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Study on triangulation network adjustment by Total Least Square Method

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Abstract: Generally, Least Squares (LS) Method treats only random errors of observation vector in adjustment function models. However, both observation vector and elements of coefficient matrix of adjustment function model contain random errors. Therefore, there is no guarantee that the result of adjustment by LS method is the global optimal solution. Total Least Square (TLS) method is a primary estimation method that treats random errors of observation vector and coefficient matrix in adjustment functional models. Since TLS method take into account both random errors of observation vector and coefficient matrix based on errors-in-variables model, it is possible to improve the accuracy compared with the result of LS method. So TLS method has been applied to different fields of science and technology including signal and image processing, computer vision,communication engineering and geodesy. However, weighted total least square (WTLS) method has been not applied in geodetic network adjustment problem compared with other fields widely. So the purpose of this paper is to summarize the algorithm of WTLS briefly and to propose an application method in adjustment of triangulation network. Key problem in application of WTLS to adjustment of geodetic network is to determine the weight matrix (or cofactor matrix) for elements of coefficient matrix in adjustment function model. In this paper proposed a method to determine cofactor matrix for errors of coefficient matrix in triangulation network, and verifies the effectiveness of suggested method through example applied to triangulation network.

Keywords: WTLS method, EIV model, cofactor matrix, adjustment function model, triangulation network adjustment

1. Introduction

Total Least Squares (TLS) method had been widespread since [8]Golub and van Loan (1980) coined the terminology of TLS firstly and demonstrated that the TLS solution could be obtained readily by singular value decomposition about 40 years ago (Golub and van Loan, 1980; van Huffel and Vandewalle, 1991). TLS method have been further systematically developed and widely applied to many science and engineering problems, namely some practical problems, such as those in signal processing, statistical calculation, regression analysis, coordinate transformations (Akyilmaz, 2007; Felus, 2004; Markovsky and van Huffel, 2005; Wang, 2016). Particularly, as TLS method considers errors both in observation vector and the coefficient matrix simultaneously, this theory is more rigorous than least square (LS) method (Felus and Burtch, 2009; Ghilant, 2006; Markovsky and van Huffel, 2007). Therefore, the TLS method based on errors-in-variables (EIV) model has been arrived as a new method on field of data processing and nowadays obtained good success in solving practical problem of geodetic science and engineering (Mastronardi et al., 2000; Neitzel, 2010; Schaffrin, 2006).

In general, the development process of TLS method seems to depend on two typical approaches to deal with data of observations (Golub and van Loan, 1980; Shen et al., 2011). The first approach of TLS is to solve by direct solution (SVD-singular value decomposition) method and second one is based on iterative solution (Lagrange multipliers) method (Schaffrin et al., 2006; Schaffrin and Wieser, 2008; Schaffrin and Felus, 2009; Shen et al., 2011; Xu, 2003; Xu,2004). SVD method is used in case of equivalent weight and iterative solution method is used in case of unequal weight in observation vector and coefficient matrix. Therefore, TLS method have been developed into more generalized weighted total least square (WTLS) method and many researchers have studied about it and its application deeply and widely (Xu, 2009; Xu and Shimada, 2000; Xu et al., 2006; Xu, 2012).

Recently most of all works on WTLS method focus on methods and applications for linear regression model fit, curved line fit and coordinate transformation (Deng, 2019; Fang, 2014; Fang, 2015; San, 2014; [20]Wang, 2016; Wang, 2019a; 2019b). But due to the lack of general method to determine cofactor matrix of coefficient matrix of function model, it is restricted in practical application. Specially, WTLS method has been less applied in adjustment problem of geodetic network than other fields. Since coefficient matrix of adjustment function model has different structure according to type of geodetic network, it is impossible to construct correlation matrix of it as general form.

The purpose of this paper is to establish the method to determine cofactor matrix of coefficient matrix of adjustment function model in geodetic network adjustment by WTLS method and improve the accuracy of geodetic control points by applying to adjustment calculation. In this paper, we describe the adjustment method based on WTLS method in triangulation network and verify effectiveness of this method through example applied to triangulation network adjustment.

So the system of paper is as follows. In second part reformulate solution algorithm to calculate the estimation of WTLS method in EIV model and then outline a formula to estimate it's accuracy. In third part describes example to demonstrate the algorithm proposed in this paper and then compare the adjustment results by WTLS method with the results by LS and TLS methods in triangulation networks.

2. Adjustment of geodetic network by WTLS

In this part reformulate the algorithm to solve the WTLS problem and then give a formula related with first order approximation to estimate the accuracy of unknown parameters in EIV model.

2.1. Parameter adjustment algorithm

EIV model in the WTLS method can be written as follows (Schaffrin and Wieser, 2008; Shen et al., 2011):

$$L - e_L = (A - E_A) \cdot \tau, \tag{1}$$

where L denotes the $m \times 1$ observation vector, τ represents the $n \times 1$ unknowns parameter vector, A represents the $m \times n$ coefficient matrix of error equation, e_L denotes the random error vector of L, and E_A denotes the random error matrix of A.

Eq. 1 can be rewritten as form of condition equations as follows:

$$A\tau - E_A\tau - L + e_L = 0. \tag{2}$$

Considering Eq. 2, the Lagrange objective function of WTLS can be written as follows:

$$\Phi(e_L, E_A, K, \tau) = e_L^T Q_L^{-1} e_L + e_A^T Q_A^{-1} e_A + 2K^T(A\tau - (\tau^T \otimes I_n)e_A - L + e_L) = \min, \tag{3}$$

where K denotes the $m \times 1$ vector of "Lagrange multipliers", Q_L, Q_A represents the cofactor matrix of L and A, and $E_A\tau = (\tau^T \otimes I_n)e_A$, $e_A = vec\,(E_A)$, in which "$vec\,(\bullet)$" denotes the operator that stacks one column of a matrix underneath the previous one, and "$\otimes$" denotes the "Kroneker–Zehfuss product" of matrices.

The solution of objective function (3) can be derived via the Euler-Lagrange necessary conditions:

$$\left.\begin{aligned} \frac{\partial\Phi}{\partial e_L} &= 2\tilde{e}_L^T Q_L^{-1} + 2\hat{K}^T = 0 \\ \frac{\partial\Phi}{\partial e_A} &= 2\tilde{e}_A^T Q_A^{-1} - 2\hat{K}^T(\hat{\tau}^T \otimes I_n) = 0 \\ \frac{\partial\Phi}{\partial K} &= A\hat{\tau} - (\hat{\tau}^T \otimes I_n)\tilde{e}_A - L + \tilde{e}_L = 0 \\ \frac{\partial\Phi}{\partial \tau} &= 2\hat{K}^T A - 2K^T \tilde{E}_A = 0 \end{aligned}\right\}, \tag{4}$$

where "$\sim$" and "$\wedge$" denotes predicted and estimated, respectively.

We can rewrite Eq. 4, as follows:

$$Q_L^{-1}\tilde{e}_L + \hat{K} = 0, \tag{5}$$

$$Q_A^{-1}\tilde{e}_A - (\hat{\tau} \otimes I_n)\hat{K} = 0, \tag{6}$$

$$A\hat{\tau} - (\hat{\tau}^T \otimes I_n)\tilde{e}_A - L + \tilde{e}_L = 0, \tag{7}$$

$$A^T \hat{K} - \widetilde{E}_A^T \hat{K} = 0. \tag{8}$$

From Eq. 5 and Eq. 6, we obtain following equations:

$$\widetilde{e}_L = -Q_L \hat{K}, \tag{9}$$

$$\widetilde{e}_A = Q_A(\hat{\tau} \otimes I_n)\hat{K}. \tag{10}$$

Substituting Eq. 9 and Eq. 10 into Eq. 7, we can obtain following:

$$\begin{gathered} A\hat{\tau} - (\hat{\tau}^T \otimes I_n)Q_A(\hat{\tau} \otimes I_n)\hat{K} - L - Q_L\hat{K} = \\ A\hat{\tau} - L - \left[Q_L + (\hat{\tau}^T \otimes I_n)Q_A(\hat{\tau} \otimes I_n)\right]\hat{K} = 0. \end{gathered} \tag{11}$$

Thus, we can represent the Lagrange multipliers K in Eq. 11 as follows:

$$\hat{K} = Q^{-1}(A\hat{\tau} - L), \tag{12}$$

where

$$Q = Q_L + (\hat{\tau}^T \otimes I_n)Q_A(\hat{\tau} \otimes I_n). \tag{13}$$

Substituting Eq. 12 into Eq. 9 and Eq. 10, respectively, we obtain as follows:

$$\widetilde{e}_L = -Q_L Q^{-1}(A\hat{\tau} - L), \tag{14}$$

$$\widetilde{e}_A = Q_A(\hat{\tau} \otimes I_n)Q^{-1}(A\hat{\tau} - L). \tag{15}$$

Substituting Eq. 12 into Eq. 8, we can represent as follows:

$$A^T Q^{-1}(A\hat{\tau} - L) - \widetilde{E}_A^T Q^{-1}(A\hat{\tau} - L) = A_1^T Q^{-1} A\hat{\tau} - A_1^T Q^{-1} L = 0 \tag{16}$$

where $A_1 = A - E_A$.

From Eq. 16, correction vector of deterministic unknowns is as follows:

$$\hat{\tau} = (A_1^T Q^{-1} A)^{-1} A_1^T Q^{-1} L. \tag{17}$$

In summation, the computation process described in Eqs. 13–17 is as follows:

Step 1. Initial values of unknown parameter τ and error matrix E_A is as follows:

$$\tau_0 = (A^T Q_L^{-1} A)^{-1} A^T Q_L^{-1} L, \quad E_{A_0} = 0.$$

Step 2. Calculate cofactor matrix Q by Eq. 13 as follows:

$$Q_i = Q_L + (\hat{\tau}_i^T \otimes I_n)Q_A(\hat{\tau}_i \otimes I_n)$$

Step 3. Calculate error vector $\widetilde{e}_L, \widetilde{e}_A$ by Eq. 14 and Eq. 15 as follows:

$$\widetilde{e}_{L_{i+1}} = -Q_L Q_i^{-1}(A\hat{\tau}_i - L)$$

$$\widetilde{e}_{A_{i+1}} = Q_A(\hat{\tau}_i \otimes I_n)Q_i^{-1}(A\hat{\tau}_i - L)$$

where i denotes the iterative number.

Step 4. Calculate correction vector τ of deterministic unknowns by Eq. 17 as follows:

$$\hat{\tau}_{i+1} = (A_1^T Q_i^{-1} A)^{-1} A_1^T Q_i^{-1} L$$

Step 5. Iterate from step 2 to step 4 until the following condition is satisfied:

$$|\hat{\tau}_{i+1} - \hat{\tau}_i| < \varepsilon. \tag{18}$$

where ε is a small positive number related with accuracy of computation.

2.2. Accuracy of parameter adjustment result

Although many algorithms have been developed to solve WTLS problems, statistical aspects of WTLS estimation have not received due attention. WTLS estimations have been proved to be weakly consistent under regularity conditions, as the number of measurements tends to infinity (Markovsky and van Huffel, 2005). However, asymptotic statistical results are not very useful in practice. Even if a WTLS estimator is known to be asymptotically unbiased, it can still be significantly biased in the case of finite samples, depending on the severity of model. The WTLS estimator of parameters in an EIV model is essentially the problem of nonlinearity.

As a result, we can naturally use nonlinear adjustment theory to analyze the nonlinear TLS estimators statistically. Here, we will investigate the statistical aspects of the nonlinear WTLS estimators in finite sample. Based on well-known knowledge, we can obtain the first-order approximation of the cofactor matrix of the WTLS estimates simply.

Considering Eq. 17, cofactor matrix of unknown parameters can be determined based on linearly approximate cofactor propagation law as follows:

$$\widetilde{Q}_\tau = (A_1^T \widetilde{Q}^{-1} A)^{-1} A_1^T \widetilde{Q}^{-1} Q_L \widetilde{Q}^{-1} A_1 (A_1^T \widetilde{Q}^{-1} A)^{-1}, \tag{19}$$

or

$$\widetilde{Q}_\tau = Q_{\tau 0} Q_L Q_{\tau 0}^T, \tag{20}$$

where

$$Q_{\tau 0} = (A_1^T \widetilde{Q}^{-1} A)^{-1} A_1^T \widetilde{Q}^{-1}.$$

Therefore, variance matrix of unknown parameters can be determined as follows:

$$D_\tau = \hat{\sigma}_0^2 \widetilde{Q}_\tau, \tag{21}$$

where $\hat{\sigma}_0^2$ is estimation of unit weight variance, and $\hat{\sigma}_0^2$ is estimated as follows:

$$\hat{\sigma}_0^2 = \frac{\widetilde{e}^T \widetilde{Q}^{-1} \widetilde{e}}{m-n}, \tag{22}$$

where

$$\widetilde{e} = \widetilde{E}_A \hat{\tau} - \widetilde{e}_L = A\hat{\tau} - L,$$

m – number of observations, n – number of unknown parameters.

3. Application of WTLS method in triangulation network adjustment

In order to apply WTLS method in adjustment of triangulation network, at first, the cofactor matrix Q of EIV model should be calculated by Eq. 13. In calculation of cofactor matrix Q, it is the most important problem to calculate the cofactor matrix Q_A of coefficient matrix. In this section, we describe the method to calculate the correlation matrix of coefficient matrix A. Then, we have adjusted the triangulation network by proposed method and analyzed the effectiveness.

3.1. Cofactor matrix of coefficients in triangulation network adjustment

Let's determine the cofactor matrix of coefficients in example of triangulation network presented in Figure 1. In Figure 1, point 6 and 7 are the given points, point 1 ∼ 5 are points to be determined and number of measurement angle is 18.

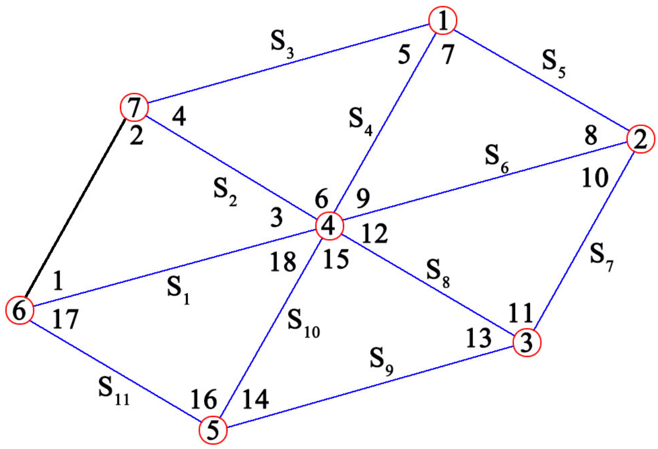

Fig. 1. Triangulation network

General form of residual equation in triangulation network with 18 angles can be written as follows:

$$v^k_{\beta_{ij}} = (a_{ki} - a_{kj})\tau_{x_k} + (b_{ki} - b_{kj})\tau_{y_k} - a_{ki}\tau_{x_i} - b_{ki}\tau_{y_i} + a_{kj}\tau_{x_j} + b\tau_{kj} + \ell^k_{\beta_{ij}}, \tag{23}$$

where k – a number of vertex forming angles, i, j – numbers of vertices forming angles in triangle (see Fig. 2), τ_{x_i}, τ_{y_i} – correction value of coordinates, $\ell^k_{\beta_{ij}} = \ell_{kj} - \ell_{ki} = \beta^{k'}_{ij} - \beta^{k^0}_{ij}$, $a_{ij} = \frac{\Delta y_{ij}}{S^2_{ij}}, b_{ij} = -\frac{\Delta x_{ij}}{S^2_{ij}}, S_{ij} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}$.

Making Eq. 23 with all angles and considering the EIV model, we can obtain as follows:

$$L - e_L = (A - E_A) \cdot \tau, \tag{24}$$

where

$$A = \begin{pmatrix} A_{1,1} & A_{1,2} & \cdots & \cdots & A_{1,9} & A_{1,10} \\ A_{2,1} & A_{2,2} & \cdots & \cdots & A_{2,9} & A_{2,10} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ A_{18,1} & A_{18,2} & \cdots & \cdots & A_{18,9} & A_{18,10} \end{pmatrix}_{18\times 10}.$$

Here, elements of coefficient matrix are represented as a function of observation angles, that is:

$$A = f(\beta_1, \beta_{,2}, \cdots, \beta_{18}). \tag{25}$$

Taking partial derivatives of Eq. 26 with relation to observation angles β_i, we can denote as follows;

$$H = \frac{\partial A}{\partial \beta} = \begin{pmatrix} \frac{\partial A_{11}}{\partial \beta_1} & \frac{\partial A_{11}}{\partial \beta_2} & \cdots & \frac{\partial A_{11}}{\partial \beta_{18}} \\ \frac{\partial A_{12}}{\partial \beta_1} & \frac{\partial A_{12}}{\partial \beta_2} & \cdots & \frac{\partial A_{12}}{\partial \beta_{18}} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial A_{110}}{\partial \beta_1} & \frac{\partial A_{110}}{\partial \beta_2} & \cdots & \frac{\partial A_{110}}{\partial \beta_{18}} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial A_{181}}{\partial \beta_1} & \frac{\partial A_{181}}{\partial \beta_2} & \cdots & \frac{\partial A_{181}}{\partial \beta_{18}} \\ \frac{\partial A_{182}}{\partial \beta_1} & \frac{\partial A_{182}}{\partial \beta_2} & \cdots & \frac{\partial A_{182}}{\partial \beta_{18}} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial A_{1810}}{\partial \beta_1} & \frac{\partial A_{1810}}{\partial \beta_2} & \cdots & \frac{\partial A_{1810}}{\partial \beta_{18}} \end{pmatrix}_{180\times 18} . \tag{26}$$

Here partial derivatives $\frac{\partial A_{ij}}{\partial \beta_k}$ should be determined. Let's consider an example to determine the partial derivatives (Fig. 2). Partial derivatives $\frac{\partial A_{1j}}{\partial \beta_1}$ $(j = 1, 2, \cdots 6)$ in relation to coefficient of correction equation of angle 1 in triangle of Figure 2 (note that 1, 2, 3 are number of angles) can be computed as follows:

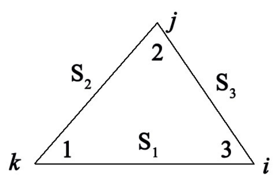

Fig. 2. Triangle

$$\frac{\partial A_{11}}{\partial \beta_1} = \frac{\sin \alpha_{ki}}{S_1} - \frac{\sin \alpha_{kj}}{S_2}$$

$$\frac{\partial A_{12}}{\partial \beta_1} = -\frac{\cos \alpha_{ki}}{S_1} + \frac{\cos \alpha_{kj}}{S_2}$$

$$\frac{\partial A_{13}}{\partial \beta_1} = \frac{\sin \alpha_{kj}}{S_2}, \quad \frac{\partial A_{14}}{\partial \beta_1} = -\frac{\cos \alpha_{kj}}{S_2}, \quad \frac{\partial A_{15}}{\partial \beta_1} = -\frac{\sin \alpha_{ki}}{S_1}, \quad \frac{\partial A_{16}}{\partial \beta_1} = \frac{\cos \alpha_{ki}}{S_1},$$

where S_i – length of triangle side (see Fig. 2).

Similarly partial derivatives $\dfrac{\partial A_{ij}}{\partial \beta_k}$ $(j = 1, 2, \cdots 6)$ in relation to all angles can be determined. Considering Eq. 25 and Eq. 26, the cofactor matrix Q_A of coefficient matrix A can be determined as follows:

$$Q_A = H Q_\beta H^T, \tag{27}$$

where Q_β is cofactor matrix of measured angles.

From Eq. 13, cofactor matrix $\widetilde{Q}$ of observation vector and coefficient matrix can be computed as follows:

$$\widetilde{Q} = Q_\beta + (\tau^T \otimes I_{18}) Q_A (\tau \otimes I_{18}). \tag{28}$$

3.2. Triangulation network adjustment by WTLS

We adjust triangulation network (Fig. 3) laid out in an area using the real observation data by WTLS method. In triangulation network presented in Figure 3, point 16 and 17 are the given points, points 1 ∼ 15 are points to be determined and number of observation angle are 66, and coordinates of given points and observation angles are presented in Table 1 and Table 2. Using given data in Table 1 and Table 2, we adjusted the triangulation network by three methods, that is, WTLS, TLS and LS methods.

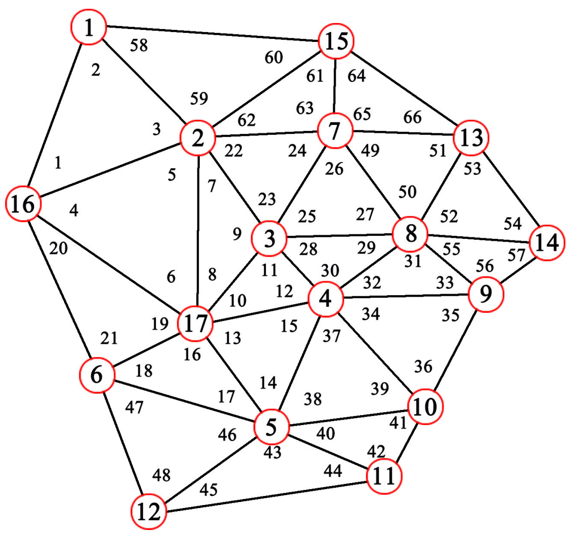

Fig. 3. Triangulation network in an area

Table 1. Coordinate of given points

Point no.	Coordinate	
	X (m)	Y (m)
16	6 749.760	–7 351.370
17	3 069.590	–2 809.280

Table 2. Observation data

Angle No.	Observation angle (° ′ ″)	Angle no.	Observation angle (° ′ ″)	Angle no.	Observation angle (° ′ ″)
1	48 13 38.0	23	58 06 8.6	45	29 49 54.1
2	58 10 6.9	24	59 38 39.8	46	59 11 38.8
3	73 36 15.6	25	61 19 56.8	47	54 03 17.4
4	62 10 55.8	26	59 00 24.2	48	66 45 6.7
5	66 14 37.2	27	59 39 35.7	49	55 26 24.6
6	51 34 30.3	28	54 54 10.5	50	59 24 56.8
7	31 54 59.3	29	40 44 39.0	51	65 08 38.3
8	35 15 2.8	30	84 21 6.0	52	68 16 14.4
9	112 49 52.2	31	93 03 25.5	53	59 16 2.6
10	41 36 29.5	32	40 21 36.7	54	52 27 38.2
11	72 49 51	33	46 35 0.6	55	38 51 9.6
12	65 33 39.0	34	54 34 53.5	56	94 38 19.6
13	70 02 28.9	35	63 41 42.8	57	46 30 32.5
14	51 53 52.2	36	61 43 22.1	58	46 29 17.2
15	58 03 45.8	37	57 04 59.0	59	89 19 51.8
16	90 47 27.6	38	62 32 16.7	60	44 10 50.7
17	38 20 39.0	39	60 22 44.0	61	49 26 11.7
18	50 51 51.5	40	36 43 46.3	62	36 39 4.8
19	70 44 0.9	41	57 07 16.7	63	93 54 44.9
20	29 34 52.2	42	86 08 59.9	64	50 45 28.7
21	79 41 7.5	43	111 17 47.0	65	91 59 46.5
22	62 15 11.3	44	38 52 25.3	66	37 14 46.1

In this case, partial derivatives with relation to observation angles β by Eq. 26 can denote as follows:

$$H = \frac{\partial A}{\partial \beta} = \begin{pmatrix} \frac{\partial A_{11}}{\partial \beta_1} & \frac{\partial A_{11}}{\partial \beta_2} & \cdots & \frac{\partial A_{11}}{\partial \beta_{66}} \\ \frac{\partial A_{12}}{\partial \beta_1} & \frac{\partial A_{12}}{\partial \beta_2} & \cdots & \frac{\partial A_{12}}{\partial \beta_{66}} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial A_{130}}{\partial \beta_1} & \frac{\partial A_{130}}{\partial \beta_2} & \cdots & \frac{\partial A_{130}}{\partial \beta_{66}} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial A_{661}}{\partial \beta_1} & \frac{\partial A_{661}}{\partial \beta_2} & \cdots & \frac{\partial A_{661}}{\partial \beta_{66}} \\ \frac{\partial A_{662}}{\partial \beta_1} & \frac{\partial A_{662}}{\partial \beta_2} & \cdots & \frac{\partial A_{662}}{\partial \beta_{66}} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial A_{6630}}{\partial \beta_1} & \frac{\partial A_{6630}}{\partial \beta_2} & \cdots & \frac{\partial A_{6630}}{\partial \beta_{66}} \end{pmatrix}_{1980\times 66}.$$

Considering Eq. 27, the cofactor matrix Q_A of coefficient matrix A can be determined as follows:

$$\underset{1980\times 1980}{Q_A} = \underset{1980\times 66}{H} \cdot \underset{66\times 66}{Q_\beta} \cdot \underset{66\times 1980}{H^T}.$$

Adjustment results are given in Table 3 and Table 4. And standard deviation of coordinates X a and Y are represented in Figure 4 and Figure 5, respectively. In Figure 3 and Figure 4 curve 1 denotes standard deviation of adjusted coordinates in WTLS method, curve 2 in TLS method and curve 3 in LS method.

Standard deviation factor of unit weight is $\sigma_{\mathrm{WTLS}} = 0.23$ in WTLS method, $\sigma_{\mathrm{TLS}} = 1.68$ in TLS method and $\sigma_{\mathrm{LS}} = 1.79$ in LS method.

As can be seen in Table 4, Figure 4 and Figure 5, the largest standard deviations of X,Y coordinates is 1.3 cm, 1.5 cm respectively in WTLS method, –6.4 cm, 7.6 cm in TLS method, –6.8 cm, 7.4 cm in LS method. Consequently, it is clear that WTLS method is superior to TLS and LS methods in raising the accuracy of adjustment results. However, differences between accuracy of adjustment results by TLS and LS methods is not so large. Because the cofactor matrix of both observation vector and coefficient matrix is assumed as unit matrix. Therefore, we should apply WTLS method after determination of cofactor matrix of both observation vector and coefficient matrix to raise the accuracy of adjustment results.

Table 3. Adjusted coordinates by three methods

Point no.	WTLS		TLS		LS	
	X (m)	Y (m)	X (m)	Y (m)	X (m)	Y (m)
1	12 104.151	–5 548.488	12 104.408	–5 548.554	12 104.487	–5 548.616
2	8 717.994	–2 751.155	8 718.164	–2 751.065	8 718.236	–2 751.054
3	5 696.258	–911.999	5 696.365	–911.8738	5 696.409	–911.832
4	3 808.487	509.959	3 808.498	510.141	3 808.506	510.210
5	–23.002	838.564	–23.100	–838.342	–23.172	–838.305
6	1 527.490	–5 304.389	1 527.480	–5 304.420	1 527.465	–5 304.461
7	8 935.259	722.412	8 935.448	722.584	8 935.554	722.645
8	5 815.230	2 689.597	5 815.318	2 689.800	5 815.371	2 689.915
9	3 970.171	4 579.740	3 970.248	4 580.016	3 970.260	4 580.181
10	527.730	3 044.974	527.755	3 045.260	527.685	3 045.387
11	–1 606.319	2 058.599	–1 606.412	2 058.869	–1 606.530	2 058.966
12	–2 703.061	–4 027.191	–2 703.319	–4 027.125	–2 703.434	–4 027.168
13	8 793.083	4 218.986	8 793.247	4 219.232	8 793.375	4 219.369
14	5 470.183	6 302.061	5 470.307	6 302.372	5 470.363	6 302.578
15	11 669.941	738.279	11 670.223	738.475	11 670.381	738.512

Table 4. Standard deviations of coordinates by three methods

Point no.	WTLS		TLS		LS	
	X (m)	Y (m)	X (m)	Y (m)	X (m)	Y (m)
1	0.009	0.008	0.045	0.048	0.047	0.050
2	0.006	0.006	0.029	0.030	0.029	0.032
3	0.005	0.005	0.024	0.022	0.024	0.022
4	0.004	0.007	0.024	0.027	0.026	0.026
5	0.006	0.008	0.032	0.027	0.032	0.029
6	0.003	0.006	0.017	0.028	0.018	0.030
7	0.008	0.008	0.044	0.039	0.043	0.041
8	0.007	0.009	0.040	0.043	0.042	0.042
9	0.010	0.013	0.050	0.059	0.053	0.057
10	0.008	0.012	0.046	0.053	0.048	0.052
11	0.011	0.011	0.055	0.052	0.056	0.053
12	0.011	0.009	0.0541	0.045	0.055	0.048
13	0.012	0.013	0.063	0.060	0.064	0.060
14	0.013	0.015	0.064	0.076	0.068	0.074
15	0.012	0.010	0.062	0.052	0.061	0.055

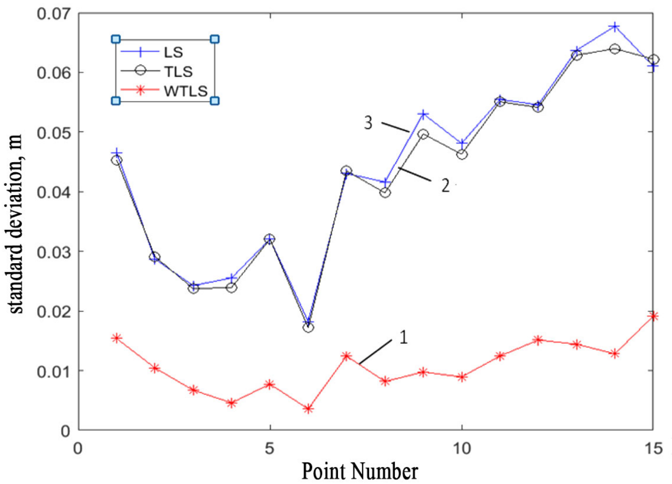

Fig. 4. Standard deviation in X coordinate

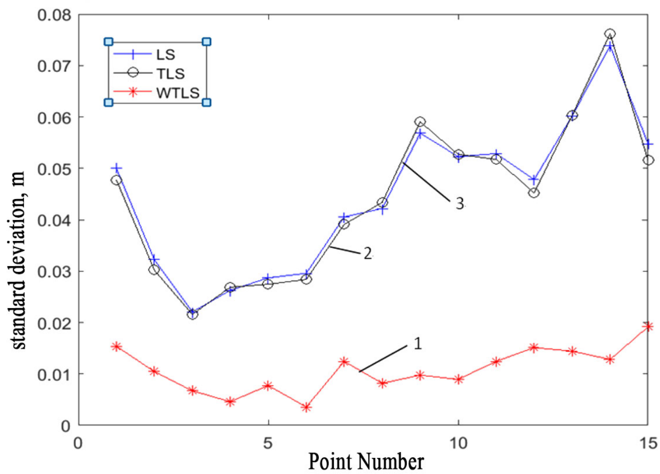

Fig. 5. Standard deviation in Y coordinate

4. Discussion and conclusions

In application of WTLS method, the most important problem is to determine the cofactor matrix of coefficient matrix of EIV model. We have discussed the methods to decide the cofactor matrix of coefficient matrix in function models of triangulation network adjustment. Since generally coefficients of function model are expressed as function of observed values in triangulation networks, it's cofactor (weight) matrix can be determined using cofactor(weight) matrix of observed values.

In this paper focuses one's attention on the use of WTLS approach for triangulation network adjustment. Firstly, adjustment algorithms by WTLS method are reformulated so as to be applied in triangulation network adjustment, secondly, we propose a method to determine the cofactor matrix of EIV model coefficients in triangulation network adjustment, thirdly, the presented WTLS method was shown to be very efficient in practice of triangulation network adjustment.

The advantage of the proposed method is to convenient for computing the cofactor matrix of coefficient matrix, i.e. we composed the algorithm to be able to calculate the coefficient matrix and its partial derivatives simultaneously. The disadvantage is to increase the storage capacity of cofactor matrix in the triangulation network of the large scale.

We suggest that research the method to determine the cofactor matrix of EIV model coefficients in order to extend the application of WTLS in various kinds of geodetic networks further. In the future research, we will propose a more general method to solve this problem.

Author contributions

Conducted the laboratory tests: J.-H.K.; wrote the paper: J.-H.K.; analyzed the data and put forward study ideas: Ch.-J.K.; conducted the experiments and compared with some methods: M.-H.R.

Data availability statement

The data used to support the findings of this study are included within the article.

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