

# Research of the process of biological wastewater treatment under conditions of uneven load of the treatment system

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**Abstract:** The main purpose of the article is to develop a multifactorial model for rapid assessment of the efficiency of biological wastewater treatment reactors. A mathematical model of the process of biological wastewater treatment has been developed based on: changes in the concentration of organic contaminants in the bioreactor over time, taking into account the uneven flow of wastewater to the treatment plant, the process of substrate entering the bioreactor (different amounts may enter at different times). The software implementation of the proposed algorithm for solving the corresponding model problem in Python is carried out. The results of computer experiments on the study of the efficiency of wastewater treatment in biological treatment reactors for different operating conditions of facilities are presented. In particular, such processes were considered with taking into account the unevenness of the load, because the maximum cleaning loads are in the morning and in the evening. The task was solved to simulate a real situation and show how cleaning takes place at the maximum load at a certain time of the day. The results obtained will be useful for calculations in the design of biological treatment facilities or in the reconstruction of existing bioreactors for their prospective operation under new operating conditions.

**Keywords:** activated sludge concentration, biological wastewater treatment reactors, mathematical model, non-uniformity conditions, organic contaminants

## INTRODUCTION

Rapid industrial progress leads to an increase in the scale of production, which negatively affects natural ecosystems. For example, due to incomplete treatment of industrial wastewater, which is an important component of production in the food, microbiological, pharmaceutical and many other industries, harmful impurities enter water ecosystems. The amount and nature of waste depend on the type of product and the company's product range. Regardless of the type of waste, all wastewater requires mandatory treatment because it contains pollutants that exceed permissible concentrations.

Depending on the concentration of pollutants in the wastewater, magnetic, mechanical, biological and other treatment systems are used to ensure compliance with regulatory

standards and reduce the harmful impact on the environment. One of the most commonly used methods is biological treatment, which uses aerobic treatment processes when additional oxygen is introduced, and anaerobic treatment without air access.

Wastewater biotreatment is an effective method of removing organic contaminants that are widely used around the world. When designing and redesigning bioreactors, it is important to quickly assess their efficiency under new operating conditions. A physical experiment for this purpose is impractical due to the long time required for the growth of biofilm and activated sludge. In addition, the experiment requires expensive equipment. Therefore, mathematical models are becoming an important tool for rapid analysis of bioreactor efficiency, taking into account modern research requirements.

Various models are used to analyse the efficiency of biological treatment plants: empirical, analytical, and numerical. However, most calculations are based on outdated empirical methods that were developed in the second half of the 20<sup>th</sup> century and do not take into account the current composition of wastewater. There is also a shortage of models that take into account the coefficient of uneven loading of the treatment system. In this regard, the development of modern methods of computer modelling of the wastewater treatment process is an extremely important and urgent task.

Biological treatment processes are based on the degradation and oxidation of organic substances by microorganisms and their ability to reproduce and die. At the same time, the activity of bacteria and microorganisms significantly depends on the temperature of the substrate and the concentration of oxygen dissolved in it. A significant factor affecting the efficiency of treatment is also the concentration of activated sludge, which is regulated by recycling and removal of excess activated sludge to maintain the vital activity of microorganisms. The main issue in the operation of aeration tanks is the development of a scheme for automatically maintaining within optimal limits the ratio between the amount of wastewater entering the aeration tanks, taking into account the coefficient of hourly unevenness, the amount of air introduced into the aeration tanks and the dose of activated sludge. Maintaining this ratio within optimal limits is the main condition for the highly efficient operation of aeration tanks to achieve the required treatment indicators (maximum permissible concentrations of treated wastewater).

To ensure acceptable levels of wastewater contamination and reduce the negative impact on the environment, various treatment systems are used, such as magnetic, mechanical, biological, and others, depending on the concentration of contaminants. Biological treatment is one of the most common methods (Ghangrekar and Shinde, 2007; Chan *et al.*, 2009; Yun *et al.*, 2019). This method includes aerobic purification processes, which involve the supply of air, and anaerobic purification, which is carried out without air. In this study, we focus on developing a mathematical model of the process of biological wastewater treatment considering changes in the concentration of organic contaminants in the bioreactor over time, changes in the concentration of activated sludge in the bioreactor over time, changes in the concentration of activated sludge in the reactor over time, and taking into account the uneven flow of wastewater to the treatment plant.

Recent scientific studies have examined the issue of biochemical wastewater treatment in aeration tanks in detail. These studies have brought a greater understanding of the dynamics of the processes, the impact of individual variable parameters, and the need to manage the output information. In addition, they helped to define the principles of designing schemes and facilities for the operation of treatment plants. For example, one of these studies (Li *et al.*, 2015) is devoted to the kinetics of sludge biodegradation under conditions of a microbial substrate that limits mesophilic anaerobic digestion. Applying appropriate kinetic models, such as Monod, Edward–Haldone, and others, mechanisms have been developed to design different types of bioreactors: batch, continuously stirred, and plunger for sludge treatment with minimal dilution. However there is a limitation in designing experiments for the Monod model that was discussed in Dette *et al.* (2005).

The mathematical model proposed in the study of Jiménez-García and Maya-Yescas (2019) takes into account the interaction of activated sludge and impurities. Using the basis of the “volume averaging method”, the authors of the model proposed and estimated the oxidation time in the liquid phase.

Developed models are widely used to monitor wastewater treatment processes. However, a study (Han *et al.*, 2023) showed that invalid or noisy data sets may not capture the dominant characteristics of treatment plants and lead to worse monitoring results. To solve this problem, a dynamic-static monitoring model of treatment plants was developed. The operating state of the treatment plants is separated by a decreasing condition distribution strategy, which can prevent the mutual influence of fluctuations between different operating conditions. The dynamic characteristics of treatment plants are extracted using a dynamic intelligent model, which is built using an interval type 2 fuzzy neural network to accurately simulate the dynamic relationship between process variables. Unreliable results caused by invalid data sets are compensated by a static statistical model that is designed to describe static properties using data sets and a similarity discrimination mechanism to perform the monitoring of wastewater treatment plants.

Soft sensing models based on artificial intelligence are widely used to monitor water quality in real time. However, the over-parameterisation of methods and the complexity of the wastewater treatment environment can lead to atypical characteristics, making it difficult to establish optimal parameters to ensure treatment accuracy. Therefore, Chang *et al.* (2023) in the article propose a soft measurement model for the intelligent adjustment of the broad study systems based on the analysis of independent components. Work by Kazemi *et al.* (2021) studies the measured parameters of the biological treatment process that provide information about the state of the system, and Xiao *et al.* (2017), Sánchez-Fernández *et al.* (2018), Wang *et al.* (2022) present new methods and devices for measuring process parameters, which is undoubtedly an important component for automating the corresponding process. Zabot *et al.* (2011) in the article performs a hybrid modeling of the xanthan gum bioproduction process making use of an artificial neural network (ANN) as a kinetic parameter estimator for the phenomenological model.

Recently, membrane separation processes gained significant attention to recover resources from waste streams, biofuels and especially wastewater to reduce potent greenhouse gases (Maaz *et al.*, 2019). Aerobic treatment of distillery and petrochemical wastewater in semifluidised bed biofilm reactor that employs liquid phase oxygen technology has been investigated by Narayanan (2012), Narayanan and Biswas (2016).

One of the control methods for biological wastewater treatment is activated sludge treatment (Brockmann *et al.*, 2021), but this process is quite complex and easily affected by various factors such as flow quality and aeration time, which can lead to unstable results. In particular in the work by Zarei *et al.* (2021), a lab-scale multi-sized packed-bed bioreactor (with non-overlapping particles in a structured arrangement) is simulated using computational fluid dynamics in order to study the bed's hydrodynamic behaviour.

There is also a set of studies that have focused on the effects of Chiral pharmaceuticals as emerging contaminants frequently detected in wastewater in sewage on biological nutrient removal

and its potential mechanism (Oehmen *et al.*, 2007; Zhang *et al.*, 2015; Wang *et al.*, 2023).

However many papers have been published on simulation of biological wastewater treatment, there is only a little information about the simulation of mass transfer and anaerobic reactions taking into account the uneven flow of wastewater to the treatment plant which is closer to real conditions.

## MATERIALS AND METHODS

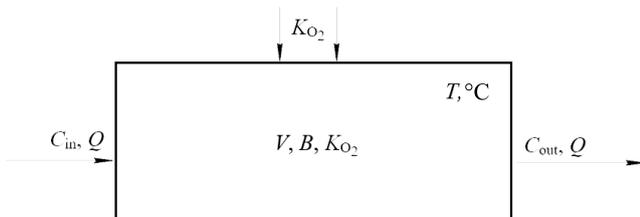
The aim of the study is to create a mathematical model that will allow us to evaluate the efficiency of the biological wastewater treatment process under different operating conditions. For this purpose, the following tasks were formulated:

- create a multi-criteria mathematical model that takes into account changes in substrate concentration in the bioreactor over time, allowing us to predict the degree of purification at peak loads;
- develop an algorithm and software package for solving the model problem;
- conduct numerical experiments and analyse the results.

The following factors will be taken into account when modelling the process of biological water treatment under conditions of unlimited oxygen regime in wastewater treatment plants:

- changes in the concentration of organic pollutants and activated sludge over time;
- the uneven flow of wastewater to the treatment plant, and the possible ingress of different amounts of substrate at different times.

We will build a mathematical model of the water treatment process in a bioreactor that takes these factors into account using the law of mass conservation. The calculation scheme is shown in Figure 1.



**Fig. 1.** Schematic diagram of the biological treatment process;  $C_{in}$  = concentration of impurities in the incoming wastewater ( $\text{mg}\cdot\text{dm}^{-3}$ ),  $C_{out}$  = concentration of pollution at the outlet of the system ( $\text{mg}\cdot\text{dm}^{-3}$ ),  $V$  = reactor volume ( $\text{m}^3$ ),  $Q$  = incoming water flow ( $\text{m}^3\cdot\text{h}^{-1}$ ),  $B$  = concentration of activated sludge in the reactor ( $\text{g}\cdot\text{dm}^{-3}$ ),  $K_{O_2}$  = oxygen concentration ( $\text{mg}\cdot\text{dm}^{-3}$ ),  $T$  = temperature ( $^{\circ}\text{C}$ ); source: own study

We consider the process of liquid purification from organic contaminants using biological bacteria. According to scientific studies (Safonyk and Bomba, 2018; Safonyk, Bomba and Tarhonii, 2018; Safonyk, Martynov and Kynytyskiy, 2019; Safonyk, Zhukovskyy and Burduk, 2020), the purification process includes the following stages: decomposition of organic pollutants by bacteria, growth and death of bacteria, formation of “young” bacteria by active sludge, and the transition of impurities to biologically non-oxidisable substances.

An equation of the type (Safonyk, Zhukovskyy and Burduk, 2020) is used to describe the process of changing the concentra-

tion of pollutants, taking into account the effect of activated sludge on the absorption of impurities:

$$\frac{\partial C}{\partial t} = v_c \frac{\partial C}{\partial x} - \beta C B + w_c + D_c \frac{\partial^2 C}{\partial x^2} \quad (1)$$

where:  $\beta$  = coefficient that takes into account the design features of the filter and the fluid flow rate;  $\beta = \frac{Q \cdot (1+k_i)}{V}$ ;  $C$  = concentration of contamination in water,  $V$  = volume of the filter,  $k_i$  = the recirculation rate of activated sludge,  $w_c$  = absorption rate of the substrate in accordance with the adequacy of the model,  $v_c$  = speed of movement of the substrate,  $D_c$  = diffusion coefficient,  $B$  = concentration of activated sludge in the reactor ( $\text{g}\cdot\text{dm}^{-3}$ ),  $t$  = time,  $x$  = coordinate along the length of the filter.

As bacteria move with the contaminant in the porous media and settle to the bottom of the filter as activated sludge, we can formulate an equation for bacterial growth, death, and transfer based on their biological oxygen demand:

$$\frac{\partial B}{\partial t} = v_B \frac{\partial B}{\partial x} + \beta B K \cdot K_B + w_B + D_B \frac{\partial^2 B}{\partial x^2} \quad (2)$$

where:  $K_B$  = coefficient of oxygen and bacteria absorption,  $w_B$  = rate of activated sludge accumulation in accordance with the adequacy of the model,  $v_B$  = rate of movement of activated sludge,  $D_B$  = diffusion coefficient,  $K$  = concentration of oxygen required to maintain the best possible absorption of contaminants by bacteria.

To achieve better efficiency and optimal conditions for the bacteria, oxygen is added to the contaminated substance. The equation describing the dynamics of the process is as follows, taking this factor into account:

$$\frac{\partial K}{\partial t} = v_K \frac{\partial K}{\partial x} + \beta K + K_K \cdot C \cdot (K_0 - K) + w_K + D_K \frac{\partial^2 K}{\partial x^2} \quad (3)$$

where:  $K_K$  = mass transfer coefficient of oxygen,  $K_0$  = concentration of oxygen saturation of water at a given temperature and pressure,  $w_K$  = rate of oxygen absorption by the substrate,  $v_K$  = rate of oxygen movement,  $D_K$  = diffusion coefficient.

Equations (1), (2), and (3) describe the change in the concentration of bacteria, contaminants, and oxygen in a porous medium. To account for the interaction between the medium and the process, coefficients can be introduced into the respective equations. This allows the processes in the reactor to be analysed as a set of interrelated influences. In real systems, there is always a lag due to certain reasons, for example, the transfer of contaminants takes time. Therefore, any change in external factors occurs only after some time has passed (lag time  $\tau > 0$ ). So, the modelling task is to take this lag into account:

$$\begin{cases} \frac{\partial C}{\partial t} = v_c \frac{\partial C}{\partial x} - \beta C B + w_c + D_c \frac{\partial^2 C}{\partial x^2}, \\ \frac{\partial B}{\partial t} = v_B \frac{\partial B}{\partial x} + \beta B K \cdot K_B + w_B + D_B \frac{\partial^2 B}{\partial x^2}, \\ \frac{\partial K}{\partial t} = v_K \frac{\partial K}{\partial x} + \beta K + K_K \cdot C \cdot (K_0 - K) + w_K + D_K \frac{\partial^2 K}{\partial x^2} \end{cases} \quad (4)$$

$$\begin{aligned} C|_{x=0} &= C^*(t), \quad B|_{x=0} = B^*(t), \quad K|_{x=0} = K^*(t), \\ \frac{\partial C}{\partial x}|_{x=l} &= 0, \quad \frac{\partial B}{\partial x}|_{x=l} = 0, \quad \frac{\partial K}{\partial x}|_{x=l} = 0, \\ C|_{t=0} &= C^*(x), \quad B|_{t=0} = B^*(x), \quad K|_{t=0} = K^*(x) \end{aligned} \quad (5)$$

where:  $l$  = length of the filter.

## RESULTS AND DISCUSSION

The solution to the model problem (Eqs. (4) and (5)) was found similarly to Safonyk, Bomba and Tarhonii (2018), Safonyk, Martynov and Kynytyskiy (2019), and Safonyk, Zhukovskyy and Burduk (2020). The developed numerical method and the corresponding algorithms were implemented in the Python software environment. Using this implementation, we obtained numerical solutions to individual problems that are part of the general boundary value problem (Eqs. (4) and (5)).

Numerical experiments were conducted with the following initial data: daily wastewater flow ( $Q_d = 10 \text{ m}^3 \cdot \text{d}^{-1}$ ), average hourly wastewater flow ( $Q_{\text{mid}} = 0.42 \text{ m}^3 \cdot \text{h}^{-1}$ ), maximum hourly wastewater flow ( $Q_{\text{max}} = 1.25 \text{ m}^3 \cdot \text{h}^{-1}$ ), coefficient of hourly irregularity of wastewater inflow ( $K = 1-3$ ), biochemical oxygen demand at the aeration tank inlet ( $BOD_{\text{in}} L_{\text{en}} = 350 \text{ mg} \cdot \text{dm}^{-3}$ ), biochemical oxygen demand at the aeration tank outlet ( $BOD_{\text{out}} L_{\text{ex}} = 13 \text{ mg} \cdot \text{dm}^{-3}$ ); dose of sludge in the aeration tank ( $a_i = 4 \text{ g} \cdot \text{dm}^{-3}$ ); specific oxidation rate ( $\rho = 6 \text{ mg} \cdot (\text{g} \cdot \text{h})^{-1}$ ); sludge ash content ( $s = 35\%$ ); working maximum depth of the aeration tank ( $H_{\text{max}} = 3.0 \text{ m}$ ); working minimum depth of the aeration tank ( $H_{\text{min}} = 1.5 \text{ m}$ ).

Duration of aeration is calculated as:

$$t_{\text{atm}} = \frac{L_{\text{en}} - L_{\text{ex}}}{a_i(1-s)\rho} = \frac{350 - 13}{4(1-0.35)6} = 21.6 \text{ h} \quad (6)$$

Maximum working volume of the aeration tank can be found based on duration of aeration:

$$V_{\text{aer}} = Q_{\text{mid}} \cdot t_{\text{atm}} = 0.42 \cdot 21.6 = 9.072 \quad (7)$$

where:  $V_{\text{aer}}$  = volume of the aeration tank,  $t_{\text{atm}}$  = duration of aeration.

The parameters of the inflow of wastewater were different at different time periods during the day:

- 6:00–9:00 (3 h) – maximum wastewater inflow at a sludge dose ( $a_i$ ) of  $4 \text{ g} \cdot \text{dm}^{-3}$ ;
- 9:00–17:00 (8 h) – minimum wastewater inflow at a sludge dose ( $a_i$ ) of  $8 \text{ g} \cdot \text{dm}^{-3}$ ;
- 17:00–22:00 (5 h) – maximum flow at a sludge dose ( $a_i$ ) of  $4 \text{ g} \cdot \text{dm}^{-3}$ ;
- 22:00–6:00 (8 h) – minimum wastewater inflow at a sludge dose ( $a_i$ ) of  $8 \text{ g} \cdot \text{dm}^{-3}$ .

Three practical scenarios were considered.

### • Scenario 1

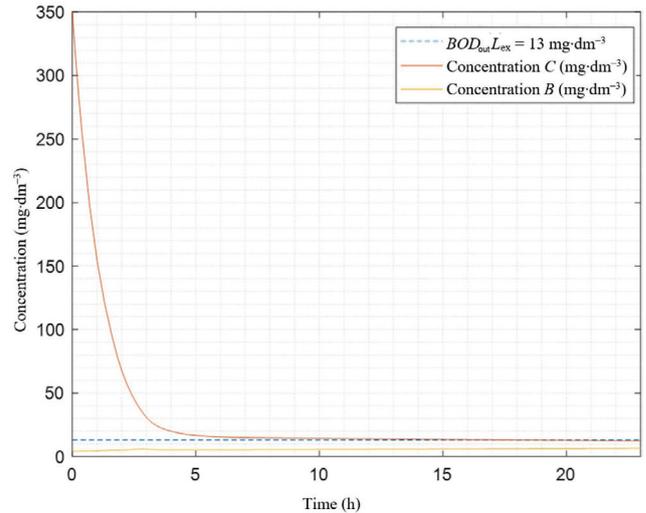
In this scenario, there is a uniform supply of wastewater to the reactor (Fig. 2).

### • Scenario 2

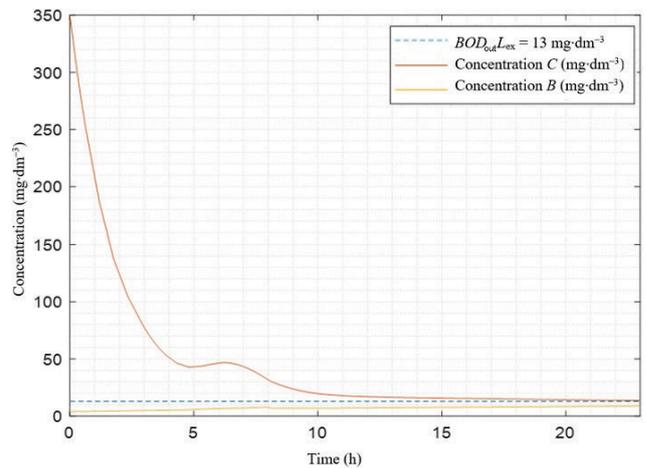
There is a uniform supply of wastewater to the reactor, but in the interval 6:00–9:00 (3 h) there is a maximum inflow of wastewater at a sludge dose ( $a_i$ ) of  $4 \text{ g} \cdot \text{dm}^{-3}$  (taking into account the daily irregularity of wastewater supply) (Fig. 3).

### • Scenario 3

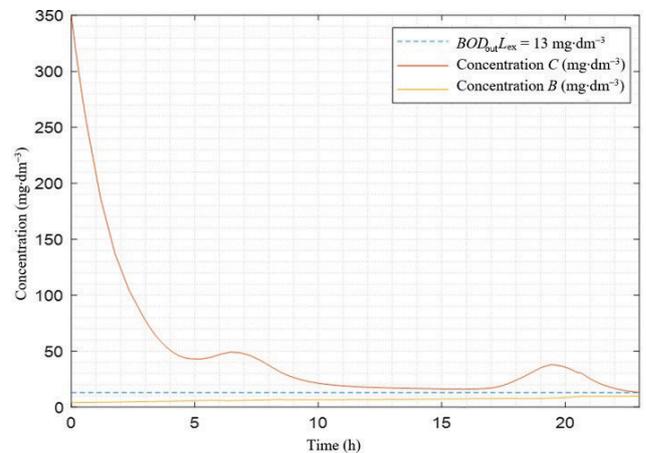
In this scenario, there is a uniform supply of wastewater to the reactor, but in the interval from 17:00 to 22:00 (5 h), the maximum inflow of wastewater occurs at a sludge dose ( $a_i$ ) of  $4 \text{ g} \cdot \text{dm}^{-3}$  (taking into account the daily irregularity of wastewater supply) (Fig. 4).



**Fig. 2.** Distribution of pollution and activated sludge concentration during the day for scenario 1;  $C$  = concentration of pollution in the filter ( $\text{mg} \cdot \text{dm}^{-3}$ ),  $B$  = concentration of activated sludge in the reactor ( $\text{mg} \cdot \text{dm}^{-3}$ ),  $BOD_{\text{out}} L_{\text{ex}}$  = biochemical oxygen demand at the aeration tank outlet; source: own study



**Fig. 3.** Distribution of pollution and activated sludge concentration during the day for scenario 2;  $C$ ,  $B$ ,  $BOD_{\text{out}} L_{\text{ex}}$  as in Fig. 2; source: own study

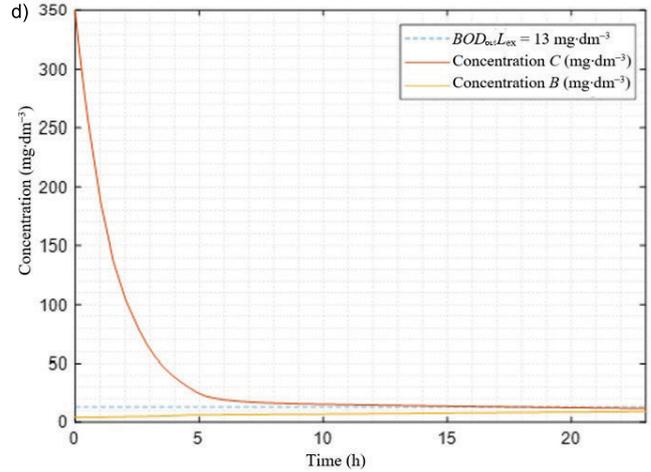
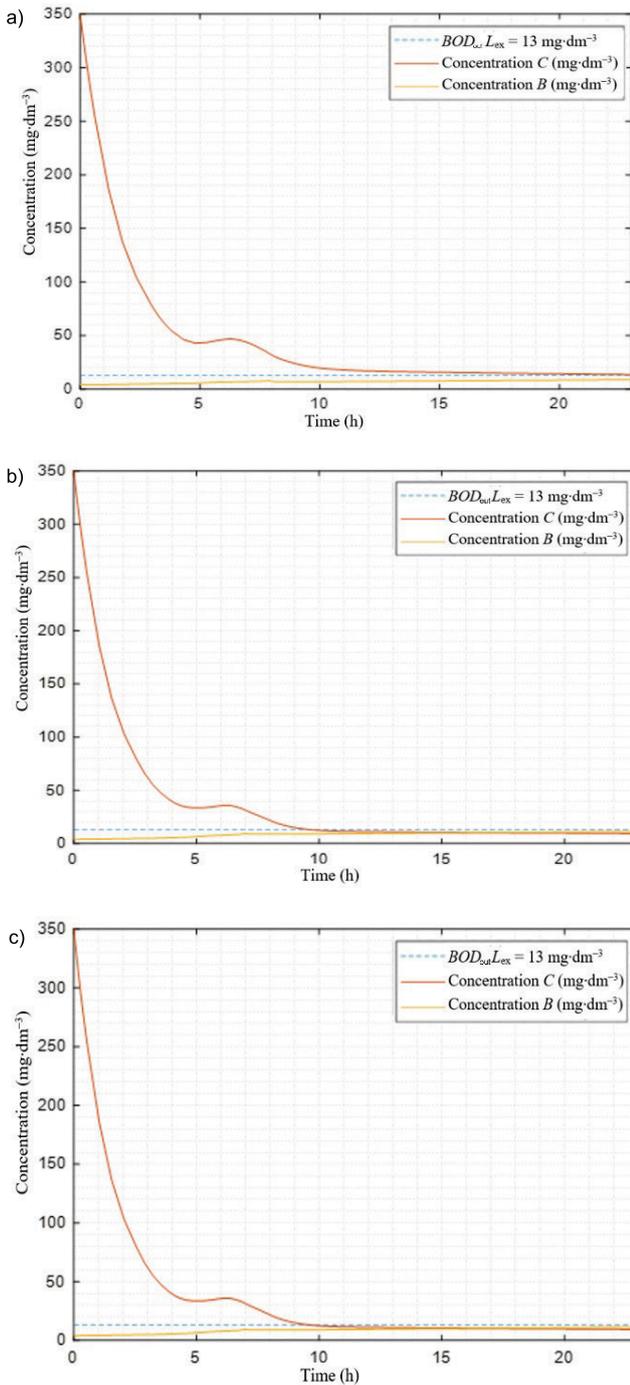


**Fig. 4.** Distribution of pollution and activated sludge concentration during the day for scenario 3;  $C$ ,  $B$ ,  $BOD_{\text{out}} L_{\text{ex}}$  as in Fig. 2; source: own study

The distribution of the concentration of pollution and activated sludge during the day for scenario 2 with an increase in air supply to the aeration zone is shown in Figure 5.

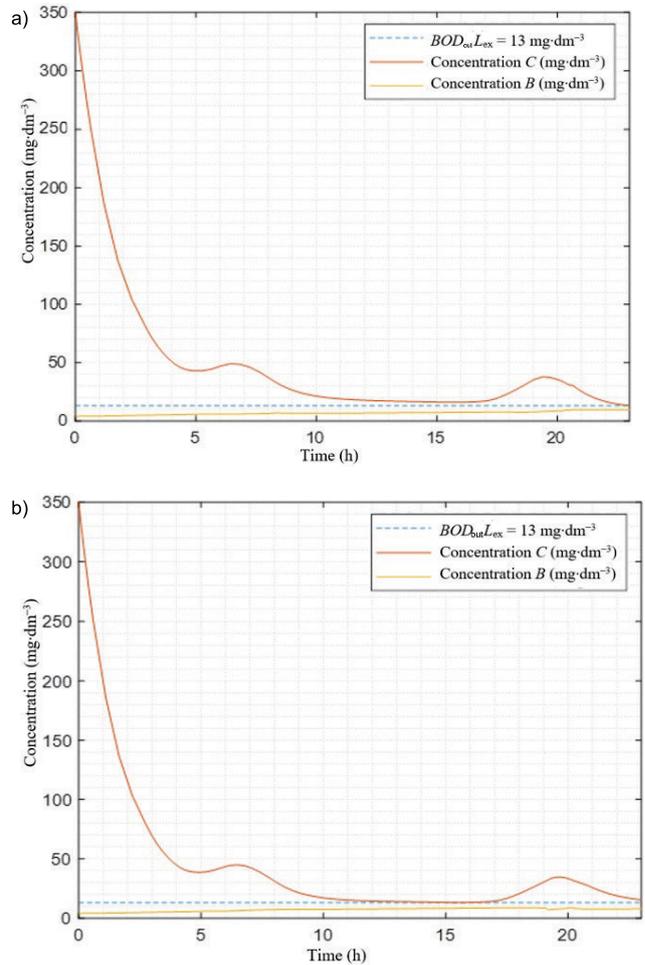
The distribution of pollution and activated sludge concentrations throughout the day under scenario 3, when the aeration zone has an increased air supply, is shown in Figure 6.

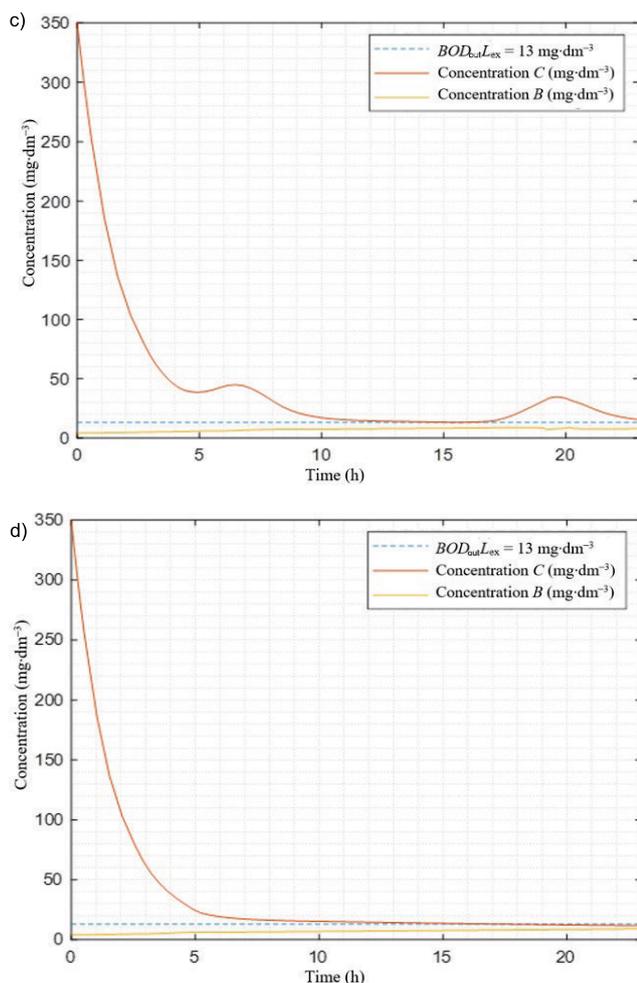
According to Figure 2, with the same wastewater supply, the steady-state mode is set to 6:00 h and the substrate concentration at the reactor outlet is  $13 \text{ mg}\cdot\text{dm}^{-3}$ . However, for scenarios 2 and 3, the reactor performance deteriorates, which leads to an increase in the substrate concentration. In the case of scenario 2, the reactor efficiency returns to 95% by 23:00 h, while scenario 3 requires additional time or other control methods to bring the reactor to a steady state.



**Fig. 5.** Distribution of pollution and activated sludge concentration during the day for the scenario with an increase in air supply to the aeration zone: a)  $K = 0.5 \text{ mg}\cdot\text{dm}^{-3}$ , b)  $K = 1.0 \text{ mg}\cdot\text{dm}^{-3}$ , c)  $K = 1.5 \text{ mg}\cdot\text{dm}^{-3}$ , d)  $K = 2.0 \text{ mg}\cdot\text{dm}^{-3}$ ;  $K$  = concentration of oxygen required to maintain the best possible absorption of contaminants by bacteria,  $C$ ,  $B$ ,  $BOD_{out}L_{ex}$  as in Fig. 2; source: own study

The regulatory indicators for scenarios 2 and 3 were achieved by controlling the process of oxygen supply to the reactor. The introduction of the proposed aeration control system not only improves the efficiency of the treatment plant but also reduces electricity consumption, as the aeration system accounts for a significant portion of the total consumption.





**Fig. 6.** Distribution of pollution and activated sludge concentration during the day for scenario 3 with an increase in air supply to the aeration zone: a)  $K = 0.5 \text{ mg}\cdot\text{dm}^{-3}$ , b)  $K = 1 \text{ mg}\cdot\text{dm}^{-3}$ , c)  $K = 1.5 \text{ mg}\cdot\text{dm}^{-3}$ , d)  $K = 2 \text{ mg}\cdot\text{dm}^{-3}$ ;  $K$  as in Fig. 5,  $C$ ,  $B$ ,  $BOD_{\text{out}}$   $L_{\text{ex}}$  as in Fig. 2; source: own study

Consideration of three work scenarios is due to a practical task, namely: in general, such processes are considered without taking into account the unevenness of the load, but in practice, this is not the case, because the maximum cleaning loads are in the morning, when the residents of the cottage town conventionally go to the toilets, and in the evening, when they return from work. Therefore, we considered three cases: without loads, loads in the morning and loads both in the morning and in the evening. Our task was to simulate the real situation and show that even with the maximum load at a specific time of the day, the treatment plants cope with their task.

## CONCLUSIONS

A mathematical model of the process of biological wastewater treatment has been developed based on: changes in the concentration of organic contaminants in the bioreactor over time, changes in the concentration of activated sludge in the bioreactor over time, changes in the concentration of activated sludge in the reactor over time, taking into account the uneven flow of wastewater to the treatment plant, the process of substrate entering the bioreactor (different amounts may enter at different

times). The solution to the corresponding model problem is found. The results of calculations of the concentration distribution of contaminants, bacteria, and oxygen during the liquid purification time for practically important operation modes of the purification system are presented. As numerical experiments have shown, the built solution correlates with real experimental data, which were taken as a basis for this system of biological wastewater treatment under conditions of uneven load. In future studies, it is proposed to develop a mathematical model that will take into account the age of activated sludge, as well as take into account such environmental parameters as the amount of air supplied to the aeration tank and the concentration of oxygen in water, etc.

## CONFLICT OF INTERESTS

All authors declare that they have no conflict of interests.

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