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The analytical framework for optimizing TCP retransmission algorithm based on adjustment of its EWMA parameters

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***Abstract*—In this paper we derive mathematical description of TCP (Transmission Control Protocol) retransmission probability based on Jacobson's smoothing algorithm that belongs to EWMA (Exponentially Weighted Moving Average) category. This description is parametrized on the probability density function (pdf) of RTT (Round Trip Time) samples and α, β – two primary parameters of Jacobson's algorithm. Although it is not a close form expression, it is formulated as an effective algorithm that let us to explicitly calculate the values of RTO (Retransmission Time Out) probability as a function of α, β and the pdf of RTT samples. We achieve the effectiveness of this approach by applying smart discretization of the state space and replacement of continuous functions with discrete approximate equivalents. In this way, we mitigate the cardinality of discrete distributions we deal with that results in linear (n+m) instead of multiplicative (n·m) growth of computational complexity. We provide the evaluation of RTO probability for a wide set of α, β parameter values and differently shaped Normal and Laplace pdfs the RTT samples are taken from. The obtained numerical results let us to draw some conclusions regarding the choice of optimal values of α, β parameters as well as the impact of pdf the RTT samples are taken from.**

***Keywords*—TCP; EWMA; Jacobson's algorithm; retransmission timeout; retransmission probability**

I. Introduction

Any communication protocol that aims at assuring reliability requires a retransmission mechanism. The crucial part of this mechanism is the algorithm for calculating the retransmission timeout (RTO) i.e. the time after which the sent portion of data is assumed to be lost and thus is retransmitted. Example of such protocols are TCP [1], SIP [2] if used over UDP [3] to mention few older ones or CoAP (Constrained Application Protocol) [4] to indicate more recent protocols. In each case, RTO mechanism impacts the performance of applications that use these protocols and that's why it is the subject of new proposals and optimization effort. For example, for CoAP a new RTO algorithm has been proposed and tested in [5]. Another example is a new variant of CoAP called CoAPEifel [6] wherein the Eifel Retransmission Timer which was originally designed for TCP has been integrated with CoAP working over UDP. For TCP protocol itself, many research works were done as well. Some authors investigated the impact of parameters of RTO algorithm e.g. EWMA parameters or minimum RTO [7] or the impact of TCP parameters like delayed acknowledgements or OS clock granularity by experiments [8]. Fewer works took an analytical approach trying to provide models to capture the dependencies between parameter values and the evaluated performance metrics of TCP [9]. Finally, some works took entirely different approach trying to improve TCP RTO mechanism by proposing new algorithm to replace well-known EWMA rule. To mention some examples, we can indicate the works: [10] where the Weighted Median concept was used and [11] where the calculation of the entropy was fundamental for RTO estimation.

Despite the long history of TCP being used as a transport by different popular applications and application protocols e.g. e-mail exchange (SMTP protocol), file transfer (FTP protocol), web browsing (HTTP), it is still under the investigation for new applications or application protocols. One of such applications is CCN (Content-Centric Networking). The authors of [12] analyzed Jacobson's algorithm to find the most appropriate setting for EWMA (Exponentially Weighted Moving Average) parameters in CCN application. They experimented with different values of minimum RTO and other parameters of RTO algorithm. In this paper we propose the analytical framework to provide more rigorous treatment of the already introduced problem that is the adjustment of RTO parameter values to achieve the optimal performance of TCP. The remaining part of this paper is organized as follows: in section 2 we state the problem that is the adjustment of RTO parameter values to achieve optimal performance. In section 3, we present the analytical framework that lets for RTO probability value and TCP performance estimation based on the given values of RTO parameters. In section 4, we provide the numerical results that let to decide about usability of the proposed framework. In section 5, we conclude the work.

II. BASIC PERFORMANCE METRICS FOR RTO ALGORITHMS

TCP retransmission algorithm works with a series of round trip time (RTT) values that represent the time from TCP segment transmission to respective TCP acknowledgement reception. The RTT values let a TCP sender to infer current network conditions and approximate the timeout it should await

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a TCP acknowledgement for any TCP segment sent. Nowadays the most widely implemented TCP retransmission timeout (RTO) algorithm is the one defined by Jacobson [13]. According to this algorithm the RTO value is calculated as a sum of a smoothed round trip time (SRTT) and a component representing a variability - see formula (1).

$$\mathrm{RTO(n)} = \mathrm{SRTT(n)} + \delta \cdot \sigma\,(\mathrm{n}) \quad (1)$$

where δ is an integer typically chosen to be 2 or 4.

A smoothed RTT (SRTT) after collecting an n-th RTT sample is computed as an exponentially weighted moving average (EWMA) according to (2):

$$\mathrm{SRTT(n)} = (1-\alpha) \cdot \mathrm{SRTT(n-1)} + \alpha \cdot \mathrm{RTT(n)} \quad (2)$$

where RTT(n) denotes the lastly measured RTT sample i.e. the n-th one, SRTT(n) and SRTT(n-1) denote the SRTT value for the last (current) and the previous RTT measurements, respectively. Parameter α is an averaging factor with default value of 1/8 [1].

The component representing the variability of the measured RTT is calculated in a very similar way - see (3):

$$\sigma(\mathrm{n}) = (1-\beta) \cdot \sigma(\mathrm{n-1}) + \beta \cdot |\mathrm{SRTT(n-1)} - \mathrm{RTT(n)}| \quad (3)$$

where σ(n) and σ(n-1) denote the σ value for the current and the previous RTT measurements, respectively. Parameter β is the averaging factor with default value of 1/4 [1].

Jacobson's algorithm tries to follow the current trend of RTT values while not being so sensitive to the sudden changes of RTT values. That's why it relies also on the previous measurements either for smoothed RTT or variability calculations. Preserving this balance between the responsiveness to changes in RTT values while keeping stable operation (without oscillations) is the crucial point. It is affected by the selection of the values of α and β parameters. More conservative approach in the sense of lower sensitivity to newest RTT values can result in underestimated SRTT and RTO values thus providing to premature (spurious) timeouts and retransmissions. On the other hand, more aggressive approach in the sense of higher sensitivity to newest RTT values can result in overestimated SRTT and RTO values thus providing to long delays before a necessary retransmission is triggered. Both cases: underestimation or overestimation of RTO values are not desired as they lead to TCP performance degradation. In the light of the above, the need for optimal selection of RTO parameter values is unquestionable. As an argument supporting this thesis we can cite a number of research works that dealt with this problem. In [12] the most appropriate settings of Jacobson's algorithm were sought to optimize the performance of TCP connections in CCN applications, Amongst the others, the values of α, β, δ, RTO_{min} parameters were examined to see its impact on TCP performance. In [7] the authors evaluated TCP performance in LAN network (short delays) with various settings of the α and β constants in the presence of varying levels of background traffic generated by conventional systems. In [9] the more rigorous approach was proposed. Instead of empirical study as in [12], [7] or [8], the authors provided an analytical model of Jacobson's algorithm to find out some dependencies between RTT statistical values drawn from Gamma probability distribution and some performance measures of RTO algorithms. These measures were chosen to be: mean reaction time of RTO algorithm expressed as an average RTO and the probability of premature retransmission of TCP RTO algorithm. Our work follows the similar approach as in [9]. Instead of empirical study that usually provides some tangible results but limited to evaluated cases, we chose an analytical approach. By providing a quite realistic model of Jacobson's algorithm that takes into account the passage of time and calculation of RTO value for consecutive RTT measurements, we believe to gather valuable insights into the characteristics of RTO algorithm and better recognition of its performance dependency on values of α and β parameters as well as the characteristics of RTT samples. Providing an analytical framework that lets for accounting of probability distribution of RTT values as one of the parameters of the model, seems to be valuable contribution in the cognitive sense of RTO algorithms as well as an element that differentiates our work from cited literature, especially from [9].

III. ANALYTICAL FRAMEWORK FOR JACOBSON'S RTO ALGORITHM

We start with a given sequence of TCP RTT samples:

$$(x_0, x_1, \dots, x_n) \quad (4)$$

and parameters α, β, δ representing the respective constants in equations (1) – (3). The values of these parameters fulfil the following conditions: 0<α<1, 0<β<1, 0<δ. The values of random variables SRTT, Dev and RTO for the consecutive RTT samples from the set (4) are denoted as SRTT, Dev and RTO and calculated using the recursive formulae (1) – (3):

$$SRTT[\mathrm{k}] = \alpha \cdot x_{k-1} + (1-\alpha) \cdot SRTT[k-1] \quad (5)$$

$$Dev[k] = \beta \cdot |SRTT[k-1] - x_{k-1}| + (1-\beta) \cdot Dev[k-1] \quad (6)$$

$$RTO[k] = SRTT[k-1] + \delta \cdot Dev[k-1] \quad (7)$$

for k=1,…,n. The initial values of SRTT and Dev random variables (for the first RTT sample) are given by:

$$SRTT[0] = \alpha \cdot x_0 \quad (8)$$

$$Dev[0] = \beta \cdot x_0 \quad (9)$$

We assume that the consecutive values of RTT samples, denoted as x_i represent the values of i.i.d (independent, identically distributed) random variables X_i, i=0,1,…,n, with the common probability density function f. We aim at defining probabilistic model M(f,α,β,δ) that lets for calculations of:

- The values: *SRTT[k], Dev[k], RTO[k]* of random variables SRTT[k], Dev[k], RTO[k] for any k value,
- Mean value of RTO random variable (denoted as E{RTO[n]}),
- Dependency of retransmission probability Prob(RTO[n] < X_n) on the values of α,β parameters,
- Possible correlations (and its visualization) between the retransmission probability and parameters (e.g. its variance, entropy) and the type of probability density function f that determines the values of consecutive RTT samples,

For this purpose we applied an approach based on the direct mapping of recursive formula for *SRTT, Dev, RTO* as the operations executed on the probability density functions of random variables SRTT[k], Dev[k], RTO[k], i.e:

$$PDF\ of\ \mathrm{SRTT}[k] = Convolution(PDF\ of\ \alpha \cdot \mathrm{X}_{k-1}, PDF\ of\ (1-\alpha) \cdot \mathrm{SRTT}[k-1]) \quad (10)$$

The above direct mapping is possible only due to an assumption about the independence of random variables X_i (in such case the probability density function of a sum of random variables equals the convolution of probability density functions of these random variables). However it leads to the need to perform n-fold convolution that results in exponential complexity of calculations of probability density functions of SRTT, Dev, RTO random variables.

One possible remedy to effectively calculate the convolution of two functions f and g it is the application of Fourier Inverse Formula i.e. calculating a convolution according to (11):

$$f * g = F^{-1}(F(f) \cdot F(g)) \quad (11)$$

where F^{-1} is calculated as a simple integral (see [14]).

The application of formula (11) and simplified calculation of F^{-1} is plausible if the functions f ang g belong to the class of Schwartz's functions (the functions with real arguments and complex values, that are infinity-times differentiable and that converge to 0 in $(+\infty,-\infty)$ domain faster than any polynomial). The presence of absolute value function in (6) results in a situation where the probability density functions of Dev and RTO random variables don't belong to Schwartz's class. Moreover, the important for us pdfs of RTT samples e.g. exponential or Laplace pdfs, do not also belong to Schwartz's class due to not being differentiable for some arguments. Therefore, straightforward application of formula (11) is not possible due to the violation of Schwartz's class conditions. On the other hand, direct application of n-fold convolution operation is not acceptable because of too high complexity of calculations of resulting pdfs. However, the lack of effective calculations of SRTT, Dev and RTO pdfs precludes the achievement of the final result that is e.g. calculation of retransmission probability. Concluding, the definition of the full analytical, probabilistic model that lets for characterization of RTT, Dev, RTO random variables is possible, but this model precludes obtaining the numerical results e.g. retransmission probability.

To alleviate the above problem, we propose to use another approach. In order to mitigate the complexity of recurrent calculations with continuous pdfs of RTT, SRTT, Dev and RTO random variables we replace continuous functions with discrete approximate equivalents. This let us to use discrete Fourier transform and Fast Fourier Transform algorithm for calculations and skips the problem of the functions not belonging to Schwartz's class. As a consequence we introduce finite discrete random variables DSRTT, DDev, DRTO in place of their continuous counterparts: SRTT, Dev, RTO.

Discrete distribution D is defined by:

- Domain of the distribution: $\mathrm{Dom(D)} = (x_0,\ldots,x_{n-1})$
- Probability distribution function Prob(D) defined in domain Dom(D): $p = (p_0,\ldots,p_{n-1})$, $\mathrm{Prob(D)}(x_i) = p_i$

Consider two discrete random variables X, Y with discrete probability distribution functions D1, D2. Then, the random variable X+Y has a discrete probability distribution D=join(D1,D2) in the following way:

$$\mathrm{Dom(D)} = \{x + y | x \in \mathrm{Dom(D1)}\ \text{and}\ y \in \mathrm{Dom(D2)}\} \quad (12)$$

$$\text{For } a \in \mathrm{Dom(D)}, \mathrm{Prob(D)}\,(a) = \sum_{x \in \mathrm{Dom(D1)}, y \in \mathrm{Dom(D2)}, x+y=a} \mathrm{Prob\ (D1)(x)} \cdot \mathrm{Prob(D2)(y)} \quad (13)$$

The above proposed approach may be applied to recursive calculations of SRTT, Dev, RTO random variables. We model the recursive equations (5) – (7) as the operations on random variables with discrete probability distributions. For example, the formula for the random variable SRTT at k-th moment (after (k-1)-th RTT sample computation), that is given by (14):

$$\mathrm{SRTT[k]} = \alpha \cdot \mathrm{X}_{k-1} + (1-\alpha) \cdot \mathrm{SRTT[k-1]} \quad (14)$$

we model in the following way. Let $D(\alpha \cdot X_{k-1})$ denotes the discrete distribution approximating probability distribution of $\alpha \cdot X_{k-1}$ random variable, i.e. discrete distribution approximating X_{k-1} that is scaled by α. Let D((1-α)·SRTT[k-1]) denotes the discrete distribution approximating probability distribution of (1-α)·SRTT[k-1] random variable, i.e. recursively computed discrete distribution DSRTT[k-1] that is scaled by 1-α.

Then, probability distribution function of DSRTT[k] is given by:

$$\text{PDF of DSRTT}[k] = join(\mathrm{D}(\alpha \cdot \mathrm{X}_{k-1}), \mathrm{D}((1-\alpha) \cdot \mathrm{SRTT[k-1]})) \quad (15)$$

where the 'join' operation is defined by (12), (13).

Let us point out some drawbacks of the above direct approach. If D1, D2 have the cardinality equal to n, m respectively, then the pessimistic estimation of the cardinality of distribution D of random variable X+Y equals n·m. In this case, the usage of the above definition for calculations of DSRTT, DDev, DRTO distributions, leads to the situation where the cardinality of final distribution domain for DRTO is of the order tens of thousands for some probability density functions the values of RTT are generated from. This in turn, will result in unacceptable burden and delay in processing final probability distributions. We solve the above problem of computation complexity by our own, proprietary algorithm. We introduce the assumption about the regularity of the approximate discrete probability distributions that we use during the analysis. We state that discrete distribution D has granularity ε if (and only if) there exists an 'a' such that:

$$\mathrm{a} \in \mathrm{R}, \mathrm{Dom(D)} = \{\mathrm{a} + \mathrm{i} \cdot \varepsilon | 0 \le \mathrm{i} \le \mathrm{n}\},\ \text{for some n} \quad (16)$$

If probability distributions D1, D2 of random variables X, Y have the same granularity ε and fulfil the condition (17):

$$\mathrm{Dom(D1)} = \{\mathrm{a} + \mathrm{i} \cdot \varepsilon | 0 \le \mathrm{i} \le \mathrm{n}\},\ \mathrm{Dom(D2)} = \{\mathrm{b} + \mathrm{j} \cdot \varepsilon | 0 \le \mathrm{j} \le \mathrm{m}\} \quad (17)$$

then discrete distribution D of random variable X+Y has the granularity also equal to ε and Dom(D) defined by (18):

$$\mathrm{Dom(D)} = \{(\mathrm{a} + \mathrm{b}) + \mathrm{k} \cdot \varepsilon | 0 \le \mathrm{k} \le \mathrm{n} + \mathrm{m}\} \quad (18)$$

Probability distribution function Prob(D) is the convolution of series (probability distribution functions) Prob(D1), Prob(D2) as defined in (19):

$$\mathrm{Prob}(D)\left((a+b)+k\cdot\varepsilon\right)=\sum_{i+j=k}\mathrm{Prob}(D1)\,(a+i\cdot\varepsilon)\cdot\mathrm{Prob}(D2)(b+j\cdot\varepsilon)\ \text{ for } k=1,..,n+m \qquad (19)$$

We denote the convolution of discrete distributions D1, D2 as:

$$D = D1 * D2 \qquad (20)$$

The calculations of this convolution might be carried out using discrete Fourier transform with FFT algorithm to speed up the operation to obtain the probability distribution function of the sum of random variables X, Y. However, the most important benefit from applying this approach is the mitigation of the cardinality of discrete distribution D that is n+m instead of n·m. For large n, m, the difference for the operation complexity (expressed as a number of convolution operations to be performed) might be even several orders of magnitude. It is itself a dramatical decline in the computation time of final probability distribution, making the numerical calculations feasible at all.

We define some common global granularity ε for all discrete probability distribution functions of DSRTT[k], DDev[k], DRTO[k] random variables. However, a new problem arises. Multiplying a random variable with discrete probability distribution by a constant (e.g. $\alpha\cdot X_{k-1}$), we scale the domain of its probability distribution, that results in the change of the granularity in this domain. This leads to the loss of homogeneity in the granularity of calculated probability distributions. As a consequence, the probability distribution function of discrete distribution D being the sum of discrete random variables X, Y with different granularities, in general won't be the convolution of the probability distribution functions of discrete random variables X and Y. All the advantages mentioned above and regarding the mitigation of the cardinality of the discrete distribution D (n+m states instead of n·m states) and the applicability of FFT algorithm, that are due to the fact of the same common and global granularity for probability distributions for all random variables involved in computations, will disappear. We solve this problem by introducing an approximate model of the information included in formula for SRTT, Dev, RTO calculations. Instead of continuous random variables, we will deal with discrete random variables denoted as: DSRTT, DDev, DRTO. This substitution is a trade of between the accuracy of results and computation feasibility of obtaining them.

For any continuous random variable Y with probability density function (pdf) g and for any granularity ε, we define discrete distribution DY with the granularity ε that approximates probability distribution of Y in the following way:

- We calculate the interval [a,b] such, that $1-\int_a^b g(x)dx < 0.001$ – i.e. the whole probability mass of pdf g is focused in [a,b] interval with the accuracy equal to 0.001,
- $\mathrm{Dom}(DY)=\{a+i\cdot\varepsilon \mid 0\le i \wedge a+i\cdot\varepsilon\le b\}$
- $\mathrm{Prob}(DY)\,(a+i\cdot\varepsilon)=\varepsilon\cdot g(a+i\cdot\varepsilon)$

The leading idea is to represent random variables SRTT, Dev, RTO as a linear combination of $X_0,\ldots,X_n$ random variables.

For k=0, …, n we express the random variable SRTT[k] as a linear combination of random variables $X_0,\ldots,X_n$ representing consecutive RTT measurements (21).

$$\mathrm{SRTT}(k)=a_{k,k}\cdot X_k+a_{k,k-1}\cdot X_{k-1}+\cdots+a_{k,i}\cdot X_i+\cdots+a_{k,1}\cdot X_1+a_{k,0}\cdot X_0 \qquad (21)$$

The derivation of $a_{k,i}$ coefficients is described in Appendix 1. Notice, all scaling operations like $\alpha\cdot X_{k-1}$ or $(1-\alpha)\cdot$SRTT[k-1] are incorporated in $a_{k,i}$ coefficients dependent on α.

For any constant a > 0, random variable a·X has pdf $g(x)=\frac{1}{a}f(x/a)$, where f is continuous pdf, common for all $X_0,\ldots,X_n$ random variables. The calculations of discrete distribution D(a·X) are accomplished by revoking to original information that is the function f in accordance to definitions outlined in bullet points above that constitute the discrete distribution DY for random variable Y=a·X, its probability density function g and, the most important, for the global, common granularity ε. In consequence we get:

$$\mathrm{DSRTT}(k)=D(a_{k,k}\cdot X_k)*D(a_{k,k-1}\cdot X_{k-1})*\ldots*D(a_{k,i}\cdot X_i)*\ldots*D(a_{k,1}\cdot X_1)*D(a_{k,0}\cdot X_0) \qquad (22)$$

According to (22), DSRTT(k) that denotes probability distribution function of DSRTT[k] random variable is defined as n-fold convolution of discrete distribution functions with the same granularity what makes the application of discrete Fourier transform possible. The same approach we apply for Dev random variable. However, in the formula for calculation of Dev, there appears an absolute value $|x_k-\mathrm{SRTT}(k-1)|$, that makes a representation of RTO random variable as a linear combination of random variables $X_0,\ldots,X_n$ more complicated. Let's assume that zero-one sequence s, $|s|\le n$, represents information about meeting (zero) or not meeting (one) the conditions (23):

$$X_i-\mathrm{SRTT}(i-1)\ge 0 \text{ for } i=1,\ldots,|s|-1 \qquad (23)$$

For zero-one sequences s, $|s|\le n$, we define Dev(s) values in the recursive way:

$$\mathrm{Dev}(s0)=\beta\cdot\left(x_{|s|+1}-\mathrm{SRTT}(|s|)\right)+(1-\beta)\cdot\mathrm{Dev}(s) \ (24)$$

$$\mathrm{Dev}(s1)=\beta\cdot(-x_{|s|+1}+\mathrm{SRTT}(|s|)+(1-\beta)\cdot\mathrm{Dev}(s)(25)$$

The values Dev(s) we represent as a linear combination:

$$\mathrm{Dev}(s)=b_{s,|s|}\cdot x_{|s|}+b_{s,|s|-1}\cdot x_{|s|-1}+\cdots+b_{s,i}\cdot x_i+\cdots+b_{s,1}\cdot x_1+b_{s,0}\cdot x_0 \qquad (26)$$

The derivation of $b_{s,i}$ coefficients is included in Appendix 2. Next, we proceed with the characterization of RTO random variable. We define value of RTO based on a given path 's' as:

$$\mathrm{rto}(s)=\mathrm{SRTT}(|s|)+\delta\cdot\mathrm{Dev}(s)=\sum_{i=0}^{|s|}\left(a_{|s|,i}+\delta\cdot b_{s,i}\right)\cdot x_i \qquad (27)$$

The value of random variable after n-th RTT measurement (RTO[n]) is defined as a rto(s) averaged over all possible paths 's' with |s|=n. On the other hand, since all applied mathematical operations are linear, we can approximate RTO[n] as a linear combination LRTO[n] of random variables $X_0,\ldots,X_n$ denoting the n consecutive RTT measurements:

$$\mathrm{LRTO}[n]=\sum_{s\in\{0,1\}^n}\mathrm{Prob}(s)\cdot\mathrm{rto}(s)=\sum_{k=0}^{n}c_k\cdot X_k \ (28)$$

The coefficients c_k are represented as the sums of $a_{n.k}$, $b_{s,k}$ coefficients averaged over appropriate paths s in a probabilistic sense:

$$c_k=\sum_{s\in\{0,1\}^n}\mathrm{Prob}(s)\cdot(a_{n.k}+\delta\cdot b_{s,k}) \qquad (29)$$

So far we are unable to give any estimation of the accuracy of approximation LRTO[n]. The probability of a path 's' (Prob(s)) we define as a multiplication of probabilities of the events:

$$X_i - SRTT(i-1) \geq 0,\ X_i - SRTT(i-1) < 0 \text{ for } i = 1, \dots, |s| - 1 \quad (30)$$

depending on the occurrence of zeros or ones on the path 's'. We calculate these probabilities (Prob(s)) by using the results of prior calculations of discrete distributions DX_i, DSRTT[i]. Linear representation LRTO[n] of RTO[n] (see formulae (28)) lets to calculate discrete distribution LDRTO(n) (of LDRTO[n] discrete random variable) that approximates pdf of LRTO[n] by the convolution of probability distribution functions with the same granularity analogously to the aforementioned calculations of the probability distribution functions DSRTT(k), k=0,…,n. Finally, our probabilistic model M(f,ε,α,β,δ), with ε being global, common granularity for probability distributions, is defined by 3 discrete probability distributions:

- DX – discrete distribution approximating the probability density function of i.i.d random variables: $X_0,\dots,X_n$
- DSRTT(0),…,DSRTT(n) – discrete distributions approximating random variables SRTT[0],…,SRTT[n]
- LDRTO(n) – discrete distribution approximating the defined above random variable LRTO[n]

and by well-defined operations on these probability distributions:

- Convolution D1*D2
- m(D) = distribution D1 defined in the following way:
  - Dom(D1)={-a | a ∈ Dom(D)}
  - Prob(D1) (a)=Prob(D)(-a) for a ∈ Dom(D1)

If D is the distribution of Y random variable, then m(D) is the distribution of -Y random variable.

All these distributions DX, DSRTT, LDRTO are dependent on parameters: f,ε,α,β,δ. The proposed model lets for calculating the aforementioned main values of interests:

- Mean value of LRTO approximated by mean value of LDRTO[n],
- The visualisation of the dependency of retransmission probability Prob(DRTO[n] < X_n) on values of α, β parameters,
- Possible correlations between retransmission probability (or equivalent measure that is retransmission probability times mean RTO: Prob(LDRTO[n] < X_n)·E{LDRTO[n]}) for different pdfs f and their parameters e.g. variance, entropy.

For example, let $D = LDRTO(n) * m(DX)$ be a discrete distribution approximating the difference of random variables LRTO[n] and X_n i.e. LRTO[n] – X_n. Then, the retransmission probability dependent on f, ε, α, β, δ we define as:

$$Prob(LDRTO[n] - X_n < 0) = \sum_{a \in Dom(D), a<0} Prob(D)(a) \quad (31)$$

IV. NUMERICAL RESULTS

The framework presented in section III lets to obtain numerical results thus allowing the investigation of the impact of α, β parameters on the retransmission probability.

Let us summarize up the algorithm.

Input:

- Parameters $\alpha, \beta, \delta = 2$
- Number of random RTT samples $n = 10$
- Global granularity $\varepsilon = 0.01$
- Probability density function f

Actions:

- For k = 0, …, n , i = 0, …, k compute coefficients $a_{k,i}$ of linear combination (21) (see Appendix 1),
- For all paths $s \in \bigcup_{k=0}^{n}\{0,1\}^k$ compute coefficients $b_{s,i}$ $i = 0, \dots, |s|$ of linear combination (26) (see Appendix 2),
- Compute $DSRTT(0), \dots, DSRTT(n)$ – discrete distributions approximating random variables SRTT[0],…,SRTT[n] by making use of Fast Fourier Transform algorithm in computation of convolutions (22),
- Compute discrete distribution DX approximating the random variable X with density f,
- For i = 0, … , n,
  - Compute discrete distribution $D_i = DX * m(DSRTT(i))$ approximating the random variable $X - SRTT[i]$
  - $q[i] = \sum_{x \in dom(D_i) \wedge x \geq 0} Prob(D_i)(x)$; probability approximating $Prob(X - SRTT[i] \geq 0)$
- Compute probabilities $Prob(s)$ of paths s by the following recursion:
  - $Prob(\varepsilon) = 1$, ε denotes an empty path here,
  - $Prob(s0) = Prob(s) \cdot q[|s|]$, $Prob(s1) = Prob(s) \cdot (1 - q[|s|])$,
- Compute coefficients $c_k = \sum_{s \in \{0,1\}^n} Prob(s)(a_{n.k} + \delta b_{s,k})$ of linear combination (28),
- Compute discrete distribution $LDRTO(n)$ approximating random variable $LRTO[n] = \sum_{k=0}^{n} c_k X_k$ by making use of Fast Fourier Transform algorithm in computation of appropriate convolutions,
- Compute discrete distribution $D = LDRTO(n) * m(DX)$ approximating the random variable $LRTO[n] - X$.

Output:

- Retransmission probability $= \sum_{x \in dom(D) \wedge x \geq 0} Prob(D)(x)$
- Mean value of RTO = $\sum_{x \in dom(LDRTO(n))} x \cdot Prob\,(LDRTO(n))(x)$

For given values of δ parameter we investigate the optimal value of retransmission probability ORP against the values of α, β parameters:

$$ORP = \min\{Prob(LDRTO[n] < X_n) \mid 0 < \alpha < 1, 0 < \beta < 1\} \quad (32)$$

We want to verify if there exists any relationship between optimal values of retransmission probability and the attributes, like variance or entropy, of different pdfs *f*. For this purpose we have made an evaluation of retransmission probability for 15 different cases of pdfs *f*. We have chosen five pdfs of Normal distribution differing in standard deviations: σ = 0.5, 1, 1.5, 2, 2.5, five pdfs of Laplace distributions differing in standard deviations: σ = 0.5, 1, 1.5, 2, 2.5, five pdfs of exponential distribution differing in λ = 0.5, 1, 1.5, 2, 2.5.

The target measure (the result) is defined to be retransmission probability (denoted on the figures as *Pr*) or retransmission probability times mean LRTO value (denoted on the figures as *Pr·avg_RTO*). Both measures depend on the value of α, β parameters denoted on the figures as *alpha*, *beta*, respectively. Since α, β parameters might take any real value in the range (0;1) we decided to discretized this range with a step of 0.1 to limit the number of possible combinations of α, β values to 9·9=81 cases.

We present the results as two-dimensional figures. The figures let for visualisation of the changes of values on α, β cartesian space. In the figures there is no information about the actual value however it is easy to observe the monotonicity of the target function. For this purpose it is enough to represent the level of the value expressed by the colour and by the size of the balls. The higher value the larger ball and the more warm colour. On the other hand, the lower value the smaller ball (might be even invisible) and the colder colour.

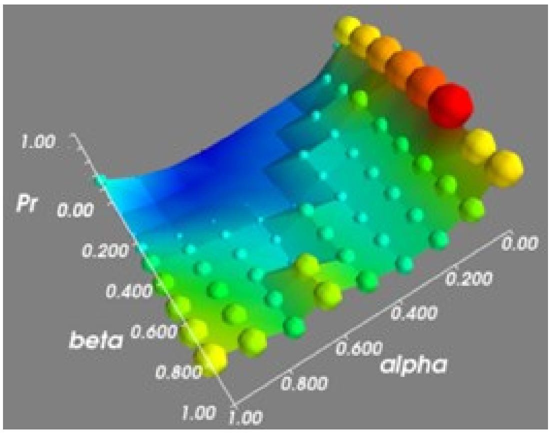

Fig. 1. The values of Pr metric calculated for RTT values taken from exponential probability distribution with λ = 1.5.

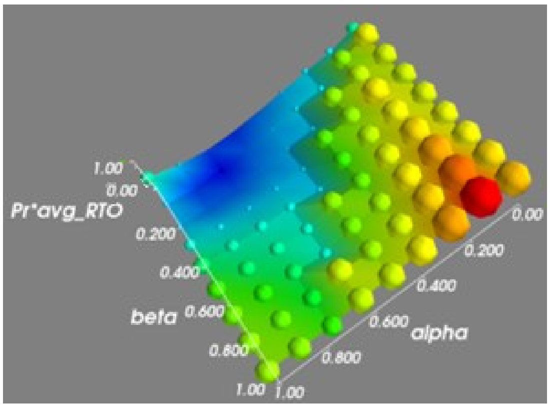

Fig. 2. The values of Pr·avg_RTO metric calculated for RTT values taken from exponential probability distribution with λ = 1.5.

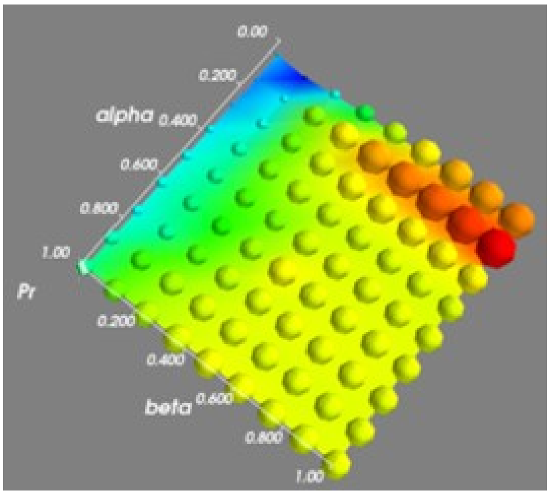

Fig. 3. The values of Pr metric calculated for RTT values taken from Laplace probability distribution with mean=2 and standard deviation σ = 1.5.

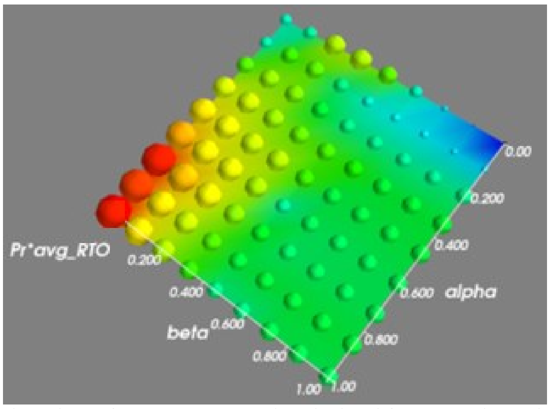

Fig. 4. The values of Pr·avg_RTO metric calculated for RTT values taken from Laplace probability distribution with mean=2 and standard deviation σ = 1.5.

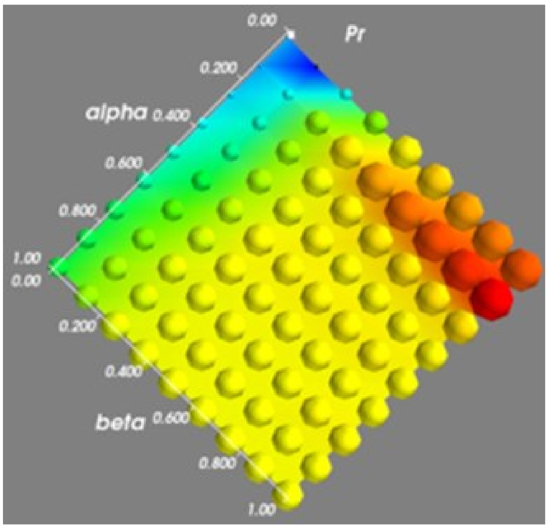

Fig. 5. The values of Pr metric calculated for RTT values taken from Normal probability distribution with mean=2 and standard deviation σ = 1.5.

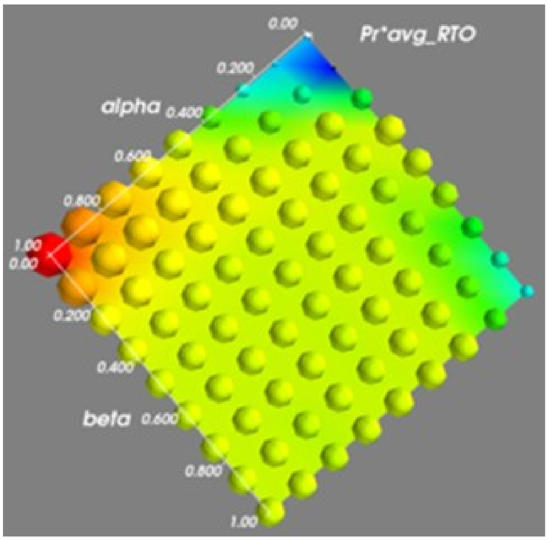

Fig. 6. The values of Pr·avg_RTO metric calculated for RTT values taken from Normal probability distribution with mean=2 and standard deviation σ = 1.5.

Above, we have presented some example results. A careful evaluation of more cases (15 different shapes of probability distributions functions like Laplace, Normal and exponential the RTT samples are drawn from) has revealed that there is always optimal pair of values of α, β parameters in the sense of minimalization of *Pr* or *Pr·avg_RTO* measures. This suggests existence of rules that govern the adjustment of α, β parameter values to provide the best performance of van Jacobsen's RTO algorithm in dependence on the probability distribution function the RTT samples are taken from. The presented framework lest for identifying these values, however not in a close-form expression but with algorithmic approach. Moreover, the numerical results revealed that the correlation between entropy of pdfs $f_1, \ldots, f_{15}$ and the optimal retransmission probability for these pdfs was at the order of 0.9 whereas the correlation with variance was at the order of 0.7. In some algorithms for the calculation of RTO value e.g. in classical van Jacobson's algorithm, the properties of a stochastic process modelling RTT values are taken into account by using its variance that is a kind of intuitive approach (Chebyshev's bound).

We state that the results of our experiments suggest the intuition for exploiting a concept of entropy while calculating the RTO values. One of the most simple suggestions for exploiting entropy concept, would be the following formulae for RTO calculation:

$$\mathrm{RTO} = \mathrm{SRTT} + \mathrm{h(e)} \cdot \mathrm{Dev} \qquad (33)$$

Formula (33) reminds classical van Jacobson's formula where a constant δ was replaced with *h(e)* denoting a function of entropy *e* of pdfs *f* modelling the RTT times. The choice of the function *h* is a separate problem. For the function $h(e) = 6 \cdot \sqrt{e}$ we performed some experiments on the RTT values artificially generated as a set of values from some arbitrary chosen probability distribution functions as well as on the RTT values taken from real network measurements. The real RTT samples usually had unneglectable autocorrelation coefficient values. In these experiments the argument *e* of *h(e)* function was calculated as entropy of histogram of 200 last RTT values. It appeared that the use of this modified RTO algorithm gave much better results in the sense of smaller number of retransmissions along the sample trajectory of RTT values than for classical van Jacobson's algorithm when RTT values were modelled as i.i.d. random variables (this i.i.d property is the fundamental assumption of our probabilistic model $M(f, \varepsilon, \alpha, \beta, \delta)$). For the RTT samples that exhibit an unneglectable autocorrelation coefficient value the situation is opposite – van Jacobson's algorithm gives better results. The honest conclusion of our work should be the statement that the right exploitation of entropy concept for RTO calculations is not straightforward task and still presents a set of challenges.

V. CONCLUSIONS

When summarizing the paper it is worth to remark that the effectiveness of proposed analytical framework relies on smart discretization of the state space and replacement of continuous functions with discrete approximate equivalents. This mitigates the cardinality of discrete distributions we deal with that results in linear (n+m) instead of multiplicative (n·m) growth of computational complexity. However, the price for circumventing the granularity homogeneity problems of discrete distributions is replacing the original RTO random variable with its linear LRTO approximation. This linearity and access to the original continuous probability density information of generating successive RTTs makes it possible to circumvent the granularity uniformity problem. In turn, the homogeneity of granulation allows the discrete Fourier transform to be used to reduce the cost of computing the discrete probability distributions we need.

The usefulness of the proposed analytical framework manifests in the fact that we were able to efficiently carry out the examination of TCP RTO probability values for a wide range of input parameter values (α, β, the shape of pdf we take RTT samples from). A simulation investigation seems infeasible in this case. This is due to the computation burden implied by the number of trajectories we would have to pass along at least 1000 times each in order to get reliable probabilities values with the accuracy of 2 digits after the dot. The number of trajectories rises geometrically with the number of possible RTT values (x) in a fashion x^n. For example, with RTT samples drawn from 10 point discrete distribution (i.e. x=10) and with n=10 (evaluation of RTO after 10 consecutive samples) would result in 10^{10} different trajectories and the requirement to pass along them $10^3 \cdot 10^{10}$ in order to collect enough data to provide reliable retransmission probability value. This is definitely infeasible in a reasonable time. Moreover, as the dependency is of geometrical nature, the problem gets worse when the number of RTT values (x) or a number of consecutive steps (n) increases. If the simulation time is not long enough or the simulations are not repeated appropriate number of times, then the results of experiments are unstable. That's why we think there is no alternative approach for the analytical investigation.

On the contrary, this analytical examination provided us valuable data to better understand the dependency of RTO probability values on the input parameters. One of the observations we notice is that the visual similarity of the graphs depends on the type of density function and not on the parameters of this density – e.g. graphs for densities with similar entropy but different types are dissimilar. We also conclude that there is always optimal pair of values of α, β parameters in the sense of minimalization of *Pr* or *Pr·avg_RTO* measures. This suggests existence of rules that govern the adjustment of α, β parameter values to provide the best performance of van Jacobsen's RTO algorithm in dependence of the probability distribution function the RTT samples are taken from.

As a final notice, we point out that the conducted analysis concerns stationary processes of independent RTT generation.

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APPENDIX 1

The derivation of coefficients $a_{k,i},\ 0 \le i \le k, k = 0, \dots, n$. We assume that the first value of smoothed RTT (SRTT) is:

$$SRTT(0) = \alpha \cdot x_0 \text{ then } a_{0,0} = \alpha. \quad (34)$$

From recursive formula (5) we have:

$$SRTT(k) = \alpha x_k + (1-\alpha) SRTT(k-1) = \alpha x_k + (1-\alpha)\left(\sum_{i=0}^{k-1} a_{k-1,i}\, x_i\right) = \alpha x_k + \sum_{i=0}^{n-1} (1-\alpha) a_{k-1,i}\, x_i. \quad (35)$$

Hence:

$$a_{k,i} = a_{k-1,i}(1-\alpha) = (1-\alpha)^{k-i}\alpha\ ;\ i = 0, \dots, k; k = 1, \dots, n \quad (36)$$

APPENDIX 2

The derivation of recursive formula for coefficients $b_{s,i}$:

$$Dev(\varepsilon) = \beta \cdot x_0 \quad (37)$$

For the sequence s , $|s| = k-1, \quad 1 \le k \le n$

$$Dev(s0) = \beta \cdot \left(x_{|s|+1} - SRTT(|s|)\right) + (1-\beta) \cdot Dev(s) \quad (38)$$

$$Dev(s1) = \beta \cdot \left(-x_{|s|+1} + SRTT(|s|)\right) + (1-\beta) \cdot Dev(s) \quad (39)$$

$$Dev(s0) = \beta\left(x_k - \sum_{i=0}^{k-1} a_{k-1,i} x_i\right) + (1-\beta) \sum_{i=0}^{k-1} b_{s,i} x_i = \beta x_k - \sum_{i=0}^{k-1} \beta a_{k-1,i} x_i + \sum_{i=0}^{k-1} b_{s,i} x_i - \sum_{i=0}^{k-1} \beta b_{s,i} x_i = \beta x_k + \sum_{i=0}^{k-1} (-\beta a_{k-1,i} + b_{s,i} - \beta b_{s,i}) x_i = \beta x_k + \sum_{i=0}^{k-1} ((1-\beta) b_{s,i} - \beta (1-\alpha)^{k-1-i} \alpha) x_i = \beta x_k + \sum_{i=0}^{k-1} ((1-\beta) b_{s,i} - \beta (1-\alpha)^{k-1-i} \alpha) x_i \quad (40)$$

$$Dev(s1) = \beta\left(-x_k + \sum_{i=0}^{k-1} a_{k-1,i} x_i\right) + (1-\beta) \sum_{i=0}^{k-1} b_{s,i} x_i = -\beta x_k + \sum_{i=0}^{k-1} \beta a_{k-1,i} x_i + \sum_{i=0}^{k-1} b_{s,i} x_i - \sum_{i=0}^{k-1} \beta b_{s,i} x_i = -\beta x_k + \sum_{i=0}^{k-1} (\beta a_{k-1,i} + b_{s,i} - \beta b_{s,i}) x_i = -\beta x_k + \sum_{i=0}^{k-1} ((1-\beta) b_{s,i} + \beta (1-\alpha)^{k-1-i} \alpha) x_i \quad (41)$$

The recursion for coefficients $b_{s,i}$:

$$b_{\varepsilon,0} = \beta,\ b_{s0,|s|+1} = \beta,\ b_{s1,|s|+1} = -\beta \quad (42)$$

$$b_{s0,i} = (1-\beta) b_{s,i} - \beta (1-\alpha)^{|s|-i} \alpha,\ i = 0, \dots, |s| \quad (43)$$

$$b_{s1,i} = (1-\beta) b_{s,i} + \beta (1-\alpha)^{|s|-i} \alpha,\ i = 0, \dots, |s| \quad (44)$$