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Research paper

The uncertainty of the calculative value of the volumetric flow rate in open channel

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Abstract: The well-known Manning formula is usually used for the calculation of the calculative volumetric flow rate in a river or open canal. The discharge depends on the geometry of the channel, i.e. the water area, the wetted perimeter and the slope, as well as on the roughness coefficients. All these quantities are determined with some uncertainty. The article proposes a methodology for calculating the uncertainty of the roughness coefficients of the riverbed and the floodplain as well as the uncertainty of the geometric dimensions of the riverbed. Then, the method of calculating the uncertainty of the calculative discharge is then given. If these uncertainties are taken into consideration in the process of discharge calculation, then, as has been demonstrated for a hypothetical river channel, the ratio of the uncertainty to the calculated value of the discharge will change from several dozen percent in case of small flows to about ten percent in case of big, flood flows. It has also been shown that the uncertainty of the roughness coefficients has the biggest influence on the uncertainty of the flow rate. The presented calculations show that in order to take into account the influence of uncertainty of linear dimensions and roughness coefficients, the engineer designing the riverbed should assume for the calculations the flow rate increased by 10% then design flow. The obtained results can be used for homogeneous flows only, which is usually assumed in practical engineering calculations.

Keywords: discharge uncertainty, flow uncertainty, open channel

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1. Introduction

In order to correctly design the regulation of the river bed, the construction of dikes, as well as other hydraulic structures, the following steps are carried out. Firstly, the flow rate is measured at the gauging station and the water level-discharge relationship is established. The measured value of the discharge we will designated Q_{meas} – measured flow. Next, a flow rate with a given probability of occurrence is estimative on the basis of a sequence of long-term observations. We will denote this flow rate by Q_{des} – designed flow. This value forms the basis for the design of river bed, dike heights etc. Once the flume has been designed, we calculate the flow rate that can be passed through it. We will denote it Q_{cal} – calculative flow. In order for a river channel or dike to be designed correctly, the condition $Q_{\mathrm{des}} < Q_{\mathrm{cal}}$ must be met. However, all three quantities are subject to uncertainty, which must be taken into account not only in the measurements (Q_{meas}) and estimations (Q_{des}) but also calculations (Q_{cal}). This means that there is a non-zero probability that $Q_{\mathrm{des}} > Q_{\mathrm{cal}}$ which in consequently means disaster.

Designing the river bed, or flood embankments, it is necessary to know the stage-discharge rating curve. It enable to design the appropriate shape of the bed, slope and a height of the flood embankment. The rating curve is determined in water gauge sections using the stage and discharge measurements. However, in the case of the designed river bed, the stage-discharge rating curve is obtained using calculations, for example using the well-known Manning's formula. But all quantities is subject to uncertainty that should be taken into account not only in the measurements, but also in the calculations.

The issue of flow uncertainty is increasingly broadly discussed in hydrology. The developed methods focus mainly on expressing the measurement uncertainty of the discharge value, e.g. [1] A review of the applied methods of uncertainty calculation was provided in [2–4]. Expressing discharge measurement uncertainty has also been included in [5]. Much attention is also forecast i. e. estimated flow [6].

However, in engineering practice, for example during the designing of levees, an element which plays an important role are not only measurements, but also the calculation of the value of the calculative discharge in a river or open canal [7]. The issue of the uncertainty of the calculative discharge value has not been discussed in scientific and technical literature until now. In the mentioned paper, Reis et al. [7] assumed an uncertainty range for the coefficient of roughness of 0.02 to 0.2 but did not state how they obtained these values. Although Dickinson [8] has already given some basic calculations of the uncertainty of the flow, the formulae for the uncertainty of the calculated rather than the measured discharge value have not yet been explicitly given in the technical literature. The present article has been devoted to the methodology of calculating the calculative uncertainty of the discharge value which will be used in the design of river beds and canals as well as flood banks and dikes. The article assumes that the designed channel has a constant, uniform flow.

2. Theoretical methods

2.1. The basic formulas in the theory of uncertainty

The notion of uncertainty is associated with a measured quantity, whereas the issue discussed in this article refers to a calculated quantity. Therefore, metrological notions referring to uncertainty require some clarification. The definitions presented below are based on the expressions included in [9]:

- uncertainty – a parameter associated with the result of a calculation; it characterizes the dispersion of the calculated value;
- standard uncertainty – uncertainty expressed in the form of standard deviation;
- expanded uncertainty – the interval around the result of a calculation, which is expected to encompass a large fraction of the distribution of the calculated value;
- expansion coefficient – a numerical coefficient used as a multiplier of combined standard uncertainty in order to obtain expanded uncertainty;
- combined uncertainty – standard uncertainty of the calculated result, determined when that result is obtained from the values of a certain number of other quantities, equal to the square root of the sum of terms which are the variances or covariances of these other quantities with weights depending on how the measurement result varies together with changes in these quantities. Combined uncertainty is calculated in the following way: if quantity y depends on x_i quantities whose uncertainties are known and are $u(x_i)$, then the uncertainty $u(y)$ is calculated according to the formula:

$$u(y) = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial y}{\partial x_i}\right)^2 u(x_i)^2} \tag{2.1}$$

In the further part of the article, standard uncertainty is going to be, for short, referred to as uncertainty.

2.2. Uncertainty of discharge

In order to calculate the value of discharge, the empirical Manning formula is commonly used. It is a modification of the Chézy formula [10]

$$Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot \sqrt{S} \tag{2.2}$$

where: Q – discharge, n – roughness coefficient, A – cross section area, R – hydraulic radius, P – wetted perimeter, S – slope

Taking into consideration that

$$R = \frac{A}{P} \tag{2.3}$$

we receive:

$$Q = \frac{1}{n} \cdot \frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}} \cdot \sqrt{S} \tag{2.4}$$

Since, there was assumed constant uniform flow therefore the hydraulic slope can be considered as slope of water surface or slop of channel bottom.

If the channel is a braided channel or main channel with one or two floodplains, then the discharge is the sum of the discharge in particular channels

$$Q = \sum_{i=1}^{k} Q_i \tag{2.5}$$

where: $k$is the number of the channels of the watercourse.

Using the dependence (2.1), the uncertainty of the discharge $u(Q)$ will be expressed with the help of the following formula:

$$u(Q) = \sqrt{\left(\frac{\partial Q}{\partial n}\right)^2 \cdot u^2(n) + \left(\frac{\partial Q}{\partial A}\right)^2 \cdot u^2(A) + \left(\frac{\partial Q}{\partial P}\right)^2 \cdot u^2(P) + \left(\frac{\partial Q}{\partial S}\right)^2 \cdot u^2(S)} \tag{2.6}$$

where: $u(Q)$ – uncertainty of the discharge, $u(n)$ – uncertainty of roughness, $u(A)$ – uncertainty of the water area, $u(P)$ – uncertainty of the wetted perimeter, $u(S)$ – uncertainty of the slope

By differentiating Eq. (2.2) we receive:

$$\frac{\partial Q}{\partial n} = \frac{\partial}{\partial n}\left(\frac{1}{n} \cdot \frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}} \cdot \sqrt{S}\right) = \frac{-1}{n^2} \cdot \frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}} \cdot \sqrt{S} = \frac{-1}{n^2} \cdot A \cdot R^{\frac{2}{3}} \cdot \sqrt{S} \tag{2.7}$$

$$\frac{\partial Q}{\partial A} = \frac{\partial}{\partial A}\left(\frac{1}{n} \cdot \frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}} \cdot \sqrt{S}\right) = \frac{5}{3} \cdot \frac{1}{n} \cdot \frac{A^{\frac{2}{3}}}{P^{\frac{2}{3}}}\sqrt{S} = \frac{5}{3} \cdot \frac{1}{n} \cdot R^{\frac{2}{3}} \cdot \sqrt{S} \tag{2.8}$$

$$\frac{\partial Q}{\partial P} = \frac{\partial}{\partial P}\left(\frac{1}{n} \cdot \frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}} \cdot \sqrt{S}\right) = -\frac{2}{3} \cdot \frac{A^{\frac{5}{3}}}{P^{\frac{5}{3}}} \cdot \frac{1}{n} \cdot \sqrt{S} = -\frac{2}{3n} \cdot R^{\frac{5}{3}} \cdot \sqrt{S} \tag{2.9}$$

$$\frac{\partial Q}{\partial S} = \frac{\partial}{\partial S}\left(\frac{1}{n} \cdot \frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}} \cdot \sqrt{S}\right) = \frac{1}{2} \cdot \frac{1}{n} \cdot \frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}} \cdot S^{-\frac{1}{2}} = \frac{1}{2n} \cdot A \cdot R^{\frac{2}{3}} \cdot \frac{1}{\sqrt{S}} \tag{2.10}$$

After substituting the obtained results into Eq. (2.5) we receive:

$$u(Q) = \sqrt{\begin{aligned}&\left(\frac{-1}{n^2} \cdot A \cdot R^{\frac{2}{3}} \cdot \sqrt{S}\right)^2 \cdot u^2(n) + \left(\frac{5}{3} \cdot \frac{1}{n} \cdot R^{\frac{2}{3}} \cdot \sqrt{S}\right)^2 \cdot u^2(A) + \\ &+\left(-\frac{2}{3n} \cdot R^{\frac{5}{3}} \cdot \sqrt{S}\right)^2 \cdot u^2(P) + \left(\frac{1}{2n} \cdot A \cdot R^{\frac{2}{3}} \cdot \frac{1}{\sqrt{S}}\right)^2 \cdot u^2(S)\end{aligned}} \tag{2.11}$$

$$u(Q) = \sqrt{\begin{aligned}&\frac{A^2 \cdot R^{\frac{4}{3}} \cdot S \cdot u^2(n)}{n^4} + \frac{25 \cdot R^{\frac{4}{3}} \cdot S \cdot u^2(A)}{9n^2} + \frac{4 \cdot R^{\frac{10}{3}} \cdot S \cdot u^2(P)}{9n^2} + \\ &+ \frac{A^2 \cdot R^{\frac{4}{3}} \cdot u^2(S)}{4n^2 \cdot S}\end{aligned}} \tag{2.12}$$

$$u(Q) = \frac{R^{\frac{2}{3}}}{6n^2 \cdot \sqrt{S}} \sqrt{\begin{aligned}&9A^2\left(4S^2 \cdot u^2(n) + n^2 \cdot u^2(S)\right) + \\ &+ 4n^2 \cdot S^2\left(25 \cdot u^2(A) + 4R^2 u^2(P)\right)\end{aligned}} \tag{2.13}$$

If the channel is a main channel with two floodplains, then the$Q$ uncertainty will be expressed by the following formula:

$$u(Q) = \sqrt{\sum_{i=1}^{k} (u(Q_i)^2} \tag{2.14}$$

2.3. Uncertainty of roughness

In engineering practice, the roughness coefficients given by Chow [11] are most commonly used for the design of canals and watercourses. The roughness coefficients have been provided mean, maximum and minimum values for appropriate types of watercourses and canals. Due to the fact that the statistical distributions of the values of these roughness coefficients are not known, it should be assumed that they are subject to uniform distribution, limited by the minimum and maximum value, i.e. distribution with probability density

$$f(x) = \frac{1}{n_{\max} - n_{\min}} \tag{2.15}$$

The variance of which is [12]:

$$\sigma^2 = \frac{(n_{\max} - n_{\min})^2}{12} \tag{2.16}$$

Standard uncertainty $u(n)$ is equal to the standard deviation σ and is equal:

$$u(n) = \sqrt{3}\frac{n_{\max} - n_{\min}}{6} \tag{2.17}$$

It should be remembered that the roughness coefficients are calculated separately for the river bed and the flood plain. However, in both of these cases, the calculation methodology is the same.

2.4. Uncertainty of distance

For engineering purposes, the distance l is calculated on the basis of the difference between the x (horizontal) and y (vertical) coordinates. If a segment is vertical, then:

$$l_x = |x_2 - x_1| \tag{2.18}$$

If a segment is horizontal, then

$$l_y = |y_2 - y_1| \tag{2.19}$$

Taking into consideration Eq. (2.1), the uncertainties of distance $u(l_x)$ and $u(l_y)$ will be expressed by the following formulas:

$$u(l_x) = \sqrt{\left(\frac{\partial l_x}{\partial x_1}\right)^2 u^2(x_1) + \left(\frac{\partial l_x}{\partial x_2}\right)^2 u^2(x_2)} \tag{2.20}$$

(2.21)
$$u\left(l_y\right) = \sqrt{\left(\frac{\partial l_y}{\partial y_1}\right)^2 u^2\left(y_1\right) + \left(\frac{\partial l_y}{\partial y_2}\right)^2 u^2\left(y_2\right)}$$

We will also assume that the uncertainties of determining all horizontal coordinates x and vertical coordinates y are equal to one another and are equal $u(x)$, i.e. that:

(2.22)
$$u\left(x\right) = u\left(x_1\right) = u\left(x_2\right) = u\left(y_1\right) = u\left(y_2\right)$$

Then the Eq. (2.21) determining the uncertainty of the length of a horizontal segment and a vertical segment take the following form:

(2.23)
$$u\left(l_x\right) = u\left(l_y\right) = \sqrt{u^2\left(x\right) + u^2\left(x\right)} = \sqrt{2} \cdot u\left(x\right)$$

If a segment is oblique, then its length l_o is calculated using the Pythagorean theorem:

(2.24)
$$l = \sqrt{l_x^2 + l_y^2}$$

Taking into consideration Eq. (2.1) and Eq. (2.22), the uncertainty of length of the oblique segment $u(l_o)$:

(2.25)
$$u\left(l_o\right) = \sqrt{\left(\frac{\partial\sqrt{l_x^2 + l_y^2}}{\partial l_x}\right)^2 u^2\left(l_x\right) + \left(\frac{\partial\sqrt{l_x^2 + l_y^2}}{\partial l_y}\right)^2 u^2\left(l_y\right)}$$

By calculating the derivative and taking into consideration Eq. (2.22) we receive:

(2.26)
$$u\left(l_o\right) = \sqrt{\left(\frac{2l_x}{2\sqrt{l_x^2 + l_y^2}}\right)^2 u^2\left(l_x\right) + \left(\frac{2l_y}{2\sqrt{l_x^2 + l_y^2}}\right)^2 u^2\left(l_y\right)} = \sqrt{\frac{l_x^2 + l_y^2}{l_x^2 + l_y^2}} \cdot \sqrt{2} \cdot u\left(x\right) = \sqrt{2} \cdot u\left(x\right)$$

2.5. Uncertainty of the water area

In engineering practice, the water area in a river is approximated using a rectangle, triangle or trapezium. The particular areas and uncertainties calculated with the help of formula (2.3) are expressed by the formulas:

– the area of the rectangle A_r is:

(2.27)
$$A_r = a \cdot h$$

therefore, the uncertainty of the area of the rectangle $u(A_r)$ is provided by the formula:

(2.28)
$$u\left(A_r\right) = \sqrt{\left(\frac{\partial a \cdot h}{\partial a}\right)^2 \cdot u^2\left(a\right) + \left(\frac{\partial a \cdot h}{\partial h}\right)^2 \cdot u^2\left(h\right)} = \sqrt{h^2 \cdot u^2\left(a\right) + a^2 \cdot u^2(h)}$$

Due to the fact that the base $a$is a horizontal segment and the altitude h is a vertical segment, after taking into consideration formula (2.22) we receive:

$$u(A_r) = \sqrt{\left(\sqrt{2}\cdot h\cdot u(x)\right)^2 + \left(\sqrt{2}\cdot a\cdot u(x)\right)^2} = \sqrt{2}\cdot u(x)\sqrt{a^2+h^2} \tag{2.29}$$

– the area of the trapezium A_{tz} is:

$$A_{tz} = \frac{a+b}{2}\cdot h \tag{2.30}$$

The uncertainty of the area of the trapezium $u(A_{tz})$ is equal:

$$u(A_{tz}) = \sqrt{\begin{array}{l}\left(\dfrac{\partial\frac{1}{2}(a+b)h}{\partial a}\right)^2\cdot u^2(a) + \left(\dfrac{\partial\frac{1}{2}(a+b)h}{\partial b}\right)^2\cdot u^2(b) + \\ + \left(\dfrac{\partial\frac{1}{2}(a+b)h}{\partial h}\right)^2\cdot u^2(h)\end{array}} = \tag{2.31}$$
$$\sqrt{\left(\frac{h}{2}\right)^2\cdot u^2(a) + \left(\frac{h}{2}\right)^2\cdot u^2(b) + \left(\frac{a+b}{2}\right)^2\cdot u^2(h)}$$

taking into consideration formula (2.22) we receive:

$$u(A_{tz}) = \sqrt{\frac{h^2}{4}\cdot 2\cdot u^2(x) + \frac{h^2}{4}\cdot 2\cdot u^2(x) + \frac{(a+b)^2}{4}\cdot 2\cdot u^2(x)} = \tag{2.32}$$
$$\sqrt{h^2 + \frac{(a+b)^2}{2}}\cdot u(x)$$

– the area of the triangle A_{tg} is:

$$A_{tg} = \frac{a}{2}\cdot h \tag{2.33}$$

Therefore, the uncertainty of the area of the triangle $u(A_{tg})$ is calculated using the formula:

$$u\left(A_{tg}\right) = \sqrt{\left(\frac{\partial\frac{1}{2}a\cdot h}{\partial a}\right)^2\cdot u^2(a) + \left(\frac{\partial\frac{1}{2}a\cdot h}{\partial h}\right)^2\cdot u^2(h)} = \tag{2.34}$$
$$\sqrt{\left(\frac{a}{2}\right)^2 u^2(a) + \left(\frac{h}{2}\right)^2 u^2(h)}$$

we take into consideration formula (2.22):

$$u\left(A_{tg}\right) = \sqrt{\frac{h^2}{4}\cdot 2u^2(x) + \frac{a^2}{4}\cdot 2u^{\;2}(x)} = \sqrt{\frac{h^2+a^2}{2}}\cdot u(x) \tag{2.35}$$

– an area composed of two figures

In the case when the water area A is composed of two figures, A_1 and A_2, the uncertainty of the area A is calculated using the formula:

$$u(A) = \sqrt{u^2\left(A_1^2\right) + u^2\left(A_2^2\right)} \tag{2.36}$$

For example, if area $A$is composed of a rectangle and a trapezium, then:

$$u(A) = \sqrt{u^2(A_r) + u^2(A_{tz})} \tag{2.37}$$

2.6. Uncertainty of the wetted perimeter

The wetted perimeter P is a broken line composed of j segments. This includes k vertical segments, m horizontal segments and$n$ oblique segments. The length of the broken line l_{bl} is calculated as the sum of the lengths of the particular segments. Let us mark the length of the i$^{\text{th}}$ vertical segment as $l_{y(i)}$, the length of the i$^{\text{th}}$ horizontal segment as $l_{x(i)}$, and the length of the i$^{\text{th}}$ oblique segment as $l_{o(i)}$. Then the length of the wetted perimeter may be written down as:

$$P = \sum_{i=1}^{k} l_{y(i)} + \sum_{i=1}^{m} l_{x(i)} + \sum_{i=1}^{n} l_{o(i)} \tag{2.38}$$

If any of the types of segments is not present, then the appropriate element of the sum disappears. In accordance with the dependence (2.1), the uncertainty of the length of the wetted perimeter $u(P)$ will be expressed using the formula:

$$u(P) = \sqrt{\sum_{i=1}^{k}\left(\frac{\partial}{\partial l_{y(i)}} l_{y(i)}\right)^2 u^2\left(l_y\right) + \left(\sum_{i=1}^{m}\frac{\partial}{\partial l_{x(i)}} l_{x(i)}\right)^2 u^2\left(l_y\right) + \left(\sum_{i=1}^{n}\frac{\partial}{\partial l_{o(i)}} l_{o(i)}\right)^2} = \sqrt{k \cdot u^2\left(l_y\right) + m \cdot u^2\left(l_x\right) + n \cdot u^2\left(l_o\right)} \tag{2.39}$$

We take into consideration formulas (2.22) and (2.26) and we receive:

$$\begin{aligned} u(P) &= \sqrt{k \cdot 2u^2(x)^2 + m \cdot 2u^2(x) + n \cdot 2u^2(x)} = \sqrt{2} \cdot u(x)\sqrt{k+m+n} \\ u(P) &= \sqrt{2} \cdot j \cdot u(x) \end{aligned} \tag{2.40}$$

2.7. Uncertainty of the slope

When designing a channel, the hydraulic gradient is assumed to be equal to the geometric gradient of the designed channel. Then hydraulic gradient S (slope) is the ratio of the water table slope Δy to the distance L on which this slope occurred

$$S = \frac{\Delta y}{L} \tag{2.41}$$

The uncertainty of the hydraulic gradient (slope) $u(S)$ is calculated using Eq. (2.1):

$$u(S) = \sqrt{\left(\frac{\partial}{\partial L}\frac{\Delta y}{L}\right)^2 u^2(L) + \left(\frac{\partial}{\Delta y}\frac{\Delta y}{L}\right)^2 u^2(\Delta y)} = \sqrt{\left(\frac{1}{L}\right)^2 u^2(\Delta y) + \left(\frac{-\Delta y}{L^2}\right)^2 u^2(L)} \tag{2.42}$$

Due to the fact that we assumed that all the distances are measured with the same uncertainty $u(x)$, therefore, taking into consideration the dependence (Eq.2.22) we receive:

$$u(S) = \frac{1}{L} \cdot u(x)\sqrt{1 + \left(\frac{-\Delta y}{L}\right)^2} = \frac{1}{L}\sqrt{1 + S^2} \cdot u(x) \tag{2.43}$$

3. Material and methods

3.1. Assumptions for the calculations

Let us assume that we are designing the channel of a lowland river (Fig. 1) together with levees. The dimensions of the designed canal are: the width at the bottom – 5.0 m, the slope of the river banks – 1:2, the depth of the river channel – 1.0 m, the width of the floodplain – 10.0 m, the height of the levee – 2.0 m, the slope of the levee – 1:3, the slope of the river channel – 0,002; and uncertainties: vertical and horizontal coordinates $u(y)$, $u(x)$ – 0.01 m, the roughness coefficient for river channel – $n_{\min} = 0.025$; $n_{\max} = 0.033$ s·m$^{-1/3}$, the roughness coefficient for the floodplain – $n_{\min} = 0.03$; $n_{\max} = 0.05$ s·m$^{-1/3}$.

The final value of the discharge uncertainty depends on these three uncertainties: the coordinates, the roughness coefficients of the river channel and the floodplain.

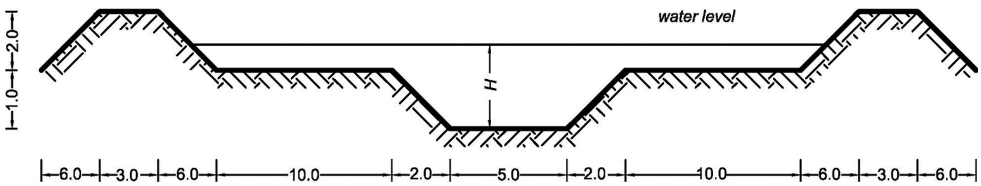

Fig. 1. The cross-section of the designed river channel. The scale of the image has not been kept

3.2. Calculations

Exemplary calculations will be carried out for the designed river channel presented in fig. 1 for the depth H of from 0 to 2.00 m with an increment of 0.1 m. The calculations were carried out using a spreadsheet. An example of the calculations for the depth $H = 1.5$ m is provided in Appendix A. Both Appendix A and the spreadsheet are available in the repository: https://doi.org/10.18150/DPPVZ4

The discharge in the river channel as well as in the floodplain was calculated according to formula (2.2), and their uncertainties were calculated on the basis of formula (2.13). The total discharge, which is the sum of the flows in the channel and in the floodplains, was calculated in accordance with formula (2.5), and its uncertainty – in accordance with formula (2.14).

The value of the influence of the uncertainties coordinates, the roughness coefficients of the river channel and the flood plain on the uncertainty of the flow rate was realized by separately increasing the partial values by 10%, 20% and 30%.

4. Results

As a result of the performed calculations, the following was obtained:

– the rating curve $Q(H)$ as a function of filling the river channel H together with the confidence interval (Fig. 2)

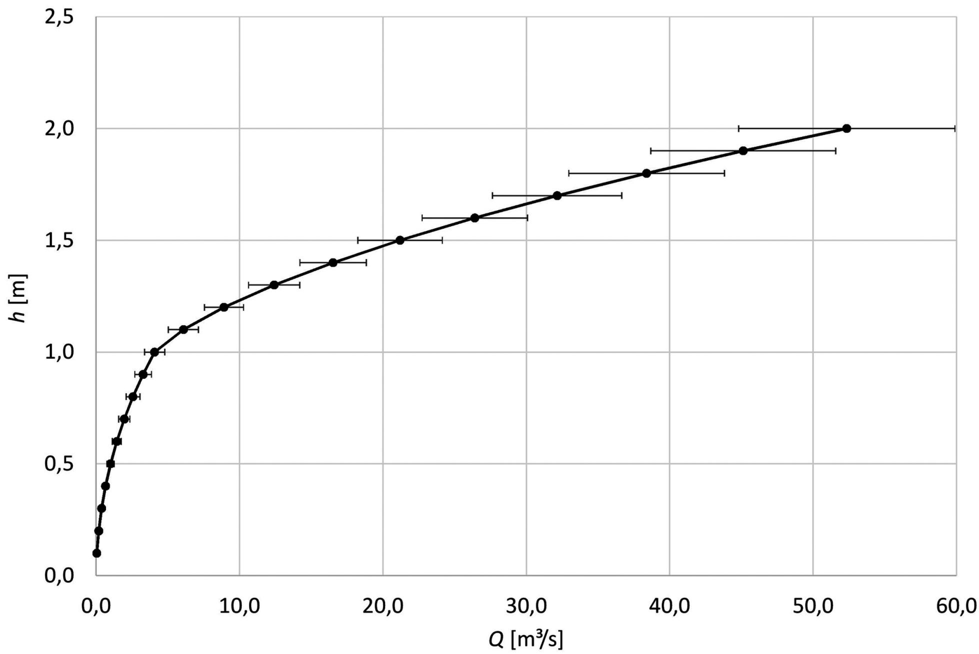

Fig. 2. The rating curve with marked intervals of expanded uncertainty

– a graph of the percentage dependence of the $u(Q)/Q$ ratio (Fig. 3)

– the value of the percentage change of the discharge uncertainty depending on the value of the percentage change of the uncertainty of roughness coefficients and of the location uncertainty (Fig. 4)

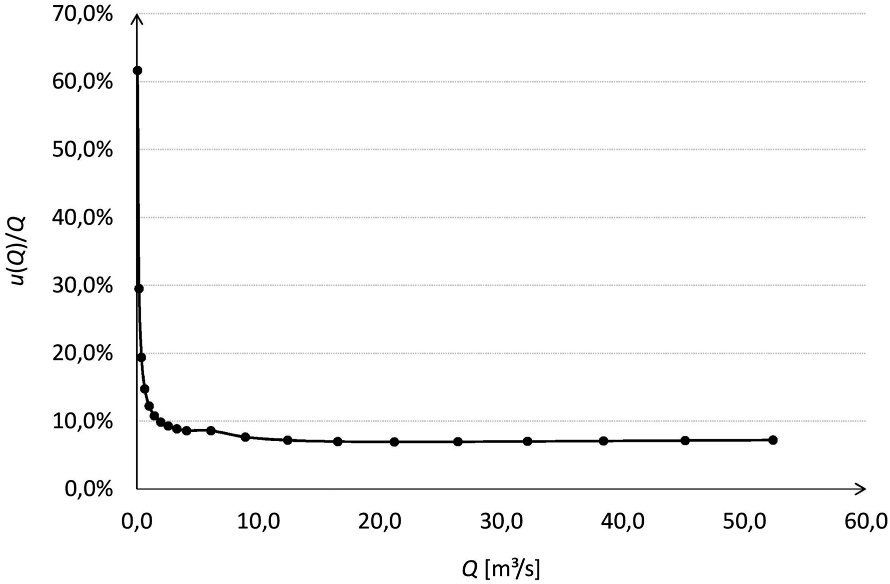

Fig. 3. A graph of the percentage dependence of the $u(Q)/Q$ ratio of the uncertainty of discharge to the calculated value of the discharge

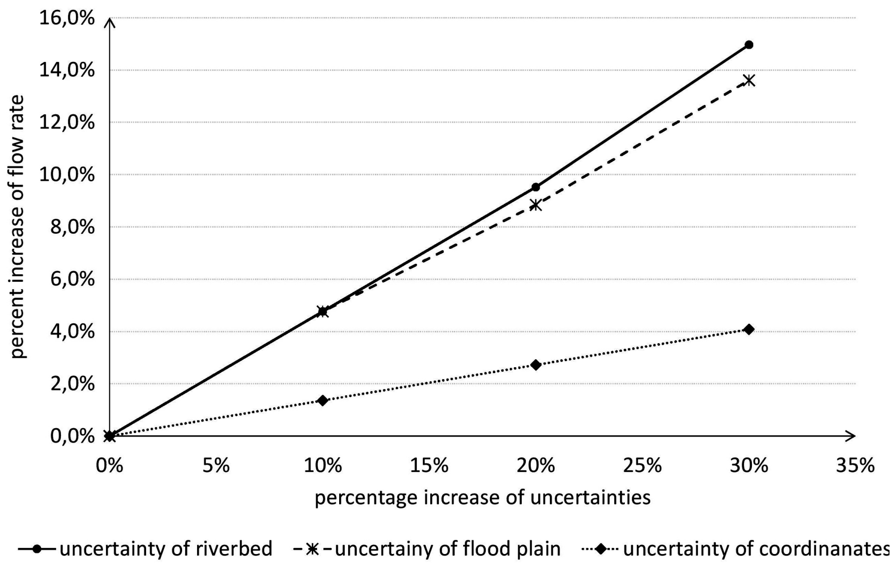

Fig. 4. The dependence of the influence of the percentage change of the uncertainty of roughness coefficients of the channel, of the floodplain, and of the uncertainty of the geometric dimensions of the river channel water area on the uncertainty of the discharge

5. Discussion

From the rating curve presented in Fig. 2 we can see that together with the increase of the discharge, there is an increase of the interval of its expanded uncertainty. At the same time, relative uncertainty calculated as the ratio of the uncertainty of discharge to the discharge $u(Q)/Q$ expressed as a percentage, decreases together with the increase of the discharge and it asymptotically heads to the value of 7.1% (Fig. 3).

As has been demonstrated in theoretical calculations, the uncertainty of the calculative value of the discharge in an open canal is impacted by two factors. The first factor is the determination of the dimensions of the channel water area. Due to the fact that the channel dimensions, adopted for the calculations of the discharge, are dimensions that are designed, this means that the uncertainty of the transverse diameter of the channel should be identical with the assumed accuracy of creation of the channel of the canal or river. In exemplary calculations it was taken into consideration that the uncertainty of distance is 1 cm. In practice, such accuracy of channel creation is impossible to achieve. However, as has been presented in fig. 4, this accuracy has the smallest influence out of the remaining factors. Nevertheless, if we even assume $u(x) = 4$ cm, then the total discharge, for the depth of 2 m $Q(H = 2\text{ m})$ will only increase or decrease by 4%. The second factor is the selection of the roughness coefficients of both, the channel as well as of the floodplain. As has been demonstrated in fig. 4, even a 10% change of the uncertainty of the roughness coefficient causes an about 5% change of the uncertainty of the total discharge.

It is obvious that the obtained value depends on the adopted geometry of the channel and on the flow conditions, i.e. the roughness coefficients and the hydraulic gradient (slope). It should also be taken into account that the hydraulic slope generally does not equal the slope of the channel bottom as it depends on the water level. Is equal for constant uniform flow only. However, this is assumed in practical engineering calculations [13]. In fact, uniform flow rarely occurs in natural or man-made channels. In fact, uniform flow rarely occurs in natural or artificial channels, as changes in channel cross-section, slope, or roughness cause depths and mean flow velocities to vary from point to point along the channel. As a result, the water surfaces are not parallel to the river bed. For this reason, the presented methodology of calculating the uncertainty of the calculative discharge should be extended to include non-uniform flow.

The presented line of reasoning does not take into consideration the uncertainty with which human chooses the values of roughness coefficients. And the uncertainty of the roughness coefficients is the main factor influencing the value of discharge uncertainty. Fig. 4 presents the influence of the percentage change of the uncertainty of roughness coefficients of the channel and of the floodplain and of the uncertainty of the geometrical dimensions of the river water area on the value of the discharge uncertainty. One can notice that the percentage change of the roughness coefficients has got an about 3.5 times bigger influence than the percentage change of the accuracy of the measurement of the channel geometry.

The proposed methodology assesses the probability P that the design riverbed or channel will be too small to pass the design flow

$$P\left(Q_{\mathrm{cal}} < Q_{\mathrm{des}}\right) = F\left(Q_{\mathrm{cal}}\right)$$

where F is the cumulative Gaussian distribution function.

Knowing Q_{cal} and its standard uncertainty u (Eq. 2.6) the probability P is:

$$F\left(Q_{\mathrm{cal}} < Q_{\mathrm{des}}\right) = \frac{1}{u\sqrt{2\pi}} \int_0^{Q_{\mathrm{cal}}} \exp\left(\frac{\left(t - Q_{\mathrm{cal}}\right)^2}{2u^2}\right) \mathrm{d}t$$

The value of this integral can easily be determined using a spreadsheet.

The paper assumes that the coefficient of roughness is constant in both the main channel and the floodplain. This is an assumption that greatly simplifies the calculation of flow uncertainty. Therefore, in order to further develop the proposed methodology, it should be complemented with a way to calculate the roughness coefficient in the case of a complex riverbed or floodplain [14].

6. Conclusions

The presented calculations show that in order to take into account the influence of uncertainty of linear dimensions and roughness coefficients, the engineer designing the riverbed should assume for the calculations the flow rate increased by 10% then design flow.

The uncertainty of the influence of roughness coefficients has 3.5 times bigger influence on the uncertainty of the flow rate than the uncertainty of the geometrical dimensions of the channel.

Data availability

All data and calculations are in the repository https://doi.org/10.18150/DPPVZ4 and may be used by other authors.

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Niepewność obliczonej wartości natężenie przepływu w kanałach otwartych

Słowa kluczowe: niepewność natężenia przepływu, kanał otwarty

Streszczenie:

Do obliczenia natężenia przepływu w rzece lub otwartym kanale zwykle stosuje się dobrze znany wzór Manninga. Przepływ zależy od geometrii koryta, nachylenia, a także od współczynników chropowatości. Wszystkie te wielkości są określone z pewną niepewnością. W hydrotechnice wyróżnić można trzy rodzaje wartości przepływu. Po pierwsze natężenie przepływu mierzone w przekrojach wodowskazowych, po drugie estymowane natężenie przepływu, na podstawie ciągów pomiarowych. Zwykle jest to przepływ minimalny lub maksymalny o zadanym prawdopodobieństwie wystąpienia, oraz projektowane natężeni przepływu. Jest to wartość obliczona podczas projektowania koryta kanału lub regulowanej rzeki. Jeżeli metoda obliczania niepewności dwóch pierwszych natężeń przepływu tj. mierzonego i estymowanego została opracowana to wzory do wyznaczania niepewności projektowanego natężenia przepływu nie były dotychczas podane explicite. W niniejszym artykule zaproponowano metodologię obliczania projektowanego natężenia przepływu. Jeśli ta niepewność zostanie uwzględniona w procesie obliczania projektowanego przepływu, to jak wykazano dla hipotetycznego koryta rzeki, stosunek niepewności do obliczonej wartości przepływu zmieni się od kilkudziesięciu procent przy małych przepływach do około dziesięciu procent w przypadku dużych przepływów powodziowych. Wykazano również, że największy wpływ na niepewność natężenia przepływu ma niepewność współczynników chropowatości. Z przedstawionych obliczeń wynika, że aby uwzględnić wpływ niepewności wymiarów liniowych i współczynników chropowatości, inżynier projektujący koryto rzeki powinien przyjąć do obliczeń natężenie przepływu zwiększone o 10% w stosunku do przepływu projektowego. Otrzymane wyniki można wykorzystać tylko dla przepływów jednorodnych, co zwykle przyjmuje się w praktycznych obliczeniach inżynierskich.

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