10.24425/acs.2024.149652

Archives of Control Sciences
Volume 34(LXX), 2024
No. 1*, pages 63–82*

A spherical fuzzy correlation coefficient based on statistical viewpoint with its applications in classification and bidirectional approximate reasoning

Abdul Haseeb GANIE and Debashis DUTTA

Spherical fuzzy sets are more powerful in modelling the uncertain situations than picture fuzzy sets, fermatean fuzzy sets, Pythagorean fuzzy sets, intuitionistic fuzzy sets, and fuzzy sets. In this paper, we first define the variance and covariance of spherical fuzzy sets. Then, using variance and covariance, we define the unique spherical fuzzy set correlation metric in line with the statistical coefficient of correlation. Two spherical fuzzy sets are correlated in both direction and strength using the provided measure of correlation. We discussed its many characteristics. We compared the measure of correlation with the current ones through linguistic variables. We established its validity by showing its application in bidirectional approximate reasoning. We also resolve a pattern identification issue in the spherical fuzzy environment using the provided correlation function, and we compare the results with several current measurements.

Key words: correlation coefficient, fuzzy set, picture fuzzy set, spherical fuzzy set, pattern recognition, bidirectional approximate reasoning

1. Introduction

Cuong and Kreinvoch [5] first proposed the idea of a picture fuzzy set (PFS) in 2013 for handling queries that call for responses of the types no, abstain, yes, and rejection. Each component of a PFS has the levels of membership, non-membership, and neutrality that define it. The sum of the membership grades in a PFS is at most one and due to this restriction, the scope of PFSs is limited. Therefore, the idea of spherical fuzzy sets (SFSs) was developed by Mahmood et al. [15] to handle the circumstances in which the sum of membership grades is

A.H. Ganie (corresponding author, e-mail: pdf_2023_ma03@nitw.ac.in) and D. Dutta (e-mail: ddutta@nitw.ac.in) are with Department of Mathematics, National Institute of Technology, Warangal 506004, Telangana, India.

Received 15.07.2023. Revised 24.01.2024.

more than one. In an SFS, the square sum of membership grades is at most one. Since the squared sum of the spherical parameters can only be at most 1.0 and decision makers can define their hesitation information independently, the unique idea of SFS gives decision makers a wider preference domain. A decision maker, for instance, might indicate their preference for one possibility over another in relation to a criterion with (0.8, 0.2, 0.4). It is obvious that the parameter sum is greater than 1, although the squared sum is only 0.84. Some spherical fuzzy (SF) measures of similarity based on cosine function and their usability in medical diagnosis and problems of classification were introduced by Wei et al. [26]. Aydogdu and Gul [2] proposed an entropy measure for SFSs with its applicability in decision-making. Applications of some novel SF similarity measures in green supplier selection and medical diagnosis were shown by Shishavan et al. [17]. Gundogdu and Kahraman [13] extended the classical analytic hierarchy process to the SF environment. Fatma and Cengiz [9] extended the VIKOR (classical VlseKriterijumska Optimizacija I Kompromisno Resenje) method to SF setting and demonstrated its application in the selection of warehouse site. Zhang et al. [28] proposed some SF aggregation operators and their application in decision-making. The correlation coefficient of SFSs is the subject of the current paper.

A coefficient of correlation is a statistical tool that can be used in a number of analytical and experimental studies. In fuzzy and its various generalizations, the correlation coefficient has been utilized in many fields like classification, decision-making, clustering, medical diagnosis, etc. Many studies [1, 4, 8, 10–12, 16, 18–20, 24] concerning the fuzzy/non-standard fuzzy functions of correlation and their applications are available in the literature.

The primary reasons we were inspired to conduct this study are listed below.

- The correlation coefficients in the SF environment have not been properly investigated.
- The current correlation measures are unable to handle the linguistic hedges properly.
- The available correlation functions are not based on statistical viewpoint.
- The applicability of correlation functions in bidirectional approximate reasoning has not been discussed yet.

The following list outlines this paper's contribution:

- We defined the variance and co-variance for SFSs.
- We proposed a novel correlation measure for SFSs based on variance and co-variance.
- We discussed the properties of the proposed correlation metric.

- We contrasted its performance with the available ones in terms of linguistic variables.
- We established its applicability in bidirectional approximate reasoning.
- We solved a classification problem in SF area with the aid of developed correlation function and compared the findings with the current compatibility functions.

The structure of the paper is as: The preliminary part of this paper is Section 2. In Section 3, we offer a novel function of correlation for SFSs along with its characteristics. An assessment of the offered SFS correlation metric with the existing compatibility metrics through linguistic hedges is shown in Section 4. In Section 5, the recommended correlation measure's application to bidirectional approximate reasoning and pattern recognition is demonstrated. Section 6 presents the conclusion along with future studies.

2. Preliminaries

Let $B = \{b_1, b_2, \ldots, b_p\}$ denote the set of universe.

Definition 1. *[27] A fuzzy set C_1 in B is given by*

$$C_1 = \{(b_t, \sigma_{C_1}(b_t)), b_t \in B\},$$

where $0 \leqslant \sigma_{C_1}(b_t) \leqslant 1$ is the grade of satisfaction of $b_t \in B$ in the set C_1.

Definition 2. *[5] A picture fuzzy set C_1 in B is given by*

$$C_1 = \{(b_t, \sigma_{C_1}(b_t), \zeta_{C_1}(b_t), \eta_{C_1}(b_t))b_t \in B\},$$

where $0 \leqslant \sigma_{C_1}(b_t) \leqslant 1$, $0 \leqslant \zeta_{C_1}(b_t) \leqslant 1$, and $0 \leqslant \eta_{C_1}(b_t) \leqslant 1$ are the grades of satisfaction, non-satisfaction, and neutrality respectively of $b_t \in B$ in the set C_1 such that $0 \leqslant \sigma_{C_1}(b_t) + \zeta_{C_1}(b_t) + \eta_{C_1}(b_t) \leqslant 1$. Also, $\theta_{C_1}(b_t) = 1 - (\sigma_{C_1}(b_t) + \zeta_{C_1}(b_t) + \eta_{C_1}(b_t))$ is the refusal degree for the element $b_t \in B$ in the set C_1.

Definition 3. *[15] A SFS C_1 in B is given by*

$$C_1 = \{(b_t, \sigma_{C_1}(b_t), \zeta_{C_1}(b_t), \eta_{C_1}(b_t))b_t \in B\},$$

where $0 \leqslant \sigma_{C_1}(b_t) \leqslant 1$, $0 \leqslant \zeta_{C_1}(b_t) \leqslant 1$, and $0 \leqslant \eta_{C_1}(b_t) \leqslant 1$ are the grades of satisfaction, non-satisfaction, and neutrality respectively of $b_t \in B$ in the set C_1 such that $0 \leqslant \sigma^2_{C_1}(b_t) + \zeta^2_{C_1}(b_t) + \eta^2_{C_1}(b_t) \leqslant 1$. Also, $\theta_{C_1}(b_t) = \sqrt{1 - (\sigma^2_{C_1}(b_t) + \zeta^2_{C_1}(b_t) + \eta^2_{C_1}(b_t))}$ is the refusal degree for the element $b_t \in B$ in the set C_1.

3. A novel SF correlation coefficient

We first give the definition of the variance and covariance of SFSs. Let $SFS(B)$ denote the set of all SFSs on B.

Definition 4. *For any $C_1 \in SFS(B)$, the variance is given by*

$$VAR(C_1) = \frac{1}{p}\sum_{t=1}^{p}\left[\begin{array}{c}\left((\sigma_{C_1}(b_t))^2 - (\overline{\sigma_{C_1}})^2\right)^2 + \left((\zeta_{C_1}(b_t))^2 - (\overline{\zeta_{C_1}})^2\right)^2 \\ + \left((\eta_{C_1}(b_t))^2 - (\overline{\eta_{C_1}})^2\right)^2\end{array}\right],$$

where $\overline{\sigma_{C_1}} = \frac{1}{p}\sum_{t=1}^{p}\sigma_{C_1}(b_t)$, $\overline{\zeta_{C_1}} = \frac{1}{p}\sum_{t=1}^{p}\zeta_{C_1}(b_t)$ *and* $\overline{\eta_{C_1}} = \frac{1}{p}\sum_{t=1}^{p}\eta_{C_1}(b_t)$.

Definition 5. *For any $C_1,\ C_2 \in SFS(B)$, the covariance is given by*

$$COV(C_1, C_2) = \frac{1}{p}\sum_{t=1}^{p}\left[\begin{array}{l}\left((\sigma_{C_1}(b_t))^2 - (\overline{\sigma_{C_1}})^2\right)\left((\sigma_{C_2}(b_t))^2 - (\overline{\sigma_{C_2}})^2\right) \\ + \left((\zeta_{C_1}(b_t))^2 - (\overline{\zeta_{C_1}})^2\right)\left((\zeta_{C_2}(b_t))^2 - (\overline{\zeta_{C_2}})^2\right) \\ + \left((\eta_{C_1}(b_t))^2 - (\overline{\eta_{C_1}})^2\right)\left((\eta_{C_2}(b_t))^2 - (\overline{\eta_{C_2}})^2\right)\end{array}\right].$$

Now, we define the correlation coefficient for SFSs.

Definition 6. *For any $C_1,\ C_2 \in B$, a correlation coefficient is given by*

$$C_{GD}(C_1, C_2) = \frac{COV(C_1, C_2)}{\sqrt{VAR(C_1) \times VAR(C_2)}}. \tag{1}$$

Now, we discuss some essential properties of the suggested correlation coefficient C_{GD} given in Eq. (1).

Theorem 1. *For any $C_1, C_2 \in SFS(B)$, we have*

(1) $C_{GD}(C_1, C_2) = C_{GD}(C_2, C_1)$,

(2) $-1 \leqslant C_{GD}(C_1, C_2) \leqslant 1$,

(3) if $C_1 = cC_2$ *for some* c, *then* $C_{GD}(C_1, C_2) = 1$.

Proof. (1)

$$
\begin{aligned}
COV(C_1,C_2) &= \frac{1}{p}\sum_{t=1}^{p}\begin{bmatrix}\left((\sigma_{C_1}(b_t))^2-(\overline{\sigma_{C_1}})^2\right)\left((\sigma_{C_2}(b_t))^2-(\overline{\sigma_{C_2}})^2\right)\\ +\left((\zeta_{C_1}(b_t))^2-(\overline{\zeta_{C_1}})^2\right)\left((\zeta_{C_2}(b_t))^2-(\overline{\zeta_{C_2}})^2\right)\\ +\left((\eta_{C_1}(b_t))^2-(\overline{\eta_{C_1}})^2\right)\left((\eta_{C_2}(b_t))^2-(\overline{\eta_{C_2}})^2\right)\end{bmatrix}\\
&= \frac{1}{p}\sum_{t=1}^{p}\begin{bmatrix}\left((\sigma_{C_2}(b_t))^2-(\overline{\sigma_{C_2}})^2\right)\left((\sigma_{C_1}(b_t))^2-(\overline{\sigma_{C_1}})^2\right)\\ +\left((\zeta_{C_2}(b_t))^2-(\overline{\zeta_{C_2}})^2\right)\left((\zeta_{C_1}(b_t))^2-(\overline{\zeta_{C_1}})^2\right)\\ +\left((\eta_{C_2}(b_t))^2-(\overline{\eta_{C_2}})^2\right)\left((\eta_{C_1}(b_t))^2-(\overline{\eta_{C_1}})^2\right)\end{bmatrix}\\
&= COV(C_2,C_1).
\end{aligned}
$$

So,

$$
\begin{aligned}
C_{GD}(C_1,C_2) &= \frac{COV(C_1,C_2)}{\sqrt{VAR(C_1)\times VAR(C_2)}}\\
&= \frac{COV(C_2,C_1)}{\sqrt{VAR(C_2)\times VAR(C_1)}} = C_{GD}(C_2,C_1).
\end{aligned}
$$

(2) Using Cauchy-Schwartz inequality, we have

$$
\begin{aligned}
(COV(C_1,C_2))^2 &= \left(\frac{1}{p}\sum_{t=1}^{p}\begin{bmatrix}\left((\sigma_{C_1}(b_t))^2-(\overline{\sigma_{C_1}})^2\right)\left((\sigma_{C_2}(b_t))^2-(\overline{\sigma_{C_2}})^2\right)\\ +\left((\zeta_{C_1}(b_t))^2-(\overline{\zeta_{C_1}})^2\right)\left((\zeta_{C_2}(b_t))^2-(\overline{\zeta_{C_2}})^2\right)\\ +\left((\eta_{C_1}(b_t))^2-(\overline{\eta_{C_1}})^2\right)\left((\eta_{C_2}(b_t))^2-(\overline{\eta_{C_2}})^2\right)\end{bmatrix}\right)^2\\
&\leqslant \frac{1}{p}\sum_{t=1}^{p}\begin{bmatrix}\left((\sigma_{C_1}(b_t))^2-(\overline{\sigma_{C_1}})^2\right)^2+\left((\zeta_{C_1}(b_t))^2-(\overline{\zeta_{C_1}})^2\right)^2\\ +\left((\eta_{C_1}(b_t))^2-(\overline{\eta_{C_1}})^2\right)^2\end{bmatrix}\\
&\times \frac{1}{p}\sum_{t=1}^{p}\begin{bmatrix}\left((\sigma_{C_2}(b_t))^2-(\overline{\sigma_{C_2}})^2\right)^2+\left((\zeta_{C_2}(b_t))^2-(\overline{\zeta_{C_2}})^2\right)^2\\ +\left((\eta_{C_2}(b_t))^2-(\overline{\eta_{C_2}})^2\right)^2\end{bmatrix}\\
&= VAR(C_1)\times VAR(C_2).
\end{aligned}
$$

So,

$$|COV(C_1, C_2)| \leqslant \sqrt{VAR(C_1) \times VAR(C_2)}$$

$$\text{or} \quad -\sqrt{VAR(C_1) \times VAR(C_2)} \leqslant COV(C_1, C_2) \leqslant \sqrt{VAR(C_1) \times VAR(C_2)}$$

$$\text{or} \quad -1 \leqslant \frac{COV(C_1, C_2)}{\sqrt{VAR(C_1) \times VAR(C_2)}} \leqslant 1$$

$$\text{or} \quad -1 \leqslant C_{GD}(C_1, C_2) \leqslant 1.$$

(3) Let $C_1 = cC_2$, then $\sigma_{C_1}(b_t) = c\sigma_{C_2}(b_t), \zeta_{C_1}(b_t) = c\zeta_{C_2}(b_t), \eta_{C_1}(b_t) = c\eta_{C_2}(b_t), \forall t = 1, 2, ..., p$.

$$\begin{aligned}
COV(C_1, C_2) &= \frac{1}{p} \sum_{t=1}^{p} \begin{bmatrix} \left((\sigma_{C_1}(b_t))^2 - (\overline{\sigma_{C_1}})^2 \right) \left((\sigma_{C_2}(b_t))^2 - (\overline{\sigma_{C_2}})^2 \right) \\ + \left((\zeta_{C_1}(b_t))^2 - (\overline{\zeta_{C_1}})^2 \right) \left((\zeta_{C_2}(b_t))^2 - (\overline{\zeta_{C_2}})^2 \right) \\ + \left((\eta_{C_1}(b_t))^2 - (\overline{\eta_{C_1}})^2 \right) \left((\eta_{C_2}(b_t))^2 - (\overline{\eta_{C_2}})^2 \right) \end{bmatrix} \\
&= \frac{1}{p} \sum_{t=1}^{p} \begin{bmatrix} \left((c\sigma_{C_2}(b_t))^2 - (c\overline{\sigma_{C_2}})^2 \right) \left((\sigma_{C_2}(b_t))^2 - (\overline{\sigma_{C_2}})^2 \right) \\ + \left((c\zeta_{C_2}(b_t))^2 - (c\overline{\zeta_{C_2}})^2 \right) \left((\zeta_{C_2}(b_t))^2 - (\overline{\zeta_{C_2}})^2 \right) \\ + \left((c\eta_{C_2}(b_t))^2 - (c\overline{\eta_{C_2}})^2 \right) \left((\eta_{C_2}(b_t))^2 - (\overline{\eta_{C_2}})^2 \right) \end{bmatrix} \\
&= \frac{c^2}{p} \sum_{t=1}^{p} \begin{bmatrix} \left((\sigma_{C_2}(b_t))^2 - (\overline{\sigma_{C_2}})^2 \right) + \left((\zeta_{C_2}(b_t))^2 - (\overline{\zeta_{C_2}})^2 \right) \\ + \left((\eta_{C_2}(b_t))^2 - (\overline{\eta_{C_2}})^2 \right) \end{bmatrix} \\
&= c^2 VAR(C_2).
\end{aligned}$$

Also,

$$\begin{aligned}
VAR(C_1) &= \frac{1}{p} \sum_{t=1}^{p} \begin{bmatrix} \left((\sigma_{C_1}(b_t))^2 - (\overline{\sigma_{C_1}})^2 \right)^2 + \left((\zeta_{C_1}(b_t))^2 - (\overline{\zeta_{C_1}})^2 \right)^2 \\ + \left((\eta_{C_1}(b_t))^2 - (\overline{\eta_{C_1}})^2 \right)^2 \end{bmatrix} \\
&= \frac{1}{p} \sum_{t=1}^{p} \begin{bmatrix} \left((c\sigma_{C_2}(b_t))^2 - (c\overline{\sigma_{C_2}})^2 \right)^2 + \left((c\zeta_{C_2}(b_t))^2 - (c\overline{\zeta_{C_2}})^2 \right)^2 \\ + \left((c\eta_{C_2}(b_t))^2 - (c\overline{\eta_{C_2}})^2 \right)^2 \end{bmatrix} \\
&= \frac{c^4}{p} \sum_{t=1}^{p} \begin{bmatrix} \left((\sigma_{C_2}(b_t))^2 - (\overline{\sigma_{C_2}})^2 \right)^2 + \left((\zeta_{C_2}(b_t))^2 - (\overline{\zeta_{C_2}})^2 \right)^2 \\ + \left((\eta_{C_2}(b_t))^2 - (\overline{\eta_{C_2}})^2 \right)^2 \end{bmatrix} \\
&= c^4 VAR(C_2).
\end{aligned}$$

So,

$$C_{GD}(C_1, C_2) = \frac{COV(C_1, C_2)}{\sqrt{VAR(C_1) \times VAR(C_2)}} = \frac{c^2 VAR(C_2)}{\sqrt{c^4 VAR(C_2) \times VAR(C_2)}} = 1.$$

4. A comparative study based on Linguistic hedges

Definition 7. *For any $C_1 \in SFS(B)$, C_1^μ is defined as*

$$C_1^\mu = \left\{ \begin{pmatrix} f_t, \left(\sigma_{C_1}(f_t) + \eta_{C_1}(f_t)\right)^\mu - \left(\eta_{C_1}(f_t)\right)^\mu, \\ \sqrt{1 - \left(1 - \left(\zeta_{C_1}(f_t)\right)^2\right)^\mu}, \left(\eta_{C_1}(f_t)\right)^\mu \end{pmatrix}, \ f_t \in B \right\}, \ \mu > 0.$$

Example 1. Consider SFS $C_1 \in B = \{f_1, f_2, f_3, f_4, f_5\}$ given as

$$C_1 = \left\{ \begin{array}{c} (f_1, 0.7, 0.1, 0.1),\ (f_2, 0.2, 0.3, 0.4),\ (f_3, 0.2, 0.1, 0.5), \\ (f_4, 0.1, 0.5, 0.2),\ (f_5, 0.2, 0.2, 0.2) \end{array} \right\}.$$

With the help of above Definition, we define the SFSs More or less LARGE $= C_1^{\frac{1}{2}}$, Not very LARGE $= (C_1^2)^c$, very very LARGE $= C_1^4$, very LARGE $= C_1^2$, and LARGE $= C_1$, as follows.

$$C_1 = \left\{ \begin{array}{c} (f_1, 0.7, 0.1, 0.1),\ (f_2, 0.2, 0.3, 0.4),\ (f_3, 0.2, 0.1, 0.5), \\ (f_4, 0.1, 0.5, 0.2),\ (f_5, 0.2, 0.2, 0.2) \end{array} \right\},$$

$$C_1^2 = \left\{ \begin{array}{c} (f_1, 0.6300, 0.1411, 0.0100), (f_2, 0.2000, 0.4146, 0.1600), \\ (f_3, 0.2400, 0.1411, 0.2500), (f_4, 0.0500, 0.6614, 0.0400), \\ (f_5, 0.1200, 0.2800, 0.0400) \end{array} \right\},$$

$$C_1^4 = \left\{ \begin{array}{c} (f_1, 0.4095, 0.1985, 0), (f_2, 0.1040, 0.5606, 0.0256), \\ (f_3, 0.1776, 0.1985, 0.0625), (f_4, 0.0065, 0.8268, 0.0016), \\ (f_5, 0.0240, 0.3881, 0.0016) \end{array} \right\},$$

$$C_1^{\frac{1}{2}} = \left\{ \begin{array}{c} (f_1, 0.5782, 0.0708, 0.3162), (f_2, 0.1421, 0.2146, 0.6325), \\ (f_3, 0.1296, 0.0708, 0.7071), (f_4, 0.1005, 0.3660, 0.4472), \\ (f_5, 0.1852, 0.1421, 0.4472) \end{array} \right\},$$

$$(C_1^2)^c = \left\{ \begin{array}{c} (f_1, 0.1411, 0.6300, 0.0100), (f_2, 0.4146, 0.2000, 0.1600), \\ (f_3, 0.1411, 0.2400, 0.2500), (f_4, 0.6614, 0.0500, 0.0400), \\ (f_5, 0.2800, 0.1200, 0.0400) \end{array} \right\}.$$

We compare our suggested method with a few other ones for estimating the correlation coefficients using these SFSs. Tables 1–3 show the results and utilize the following notations.

Not very LARGE: N.V.L., more or less LARGE: M.L.L., very very LARGE: V.V.L., very LARGE: V.L., LARGE: L.

Table 1: Correlation coefficients using C_{UGMJA1} [25]

	M.L.L.	L.	V.L.	V.V.L.	N.V.L.
M.L.L.	1.0000	0.8991	0.6519	0.4339	0.4768
L.	0.8991	1.0000	0.9045	0.7167	0.5995
V.L.	0.6519	0.9045	1.0000	0.9307	0.5866
V.V.L.	0.4339	0.7165	0.9307	1.0000	0.4950
N.V.L.	0.4768	0.5995	0.5866	0.4950	1.0000

Table 2: Correlation coefficients using C_{UGMJA2} [25]

	M.L.L.	L.	V.L.	V.V.L.	N.V.L.
M.L.L.	1.0000	0.8361	0.6476	0.3822	0.4737
L.	0.8361	1.0000	0.8467	0.5870	0.5612
V.L.	0.6476	0.8467	1.0000	0.8144	0.5866
V.V.L.	0.3822	0.5870	0.8144	1.0000	0.4332
N.V.L.	0.4737	0.5612	0.5866	0.4332	1.0000

Table 3: Correlation coefficients using C_{GD}

	M.L.L.	L.	V.L.	V.V.L.	N.V.L.
M.L.L.	1.0000	0.9129	0.6862	0.4026	–0.1484
L.	0.9129	1.0000	0.8947	0.6006	–0.2789
V.L.	0.6862	0.8947	1.0000	0.8710	–0.3732
V.V.L.	0.4026	0.6006	0.8710	1.0000	–0.3481
N.V.L.	–0.1484	–0.2789	–0.3732	–0.3481	1.0000

A coefficient of correlation C should satisfy the following conditions due to the characterization of linguistic variables.

$$\begin{gathered} C(M.L.L., V.V.L.) < C(M.L.L., V.L.) < C(M.L.L., L.) \\ C(V.V.L., M.L.L.) < C(V.V.L., L.) < C(V.V.L., V.L.) \\ C(L., V.V.L.) < C(L., V.L.) < C(L., M.L.L.) \\ C(V.L., M.L.L.) < C(V.L., V.V.L.) < C(V.L, L.) \end{gathered} \tag{2}$$

From Tables 1–3, we find out that the correlation coefficients C_{UGMJA1} [25] and C_{UGMJA2} [25] do not satisfy all the conditions of Eq. (2) whereas our suggested correlation fulfills all the required conditions as shown in Figs. 1–4. Also because

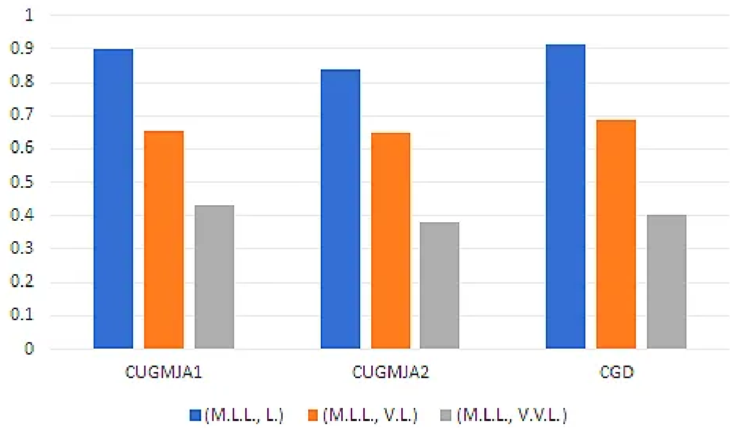

Figure 1: Condition first of Eq. (2)

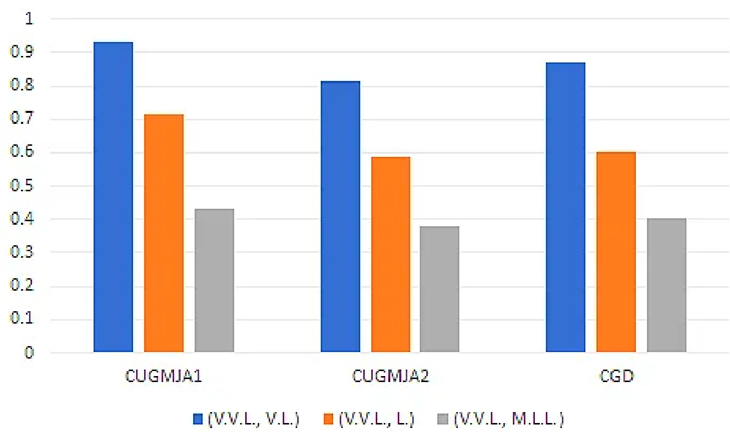

Figure 2: Condition second of Eq. (2)

of the characterization of linguistic variables, the correlation of N.V.L. with all other linguistic hedges should be negative and this is shown by only the offered correlation measure.

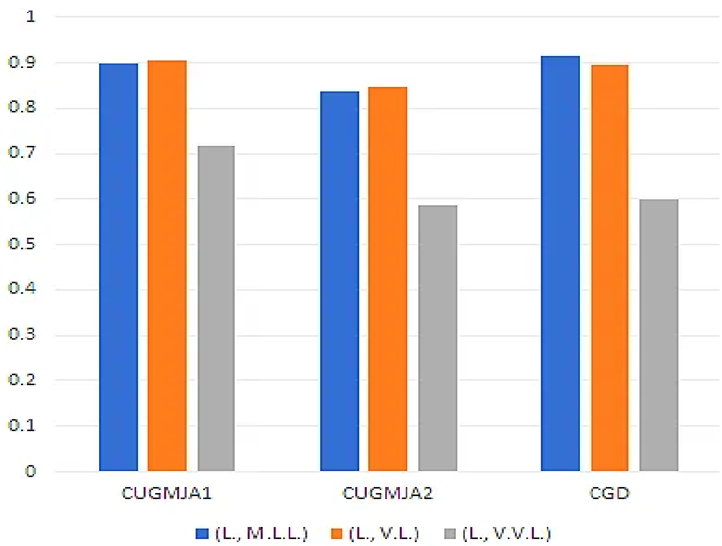

Figure 3: Condition third of Eq. (2)

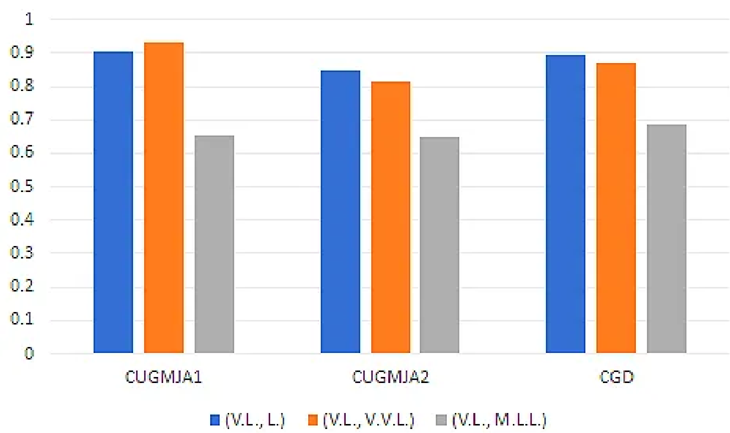

Figure 4: Condition fourth of Eq. (2)

5. Applications

Here we establish the utility of the developed metric in bidirectional approximate reasoning and pattern investigation.

5.1. Bidirectional approximate reasoning

Here, we apply the proposed correlation function to a bidirectional approximative reasoning system for SFSs to check the accuracy of the correlation function. Let us start by taking a look at the single-input, single-output, forward approximate reasoning scheme:

$$\begin{aligned}
P_1 &: \text{ If } A \text{ is } C_1, \text{ then } E \text{ is } D_1.\\
P_2 &: \text{ If } A \text{ is } C_2, \text{ then } E \text{ is } D_2.\\
P_3 &: \text{ If } A \text{ is } C_3, \text{ then } E \text{ is } D_3.\\
&\dots\\
P_p &: \text{ If } A \text{ is } C_p, \text{ then } E \text{ is } D_p.\\
\text{Fact} &: \qquad A \text{ is } C^*.\\
\text{Consequence} &: \qquad E \text{ is } D^*.
\end{aligned}$$

In this model, C_t and C^* are SFSs of the Universe $B = \{b_1, b_2, \dots, b_p\}$ and $P_t (1 \leqslant t \leqslant p)$ is the t-th output rule and D_t and D^* are SFSs of the Universe $F = \{f_1, f_2, \dots, f_q\}$. With the help of the Chen et al. [3], we have the following: $C_{GD}(C_1, C^*) = l_1 \implies$ "E is D_1^*" is the derived result of the Rule P_1, where $C_{GD}(C_1, C^*)$ indicates the correlation between C_1 and C^*, and

$$D_1^* = |l_1| \times D_1 = \left\{ \begin{array}{c} \left(f_1, |l_1|\sigma_{D_1}(f_1), 1 - |l_1| + |l_1|\zeta_{D_1}(f_1), |l_1|\eta_{D_1}(f_1)\right), \\ \left(f_2, |l_1|\sigma_{D_1}(f_2), 1 - |l_1| + |l_1|\zeta_{D_1}(f_2), |l_1|\eta_{D_1}(f_2)\right), \dots, \\ \left(f_q, |l_1|\sigma_{D_1}(f_q), 1 - |l_1| + |l_1|\zeta_{D_1}(f_q), |l_1|\eta_{D_1}(f_q)\right). \end{array} \right\}$$

$C_{GD}(C_2, C^*) = l_2 \implies$ "E is D_2^*" is the desired result of the Rule P_2, where

$$D_2^* = |l_2| \times D_2 = \left\{ \begin{array}{c} \left(f_1, |l_2|\sigma_{D_2}(f_1), 1 - |l_2| + |l_2|\zeta_{D_2}(f_1), |l_2|\eta_{D_2}(f_1)\right), \\ \left(f_2, |l_2|\sigma_{D_2}(f_2), 1 - |l_2| + |l_2|\zeta_{D_2}(f_2), |l_2|\eta_{D_2}(f_2)\right), \dots, \\ \left(f_q, |l_2|\sigma_{D_2}(f_q), 1 - |l_2| + |l_2|\zeta_{D_2}(f_q), |l_2|\eta_{D_2}(f_q)\right). \end{array} \right\}$$

$C_{GD}(C_p, C^*) = l_p \implies$ "E is D_p^*" is the desired result of the Rule P_p, where

$$D_p^* = |l_p| \times D_p = \left\{ \begin{array}{c} \left(f_1, |l_p|\sigma_{D_p}(f_1), 1 - |l_p| + |l_p|\zeta_{D_p}(f_1), |l_p|\eta_{D_p}(f_1)\right), \\ \left(f_2, |l_p|\sigma_{D_p}(f_2), 1 - |l_p| + |l_p|\zeta_{D_p}(f_2), |l_p|\eta_{D_p}(f_2)\right), \dots, \\ \left(f_q, |l_p|\sigma_{D_p}(f_q), 1 - |l_p| + |l_p|\zeta_{D_p}(f_q), |l_p|\eta_{D_p}(f_q)\right). \end{array} \right\}.$$

So, the desired result of the approximate reasoning method is "E is D^*", where

$$D^* = D_1^* \cup D_2^* \cup \ldots \cup D_p^*$$
$$= \left\{ \begin{array}{c} \left(f_1, \max_t \left(|l_t|\sigma_{D_t}(f_1)\right), \min_t \left(1 - |l_t| + |l_t|\zeta_{D_t}(f_1)\right), \min_t \left(|l_t|\eta_{D_t}(f_1)\right)\right) \\ \left(f_2, \max_t \left(|l_t|\sigma_{D_t}(f_2)\right), \min_t \left(1 - |l_t| + |l_t|\zeta_{D_t}(f_2)\right), \min_t \left(|l_t|\eta_{D_t}(f_2)\right)\right) \\ \left(f_q, \max_t \left(|l_t|\sigma_{D_t}(f_q)\right), \min_t \left(1 - |l_t| + |l_t|\zeta_{D_t}(f_q)\right), \min_t \left(|l_t|\eta_{D_t}(f_q)\right)\right) \end{array} \right\},$$

and $\cup$ denotes the union operator between SFSs, $1 \leqslant t \leqslant p$.

Example 2. Let us have a look at the following SFS-based forward approximation reasoning system.

$$\begin{aligned} P_1 &: \text{If } A \text{ is } C_1, \text{ then } E \text{ is } D_1. \\ P_2 &: \text{If } A \text{ is } C_2, \text{ then } E \text{ is } D_2. \\ P_3 &: \text{If } A \text{ is } C_3, \text{ then } E \text{ is } D_3. \\ \text{Fact} &: A \text{ is } C^*. \\ \text{Consequence} &: E \text{ is } D^*. \end{aligned}$$

In this system

$$\begin{aligned} C_1 &= \{(b_1, 0.2, 0.1, 0.4), (b_2, 0.3, 0.5, 0.1), (b_3, 0.7, 0, 0.2)\}, \\ C_2 &= \{(b_1, 0.5, 0.1, 0.3), (b_2, 0.8, 0, 0.1), (b_3, 0.4, 0.5, 0)\}, \\ C_3 &= \{(b_1, 0.7, 0.2, 0.1), (b_2, 1, 0, 0), (b_3, 0.1, 0.3, 0.5)\}, \\ C^* &= \{(b_1, 0.8, 0, 0.1), (b_2, 0.4, 0.5, 0), (b_3, 0.2, 0.3, 0.3)\}, \\ D_1 &= \{(f_1, 0.1, 0.3, 0.4), (f_2, 0.1, 0.4, 0.2), (f_3, 0.3, 0.4, 0.2)\} \\ D_2 &= \{(f_1, 0.1, 0.9, 0), (f_2, 0, 0.5, 0), (f_3, 0.1, 0, 0.1)\}, \\ D_3 &= \{(f_1, 0.3, 0.1, 0.4), (f_2, 0.5, 0.3, 0.1), (f_3, 0.4, 0, 0.3)\}. \end{aligned}$$

Using the suggested correlation metric, we have $C_{GD}(C_1, C^*) = -0.3071$, $C_{GD}(C_2, C^*) = -0.1073$, and $C_{GD}(C_3, C^*) = 0.2335$. So, we obtain

$$D_1^* = \left\{ \begin{array}{c} (f_1, 0.0307, 0.7850, 0.1228), (f_2, 0.0307, 0.8158, 0.0614), \\ (f_3, 0.0921, 0.8158, 0.0614) \end{array} \right\}$$

$$D_2^* = \left\{(f_1, 0.0104, 0.9896, 0), (f_2, 0, 0.9482, 0), (f_3, 0.0104, 0.8963, 0.0104)\right\}$$

$$D_3^* = \left\{ \begin{array}{c} (f_1, 0.0700, 0.7899, 0.0934), (f_2, 0.1167, 0.8366, 0.0233), \\ (f_3, 0.0934, 0.7665, 0.0700) \end{array} \right\}.$$

So,

$$D^* = D_1^* \cup D_2^* \cup \ldots \cup D_p^* = \left\{ \begin{matrix} (f_1, 0.0700, 0.7850, 0), (f_2, 0.1167, 0.8158, 0), \\ (f_3, 0.0934, 0.7665, 0.0104) \end{matrix} \right\}.$$

Now, using the suggested correlation metric C_{GD}, we have $C_{GD}(D_1, D^*) = 0.0612, C_{GD}(D_2, D^*) = 0.1471$, and $C_{GD}(D_3, D^*) = 0.4562$.

The results indicate that $C_{GD}(C_1, C^*) < C_{GD}(C_2, C^*) < C_{GD}(C_3, C^*)$ and $C_{GD}(D_1, D^*) < C_{GD}(D_2, D^*) < C_{GD}(D_3, D^*)$, i.e., $C_{GD}(C_1, C^*)$ and $C_{GD}(C_3, C^*)$ have the smallest and maximum values respectively among the values of $C_{GD}(C_1, C^*)$, $C_{GD}(C_2, C^*)$, and $C_{GD}(C_3, C^*)$, whereas D_1 and D_3 share the least and greatest correlation with D^* respectively (see Fig. 5).

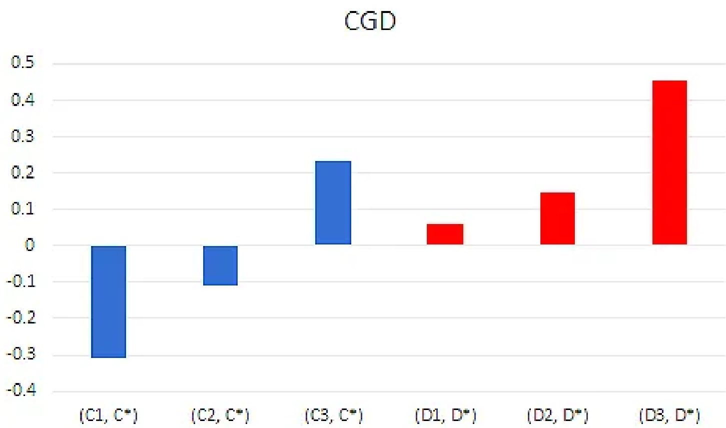

Figure 5: Forward approximate reasoning

On the other hand, consider the following backward approximation-based reasoning model:

$$\begin{aligned} P_1:&\quad \text{If } A \text{ is } C_1, \text{ then } E \text{ is } D_1. \\ P_2:&\quad \text{If } A \text{ is } C_2, \text{ then } E \text{ is } D_2. \\ P_3:&\quad \text{If } A \text{ is } C_3, \text{ then } E \text{ is } D_3. \\ &\ldots \\ P_p:&\quad \text{If } A \text{ is } C_p, \text{ then } E \text{ is } D_p. \\ \text{Fact}:&\quad\quad\quad E \text{ is } D^*. \\ \text{Consequence}:&\quad\quad\quad A \text{ is } C^*. \end{aligned}$$

In this model, C_t and C^* are SFSs of the Universe $B = \{b_1, b_2, \dots, b_p\}$ and $P_t (1 \leqslant t \leqslant p)$ is the t-th output rule and D_t and D^* are SFSs of the Universe $F = \{f_1, f_2, \dots, f_q\}$. With the help of the Chen et al. [3], we have the following: $C_{GD}(D_1, D^*) = k_1 \implies$ "A is C_1^*" is the derived result of the Rule P_1, where $C_{GD}(D_1, D^*)$ indicates the correlation between D_1 and D^*, where

$$C_1^* = |k_1| \times C_1 = \left\{ \begin{matrix} \left(b_1, |k_1|\sigma_{C_1}(b_1), 1 - |k_1| + |k_1|\zeta_{C_1}(b_1), |k_1|\eta_{C_1}(b_1)\right), \\ \left(b_2, |k_1|\sigma_{C_1}(b_2), 1 - |k_1| + |k_1|\zeta_{C_1}(b_2), |k_1|\eta_{C_1}(b_2)\right), \dots, \\ \left(b_q, |k_1|\sigma_{C_1}(f_q), 1 - |k_1| + |k_1|\zeta_{D_1}(f_q), |k_1|\eta_{D_1}(f_q)\right). \end{matrix} \right\}$$

$C_{GD}(D_2, D^*) = k_2 \implies$ "A is C_2^*" is the desired result of the Rule P_2, where

$$C_2^* = |k_2| \times C_2 = \left\{ \begin{matrix} \left(b_1, |k_2|\sigma_{C_2}(f_1), 1 - |k_2| + |k_2|\zeta_{C_2}(b_1), |k_2|\eta_{C_2}(b_1)\right), \\ \left(b_2, |k_2|\sigma_{C_2}(b_2), 1 - |k_2| + |k_2|\zeta_{C_2}(b_2), |k_2|\eta_{C_2}(b_2)\right), \dots, \\ \left(b_q, |k_2|\sigma_{C_2}(b_q), 1 - |k_2| + |k_2|\zeta_{C_2}(b_q), |k_2|\eta_{C_2}(b_q)\right). \end{matrix} \right\}$$

$C_{GD}(D_p, D^*) = k_p \implies$ "A is C_p^*" is the desired result of the Rule P_p, where

$$C_p^* = |k_p| \times C_p = \left\{ \begin{matrix} \left(b_1, |k_p|\sigma_{C_p}(b_1), 1 - |k_p| + |k_p|\zeta_{C_p}(b_1), |k_p|\eta_{C_p}(b_1)\right), \\ \left(b_2, |k_p|\sigma_{C_p}(b_2), 1 - |k_p| + |k_p|\zeta_{C_p}(b_2), |k_p|\eta_{C_p}(b_2)\right), \dots, \\ \left(b_q, |k_p|\sigma_{C_p}(b_q), 1 - |k_p| + |k_p|\zeta_{C_p}(b_q), |k_p|\eta_{C_p}(b_q)\right) \end{matrix} \right\}$$

So, the desired result of the approximate reasoning method is "A is C^*", where

$$\begin{aligned} C^* &= C_1^* \cup C_2^* \cup \dots \cup C_p^* \\ &= \left\{ \begin{pmatrix} \left(b_1,\ \max\limits_t(|k_t|\sigma_{C_t}(b_1)),\ \min\limits_t\left(1-|k_t|+|k_t|\zeta_{C_t}(b_1)\right),\ \min\limits_t\left(|k_t|\eta_{C_t}(b_1)\right)\right) \\ \left(f_2,\ \max\limits_t(|k_t|\sigma_{C_t}(b_2)),\ \min\limits_t\left(1-|k_t|+|k_t|\zeta_{C_t}(b_2)\right),\ \min\limits_t\left(|k_t|\eta_{C_t}(b_2)\right)\right) \\ \left(b_q,\ \max\limits_t(|k_t|\sigma_{C_t}(b_q)),\ \min\limits_t\left(1-|k_t|+|k_t|\zeta_{C_t}(b_q)\right),\ \min\limits_t\left(|k_t|\eta_{C_t}(b_q)\right)\right) \end{pmatrix} \right\}, \end{aligned}$$

and $\cup$ denotes the union operator between SFSs, $1 \leqslant t \leqslant p$.

Example 3. Let us have a look at the following SFS-based backward approximation reasoning system.

$$\begin{aligned} P_1&: \text{ If } A \text{ is } C_1, \text{ then } E \text{ is } D_1. \\ P_2&: \text{ If } A \text{ is } C_2, \text{ then } E \text{ is } D_2. \\ P_3&: \text{ If } A \text{ is } C_3, \text{ then } E \text{ is } D_3. \\ \text{Fact}&: \qquad\qquad E \text{ is } D^*. \\ \text{Consequence}&: \qquad\qquad A \text{ is } C^*. \end{aligned}$$

In this system

$$
\begin{aligned}
C_1 &= \{(b_1, 0.2, 0.7, 0.1), (b_2, 0, 0.1, 0), (b_3, 0.5, 0.3, 0.1)\},\\
C_2 &= \{(b_1, 0.4, 0.1, 0.2), (b_2, 0.1, 0.5, 0.3), (b_3, 0.7, 0.1, 0.2)\},\\
C_3 &= \{(b_1, 0.1, 0.5, 0.3), (b_2, 0.1, 0.6, 0.1), (b_3, 0, 0.5, 0.4)\},\\
D_1 &= \{(f_1, 0.4, 0.1, 0.3), (f_2, 0.3, 0.5, 0.1), (f_3, 0.4, 0.1, 0.3)\},\\
D_2 &= \{(f_1, 0, 0.9, 0.1), (f_2, 0, 0.5, 0.1), (f_3, 0.1, 0.3, 0.5)\},\\
D_3 &= \{(f_1, 0.3, 0.4, 0.1), (f_2, 0.2, 0.4, 0.3), (f_3, 0.5, 0.1, 0.2)\},\\
D^* &= \{(f_1, 0.6, 0.1, 0.1), (f_2, 0.5, 0.4, 0), (f_3, 0.3, 0.3, 0.2)\}.
\end{aligned}
$$

Using the suggested correlation metric, we have $C_{GD}(D_1, D^*) = 0.3919$, $C_{GD}(D_2, D^*) = -0.2714$, and $C_{GD}(D_3, D^*) = -0.4876$. So, we obtain

$$
C_1^* = \left\{ \begin{array}{c} (b_1, 0.0784, 0.8824, 0.0392), (b_2, 0, 1, 0), \\ (b_3, 0.1960, 0.7256, 0.0392) \end{array} \right\},
$$

$$
C_2^* = \left\{ \begin{array}{c} (b_1, 0.1086, 0.7557, 0.0543), (b_2, 00271, 0.8643, 0.0814), \\ (b_3, 0.1900, 0.7557, 0.0543) \end{array} \right\},
$$

$$
C_3^* = \left\{ \begin{array}{c} (b_1, 0.0488, 0.7562, 0.1463), (b_2, 0.0488, 0.8050, 0.0488), \\ (b_3, 0, 0.7562, 0.1950) \end{array} \right\}.
$$

So,

$$
\begin{aligned}
C^* &= C_1^* \cup C_2^* \cup \ldots \cup C_p^* \\
&= \left\{ \begin{array}{c} (b_1, 0.1086, 0.7557, 0.0392), (b_2, 0.0488, 0.8050, 0), \\ (b_3, 0.1960, 0.7256, 0.0392) \end{array} \right\}.
\end{aligned}
$$

Now, using the suggested correlation metric C_{GD}, we have
$C_{GD}(C_1, C^*) = 0.9783$, $C_{GD}(C_2, C^*) = 0.6755$, and $C_{GD}(C_3, C^*) = 0.5590$.

The results indicate that $C_{GD}(C_1, C^*) > C_{GD}(C_2, C^*) > C_{GD}(C_3, C^*)$ and $C_{GD}(D_1, D^*) > C_{GD}(D_2, D^*) > C_{GD}(D_3, D^*)$, i.e., $C_{GD}(C_1, C^*)$ and $C_{GD}(C_3, C^*)$ have the greatest and smallest values respectively among the values of $C_{GD}(C_1, C^*)$, $C_{GD}(C_2, C^*)$, and $C_{GD}(C_3, C^*)$, whereas D_1 and D_3 shares the greatest and least correlation with D^* respectively (see Fig. 6).

Together, the outcomes of Examples 2 and 3 allow us to draw the conclusion that the suggested strategy works well for approximate reasoning.

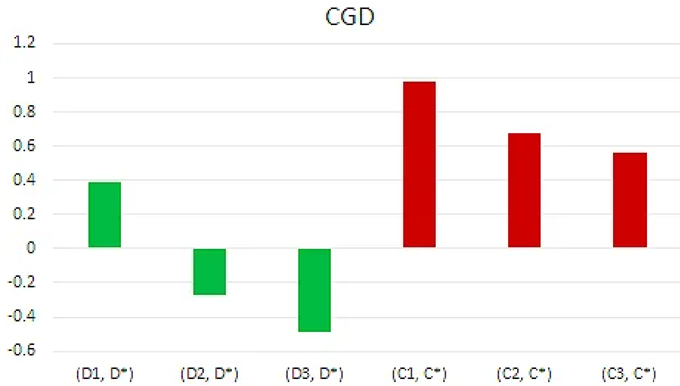

Figure 6: Backward approximate reasoning

5.2. Pattern investigation

Here, we establish how to use the offered SFS correlation metric to solve classification-related problems. Utilizing numerous criteria of compatibility, like "correlation", "distance", "similarity", etc., enables pattern analysis to categorize an unexplained pattern into one of the recognized patterns. We contrast our results with several compatibility measurements as well.

In the below example, we will answer a classification-related problem.

Example 4. [11] Let C_k, $k = 1, 2, 3$ and C be some patterns expressed in terms of SFSs as:

$$C_1 = \left\{ \begin{array}{c} (f_1, 0.4, 0.3, 0.1), (f_2, 0.5, 0.3, 0.2), (f_3, 0.4, 0.3, 0), \\ (f_4, 0.7, 0, 0.2), (f_5, 0.6, 0.1, 0.1) \end{array} \right\},$$

$$C_2 = \left\{ \begin{array}{c} (f_1, 0.7, 0.1, 0.1), (f_2, 0.2, 0.3, 0.4), (f_3, 0.2, 0.1, 0.5), \\ (f_4, 0.1, 0.5, 0.2), (f_5, 0.3, 0.3, 0.3) \end{array} \right\},$$

$$C_3 = \left\{ \begin{array}{c} (f_1, 0.1, 0.3, 0.4), (f_2, 0.4, 0.3, 0.1), (f_3, 0.3, 0.4, 0.2), \\ (f_4, 0.2, 0.5, 0.3), (f_5, 0.5, 0.3, 0.1) \end{array} \right\}, \text{and}$$

$$C = \left\{ \begin{array}{c} (f_1, 0.6, 0.2, 0.1), (f_2, 0.3, 0.4, 0.2), (f_3, 0.4, 0.3, 0.2), \\ (b_4, 0.7, 0.1, 0), (f_5, 0.4, 0.2, 0.2) \end{array} \right\}.$$

The task is to verify which pattern C_t, $t = 1, 2, 3$ shares the maximum similarity with C. For this purpose, we combine the offered SFS correlation measurement with the currently known compatibility functions. The computed results are displayed in Table 4. The most of functions of compatibility along with the developed SFS metric, make it clear that C should be assigned to C_1 (see Table 4). After finding out the pattern to which C belongs, we compute the Confidence degree (CD) [14] of each compatibility function as

$DoC = \sum_{k=1,k\neq j}^{m} |COR(C_k, C) - COR(C_j, C)|$, where COR is any measure of comparison like correlation measure, distance measure, similarity measure, etc. and C_j is the pattern to which C belongs. In comparison to the current picture fuzzy and SF compatibility measures, as shown in Fig. 7, we find that the CD of the proposed SF correlation coefficient is very high.

Table 4: Values of various PF/SF compatibility tests calculated with reference to Example 4

Compatibility measure	(C_1, C)	(C_2, C)	(C_3, C)	Result
D_1 [6]	0.1000	0.1867	0.1933	C_1
D_2 [6]	0.1000	0.1833	0.1929	C_1
D_3 [6]	0.2000	0.2600	0.2600	C_1
D_4 [6]	0.0894	0.1428	0.1456	C_1
D_5 [7]	0.2000	0.3000	0.3400	C_1
D_6 [7]	0.4000	0.6557	0.7071	C_1
D_7 [7]	0.1789	0.2933	0.3162	C_1
D_8 [7]	0.2000	0.2800	0.3000	C_1
D_9 [21]	0.4000	0.6000	0.6800	C_1
D_{10} [21]	0.1265	0.2074	0.2236	C_1
D_{11} [21]	0.0500	0.0650	0.0650	C_1
D_{12} [23]	0.3750	0.5154	0.4755	C_1
D_{13} [23]	0.3491	0.3951	0.3880	C_1
D_{14} [23]	0.1250	0.1872	0.1775	C_1
D_{15} [23]	0.1955	0.2268	0.2232	C_1
K_1 [22]	0.9168	0.7625	0.7138	C_1
K_2 [22]	0.8838	0.7500	0.6739	C_1
C_{GD} (Proposed)	0.5014	0.2235	-0.5555	C_1

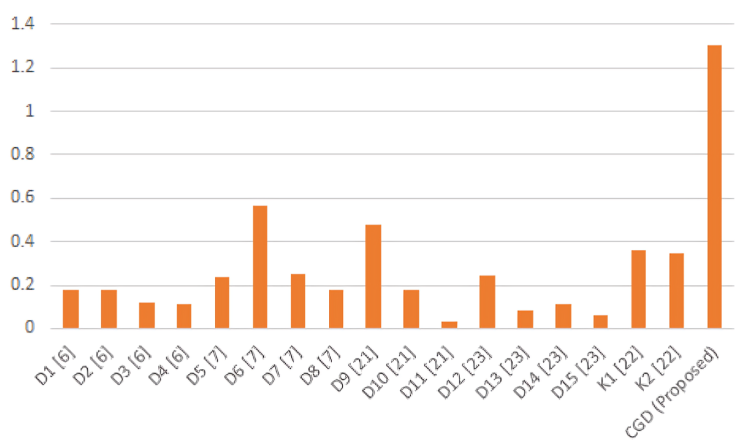

Figure 7: Confidence Degree of various picture fuzzy/SF compatibility measures

6. Conclusion

The SFS correlation coefficient proposed in this paper has shown both the level of association and the correlation degree between SFSs. The offered correlation metric has handled the linguistic hedges properly and the existing ones have led to unsatisfactory results. In bidirectional approximate reasoning, the suggested metric has given accurate results. Also, the suggested SF correlation coefficient has given satisfactory results in pattern investigation and has a very high CD than some available comparison measures. In the future, we will discuss its application in decision-making, clustering, medical diagnosis, etc.

References

[1] E.P. Augustine: Novel correlation coefficient for intuitionistic fuzzy sets and its application to multi-criteria decision-making problems. *International Journal of Fuzzy System Applications*, **10**(2), (2021), 39–58, DOI: 10.4018/IJFSA.2021040103

[2] A. Aydogdu and S. Gul: A novel entropy proposition for spherical fuzzy sets and its application in multiple attribute decision-making. *International Journal of Intelligent Systems*, **35**(9), (2020), 1354–1374, DOI: 10.1002/int.22256

[3] S.M. Chen, W.H. Hsiao and W.T. Jong: Bidirectional approximate reasoning based on interval-valued fuzzy sets. *Fuzzy Sets and Systems*, **91**(3), (1997), 339–353, DOI: 10.1016/S0165-0114(97)86594-310.1016/ S0165-0114(97)86594-3

[4] D.A. Chiang and N.P. Lin: Correlation of fuzzy sets. *Fuzzy Sets and Systems*, **102**(2), (1999), 221–226, DOI: 10.1016/S0165-0114(97)00127-9

[5] B.C. Cuong and V. Kreinovich: Picture fuzzy sets-a new concept for computational intelligence problems. In *2013 third World Congress on Information and Communication Technologies (WICT 2013)*, 1-6. IEEE, Hanoi, Vietnam (2013), DOI: 10.1109/WICT.2013.7113099

[6] N.V. Dinh and N.X. Thao: Some measures of picture fuzzy sets and their application in multi-attribute decision-making. *International Journal of Mathematical Sciences and Computing*, **4**(3), (2018), 23–41. DOI: 10.5815/ijmsc.2018.03.03

[7] P. Dutta: Medical diagnosis based on distance measures between picture fuzzy sets. *International Journal of Fuzzy System Applications*, **7**(4), (2018), 15–36. DOI: 10.4018/IJFSA.2018100102

[8] P.A. Ejegwa: Generalized triparametric correlation coefficient for Pythagorean fuzzy sets with application to MCDM problems. *Granular Computing*, **6**(3), (2021), 557–566. DOI: 10.1007/s41066-020-00215-5

[9] K.G. Fatma and K. Cengiz: A novel VIKOR method using spherical Fuzzy Sets and its application to warehouse site selection. *Journal of Intelligent & Fuzzy Systems*, **37**(1), (2019), 1197–1211. DOI: 10.3233/JIFS-182651

[10] A.H. Ganie: Applicability of a novel Pythagorean fuzzy correlation coefficient in medical diagnosis, clustering, and classification problems. *Computational and Applied Mathematics*, **41**(8), (2022), 410. DOI: 10.1007/s40314-022-02108-6

[11] A.H. Ganie, S. Singh and P.K. Bhatia: Some new correlation coefficients of picture fuzzy sets with applications. *Neural Computing and Applications*, **32**(16), (2020), 12609–12625. DOI: 10.1007/s00521-020-04715-y

[12] T. Gerstenkorn and J. Manko: Correlation of intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, **44**(1), (1991), 39–43. DOI: 10.1016/0165-0114(91)90031-K

[13] F.K. Gundogdu and C. Kahraman: A novel spherical fuzzy analytic hierarchy process and its renewable energy application. *Soft Computing*, **24**(6), (2020), 4607–4621. DOI: 10.1007/s00500-019-04222-w

[14] A.G. Hatzimichailidis, G.A. Papakostas and V.G. Kaburlasos: A novel distance measure of intuitionistic fuzzy sets and its application to pattern recognition problems. *International Journal of Intelligent Systems*, **27**(4), (2012), 396–409. DOI: 10.1002/int.21529

[15] T. Mahmood, K. Ullah, Q. Khan and N. Jan: An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets. *Neural Computing and Applications*, **31**(11), (2019), 7041–7053. DOI: 10.1007/s00521-018-3521-2

[16] S. Sharma and S. Singh: On some generalized correlation coefficients of the fuzzy sets and fuzzy soft sets with application in cleanliness ranking of public health centres. *Journal of Intelligent & Fuzzy Systems*, **36**(4), (2019), 3671–3683. DOI: 10.3233/JIFS-181838

[17] S.A. Shishavan, F.K. Gündogdu, E. Farrokhizadeh, Y. Donyatalab and C. Kahraman: Novel similarity measures in spherical fuzzy environment and their applications. *Engineering Applications of Artificial Intelligence*, **94** (2020), 103837. DOI: 10.1016/j.engappai.2020.103837

[18] S. Singh and A.H. Ganie: On some correlation coefficients in Pythagorean fuzzy environment with applications. *International Journal of Intelligent Systems*, **35**(4), (2020), 682–717. DOI: 10.1002/int.22222

[19] S. Singh and A.H. Ganie: On a new picture fuzzy correlation coefficient with its applications to pattern recognition and identification of an investment sector. *Computational and Applied Mathematics*, **41**(1), (2022), 8. DOI: 10.1007/s40314-021-01699-w

[20] S. Singh and A.H. Ganie: Some novel q-rung orthopair fuzzy correlation coefficients based on the statistical viewpoint with their applications. *Journal of Ambient Intelligence and Humanized Computing*, **13**(4), (2022), 2227–2252. DOI: 10.1007/s12652-021-02983-7

[21] P. Singh, N.K. Mishra, M. Kumar, S. Saxena and V. Singh: Risk analysis of flood disaster based on similarity measures in picture fuzzy environment. *Afrika Matematika*, **29**(7-8), (2018), 1019–1038. DOI: 10.1007/s13370-018-0597-x

[22] P. Singh: Correlation coefficients for picture fuzzy sets. *Journal of Intelligent & Fuzzy Systems*, **28**(2), (2015), 591–604. DOI: 10.3233/IFS-141338

[23] L.H. Son: Measuring analogousness in picture fuzzy sets: from picture distance measures to picture association measures. *Fuzzy Optimization and Decision Making*, **16**(3), (2017), 359–378. DOI: 10.1007/s10700-016-9249-5

[24] N.X. Thao, M. Ali and F. Smarandache: An intuitionistic fuzzy clustering algorithm based on a new correlation coefficient with application in medical diagnosis. *Journal of Intelligent & Fuzzy Systems*, **36**(1), (2019), 189–198. DOI: 10.3233/JIFS-181084

[25] K. Ullah, H. Garg, T. Mahmood, N. Jan and Z. Ali: Correlation coefficients for T-spherical fuzzy sets and their applications in clustering and multi-attribute decision making. *Soft Computing*, **24** (2020), 1647–1659. DOI: 10.1007/s00500-019-03993-6

[26] G. Wei, J. Wang, M. Lu, J. Wu and C. Wei: Similarity measures of spherical fuzzy sets based on cosine function and their applications. *IEEE Access*, **7** (2019), 159069–159080. DOI: 10.1109/ACCESS.2019.2949296

[27] L.A. Zadeh: Fuzzy sets. *Information and Control*, **8**(3), (1965), 338–353. DOI: 10.1016/S0019-9958(65)90241-X

[28] H. Zhang, Q. Cai and G. Wei: Spherical fuzzy power partitioned Maclaurin symmetric mean operators and their application in multiple attribute group decision making. *Archives of Control sciences*, **33**(1), (2023), 179–238. DOI: 10.24425/acs.2023.145119