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Research paper

Practical problems of dynamic similarity criteria in fluid–solid interaction at different fluid–solid relative motions

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Abstract: The work concerns dynamic similarity criteria of various phenomena occurring in hydraulics and fluid dynamics originally derived from ratios of forces and forces moments affecting these phenomena. The base of dynamic similarity criteria formulations and considerations is A. Flaga's method and procedure for determining dynamic similarity criteria in different issues of fluid–solid interactions i.e. at different fluid–solid relative motions. The paper concerns the determination and analysis of dynamic similarity criteria for various practical problems encountered mainly in hydraulics and fluid dynamics at steady, smooth fluid onflow in front of a solid. Moreover, the cases of mechanically induced vibrations of a body in a stationary fluid moving with constant velocity in front of the body have been presented. Assuming authorial method and procedure for determining dynamic similarity criteria, its have been presented and analysed in the paper both well known similarity numbers obtained in another way (e.g. from dimensional analysis or differential equations for particular problems – as Reynolds, Froude, Euler, Cauchy, Strouhal, Mach numbers) – as well as several new similarity numbers encountered in different fluid solid interaction problems (e.g. new forces and moments coefficients encountered in problems of vibrating solid bodies in fluids).

Keywords: dynamic similarity criteria, fluid–solid interaction, fluid–solid relative motions

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1. Introduction

The paper concerns the determination and analysis of dynamic similarity criteria for various practical problems encountered mainly in hydraulics and fluid dynamics at steady, smooth fluid onflow in front of a solid.

Moreover, the cases of mechanically induced vibrations of a body in a stationary fluid moving with constant velocity in front of the body have been presented.

Dynamic similarity criteria have been originally derived from ratios of forces and forces moments affecting considered phenomena. The base of dynamic similarity criteria formulations and considerations is A. Flaga's method and procedure for determining dynamic similarity criteria in different issues of fluid–solid interactions i.e. at different fluid–solid relative motions [1].

Assuming consequently these method and procedure, a comprehensive statement of dynamic similarity criteria in fluid-body interaction with various relative motions of body and fluid have been achieved.

There are many different ways of formulating and defining similarity criteria in engineering. Dimensional analysis and theory of similarity of physical phenomena occurring in various issues of engineering, can be found e.g. in the following books, monographs and publications devoted to:

– Hydro-mechanics, hydraulics and fluid dynamics i.e. e.g.:
  - similarity, dimensional analysis and critical numbers at different fluid–solid relative motions [2],
  - similarity criteria for fluid flows in conduits, fluid flows in channels, floating objects, in turbomachinery [3],
  - similarity criteria and modelling in fluid mechanics [4],
  - similarity and dimensional analysis in mechanics [5];
– Wind engineering and aerodynamics of buildings and structures i.e., e.g.:
  - similarity numbers important in problems of wind action on building structures encountered in their design [6],
  - original similarity criteria elaborated and analyzed by Flaga A. and Flaga et al. for different special issues e.g.: wind vortex-induced excitation and vibration of slender structures [7]; resistant of freight railway vehicles to roll-over in strong winds [8]; linear building objects at aerodynamic and gravitational actions [9]; wind tunnel model tests of two free-standing lighting protection masts [10]; wind tunnel model tests of wind action on the chimney with grid-type curtain structure [11]; sectional model of power line free-cable bundlet conductors at their aeroelastic vibrations [12]; relation between shape and phenomenon of flutter of bridge deck-line bluff bodies [13],
  - similarity criteria in aeroelastic model tests of flutter phenomenon for cable-stayed bridges [14] and suspension bridges [15],
  - similarity methods in engineering dynamics: theory and practice of scale modelling [16];
– Different issues of structural mechanics, engineering dynamics, thermomechanics, aircrfat flowing qualities i.e., e.g.:
  - problems of scalling, self-similarity and intermediate asymptotes [17],

  - hull girder ultimate strength assessment based on experimental results and the dimensional theory which maintains the first-order similarity between the model and real structures. [18],
  - design of scaled down model for structural vibration analysis of a tower crane mast by using similitude theory as well as numerical modal analyses for prototype and two scaled models [19],
  - dimensional and similitude analysis of stiffened panels under longitudinal compression considering buckling behaviours [20],
  - similarity criteria for thin-walled cylinders subjected to coupled thermo-mechanical loads including thermo-elastoplastic failure behaviors predicted by numerical model validated by experiments [21],
  - flying qualities criteria for scaled-model aircraft based on similarity theory taking into account relations between configuration parameters, control law parameters, and flight condition parameters [22];
- Environmental engineering i.e. e.g.:
  - environmental effects on buildings, structures and people: actions, influences, interactions, discomfort [23],
  - similarity criteria and problems of their fulfilment in different issues of environmental engineering [24];
- Wind turbines i.e., e.g.:
  - original similarity criteria for authorial models of different types of vertical axis and horizontal axis wind rotors [25],
  - modelling and the performance of different wind turbines, e.g. diffuser augmented wind turbine, stepped blade wind turbine, vertical axis wind turbine [26], cross-flow wind turbine above windbreak fence [27],
  - modelling and experimental investigations of the performance of a cross-flow wind turbine with and without diffuser [28];
- Snow engineering i.e., e.g.:
  - snow load distributions on different stadium roofs [29],
  - aerodynamic and aeroelastic similarity criteria for wind tunnel model tests of overhead power lines in triangular configuration under different icing conditions [30],
  - similarity criteria for wind tunnel model tests of snow precipitation and snow redistribution on rooftops [31], terraces and in the vicinity of high rise buildings [32].

A dimensional analysis can be carried out e.g. in relation to general functional relationships describing the phenomenon [33,34]. After carrying out dimensional analysis on these functional relationships, in obtained dimensionless functional relationships appear set of nondimensional numbers – being monomials created from main dimension or/and dimensional quantities (i.e. variables and parameters) characterizing this phenomenon – which constitute the respective similarity criteria.

A dimensional analysis can also be carried out by considering the ratios of forces or moments of forces describing a given dynamic problem [1]. Because the dimensions of such quantities are similar, hence their ratios are dimensionless and as such ones constitute specific criteria/numbers of dynamic similarity of the analyzed issues. That is what this work is about.

2. The ratio of global inertia force to global viscous force for fluid flows and flows past objects – Reynolds number Re

In practical applications of fluid flows or flows past a solid body in a space domain of characteristic dimension D_f, there are several situations when global inertia and viscous forces are the most important. In such cases, the velocity field and the pressure field in this domain are strictly dependent on the ratio of the global inertia force to the global viscous force. Let us consider several examples.

In the flow of a fluid through a completely filled pipe, gravity does not affect the flow pattern. It is also obvious that capillarity is of no practical importance, and hence the significant forces are inertia and fluid friction due to viscosity. The same is true for an airplane traveling at speeds below the one at which air compressibility is appreciable. Similarly, for a submarine submerged deep enough so that it does not produce waves on the surface, the only forces involved are those of friction and inertia.

Considering the ratio of inertia forces to viscous forces, the parameter obtained is called the Reynolds number Re in honour of Osborne Reynolds, who presented it in a publication of his experimental work in 1882 [4].

The same criterial number was obtained ten years later by Lord Rayleigh, who developed the theory of dynamic similarity [5]. The ratio of these two forces is:

$$F_{vf}^{*if} = \frac{F_{if}}{F_{vf}} \triangleq Re = \frac{\rho_f V_f^2 D_f^2}{\mu_f V_f D_f} = \frac{\rho_f V_f D_f}{\mu_f} = \frac{V_f D_f}{\nu_f} \tag{2.1}$$

$\nu_f = \mu_f/\rho_f$ – kinematic viscosity coefficient; $\triangleq$ – sign indicating that the respective relationships are equal or equivalent with regard to the dimensions.

For any consistent system of units, Re is a dimensionless number. The linear dimension D_f may be any length that is significant in the flow pattern. Thus, for a pipe completely filled, it might be either the diameter or the radius (e.g. hydraulics radius), and the numerical value of Re will vary accordingly.

If two systems, such as a prototype and its model (e.g. two pipelines with different fluids), are to be dynamically equivalent so far as inertia and viscous friction are concerned, they must both have the same value of Re. For the same fluid in both cases, the equation shows that a high velocity must be used with a model of small linear dimensions. It is also possible to compare the action of fluids of very different viscosities provided only that D_f and V_f are chosen so as to give the same value of Re.

Because surface pressures acting on a solid body immersed in a fluid depend on the Re number, aerodynamic or hydrodynamic forces and moments resulting from it are also Reynolds number dependent.

In the case of flow past a solid body of a characteristic dimension D_s, it is usually assumed that $D_f = D_s = D$.

If the Reynolds numbers of a prototype and its model are the same, the expressions for the scales of velocity k_V, time k_t, acceleration k_a, force k_F and pressure k_p should be as follows:

$$\text{Re} = \frac{D_M V_M}{\nu_M} = \frac{D_P V_P}{\nu_P} \tag{2.2}$$

$$k_V = \frac{V_P}{V_M} = \frac{D_M \nu_P}{D_P \nu_M} = \frac{k_\nu}{k_D} \tag{2.3}$$

$$k_t = \frac{k_D}{k_V} = \frac{k_D^2}{k_\nu} \tag{2.4}$$

$$k_a = \frac{k_V}{k_t} = \frac{k_\nu}{k_D^3} \tag{2.5}$$

$$k_F = k_\rho k_V^2 k_D^2 = k_\rho k_\nu^2 \tag{2.6}$$

$$k_p = k_F k_D^{-2} = k_\rho k_\nu^2 k_D^{-2} \tag{2.7}$$

Illustrative example.

A rigid, axially symmetrical body of length $D_p = 3.0$ m is to be towed deep under water of temperature 15° with velocity $V_p = 4\ \text{m·s}^{-1}$. In order to determine the force required for towing, a model of length $D_M = 0.6$ m has been created and tested in a wind tunnel at air velocity 20 m/s and temperature 15°. The absolute viscosity of water in such conditions is 62.5 times greater than the viscosity of air.

The air drag force equal to 100 N has been determined on the basis of the tests in the wind tunnel.

The adequate similarity criterion in the analysed case is the Reynolds criterion.

The model has been executed in the scale: $k_D = 3.0/0.6 = 5$. The velocity and absolute viscosity scales have also been determined: $k_V = 4/20 = 0.2$; $k_\mu = 62.5$. Since the relation $\nu = \mu/\rho$ results in $k_\nu = k_\mu/k_\rho$, the condition $\text{Re}_P = \text{Re}_M$ may be satisfied by adequate selection of viscosity scale $k_\nu = \dfrac{\nu_P}{\nu_M}$. This quantity may be influenced by changing air pressure in model tests. Thus the necessity to use a pressurised wind tunnel becomes evident. Let us first calculate the required air pressure in the tunnel.

The equality of Reynolds numbers results in the following dependency for scales of relevant quantities:

$$\frac{k_\rho k_V k_D}{k_\mu} = 1 \rightarrow k_\rho = \frac{k_\mu}{k_V k_D} = \frac{62.5}{0.2 \cdot 5} = 62.5$$

Given the water density: $\rho_P = 1000\ \text{kg/m}^3$, the required air density is:

$$\rho_M = \frac{\rho_P}{k_\rho} = \frac{1000}{62.5} = 16\ \text{kg} \cdot \text{m}^{-3}$$

In temperature equal to 15°, the same air density will be obtained under the pressure of 1.2 MPa. Hence, the model should be tested in a pressurised wind tunnel under precisely this pressure.

We use formula (2.6) to calculate the scale of forces:

$$k_F = k_\rho k_V^2 k_D^2 = 62.5 \cdot 0.2^2 \cdot 5^2 = 62.5$$

Finally, we arrive at the conclusion that towing the object under water requires force:

$$F_P = k_F F_M = 62.5 \cdot 100 = 6250\ \text{N} = 6.25\ \text{kN}$$

3. The ratio of global inertia force to global gravity force – Froude number Fr

A typical phenomenon related to the action of gravity is creation of waves on the free surface of a fluid. They are the source of wave resistance which meets each body sliding on this surface or floating partially immersed. Obviously, in both cases, viscous resistance also has its role in the total resistance.

Considering global inertia and gravity forces alone, a ratio is obtained called a Froude number Fr in honour of William Froude, who experimented with flat plates towed lengthwise through water in order to estimate the resistance of ships due to wave action [4]. The ratio of global inertia force to gravity force is:

$$F_{gf}^{if} = \frac{F_{if}}{F_{gf}} \triangleq \mathrm{Fr} = \frac{\rho_f V_{rel}^2 D_f^2}{g \rho_f D_f^3} = \frac{V_{rel}^2}{g D_f}; \quad V_{rel} = \begin{cases} V_s \\ V_t \\ V_s - V_t \end{cases} \tag{3.1}$$

In this case, the characteristic dimension D_f is usually taken as a reservoir depth D_d or a body dimension D_s.

Although it is sometimes defined as the Froude number, it is more common to use the square root so as to have V_{rel} in the first power, as in the Reynolds number. Thus the Froude number is:

$$\mathrm{Fr} = \frac{V_{rel}}{\sqrt{g D_f}} \tag{3.2}$$

Systems involving gravity and inertia forces are [4]: the wave action induced by a ship, the flow of water in open channels, the forces of a stream on a bridge pier, the flow over a spillway, the flow of a stream from an opening, and other cases where gravity is a dominant factor.

A comparison of relationships (2.1) and (3.2) shows that the two cannot be satisfied at the same time with the fluid of the same viscosity, since one requires that the velocity vary inversely as D_f, while the other requires it to vary directly as $\sqrt{D_f}$. In this sense, the Reynolds and Froude criteria must be deemed mutually contradictory. If both friction and gravity are involved, it is necessary to decide which of the two factors is more important or more useful. In the case of a ship [4], towing of a model will give the total resistance, from which must be subtracted the empirically computed skin friction in order to determine the wave-making resistance, and the latter may be smaller than the former. But for the same Froude number, the wave-making resistance of the full-size ship may be determined from this result. A computed skin friction for the ship is then to be added to this value to give the total ship resistance.

If we need to find the resistance of a ship to be designed, we may also proceed in the following way: we may determine it on the grounds of model tests performed at relevant velocity V_{Fr}, and the correction accommodating the influence of viscosity is then estimated by extrapolation of results obtained with several models in various scales.

Transferring the results of measurements obtained with a model onto the object, with the Froude number preserved, we use the following formulas for scales:

– of velocity

$$k_V = \sqrt{k_D} \tag{3.3}$$

– time (e.g. wave period)

$$k_t = k_D k_V^{-1} = \sqrt{k_D} \tag{3.4}$$

– forces

$$k_P = k_\rho k_V^2 k_D^2 = k_\rho k_D^3 \tag{3.5}$$

In the flow of water in open channels [5], fluid friction is a factor, as well as gravity and inertia, and, apparently, we face the same difficulty here. However, for flow in an open channel there is usually fully developed turbulence, so that the hydraulic friction loss is exactly proportional to V_f^2, as will be shown later. Thus, fluid friction in open channels is independent of the Reynolds number, with rare exceptions, and, therefore, it is a function of the Froude number alone.

The only way to satisfy Eqs. (2.1) and (3.2) for both the prototype and its model is to use fluids of very different viscosities in the two cases. Sometimes this can be done, but often it is either impractical or impossible.

For the computation of Fr, the length D must be some linear dimension that is significant in the flow pattern. For a ship, it is commonly taken as the length at the waterline. For an open channel, it is taken as the depth of flow.

Illustrative example (comp. [5]).

A 1:50 model of a boat has a wave resistance of 0.02 when operating at 1.0 m/s. Let us determine the corresponding prototype wave resistance, the engine power requirement for the prototype and the velocity to which this test corresponds in the prototype.

Gravity and inertia forces predominate; hence the Froude criterion is applicable:

$$\mathrm{Fr}_P = \mathrm{Fr}_M \rightarrow \left(\frac{V}{\sqrt{gD}}\right)_P = \left(\frac{V}{\sqrt{gD}}\right)_M \rightarrow \frac{V_P^2}{D_P} = \frac{V_M^2}{D_M}$$

Thus:

$$\frac{V_P^2}{V_M^2} = k_V^2 = \frac{D_P}{D_M} = k_D = 50$$

Since:

$$k_F = k_\rho k_V^2 k_D^2 = k_D^3$$

therefore:

$$F_P = k_D^3 F_M = 50^3 \cdot 0.02 = 2500\ \mathrm{N} = 2.5\ \mathrm{kN}$$

$$V_P = \sqrt{k_D} \cdot V_M = \sqrt{50} \cdot 1 = 7.1\ \mathrm{m/s}$$

$$P_P = F_P \cdot V_P = 2.5 \cdot 7.1 = 17.75\ \mathrm{kNm/s} = 17.75\ \mathrm{kW}$$

4. The ratio of global wave resistance force for a floating body (e.g. ship) to global fluid inertia force

Let us consider the problem of wave resistance of a body moving on the surface of a wavy fluid which otherwise remains stationary. Let the body of characteristic dimension D_s move with velocity V_s, whereas the surface wave running towards the body has the following parameters: V_w – wave velocity; A_w – wave amplitude; λ_w – wave length.

Let us define the similarity number resulting in this case from the relation of the global wave resistance force F_w to the global fluid inertia force F_{if}:

$$F_{if}^{w} = \frac{F_w}{F_{if}} \triangleq \pi_{if}^{w} = \frac{A_w \lambda_w^{-1} \rho_f V_w^2 D_s^2}{\rho_f V_s^2 D_s^2} = \frac{A_w}{\lambda_w} \cdot \left(\frac{V_w}{V_s}\right)^2 \tag{4.1}$$

It may be seen from the above relations that the characteristic numbers here are the following numbers: A_w/λ_w and V_w/V_s.

5. The ratio of the solid/particle local surface pressure forces to the fluid local inertia forces – Euler number Eu, pressure coefficient *Cp*

A dimensionless quantity related to the ratio of the local surface pressure forces $F_{\Delta po}$ to the local inertia forces F_{if} is known as the Euler number Eu. It is expressed in a variety of ways, one form being:

$$F_{if}^{\Delta p_o} = \frac{F_{\Delta po}}{F_{if}} \triangleq \mathrm{Eu} = \frac{\Delta p_o d^2}{\rho_f V_f^2 d^2} = \frac{\Delta p_o}{\rho_f V_f^2} \tag{5.1}$$

where: $d = d_f = d_s = d_p$.

In aerodynamics and hydrodynamics of various engineering objects, the Eu number is frequently replaced by the so-called pressure coefficient C_p, also dimensionless, defined in the following way:

$$C_p = \frac{\Delta p_o}{\frac{1}{2}\rho_f V_f^2} = \frac{\Delta p_o}{q_f} \tag{5.2}$$

where: $q_f = \frac{1}{2}\rho_f V_f^2$ – so-called stream / wind velocity pressure or dynamic pressure.

Both Eu and C_p numbers are important criteria of dynamic similarity, but mostly in incompressible flow, because pressure and density are treated here as independent parameters. Both these numbers depend primarily on the Re number.

In dynamics of gases – when a gas flow is accompanied by small pressure changes – it is more customary to use the so-called pressure coefficient C_p defined in the following way:

$$C_p = \frac{\Delta p_f}{\frac{1}{2}\overline{\rho} V_f^2} \tag{5.3}$$

where: $\overline{\rho}$ signifies average density within a pressure range $\left(p_f, -\Delta p_f; p_f + \Delta p_f\right)$.

If only pressure and inertia influence the flow, the Euler number for any boundary form will remain constant. However, if other parameters (viscosity, compressibility, gravity etc.) cause the flow pattern to change, Eu will also change.

6. The ratio of local surface pressure forces to local surface viscosity forces

In this case, the similarity number takes the form:

$$F_{vo}^{\Delta po} = \frac{F_{\Delta po}}{F_{vo}} \triangleq \frac{\Delta p_o d_o^2}{\mu_f V_f d_o} = \frac{\Delta p_o d_o}{\mu_f V_f} = \frac{C_p \cdot \frac{1}{2}\rho_f V_f^2 d_o}{\mu_{fs} V_f} = \frac{1}{2} C_p \cdot \mathrm{Re} \tag{6.1}$$

Since the values of the pressure coefficient are numbers of the order of 1, the above relation indicates that local viscosity forces for Reynolds numbers greater than 10 (definite majority of cases in applied aerodynamics and hydrodynamics) are generally negligibly small in comparison to local surface pressure forces. The exception are very slow flows past bodies of small dimensions (e.g. material particles or microparticles).

For a solid body moving at velocity V_s in a reservoir of stationary fluid, V_f should be replaced by V_s in the above relation.

7. The ratios of the solid/particle global surface pressure forces and moments to the fluid global inertia forces and moments – forces and moments aerodynamic/ hydrodynamic coefficients C_{oj} and C_{moj}

– Aerodynamic or hydrodynamic forces coefficients:

$$F_{if}^{\Delta poj} = \frac{F_{\Delta poj}}{F_{if}} \triangleq \frac{\Delta_{po}^* \cdot C_{oj}^* D_o^2}{\rho_f V_f^2 D_o^2} = 2C_p^* C_{oj}^* = C_{oj}^{**}\left(\mathrm{Re}, \left(\check{G}\right)\right) \tag{7.1}$$

where: $\left(\check{G}\right)$ – a set of dimensionless geometrical parameters characterising the geometry of the solid body/material particle.

In practical applications, the above relation is rewritten to the following form:

(7.2) $$F_{\Delta poj} = \frac{1}{2}\rho_f V_f^2 D_o^2 C_{oj}; \quad C_{oj} = C_{oj}\left(\text{Re}, \left(\check{G}\right)\right)$$

where: C_{oj} – aerodynamic / hydrodynamic coefficients of the body / particle forces: C_x – resistance coefficient, C_y, C_z – coefficients of side and vertical forces.

– Aerodynamic or hydrodynamic moments coefficients:

(7.3) $$M_{if}^{\Delta poj} = \frac{M_{\Delta poj}}{M_{if}} \triangleq \frac{\Delta_{po}^* \cdot C_{moj}^* D_o^3}{\rho_f V_f^2 D_o^3} = 2C_p^* C_{moj}^* = C_{moj}^{**}\left(\text{Re}, \left(\check{G}\right)\right)$$

(7.4) $$M_{\Delta poj} = \frac{1}{2}\rho_f V_f^2 D_o^3 C_{moj}; \quad C_{moj} = C_{moj}\left(\text{Re}, \left(\check{G}\right)\right)$$

where: C_{moj} – aerodynamic / hydrodynamic coefficients of the body / particle moments: C_{mx} – rotation moment coefficient; C_{my} – pitch moment coefficient; C_{mz} – yaw moment coefficient.

8. The ratios of the solid/particle global surface roughness/ friction forces and moments to the fluid global inertia forces and moments – aerodynamic/hydrodynamic forces and moments coefficients C_{roj} and C_{mroj} of surface roughness/friction

The surface roughness of a solid may significantly influence the values of aerodynamic/hydrodynamic forces and moments exerted on the solid by fluid. In such case, other characteristic numbers related to the solid's surface roughness must also be taken into account alongside the Reynolds number, e.g. defined as:

– Forces surface roughness/friction coefficients:

(8.1) $$F_{if}^{roj} = \frac{F_{roj}}{F_{if}} \triangleq \frac{\mu_{ro}^* \cdot \Delta_{po}^* \cdot C_{roj}^* D_o^2}{\rho_f V_f^2 D_o^2} = 2C_p \mu_{ro}^* C_{roj}^* = C_{roj}^{**}\left(\text{Re}, \left(\check{G}\right)\right)$$

(8.2) $$F_{roj} = \frac{1}{2}\rho V_f^2 D_o^2 C_{roj}; \quad C_{roj} = C_{roj}\left(\text{Re}, \left(\check{G}\right)\right)$$

where: C_{roj} – forces surface roughness coefficients of a solid/particle;

– Moments surface roughness/friction coefficients:

(8.3) $$M_{if}^{roj} = \frac{M_{roj}}{M_{if}} \triangleq \frac{\mu_{ro}^* \cdot \Delta_{po}^* \cdot C_{mroj}^* D_o^3}{\rho_f V_f^2 D_o^3} = 2C_p^* \mu_{ro}^* C_{mroj}^* = C_{mroj}^{**}\left(\text{Re}, \left(\check{G}\right)\right)$$

(8.4) $$M_{roj} = \frac{1}{2}\rho V_f^2 D_o^3 C_{mroj}; \quad C_{mroj} = C_{mroj}\left(\text{Re}, \left(\check{G}\right)\right)$$

where: C_{mroj} surface roughness / friction moments coefficients of a solid / particle.

In the case of a moving solid body in a reservoir of otherwise stationary fluid, fluid velocity V_f should be replaced by the solid/particle velocity V_o.

The roughness of a model should be scaled down in the same ratio as the other linear dimensions, which means that a small model should have surfaces that are much smoother than those in its prototype. However, this requirement imposes a limit on the scale that can be used if true geometric similarity is to be had. Yet, in the case of a river model with a vertical scale larger than the horizontal scale, it may be necessary to make the model surface rough in order to simulate the flow conditions in the actual stream [5]. As any distorted model lacks the proper similitude, no simple rule can be given for this; the roughness should be determined by trial and error until the flow conditions are judged to be typical of those in the prototype.

9. The ratio of local inertia forces to local elastic forces of fluid – Cauchy number Ca, Mach number Ma

Where compressibility of a fluid is important, it is necessary to consider the ratio of the fluid local inertia forces F_{if} to the fluid local elastic forces F_{ef}, called the Cauchy number Ca.

Thereby:

$$F_{ef}^{if} = \frac{F_{if}}{F_{ef}} \triangleq \mathrm{Ca} = \frac{\rho_f V_f^2 d_f^2}{K_f d_f^2} = \frac{\rho_f V_f^2}{K_f} \tag{9.1}$$

Taking into account that the acoustic wave velocity C_f (or celerity) in the medium in question is:

$$c_f = \sqrt{\frac{K_f}{\rho_f}} \tag{9.2}$$

The Cauchy number can also be obtained by:

$$\mathrm{Ca} = \frac{V_f^2}{c_f^2} = \left(\frac{V_f}{c_f}\right)^2 = \mathrm{Ma}^2 \tag{9.3}$$

where: Ma – the so-called Mach number, named in honour of the Austrian scientist Mach.

If Ma is less than 1, the flow is called subsonic; if it is equal to 1, the flow is sonic; if it is greater than 1, the flow is called supersonic; and for extremely high values of Ma the flow is called hypersonic.

It is possible to satisfy the Mach and Reynolds criteria simultaneously in two air flows of the same temperature T. Hence, it is possible to satisfy two equations:

$$\frac{k_V}{k_c} = 1, \quad \frac{k_D k_V}{k_\nu} = 1 \quad \text{for} \quad k_T = 1 \tag{9.4}$$

Given the fact that the kinematic viscosity scale $k_\nu = k_\mu / k_\rho$, having eliminated k_V, we obtain the following from formula (9.4):

$$\frac{k_\rho k_D k_c}{k_\mu} = 1 \tag{9.5}$$

Dynamic viscosity of fluids is strongly dependent on temperature, with proportionality $\mu \sim T$ valid for air, so $k_\mu = k_T$. The relevant dependency for sound velocity takes the form $c \sim \sqrt{T}$, so $k_c = \sqrt{k_T}$. Substituting these relationships in (9.4) gives $k_\rho k_D / \sqrt{k_T} = 1$, or – for $k_T = 1$:

$$k_\rho k_D = 1 \tag{9.6}$$

The non-contradictory character of the Mach and Reynolds criteria for two air flows of the same temperature has thus been demonstrated. In laboratory practice, the air density in a model flow, and thus the density scale k_ρ, may be altered within a fairly large range by changing pressure p. Therefore, aerodynamic tests are then performed in pressurised tunnels.

For example [5], when modelling a subsonic airplane in a wind tunnel, it is commonly necessary to conduct the test under high pressure in order to satisfy the Reynolds criterion without introducing compressibility effects. Suppose e.g. that $k_D = D_P / D_M = 20$. If viscosity μ and density ρ of the air were the same in the model and prototype, then to satisfy Reynolds' criterion, $V_M = 20 V_P$. For an airplane operating at normal speed, this would make the model Mach number much greater than one, and compressibility effects would invalidate the behaviour of the model. If, however, the test were conducted under the pressure of 20 atm with identical model and prototype temperatures, $\rho_M = 20 \rho_P$ and $\mu_M \approx \mu_P$ since viscosity of air changes very little with pressure (or density). In this case, the model should be operated at a velocity equal to that of the prototype in order for the Reynolds numbers to be the same.

On the other hand, for small velocities of gas flow (approximately for Ma < 0.3), the similarity of pressure fields is obtained by satisfying the Euler criterion.

10. Ratios of global forces and characteristic numbers resulting therefrom in the case of moderately fast or fast rising or falling of solid bodies or material particles immersed in a stationary fluid

The decisive factors in the considered case are the following: the body/particles weight forces F_{go}, the buoyant forces exerted by fluid on the body/particle F_{bo} and the fluid inertia forces F_{if}. Given the above, we may introduce other characteristic numbers defined as follows:

$$F_{if}^{gb} = \frac{F_{gb}}{F_{if}} \triangleq \begin{cases} \dfrac{(\rho_o - \rho_f)\, g D_o^3}{\rho_f V_o^2 D_o^2} = \dfrac{\Delta\rho}{\rho_f} \cdot \dfrac{g D_o}{V_o^2} \\ \dfrac{(\rho_o - \rho_f)\, V_t^2 D_o^2}{\rho_f V_o^2 D_o^2} = \dfrac{\Delta\rho}{\rho_f} \cdot \left(\dfrac{V_t}{V_o}\right)^2 \end{cases} \tag{10.1}$$

Three new characteristic numbers have appeared in the above relations:

– dimensionless parameter of mass

$$\pi_\rho = \frac{\Delta\rho}{\rho_f} \tag{10.2}$$

– dimensionless parameter of velocity

$$V_o^t = \frac{V_t}{V_o} \tag{10.3}$$

– Froude's number (or its reverse) referred to the body / particle

$$\mathrm{Fr}_o = \frac{V_o}{\sqrt{gD_o}} \tag{10.4}$$

If we assume the characteristic velocity V_o as the terminal velocity, then $V_o^t = 1$, and

$$\mathrm{Fr}_o = \mathrm{Fr}_t = \frac{V_t}{\sqrt{gD_o}} \tag{10.5}$$

11. Ratios of forces and characteristic numbers resulting therefrom in cases of solid bodies vibrating in fluids

Let us consider this problem using the example of a solid body translational-rotational vibrations of three degrees of freedom: translational ξ, ζ, and rotational ε, immersed first in a stationary and then in a moving fluid. The adopted model of the vibrating body is illustrated in Fig. 1. It is an elastically and viscously supported rigid body. We assume that the excitation of the system has its static (mean) components – e.g. resulting from gravitation – and dynamic (fluctuating) components. We shall consider solely the dynamic components of excitation, the ones that generate vibrations of the system around its static balance position.

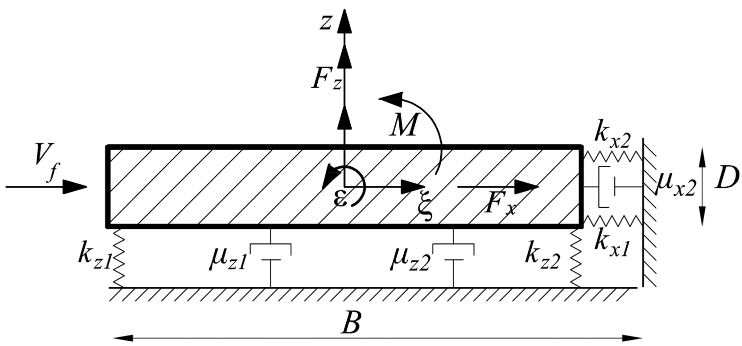

Fig. 1. A mass-spring-dashpot system in a flowing fluid

11.1. The case of induced vibrations of a solid body in a stationary fluid (outside the fluid-body contact area) – $V_f = 0$

The following sets of components of global forces and moments must be taken into account in the considered case:

– the ones related to the elastic properties of the body supports

$$(F_{es}) = (F_{kx}, F_{kz}, M_{k\varepsilon}) \tag{11.1}$$

– the ones related to the damping (viscous) properties of the body supports

$$(F_{ds}) = \left(F_{\mu x}, F_{\mu z}, M_{\mu\varepsilon}\right) \tag{11.2}$$

– the ones related to the damping (viscous) properties of the fluid surrounding the body at their relative motion

$$\left(F_{dsf}\right) = \left(F_{\mu xf}, F_{\mu zf}, M_{\mu\varepsilon f}\right) \tag{11.3}$$

It is assumed that the resultant forces and the viscous damping moment of vibrations are as follows:

$$(F_{d\Sigma}) = \left(F_{\mu x} + F_{\mu xf}; F_{\mu z} + F_{\mu zf}; M_{\mu\varepsilon} + M_{\mu\varepsilon f}\right) \tag{11.4}$$

– the ones related to the body inertia

$$(F_{is}) = (F_{ix}, F_{iz}, M_{i\varepsilon}) \tag{11.5}$$

– the ones related to the added fluid mass

$$\left(F_{if}\right) = \left(F_{ixf}, F_{izf}, M_{i\varepsilon f}\right) \tag{11.6}$$

It is assumed that the resultant forces and the body inertia forces moment are as follows:

$$(F_{i\Sigma}) = \left(F_{ix} + F_{ixf}; F_{iz} + F_{izf}; M_{i\varepsilon} + M_{i\varepsilon f}\right) \tag{11.7}$$

– the ones related to the induction of the system vibrations

$$(F_a) = (F_x, F_z, M) \tag{11.8}$$

while:

$$F_x = F_x(t) = F_{ox}\check{F}_x(t; (G_x)) = F_{ox}\check{F}_x^*\left(\check{t}; \left(\check{G}_x\right)\right) \tag{11.9}$$

$$F_z = F_z(t) = F_{oz}\check{F}_z(t; (G_z)) = F_{oz}\check{F}_z^*\left(\check{t}; \left(\check{G}_z\right)\right) \tag{11.10}$$

$$M = M(t) = M_o\check{M}(t; (G_\varepsilon)) = M_o\check{M}^*\left(\check{t}; \left(\check{G}_\varepsilon\right)\right) \tag{11.11}$$

where: F_{ox}, F_{oz}, M_o – excitation amplitudes (or other characteristic dimensional quantities of excitation); t – time; $\check{t} = \dfrac{V_z t}{B}$ – dimensionless time; (G_x), (G_z), (G_ε); $\left(\check{G}_x\right)$, $\left(\check{G}_z\right)$, $\left(\check{G}_\varepsilon\right)$ – dimensional and dimensionless sets of excitation parameters; $\check{F}_x^*(\ldots)$, $\check{F}_z^*(\ldots)$, $\check{M}^*(\ldots)$ – dimensionless functions of dimensionless time and dimensionless excitation parameters which constitute specified function similarity criteria for excitation.

Parameters characterising mechanical properties of the considered problem may be classified in the following way:

– elastic properties: elastic supports rigidity

(11.12) $$(k_s) = (k_x, k_z, k_\varepsilon)$$

– damping properties: body and fluid viscous damping coefficients

(11.13) $$(\mu_s) = (\mu_x, \mu_z, \mu_\varepsilon)\,; \quad \left(\mu_f\right) = \left(\mu_{xf}, \mu_{zf}, \mu_{\varepsilon f}\right)$$

– inertia properties: body and fluid masses and mass inertia moments

(11.14) $$(m_s) = (m, m, I)\,; \quad \left(m_f\right) = \left(m_{xf}, m_{zf}, I_f\right)$$

– excitation characteristics: excitation amplitudes

(11.15) $$(F_{os}) = (F_{ox}, F_{oz}, M_o)$$

– body characteristic dimensions

(11.16) $$(D_s) = (D, B, L)\,;\ \Omega_s$$

where: D, B – body cross-sectional dimensions; L – body length; Ω_s – body volume,

– body and fluid mass densities

(11.17) $$(\rho) = \left(\rho_s, \rho_f\right)$$

while:

(11.18) $$\rho_s \Omega_s = m$$

– body characteristic velocity

(11.19) $$V_s = V_z$$

In our further considerations we shall determine the ratios between components of relevant forces and moments of forces with reference to one of the component forces so that they become dimensionless quantities. The dimensionless numbers related to them will be dynamic similarity numbers adequate for the analysed problem. We shall adopt the vertical component of the body inertia force and the added fluid mass as the reference force, i.e.:

(11.20) $$F_{iz\Sigma} = F_{iz} + F_{izf}$$

However, as the dimensional base, we shall adopt a three-item set: (m_Σ, V_z, B), where:

(11.21) $$m_\Sigma = m + m_{zf}$$

We shall then obtain the following relationships:

– Characteristic numbers related to the body and fluid inertia

$$F^{ix\Sigma}_{iz\Sigma} = \frac{F_{ix\Sigma}}{F_{iz\Sigma}} \triangleq \pi^{ix\Sigma}_{iz\Sigma} = \frac{\left(m + m_{xf}\right) V_z^2 B^{-1}}{\left(m + m_{zf}\right) V_z^2 B^{-1}} = \frac{1 + \dfrac{m_{xf}}{m}}{1 + \dfrac{m_{zf}}{m}} \tag{11.22}$$

$$F^{i\varepsilon\Sigma}_{iz\Sigma} = \frac{F_{i\varepsilon\Sigma}}{F_{iz\Sigma}} = \frac{M_{i\varepsilon}}{BF_{iz\Sigma}} \triangleq \pi^{i\varepsilon\Sigma}_{iz\Sigma} = \frac{\left(I + I_f\right) V_z^2 B^{-2}}{\left(m + m_{zf}\right) V_z^2} = \frac{I_\Sigma}{m_{z\Sigma} B^2} \tag{11.23}$$

Let us introduce the following additional designations and dependencies:

$$m_{xf} = \rho_f \Omega_s \varkappa_x = m_f \varkappa_x \tag{11.24}$$

$$m_{zf} = \rho_f \Omega_s \varkappa_z = m_f \varkappa_z \tag{11.25}$$

$$I = mB^2 \varkappa = \rho_s \Omega_s B^2 \varkappa \tag{11.26}$$

$$I_f = \rho_f \Omega_s B^2 \varkappa_\varepsilon \tag{11.27}$$

Then the characteristic numbers $\pi^{ix\Sigma}_{iz\Sigma}$ and $\pi^{i\varepsilon\Sigma}_{iz\Sigma}$ may also be expressed as:

$$\pi^{ix\Sigma}_{iz\Sigma} = \frac{1 + \dfrac{\rho_f}{\rho_s} \varkappa_x}{1 + \frac{\rho_f}{\rho_s} \varkappa_z} \tag{11.28}$$

$$\pi^{i\varepsilon\Sigma}_{iz\Sigma} = \frac{\left(\varkappa + \dfrac{\rho_f}{\rho_s} \varkappa_\varepsilon\right)}{\left(1 + \dfrac{\rho_f}{\rho_s} \varkappa_z\right)} \tag{11.29}$$

If we assume that the dimensionless quantity $\varkappa$ may be calculated for a given body geometry, while the dimensionless quantities: $\varkappa_x$, $\varkappa_z$, $\varkappa_\varepsilon$ may be estimated theoretically or determined experimentally for selected body geometries, the only characteristic number left here is the following number:

$$\pi_{\rho fs} = \frac{\rho_f}{\rho_s} \tag{11.30}$$

– Characteristic numbers related to the solid body supports elasticity

$$F^{kx}_{iz\Sigma} = \frac{F_{kx}}{F_{iz\Sigma}} \triangleq \pi^{kx}_{iz\Sigma} = \frac{k_x \cdot B}{\left(m + m_{zf}\right) V_z^2 B^{-1}} \tag{11.31}$$

$$F^{kz}_{iz\Sigma} = \frac{F_{kz}}{F_{iz\Sigma}} \triangleq \pi^{kz}_{iz\Sigma} = \frac{k_z B}{\left(m + m_{zf}\right) V_z^2 B^{-1}} \tag{11.32}$$

$$F^{k\varepsilon}_{iz\Sigma} = \frac{F_{k\varepsilon}}{F_{iz\Sigma}} = \frac{M_{k\varepsilon}}{BF_{iz\Sigma}} \triangleq \pi^{k\varepsilon}_{iz\Sigma} = \frac{k_\varepsilon}{\left(m + m_{zf}\right) V_z^2} \tag{11.33}$$

Let us introduce further designations and dependencies:

$$\frac{k_x}{m + m_{xf}} = \omega_x^2 \tag{11.34}$$

$$\frac{k_z}{m + m_{zf}} = \omega_z^2 \tag{11.35}$$

$$\frac{k_\varepsilon}{I + I_f} = \omega_\varepsilon^2 \tag{11.36}$$

where: $\omega_x, \omega_z, \omega_\varepsilon$ – so-called circular frequencies of translational and rotational normal vibrations of the analysed system.
Additionally, let us express the characteristic velocity of a solid body V_z as:

$$V_z = \omega_z B \tag{11.37}$$

Then, the last three characteristic numbers may be expressed as:

$$\pi_{iz\Sigma}^{kx} = \left(\frac{\omega_x}{\omega_z}\right)^2 \frac{1 + \frac{\rho_f}{\rho_s}\varkappa_x}{1 + \frac{\rho_f}{\rho_s}\varkappa_z} \tag{11.38}$$

$$\pi_{iz\Sigma}^{kz} = 1 \tag{11.39}$$

$$\pi_{iz\Sigma}^{k\varepsilon} = \left(\frac{\omega_\varepsilon}{\omega_z}\right)^2 \frac{\left(\varkappa + \frac{\rho_f}{\rho_s}\varkappa_\varepsilon\right)}{\left(1 + \frac{\rho_f}{\rho_s}\varkappa_z\right)} \tag{11.40}$$

As may be seen, the new characteristic numbers here are the following:

$$\pi_{\omega xz} = \frac{\omega_x}{\omega_z}; \quad \pi_{\omega\varepsilon z} = \frac{\omega_\varepsilon}{\omega_z} \tag{11.41}$$

– Characteristic numbers related to viscous damping of the body and fluid

$$F_{iz\Sigma}^{\mu x\Sigma} = \frac{F_{\mu x\Sigma}}{F_{iz\Sigma}} \triangleq \pi_{iz\Sigma}^{\mu x\Sigma} = \frac{\left(\mu_x + \mu_{xf}\right) V_z}{\left(m + m_{zf}\right) V_z^2 B^{-1}} = \frac{\mu_{x\Sigma}}{m_{z\Sigma}\omega_z} = \frac{\mu_{x\Sigma}}{m_{x\Sigma}} \cdot \frac{m_{x\Sigma}}{m_{z\Sigma}} \cdot \frac{1}{\omega_z} \tag{11.42}$$

$$F_{iz\Sigma}^{\mu z\Sigma} = \frac{F_{\mu z\Sigma}}{F_{iz\Sigma}} \triangleq \pi_{iz\Sigma}^{\mu z\Sigma} = \frac{\left(\mu_z + \mu_{zf}\right) V_z}{\left(m + m_{zf}\right) V_z^2 B^{-1}} = \frac{\mu_{z\Sigma}}{m_{z\Sigma}} \cdot \frac{1}{\omega_z} \tag{11.43}$$

$$F_{iz\Sigma}^{\mu\varepsilon\Sigma} = \frac{F_{\mu\varepsilon\Sigma}}{F_{iz\Sigma}} \triangleq \pi_{iz\Sigma}^{\mu\varepsilon\Sigma} = \frac{\left(\mu_\varepsilon + \mu_{\varepsilon f}\right) V_z B^{-1}}{B\left(m + m_{zf}\right) V_z^2 B^{-1}} = \frac{\mu_{\varepsilon\Sigma}}{m_{z\Sigma}} \cdot \frac{1}{B^2\omega_z} = \tag{11.44}$$

$$= \frac{\mu_{\varepsilon\Sigma}}{I_\Sigma} \cdot \frac{I_\Sigma}{m_{z\Sigma}} \cdot \frac{1}{B^2\omega_z}$$

Let us introduce further designations and dependencies:

$$\frac{\mu_{x\Sigma}}{m_{x\Sigma}} = 2\gamma_x\omega_x \tag{11.45}$$

$$\frac{\mu_{z\Sigma}}{m_{z\Sigma}} = 2\gamma_z\omega_z \tag{11.46}$$

$$\frac{\mu_{\varepsilon\Sigma}}{I_\Sigma} = 2\gamma_\varepsilon\omega_\varepsilon \tag{11.47}$$

The characteristic numbers (11.42), (11.43) and (11.44) will then adopt the following form:

$$\pi_{iz\Sigma}^{\mu x\Sigma} = 2\gamma_x \left(\frac{\omega_x}{\omega_z}\right) \frac{\left(1 + \frac{\rho_f}{\rho_s}\varkappa_x\right)}{\left(1 + \frac{\rho_f}{\rho_s}\varkappa_z\right)} \tag{11.48}$$

$$\pi_{iz\Sigma}^{\mu z\Sigma} = 2\gamma_z \tag{11.49}$$

$$\pi_{iz\Sigma}^{\mu\varepsilon\Sigma} = 2\gamma_\varepsilon \left(\frac{\omega_\varepsilon}{\omega_z}\right) \frac{\left(\varkappa + \frac{\rho_f}{\rho_s}\varkappa_\varepsilon\right)}{\left(1 + \frac{\rho_f}{\rho_s}\varkappa_z\right)} \tag{11.50}$$

Dimensionless damping coefficients: $\gamma_x, \gamma_z, \gamma_\varepsilon$ are called critical damping ratios and they characterise the damping ratio (amplitude decrease) of damped translational and rotational normal vibrations of the system under consideration.

– Characteristic numbers related to excitation amplitudes

$$F_{iz\Sigma}^{ox} = \frac{F_{ox}}{F_{iz\Sigma}} \triangleq \pi_{iz\Sigma}^{ox} = \frac{F_{ox}}{\left(m + m_{zf}\right) V_z^2 B^{-1}} = \frac{F_{ox}}{k_z B} = \frac{F_{ox}}{k_x B} \cdot \frac{k_x}{k_z} = \frac{\xi_{st}}{B} \cdot \frac{k_x}{k_z} \tag{11.51}$$

$$F_{iz\Sigma}^{oz} = \frac{F_{oz}}{F_{iz\Sigma}} \triangleq \pi_{iz\Sigma}^{oz} = \frac{F_{oz}}{\left(m + m_{zf}\right) V_z^2 B^{-1}} = \frac{F_{oz}}{k_z B} = \frac{\zeta_{st}}{B} \tag{11.52}$$

$$\begin{aligned} F_{iz\Sigma}^{o\varepsilon} &= \frac{M_o}{BF_{iz\Sigma}} \triangleq \pi_{iz\Sigma}^{o\varepsilon} = \frac{M_o}{B\left(m + m_{zf}\right) V_z^2 B^{-1}} = \\ &= \frac{M_o}{k_z B^2} = \frac{M_o}{k_\varepsilon} \cdot \frac{k_\varepsilon}{k_z B^2} = \varepsilon_{st} \cdot \frac{k_\varepsilon}{k_z B^2} \end{aligned} \tag{11.53}$$

where: $\xi_{st}, \zeta_{st}, \varepsilon_{st}$ – static translational displacements and rotation angle from the amplitudes of relevant excitations.

11.2. The case of induced vibrations of a body in a fluid moving at velocity V_f

Additional relevant ratios of aerodynamic / hydrodynamic forces and moment of force to inertia forces and moment of force as well as the similarity numbers resulting therefrom will then be as follows:

$$F_{iz\Sigma}^{\Delta psx} = \frac{F_{\Delta psx}}{F_{iz\Sigma}} \triangleq \pi_{iz\Sigma}^{\Delta psx} = \frac{\rho_f V_f^2 B^2 C_x}{\left(m + m_{zf}\right) V_z^2 B^{-1}} = \frac{\rho_f V_f^2 B}{k_z} \cdot C_x \tag{11.54}$$

$$F_{iz\Sigma}^{\Delta psz} = \frac{F_{\Delta psz}}{F_{iz\Sigma}} \triangleq \pi_{iz\Sigma}^{\Delta psz} = \frac{\rho_f V_f^2 B^2 C_y}{\left(m + m_{zf}\right) V_z^2 B^{-1}} = \frac{\rho_f V_f^2 B}{k_z} \cdot C_y \tag{11.55}$$

$$F_{iz\Sigma}^{\Delta ps\varepsilon} = \frac{M_{\Delta ps\varepsilon}}{BF_{iz\Sigma}} \triangleq \pi_{iz\Sigma}^{\Delta ps\varepsilon} = \frac{\rho_f V_f^2 B^3 C_m}{B\left(m + m_{zf}\right) V_z^2 B^{-1}} = \frac{\rho_f V_f^2 B}{k_z} \cdot C_m \tag{11.56}$$

As we see, another characteristic number $\rho V_f^2 B/k_z$ has appeared here, alongside aerodynamic coefficients C_x, C_y, C_m, which may also be expressed in a different way, namely:

$$\frac{\rho_f V_f^2 B}{k_z} = \frac{\rho_f V_f^2 B}{m_\Sigma f_z^2 4\pi^2} = \frac{\rho_f D^2}{\frac{m_\Sigma}{L}} \cdot \frac{B}{L} \cdot \left(\frac{V_f}{f_z D}\right)^2 \cdot \frac{1}{4\pi^2} = M_\rho \cdot \lambda_B \cdot (V_r)^2 \cdot \frac{1}{4\pi^2} \tag{11.57}$$

where the newly obtained characteristic numbers have been named and defined in the following way:

– dimensionless mass parameter

$$M_\rho = \frac{\rho_f D^2}{\frac{m_\Sigma}{L}} = \frac{\rho_f D^2}{m_\Sigma^*} \tag{11.58}$$

where: $m_\Sigma^* = \dfrac{m_\Sigma}{L}$ – summary mass density per body length unit

– body slenderness

$$\lambda_B = \frac{B}{L} \tag{11.59}$$

– reduced velocity

$$V_r = \frac{V_f}{f_z D} = \frac{1}{\frac{f_z D}{V_f}} = \frac{1}{\mathrm{St}_k} \tag{11.60}$$

where: St_k – the kinematic Strouhal number.

11.3. Dynamic response of a system and the related characteristic numbers

The dynamic response of the considered system are two translational displacements ξ and ζ and the rotation (torsion) angle ε. They are dependent on time t and on the set of all the dimensionless parameters of the system and its excitation, which could be e.g. the characteristic numbers discussed above. Designating the sets of these parameters as $\left(\check{X}\right)$, $\left(\check{Z}\right)$ and $\left(\check{E}\right)$, respectively, similarity relations for the system response are represented by the following dimensionless function dependencies:

$$\check{\xi} = \frac{\xi}{B} = \xi\left(\check{t}; \left(\check{X}\right)\right); \quad \check{\zeta} = \frac{\zeta}{B} = \check{\zeta}\left(\check{t}; \left(\check{Z}\right)\right); \quad \varepsilon = \check{\varepsilon} = \check{\varepsilon}\left(\check{t}; \left(\check{E}\right)\right) \tag{11.61}$$

These relations enable transferring the test results of the considered system model response and its excitation provided that at least the most important characteristic numbers discussed above have been satisfied.

12. Criteria of periodical phenomena similarity – Strouhal (synchronicity) number St

12.1. Vortex shedding and vortex excitation

Vortex shedding, as well as its related vortex excitation, of slender structural elements or structures of longitudinal axis situated normal to flow, are more or less periodical phenomena. Let us consider the relation of two global forces per length unit of a slender structure, namely: the global amplitude of vortex excitation force F^v_{os} and the global force of the onflowing fluid inertia F_{if}. Let the frequency of vortex shedding f^v be the parameter of vortex excitation translational force amplitude. Assuming that the remaining parameters remain the same as previously, we obtain:

$$F_{if}^{vos} = \frac{F^v_{os}}{F_{if}} \triangleq \pi_{if}^{vos} = \frac{f^v \rho_f V_f D_s^2}{\rho_f V_f^2 D_s} = \frac{f^v D_s}{V_f} = \mathrm{St}_v \tag{12.1}$$

Since the vortex shedding frequency and the configuration in which they follow the body (the so-called vortex trail) depend on the shadowing effect /hydrodynamic trail left by the body, and this in turn depends on Re number, it must in general be assumed that:

$$\mathrm{St} = \mathrm{St}\left(\mathrm{Re}; \left(\check{G}\right)\right) \tag{12.2}$$

Where the vortex shedding is of resonant character (i.e. $f^v = f_y$, where f_y is the frequency of the body normal translational oscillations), we may write:

$$V_c^v = \frac{f_y D_s}{\mathrm{St}} \tag{12.3}$$

12.2. Rotating turbines, propellers, screw propellers etc.

Let us consider the relation of two global forces of fluid inertia per length unit of a rotating blade of a rotor / propeller / screw propeller, i.e. circumferential force $F_{if\theta}$, lying on the rotor surface, normal to the blade, and axial force F_{ifx}, parallel to the inflowing fluid stream velocity V_f (or relative axial velocity of the fluid stream and the moving rotor). Let the blade circumferential velocity V_θ be the inertia force parameter $F_{if\theta}$, while:

$$V_\theta = \omega R = 2\pi n R \tag{12.4}$$

where: ω – rotation angular velocity; R – rotor radius, n – rotation frequency (i.e. the number of rotations per time unit). Let us assume that the remaining parameters are the same as previously. We will then have the following:

$$F_{ifx}^{if\theta} = \frac{F_{if\theta}}{F_{ifx}} \triangleq \pi_{ifx}^{if\theta} = \frac{V_\theta \rho_f V_f D_s}{\rho_f V_f^2 D_s} = \frac{V_\theta}{V_f} = \frac{\omega R}{V_f} = 2\pi \frac{nR}{V_f} \tag{12.5}$$

The characteristic numbers obtained here are called:

– tip speed ratio

$$Z = \frac{\omega R}{V_f} \tag{12.6}$$

– Strouhal (or synchronicity) number

$$\mathrm{St}_n = \frac{nR}{V_f} \tag{12.7}$$

In propeller or screw propeller model tests, it is assumed that the characteristic dimension is the circumscribed circle diameter D. If so, then $\mathrm{St}_n = \frac{nD}{V_f}$, although it is more customary to use the reverse of this number: $\frac{1}{\mathrm{St}_n} = \lambda$, called the propeller or screw propeller advance. For screw propellers $\lambda = 0.03 \div 3$, and for propellers $\lambda = 0.1 \div 2$.

13. Summarizing conclusions

At the end of this paper, the following summarizing conclusions can be drawn:

1. The similarity laws enable carrying out experiments with a convenient fluid, such as water or air, for example, and then applying the results to a fluid which is less convenient to work with, such as gas, steam, or oil.
2. In both hydraulics and aeronautics, valuable results can be obtained at a minimum cost by tests made with small-scale models of the full-size apparatus. The laws of similitude make it possible to determine the performance of the prototype, i.e. the full-size device, from tests made with the model. It is not necessary to use the same fluid for the model and the prototype. Neither must the model necessarily be smaller than the prototype. Thus the flow of water at the entrance to a small centrifugal-pump runner might be investigated by the flow of air at the entrance to a large model of the runner. It should be emphasized that the model need not necessarily be different in size from the prototype. In fact, it may be the same device, the variables in this case being the velocity and the physical properties of the fluid.
3. A few other examples where models may be used are: ships in towing basins, airplanes in wind tunnels, hydraulic turbines, centrifugal pumps, spillways of dams, river channels, and the study of such phenomena as the action of waves and tides on beaches, soil erosion, and the transportation of sediment.
4. In the use of models, it is essential that the fluid velocity should not be too low, as it may not produce laminar flow where in the prototype the flow is turbulent. Additionally, the conditions in the model should not be such that would make surface tension important if such conditions do not exist in the prototype. For example, the depth of water flowing over the crest of a model spillway should not be too low.

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Praktyczne problemy kryteriów podobieństwa dynamicznego w zagadnieniach interakcji płyn–ciało stałe przy różnych ich ruchach względnych

Słowa kluczowe: kryteria podobieństwa dynamicznego, interakcja płyn–ciało stałe, ruchy względne płyn–ciało stałe

Streszczenie:

Praca dotyczy kryteriów podobieństwa dynamicznego różnych zjawisk zachodzących w hydraulice i dynamice płynów, oryginalnie wyprowadzonych ze stosunków siłi momentów siłwpływających na te zjawiska. Podstawą formułowania i rozważań dotyczących kryteriów podobieństwa dynamicznego jest metoda i procedura Andrzeja Flagi dotycząca wyznaczania kryteriów podobieństwa dynamicznego w różnych zagadnieniach interakcji płyn-ciało stałe, tj. przy różnych względnych ruchach płynu i ciała stałego. Praca dotyczy wyznaczania i analizy kryteriów podobieństwa i analizy różnych praktycznych problemów spotykanych głównie w hydraulice i mechanice płynów przy ustalonym bezturbulencyjnym napływie płynu przed ciałem stałym. Ponadto przedstawiono przypadki drgań ciała stałego wymuszonych mechanicznie przy stacjonarnym ruchu płynu ze stałą prędkością przed ciałem stałym. Przyjmując autorską metodę i procedurę wyznaczania kryteriów podobieństwa dynamicznego, w pracy przedstawiono i analizowano zarówno znane liczby kryterialne otrzymane na innej drodze (np. z analizy wymiarowej czy równań różniczkowych danego zagadnienia – jak liczby: Reynoldsa, Froude'a, Eulera, Cauchy'ego, Strouhala, Macha) – ale także wiele nowych liczb kryterialnych występujących w róznych zagadnieniach interakcji płyn – ciało stałe (np. nowe współczynniki siłi momentów aerodynamicznych występujących w zagadnieniach drgań ciałstałych w płynach).

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