym rozwiązaniem w zakresie rozkładu lokalnych naprężeń okazał się opis połączeń fikcyjnymi elementami powłokowymi w przypadku modelu No. 4. Takie podejście niewiele zwiększa liczbę niewiadomych, a urealnia wyniki naprężeń w rejonach połączeń w stosunku do pełnego modelowania bryłowymi elementami skończonymi.

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