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Research paper

The system reliability of steel trusses with correlated variables

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Abstract: The paper focuses on the system reliability of steel trusses with correlated variables. The correlation between bearing capacities of bars was considered. Two static truss schemes were considered. Nodal forces were the only load. The Finite Element Method analysis was conducted in Robot Structural Analysis program. To conduct system reliability analysis it is essential to find cut-sets, it was realized by stiffness matrix spectral analysis. Then reliability analysis was performed in Sysrel module of Strurel computing environment. First Order Reliability Method was used as the base, Subset Simulation method was used to check the correctness of the results. The sensitivity analysis of reliability index enabled the authors to draw conclusions, which variables have the greatest influence on the reliability of the structure. The effects of actions and bearing capacities were assumed to be the only random variables and that the excessing the bearing capacities of bars is the only way the structure can get into failure area.

Keywords: First Order Reliability Method (FORM), sensitivity analysis, steel truss, system reliability analysis, Subset Simulation, Sysrel program

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1. Introduction

Reliability theory is a well-established field of science. There are numerous scientific books and publication about the reliability and its basic methods, which are some kind of "classic". One of the earliest algorithms for the calculation of structural reliability under combined loading was formulated in 1978 by Rackwitz and Flessler [1]. The introduction of the Hasofer-Lind index, as a reliability measurement, was the biggest step in the development of reliability theory [2]. This index practically displaced the previously used Cornell index, which the basic weakness was the lack of the invariance [3]. The introduction of Hasofer-Lind algorithm results in the more effective way of using First and Second Order Reliability Method [4]. A new approach to calculating the failure probability is carried by the stochastic finite element method (SFEM). An interesting example of the use of SFEM in relation to steel lattice towers is presented in [5,6]. Another trend is using the response surface methods (RSM) [7]. The application of the method in the design of composite panels is presented in works [8, 9]. The Monte Carlo method is effectively combined with artificial neural networks [10, 11]. The presented papers usually describe only one limit state. A different approach involves system reliability analysis, which gives the possibility of combining several limit functions. In the case of system reliability analysis, the biggest problem is the amount of failure modes, what influence on the computational cost of the analysis. Therefore, some redundant strategies are developed. Kim et al. proposed selective searching technique [12]. Safari considered redundancy strategies for multi-objective reliability optimization [13]. The interesting approach to the problem is presented in [14], where the authors use a K-mixed redundancy strategy. Initially, the mathematical formulation for calculating the reliability of the K-mixed strategy is investigated, and then its power and efficiency are evaluated against different test problems and a famous benchmark problem.

The reliability methods are useful for the issues related to different technical problems, connected with civil engineering. Siacara et al. showed how accurate and efficient reliability analyses of geotechnical installations can be performed by directly coupling geotechnical software with a reliability solver [15]. Ontiveros-Péreza S.P. and Miguel L.F.F. proposed a methodology for reliability-based optimum design of multiple tuned mass dampers (MTMD) for installation in buildings situated in seismic regions [16]. The authors of the presented paper have been investigating since few years the reliability of steel structures under fire conditions. The comparison of different reliability methods for steel structures subjected to fire was analysed in [17]. The other paper [18] concerns the influence of the type of support on the reliability of steel truss subjected to fire. The authors also made an analysis of the influence of the randomness of buckling coefficient, defined according to the accident situation, on the reliability of the structure [19]. The interesting application of reliability methods is presented in [20], where artificial neural networks were used to identify parameters of elastic-plastic material parameters. The probabilistic assessment of load-bearing capacity and reliability for different STM of beams loaded with a torsional and bending moment is presented in the paper [21]. Three beams having different reinforcement arrangement obtained on the basis of STM but the same overall geometry and loading pattern were analysed. Stochastic modelling of this beams were performed in order to assess probabilistic load-bearing capacity. During the randomization of variables the Monte Carlo simulation with the reduce the number of simulations the Latin Hypercube Sampling (LHS) method was applied. An original simplified procedure to estimate the remaining service time of corroded shell of an on-the-ground steel tank used to store liquid fuels is presented in the paper [22]. The proposed algorithm is based on fully probabilistic considerations, and those, according to Authors' opinion, by their nature lead to more reliable, and at the same time, objective estimates.

The following paper focuses on the reliability of steel trusses with correlated random variables. The basic research methods, including system reliability analysis combined with First Order Reliability Method (FORM) [1–4, 23] or with Subset Simulation [24–26] are discussed in the following part of the paper. These methods were used by authors during the reliability computation, which were performed in Sysrel module of Strurel program [27]. Strurel is a set of programs dedicated to the reliability analysis. It consists of four modulus, including: Comrel (for time-invariant and time-variant element reliability analysis), Sysrel (for system reliability analysis), Costrel (for reliability-based design optimization), and Statrel (for statical analysis and simulation). The Strurel program was created by the group of scientists from the Technische Universität München (TUM). It has been developed and tested for more than 30 years.

1.1. System reliability analysis

The inclusion of one limit state function in reliability analysis means that we are only able to estimate the reliability of a structure for one exceeded limit state. Even for simple structures, safe design requires the analysis of at least several limit-state functions. Typically, limit states related to the strength, stability and serviceability limit states should be checked. Furthermore, estimating the probability of failure requires consideration of the correlation between the limit states. Combining localized events (limit state functions) together allows one to define the failure system. There are two basic types of failure systems: series and parallel. A **series system** is in a failure state when one of its elements is in a failure state. The load capacity of the series system is equal to the capacity of the weakest element. This is called "the principle of the weakest link in the chain". The graphical interpretation of the series system is presented in Fig. 1a. The failure area of the series system (Ω_f) and the failure probability (P_f) can be written as follows:

$$\Omega_f = \bigcup_{i=1}^{m} \{g_i(\boldsymbol{X}) \leq 0\} \quad P_f = P\left[\bigcup_{i=1}^{m} \{g_i(\boldsymbol{X}) \leq 0\}\right] \tag{1.1}$$

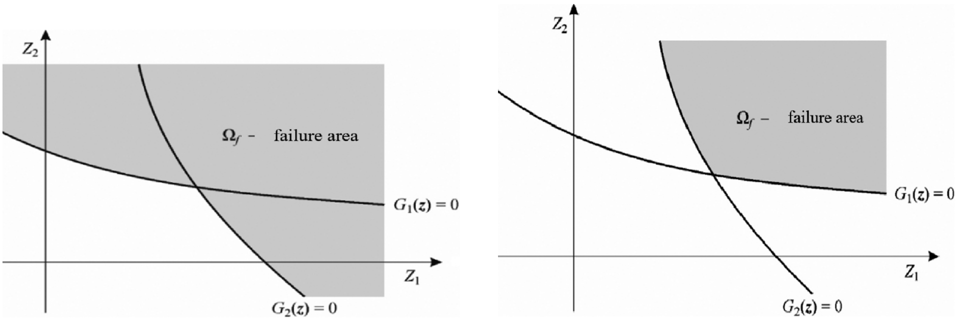
where: m – number of elements of the system, g_i – limit state function of i-th element, $\boldsymbol{X}$ – vector of random variables.

Statically determinate structures correspond to the series systems. The statically determinate truss can be an example. Failure of even one of the truss bars will transform the truss into a mechanism.

In the case of a **parallel system**, the structure remains reliable as long as at least one element is reliable. The graphical interpretation of series system is presented in Fig. 1b.

The area of failure of a parallel system and failure probability can be written as follows:

$$\Omega_f = \bigcap_{i=1}^{m} \{g_i(\boldsymbol{X}) \leq 0\} \qquad P_f = P\left[\bigcap_{i=1}^{m} \{g_i(\boldsymbol{X}) \leq 0\}\right] \tag{1.2}$$



(a) (b)

Fig. 1. The failure area of the (a) series system, (b) parallel system in the standardized Gaussian space

Complex building structures can be modelled by **mixed systems** [28]. In the case of statically indeterminate trusses, it is usually possible to define several groups of bars, which failure leads to various forms of the destruction of the entire structure. The failure model for each group of bars is the parallel system. These single models of failure, connected in a series way, create the whole system. Sets of failure elements that make up parallel systems are called cut-sets. The area of failure of a parallel series system and failure probability can be presented as follows:

$$\Omega_f = \bigcup_{j=1}^{n_p} \bigcap_{i=1}^{m_j} \left\{g_{ij}(\boldsymbol{X}) \leq 0\right\} \qquad P_f = P\left[\bigcup_{j=1}^{n_p} \bigcap_{i=1}^{m_j} \left\{g_{ij}(\boldsymbol{X}) \leq 0\right\}\right] \tag{1.3}$$

where: n_p – number of parallel systems connected in series, m_j – number of elements in the j-th parallel system.

In general, the probability of system failure can be written as the integral of the probability density function of random parameters of structure $f(\boldsymbol{X})$:

$$P_f = \int_{\Omega} f f(x)dx = \int_{R}^{n} I_{\Omega_f}(x) f(x)dx \tag{1.4}$$

where $I_{\Omega_f}(x)$ is a characteristic function of the failure area.

Using the probability of a system failure, the reliability index is defined in the same way as in the case of element reliability:

$$\beta = -\Phi^{-1}\left(P_f\right) \tag{1.5}$$

In the case of parallel-series system the biggest challenge is to correctly identify minimal cut-sets, i.e., minimal sets of causative components, which failure results in the failure of the entire structure. In the research performed so far, the authors have carried out this task through spectral analysis of the linear stiffness matrix [29]. In this approach, only the geometry of the truss and the boundary conditions were considered. A large number of cut-sets identified in this way is insignificant from the computational point of view. These bars that experienced the lowest stress do not affect the reliability index that was ultimately obtained for the entire structure. This knowledge enabled to develop the of a cut-set algorithm that rejects these negligible cut-sets and is much faster [30]. The static analysis of the structure was conducted using the Robot Structural Analysis program. After getting all reduced cut-sets the FORM method in the Sysrel module of the Strurel computing environment [27] was used.

1.2. System reliability analysis with FORM

The approximation of the system failure probability with the FORM method starts from transformation all limit state functions into standard normal space:

$$g_i(\boldsymbol{X}), i = 1, 2, \ldots, m \rightarrow G_i(\boldsymbol{X}) i = 1, 2, \ldots, m \tag{1.6}$$

For series systems, each of the limit state functions is linearized by expanding into a Taylor series around the design point while preserving the linear terms:

$$\beta_i - \boldsymbol{\alpha_i Z} = 0 \tag{1.7}$$

where: $\beta_i = \left\| z_i^* \right\|$ – the distance from the origin to the design point z_i^*,

$\alpha_i = \dfrac{-\nabla G\left(z_i^*\right)}{\left\|\nabla G\left(z_i^*\right)\right\|}$ – the normalized negative gradient vector at the design point.

Then the new random variables $Y_i, i = 1, 2, \ldots, m$, which are defined as functions of $Z = [Z_1, Z_2, \ldots., Z_n]$ vector are introduced:

$$Y_i = \boldsymbol{\alpha_i Z}, \quad i = 1, 2, \ldots, m \tag{1.8}$$

Additionally, the correlation between the Y_i, Y_j can be described by r_{ij}:

$$r_{ij} = \alpha_i \alpha_j^T, \quad i = 1, 2, \ldots, m, \; j = 1, 2, \ldots, m \tag{1.9}$$

Then, the i-th failure event of element can be written as:

$$F_i = \{\beta_i \leq Y_i\}. \tag{1.10}$$

Finally, the probability of a series system failure can be computed as follows:

$$P\left[\bigcup_{i=1}^{m} (\beta_i \leq Y_i)\right] = 1 - P\left[\bigcap_{i=1}^{m} (Y_i < \beta_i)\right] = 1 - \Phi_m(B, R), \tag{1.11}$$

where: $B = [\beta_1, \beta_2, \ldots, \beta_m]^T$ – vector of system elements reliabilities' indices, $\boldsymbol{R}$ – correlation matrix of r_{ij}.

For parallel systems, the linearization is carried out at a so-called common design point and does not apply to all functions, but only to the active functions. The active functions mean those limit functions which value at a common design point is equal to zero. These functions are linearized not at their design points (as is the case of series systems), but at a common design point (Fig. 2).

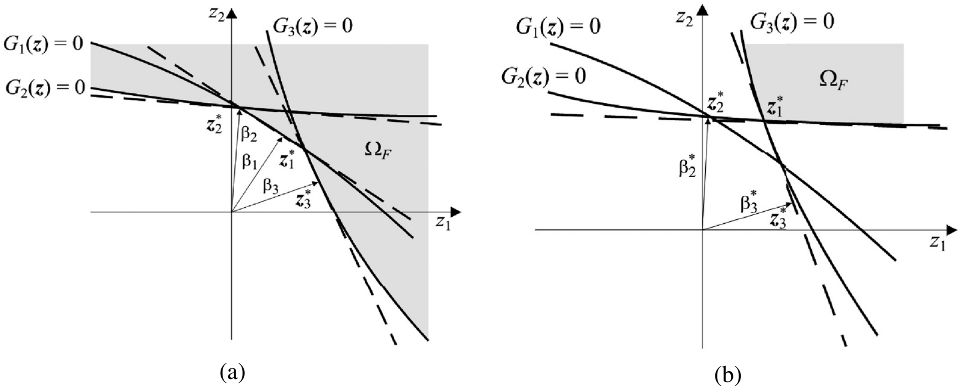


Fig. 2. Linearization of the limit state functions in the case of the a) series system, b) parallel system

The probability of a parallel system failure can be computed as follows:

$$P\left[\bigcap_{i=1}^{m_A}\left(\beta_i^* \le Y_i^*\right)\right] = P\left[\bigcap_{i=1}^{m_A}\left(Y_i^* < -\beta_i^*\right)\right] = \Phi_m\left(-B^*, R^*\right), \tag{1.12}$$

where:m_A – number of active functions, $\boldsymbol{B}^*$ – vector of the reliabilities indices of the elements of the system, but only active functions, $\boldsymbol{R}^*$ – correlation matrix, but only for active functions.

An extremely valuable advantage of the FORM method is the ability to compute the sensitivity of the reliability index to the change of any parameters of the task. In the paper, the sensitivity of the reliability index with respect to changes in the random variables is defined by α vector. The sensitivity of the reliability index to the coordinates of the design point $z*$ has the following form:

$$\alpha_i = \left.\frac{\partial\beta}{\partial Z_i}\right|_{z=z*} \qquad i = 1, \ldots, n \tag{1.13}$$

The α vector can be interpreted as a relative measurement of importance of the standardised variables. The higher the value of α_i, the greater the sensitivity of the reliability index to this variable. The negative value means that an increase in the value of the variable will result in a decrease in the reliability index β. A positive value indicates an increase in the reliability index β with an increase in the value of the variable.

1.3. Subset Simulation

Reliability analysis of building structures is often characterized by very low failure probabilities. This phenomenon, expected by designers of building structures, poses a considerable challenge in numerical calculations. The Monte Carlo method is ineffective in the case of computing the low probabilities. The problem of very low probabilities of failure was solved by Siu-Kui Au and James L. Beck [26].The authors proposed a new method for computing the probability of failure called "subset simulation". The basic idea of ??the method is to express the failure probability as a product of the greatest conditional failure probabilities by introducing intermediate failure events. With the right choice of conditional events, the conditional failure probabilities can be large enough to be estimated by simulation with a small number of samples. The original problem of computing a low probability of failure is limited to computing a series of conditional probabilities that can be easily and efficiently estimated by simulation. Conditional probabilities, however, cannot be efficiently estimated using the standard Monte Carlo procedure [31]. Therefore, their estimation is performed using the Markov MCS chain simulation technique based on the algorithm of the modified Metropolis method [32]. The proposed method is insensitive to the number of random parameters and efficient in computing low probabilities. The estimator of the failure probability we can be described as follows:

$$P_f = P(F_m) = P\left(\bigcap_{i=1}^{m} F_i\right) = P\left(F_m | \bigcap_{i=1}^{m-1} F_i\right) \cdot P\left(\bigcap_{i=1}^{m-1} F_i\right) = P(F_m =| F_{m-1}) \cdot P\left(\bigcap_{i=1}^{m-1} F_i\right) = P(F_1) \prod_{i=1}^{m-1} P(F_{i+1} | F_i) \tag{1.14}$$

1.4. Correlation between random variables

In the presented paper the correlation between the values of bearing capacities of all elements is taken into account. These values are correlated because the structural members within a structure are subjected to common influencing factors, such as the quality of the manufacturer, environmental conditions, and quality control procedures.

In order to simplify the considerations, it is assumed that we have only two random variables X and Y. Correlation is the relationship between two random variables X and Y. Let X and Y be two random variables with mean values $\mu_x = E(X)$ and $\mu_y = E(Y)$ with standard deviations σ_x and σ_y, respectively. The variance of the two random variables is defined as follows:

$$V(X+Y) = E[(X+Y) - E(X+Y)]^2 = E[(X - E(X)) + (Y - E(Y))]^2 = E(X - E(X))^2 + 2E[(X - E(X))(Y - E(Y))] + E(Y - E(Y))^2 \tag{1.15}$$

and the covariance of X and Y is defined as follows:

$$Cov(X,Y) = E[(X - E(X))(Y - E(Y))] = E\left[(X - \mu_x)\left(Y - \mu_y\right)\right] \tag{1.16}$$

The measurement of the correlation between the random variables X, Y is defined as the correlation coefficient which is the quotient of covariance and variance:

$$\rho_{XY} = \frac{Cov\,(X, Y)}{\sigma_x \sigma_y} \tag{1.17}$$

The properties of the correlation coefficient are as follows:

– the correlation coefficient takes the values from the following range:

$$-1 \leq \rho_{XY} \leq 1$$

– if $\rho_{XY} = 0$, then X and Y are not correlated (linearly),
– if $|\rho_{XY}| \cong 1$, then X and Y are linearly correlated,
– if $Cov\,(X, Y) = E\left[(X - \mu_x)\left(Y - \mu_y\right)\right] = E\,(XY) - \mu_y E\,(X) - \mu_x E\,(Y) + \mu_x \mu_y$

$$= E\,(XY) - \mu_x \mu_y = 0$$

then $E\,(XY) = \mu_x \mu_y$

In the case when a set of n random variables, the covariance matrix can be defined as follows:

$$[C] = \begin{bmatrix} Cov\,(X_1, X_1) & Cov\,(X_1, X_2) & \ldots & Cov\,(X_1, X_n) \\ Cov\,(X_2, X_1) & Cov\,(X_2, X_2) & \ldots & Cov\,(X_2, X_n) \\ . & . & \ldots & . \\ . & . & \ldots & . \\ . & . & \ldots & . \\ Cov\,(X_n, X_1) & Cov\,(X_n, X_2) & \ldots & Cov\,(X_n, X_n) \end{bmatrix} \tag{1.18}$$

2. Results and discussion

The purpose of the article was to check the influence of taking into account the correlation between random variables, which described bearing capacities of the statically indeterminate trusses on the failure probability. To make the comparison easier, it was assumed that there are two random variables: bearing capacity and effect of actions. Such an assumption allowed the authors to make some comparisons, but it should be underlined that, in fact, it is difficult to decide which parameters affecting bearing capacity or effect of actions should be treated as random. The only considered way to transform the structure into mechanism was by exceeding the bearing capacity in single members. Reliability of the nodes were not considered. The FORM method was used as the starting point, and the Subset Simulation method was used to verify the correctness of computation. There were two different trusses analysed (Figs. 3 and 7). The nodal forces, shown in the mentioned figures, were the only load. The limit state function (g) for each element was defined as the difference between the bearing capacity (N) and the effect of action (E), that is:

$$g_i = N_i - E_i \tag{2.1}$$

where i is the number of the i-th element.

In the considerations, the following assumptions were made.

Assumptions related to mechanical issues:

– geometric relationships are linear,
– the strains remain within a range that allows the use of linear constitutive relationships,
– structure is protected against the loss of stability from the plane.

Assumptions connected with the reliability analysis:

– effects of actions and bearing capacities of bars are treated as the random variables,
– random variables are assumed to have normal distribution,
– the coefficient of variation of the cross-section area and of the yield strength are assumed to be equal to $\nu_A = 6\%$ and $\nu_{fy} = 8\%$, respectively; therefore the coefficient of variation of the bearing capacities as the function of the mentioned variables (with normal distribution) is approximated as follows:

$$\nu_N = \sqrt{\nu_A^2 + \nu_{fy}^2} = \sqrt{0.06^2 + 0.08^2} = 0.1 = 10\% \tag{2.2}$$

– coefficient of variation of effects of actions is assumed to be equal to 6%,
– random variables, which describes bearing capacities are correlated, while effects of actions are independent.

It has to be underlined that elements that reliabilities was equal to 1.0 was neglected in the computation.

2.1. Example 1: Truss 1

The truss shown in Fig. 3 is the first example. The profiles of the elements are presented in Table 1. In this case, only the reliabilities of elements 11, 13, 14, 16 were lower than 1. So, only they were considered while searching cut-sets. The reliabilities of elements were calculated on the base of bearing capacity and effect of actions in single members (Fig. 4). The obtained reliability system is presented in Fig. 5.

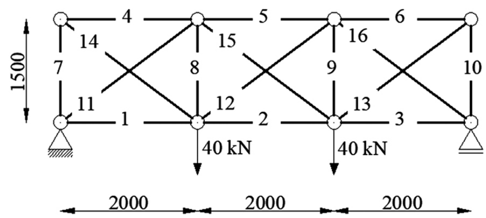


Fig. 3. Truss A: static scheme with numeration of truss bars

Computing the reliability of the system above (Fig. 5) first has to be done in parallel way, then in series. Reliabilities of the pairs of elements connected in parallel are computed as follows:

$$R_I = 1 - (1 - R_{11}) \cdot (1 - R_{14}) \tag{2.3}$$

$$R_{II} = 1 - (1 - R_{13}) \cdot (1 - R_{16}) \tag{2.4}$$

Table 1. Profiles of truss A elements

	Bottom chord	Top chord and external posts	Cross-braces and intermediate posts
Elements' numbers	1–3	4–7, 10	8, 9, 11–16
Profiles	IPE 100	IPE 180	SHS 40 × 40 × 4

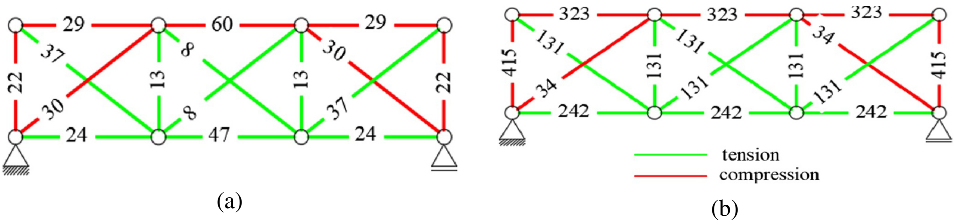


Fig. 4. Truss A: a) effect of actions, b) bearing capacities

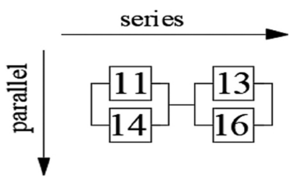


Fig. 5. Reliability system for truss A

Then, the reliability of the whole structure (R) is computed in the series way:

$$R = R_I \cdot R_{II} \tag{2.5}$$

In fact, the computation presented above was performed using Sysrel program. The results according to different values of the coefficient of correlation are presented in Table 2.

Table 2. Reliability indices for truss A with different coefficients of correlation

Reliability analysis method	Coefficient of correlation			
	$\rho = 0$	$\rho = 0.2$	$\rho = 0.5$	$\rho = 0.7$
FORM	7.056	6.854	6.854	6.850
Subset Simulation	7.092	7.002	6.972	6.855

Using the FORM method in the SYSREL program, it is possible to get information about α values. The higher α value for some variable, the greater the influence of this variable on the reliability of the structure. For truss A without correlation the bearing capacities of elements 13, 14 had the highest influence on the reliability index (Fig. 6a). Then with an increase of correlation between the variables corresponding to bearing capacities the influence of bearing capacity of elements 11, 16 became more noticeable (Fig. 6b–6d).

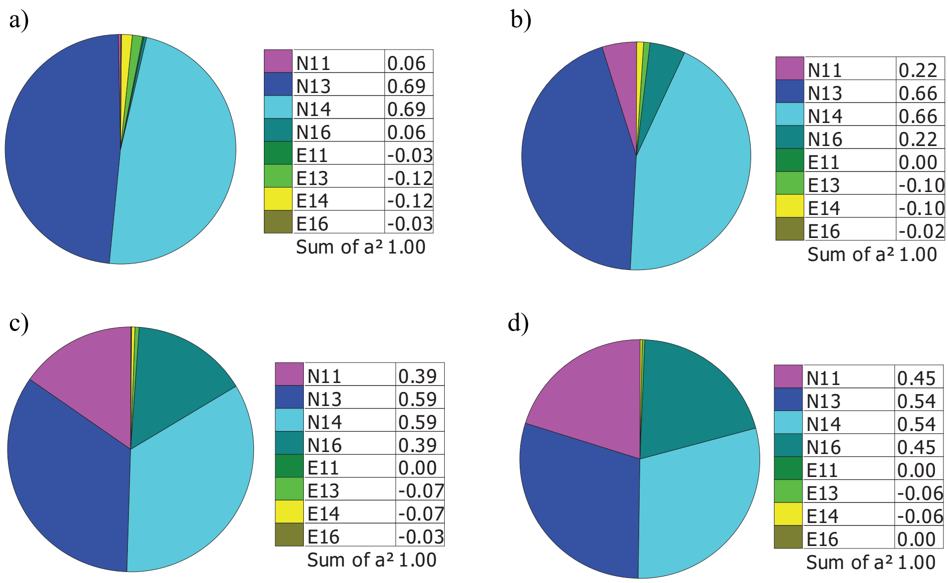


Fig. 6. Representative alphas for truss A: (a) without correlation, (b) with the correlation $\rho = 0.2$, (c) with the correlation $\rho = 0.5$, (d) with the correlation $\rho = 0.7$

2.2. Example 2: Truss B

The following example is a slightly more complex truss (Fig. 7, Table 3) compared with truss A, but the basic assumptions are the same.

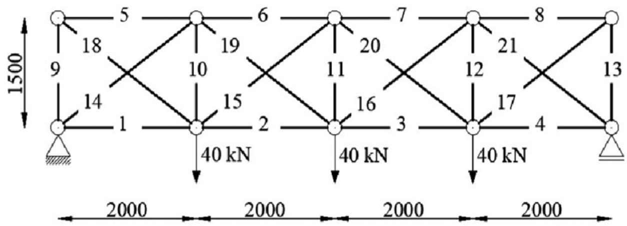


Fig. 7. Truss B: static scheme with numeration of truss bars

In this case, more elements had reliability lower than 1.0, compared with truss A, so the reliability system was slightly more complicated. This system is shown in Fig. 8. The effect of actions and bearing capacities are presented in Fig. 9.

The reliability indices obtained for truss B with different coefficients of correlation are set together in Table 4.

Table 3. Profiles of truss B elements

	Bottom chord	Top chord and external posts	Cross-braces and intermediate posts
Elements' numbers	1–4	5–9, 13	10, 12, 14–21
Profiles	IPE 100	IPE 180	SHS 50 × 50 × 5

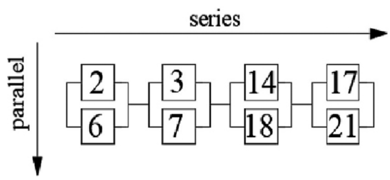
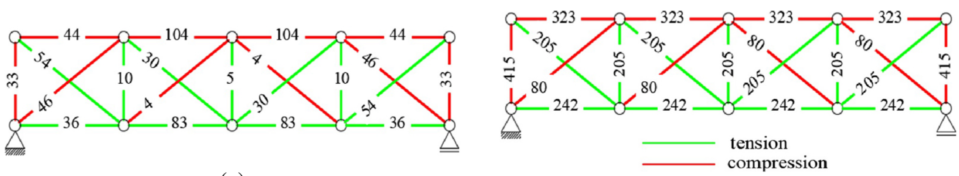


Fig. 8. Reliability system for truss B



(a) (b)

Fig. 9. Truss B: a) effect of actions, b) bearing capacities

Table 4. Reliability indices for truss B with different coefficient of correlation

Reliability analysis method	Coefficient of correlation			
	$\rho = 0$	$\rho = 0.2$	$\rho = 0.5$	$\rho = 0.7$
FORM	8.223	7.653	7.094	6.853
Subset Simulation	8.314	7.517	7.084	6.854

Figures 10a– 10d presents α values for different variables depending on the coefficient of correlation between the bearing capacities. It is clearly visible that in the case of truss B the influence of the bearing capacities is very sensitive to correlation. Without correlation (Fig. 10a), the variables corresponding to the bearing capacities of 17 and 18 elements are definitely more influential compared to one another. Figures 10b– 10d indicate that with the increase of coefficient of correlation the influence of another variables becomes more significant. For a coefficient equal to 0.7 all variables connected with bearing capacities are almost the same influential.

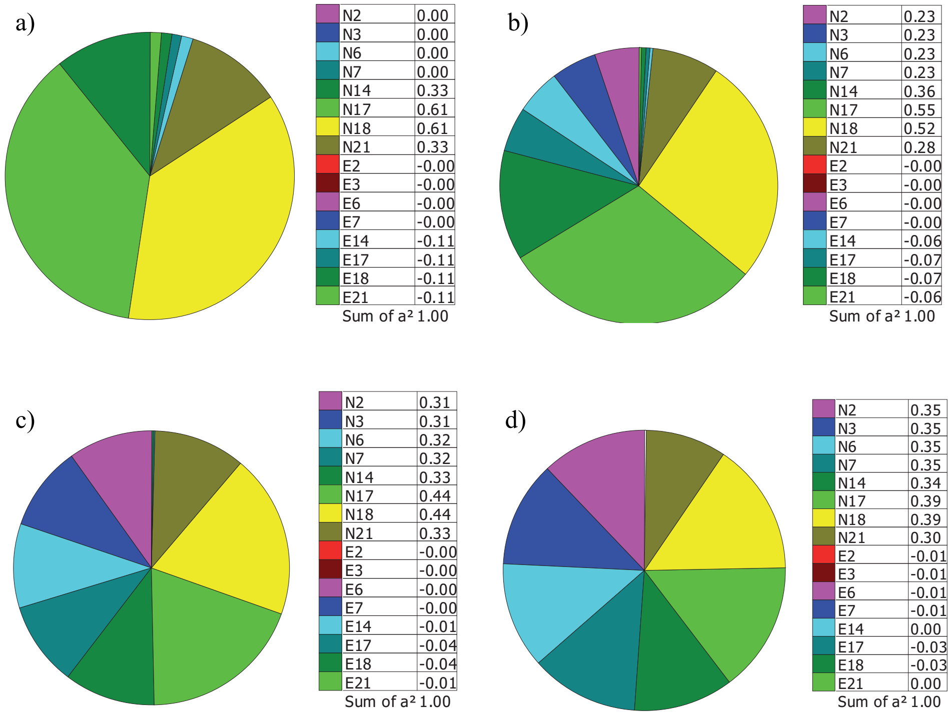


Fig. 10. Representative alphas for truss B: (a) without correlation, (b) with the correlation $\rho = 0.2$, (c) with the correlation $\rho = 0.5$, (d) with the correlation $\rho = 0.7$

3. Conclusions

The examples presented above indicated that in the case of the structures corresponding to mixed system taking into account correlation between variables has significant influence on the structure reliability index. The higher coefficient of correlation the lower reliability index. In the paper the leading reliability method was FORM, the Subset Simulation was used to check the correctness of the results. The results obtained by both methods are very similar, but it should be emphatically underlined that Subset Simulation is appropriate if the reliability indices are high (i.e. probability of failure is small), such situation occurred in the article (see Tables 2 and 4). In the opposite situation (small reliability indices) Subset Simulation could not work so well. All analysed examples indicated that the values connected with bearing capacity (N) have more significant influence on the structure's reliability than those connected with effect of action (E) (Fig. 6a–6d, Fig. 10a–10d). Comparing the results obtained for the truss A with the truss B it is easy to notice that the way of transforming the structure into mechanism could be completely different. This is strictly connected with the force distribution. For the truss A cross-braces are the most stressed elements, so they would decide about the failure of the structure regardless of the degree of correlation between variables. For a slightly more complex structures (truss B) there is bigger amount of causative elements. Without taking correlation into account or in the case of weak correlation (Fig. 10a, 10b) the cross-braces are these elements which decide about the failure of the entire structure. But, with the higher coefficient of correlation, the influence of bearing capacity of both bottom and top chord's elements on the reliability index became more significant. Furthermore, it is clearly visible that the higher coefficient of correlation, the lower reliability index, i.e. the greater failure probability (Tab. 3 and 4). To sum up, it should be underlined that it is worth to take into account correlation between variables during reliability analysis, because it can have quite high influence on the obtained results. The authors would like to inform that the presented article is the part of wider research, which are conducted to find some dependencies during reliability analysis of steel structure. The main finding of the article is the fact, that the correlation between random variables should be taken into account, but there are still some problems to be solved. The main problem is the appropriate values of coefficient of correlation and the definition of correlated variables. This issue will be investigated by the authors in the further scientific work.

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Systemowa analiza niezawodności stalowych kratownic ze skorelowanymi zmiennymi

Słowa kluczowe: kratownice stalowe, systemowa analiza niezawodności, analiza wrażliwości, FORM, Subset Simulation, Sysrel

Streszczenie:

W artykule przedstawiono analizę wpływu korelacji między zmiennymi losowymi na uzyskiwany wskaźnik niezawodności, z wykorzystaniem analizy systemowej. W pracy uwzględniono kratownice stalowe o dwóch różnych geometriach, w obu przypadkach były to konstrukcje statycznie niewyznaczalne. Założono, że jedynym obciążeniem działającym na kratownice są siły [skupione, zaczepione w węzłach pasa dolnego. W pierwszym kroku przeprowadzono obliczenia statyczno-wytrzymałościowe w programie Robot Structural Analysis, uzyskując wartości nośności oraz efektu oddziaływań dla poszczególnych prętów kratownic. Jako zmienne losowe przyjęto efekt oddziaływań (E) oraz nośności poszczególnych elementów (N), o współczynnikach zmienności odpowiednio 6% i 10%. Założono, że wszystkie zmienne mają rozkład normalny. Jako kryterium zniszczenia elementu przyjęto przekroczenie nośności w prętach. Po oszacowaniu niezawodności poszczególnych elementów określono wszystkie możliwe schematy zniszczenia (cut-sets).W tym celu wykorzystano analizę spektralną macierzy sztywności konstrukcji, z wykorzystaniem informacji o niezawodnościach elementów. Elementy o niezawodności równej 1.0 były pomijane, gdyż nie mają one wpływu na niezawodność konstrukcji. Porównując wyniki uzyskane dla kratownicy A z kratownicą B łatwo zauważyć, że sposób przekształcenia konstrukcji w mechanizm dla każdej z nich jest zupełnie inny. Jest to ściśle związane z różną redystrybucją sił wewnętrznych. Znając modele niezawodnościowe kratownic wykonano obliczenia w programie Sysrel środowiska obliczeniowego Strurel. Strurel to zestaw programów do analizy niezawodności. W jego skład wchodzą Comrel (program do analizy niezawodności elementu), Sysrel (program do analizy niezawodności systemu), Costrel (program do optymalizacji w oparciu o niezawodność) oraz Statrel (program do analizy statystycznej i symulacji). Program Strurel został stworzony przez grupę naukowców z Technische Universt?t München (TUM). Jest rozwijany i testowany od ponad 30 lat. W pracy jako metodę wiodącą wykorzystano First Order Reliability Method(FORM). W celu weryfikacji poprawności uzyskanych wyników zastosowano metodę Subset Simulation (symulacji podzbiorów). Obliczenia dla każdej z analizowanych kratownic przeprowadzono dla różnych współczynników korelacji ($\rho = 0$, $\rho = 0,2$, $\rho = 0,5$, $\rho = 0,7$). Korelacja zachodziła między zmiennymi losowymi, związanymi z nośnością prętów kratownic. Podczas obliczeń w programie Sysrel, z wykorzystaniem metody FORM możliwa jest analiza wrażliwości wskaźnika niezawodności, która pozwala śledzić w jakim stopniu poszczególne zmienne wpływają na otrzymywane prawdopodobieństwo awarii konstrukcji. Przeprowadzone analizy pozwoliły określić jaki jest wpływ uwzględnienia korelacji między zmiennymi na wskaźnik niezawodności konstrukcji. Wyraźnie widać, że im wyższy współczynnik korelacji, tym niższy wskaźnik niezawodności, czyli większe prawdopodobieństwo awarii (tab. 3 i 4). Podsumowując, należy podkreślić, że podczas analizy niezawodności warto uwzględniać korelację między zmiennymi, ponieważ może ona mieć duży wpływ na uzyskiwane wyniki. Autorzy informują, że prezentowany artykuł jest częścią szerszych badań, które są prowadzone w celu znalezienia pewnych zależności podczas analizy niezawodności konstrukcji stalowych. Przeprowadzona analiza wykazuje, że korelacja między zmiennymi losowymi powinna być brana pod uwagę podczas analizy niezawodności konstrukcji. Nie jest to jednak zadanie proste, gdyż najbardziej problematyczny jest odpowiedni dobór wartości współczynnika korelacji oraz wybór zmiennych skorelowanych. Kwestia ta zostanie zbadana przez autorów w dalszej pracy.

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