Co-published by
Institute of Fluid-Flow Machinery
Polish Academy of Sciences
Committee on Thermodynamics and Combustion
Polish Academy of Sciences

http://www.imp.gda.pl/archives-of-thermodynamics/

ARCHIVES OF THERMODYNAMICS

Phase change material selection for energy storage units using a simple and effective decision-making method

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Received: 12.05.2024; revised: 17.06.2024; accepted: 21.06.2024

Abstract

This study provides a simple and effective decision-making method to choose the best phase-change material for different energy storage applications. Three case studies are provided to demonstrate the proposed decision-making method. The first case study addresses the problem of best phase-change material selection for a domestic water heating latent heat storage system by considering 15 different phase-change materials and 8 selection attributes; the second case study addresses the problem of selecting the best phase-change material for a triple tube heat exchanger unit by considering 12 different phase-change materials and 5 selection attributes; the third case study addresses the problem of best phase-change material selection for latent heat thermal energy storage within the walls of Trombe to enhance performance considering 11 phase-change materials and 4 selection attributes. The results of the proposed decision-making method are compared with those of other well-known multi-attribute decision-making methods. The proposed method is shown to be simple to implement, providing a logical way for allocating weights to the selection attributes and adaptable to phase-change material selection problems in different energy storage contexts.

Keywords: Energy storage; Phase-change material selection; Selection attributes; Decision-making method.

Vol. 45(2024), No. 3, 67–79; doi: 10.24425/ather.2024.151217

Cite this manuscript as: Rao, R.V. (2024). Phase change material selection for energy storage units using a simple and effective decision-making method. *Archives of Thermodynamics*. 45 (3), 67−79.

1. Introduction

A new area of research that connects energy generation and consumption is thermal energy storage. Phase change materials with high energy storage density and isothermal working qualities are particularly significant in latent heat storage units. The effective and efficient heat storage of the thermal energy storage system depends on the use of phase change material (PCM). Finding a compromise between competing PCM selection attributes is usually necessary when choosing the best PCM to meet specific requirements. Thermal properties (e.g., latent heat of transition, thermal conductivity, specific heat, thermal stability, etc.); physical properties (e.g., density, volume change, vapour pressure); chemical properties (e.g., recycle, toxicity, flammability); kinetic properties (e.g., supercooling, phase separation); economic performance (e.g., cost); and certain managerial considerations are among the important requirements that must be met. All these requirements are considered as the PCM selection attribu-

Nomenclature

Cp – specific heat, kJ/(kg K)
K – thermal conductivity, W/(m K)
LH – latent heat of fusion, kJ/kg
t – time, min
T – temperature, °C

Greek symbols
ρ – density, kg/m^3

Subscripts and Superscripts
i – attribute
j – alternative
l – liquid
s – solid

Abbreviations and Acronyms
AHP – analytic hierarchy process
BWM – best-worst method
CoCoSo – combined compromise solution
COPRAS– complex proportional assessment
CRITIC – criteria importance through intercriteria correlation
EDAS – evaluation based on distance from average solution
EXPROM2– extension of PROMETHEE
MADM – multi-attribute decision-making
MEREC – method based on the removal effect of criteria
MOORA – multi-objective optimization of ratio analysis
MULTIMOORA – multi-objective optimization of ratio analysis plus the full multiplicative form
PCM – phase change material
PROMETHEE – preference ranking organization method for enrichment evaluations
TOPSIS – technique for order preference by similarity to ideal solution
VIKOR – višekriterijumsko kompromisno rangiranje
WASPAS– weighted aggregated sum product assessment
WPM – weighted product method

tes. As a number of PCM materials are available in the market, selecting the right PCM for a particular application becomes difficult and challenging. No single PCM can possess all the required properties and characteristics and hence selection of a best PCM for a given energy storage application is considered as a multi-attribute decision-making (MADM) problem.

Any MADM method for PCM selection involves the (i). PCM alternatives, (ii). PCM selection attributes, (iii). weights of importance assigned to the PCM selection attributes, and (iv). performance data of the PCM alternatives corresponding to the selection attributes. The chosen MADM method process the given data keeping in view of these four components and suggests the best PCM for the given energy storage application for optimal storage performance. The person making the decision (known as decision-maker) considers the importance of each selection attribute for the particular application based on his/her expertise and professional judgment.

Over the past ten years, researchers used several MADM methods to establish reliable methodologies for selecting the best PCMs for certain applications [1–16]. It is observed from the literature review on PCM selection that the researchers used different MADM methods. Even a particular MADM method like TOPSIS (technique for order preference by similarity to ideal solution) was used by the researchers for different applications. Two or more MADM methods were also used for a given application by many researchers in their works. Some of the widely used MADM methods for PCM selection were: technique for order preference by similarity to ideal solution (TOPSIS) [1–3,5,7–12,15,16], višekriterijumsko kompromisno rangiranje (VIKOR) [2,11,16], multi-objective optimization of ratio analysis (MOORA) [6,7], multi-objective optimization of ratio analysis plus the full multiplicative form (MULTIMOORA) [6], complex proportional assessment (COPRAS) and weighted aggregated sum product assessment (WASPAS) [11], preference ranking organization method for enrichment evaluations (PROMETHEE) [2], extension of PROMETHEE (EXPROM2) [16], evaluation based on distance from average solution (EDAS) [7], combined compromise solution (CoCoSo) [6], weighted product method (WPM) [14], etc. For obtaining the weights of importance of the PCM selection attributes, the methods like analytic hierarchy process (AHP) and fuzzy AHP [2], entropy method [3], best-worst method (BWM) [6], criteria importance through intercriteria correlation (CRITIC) [7], method based on the removal effect of criteria (MEREC) [11], range analysis [15], compromise weights approach [16], etc. were used by the researchers and those weights were utilized in the MADM methods for processing the data. Fuzzy scales were also used for converting the qualitative attributes into quantitative ones [2,16]. However, the fuzzy logic uses different membership functions and defuzzification methods and application of these functions and approaches may produce different results [17]. It is also observed that TOPSIS method is the widely used method by the researchers for PCM selection.

An important observation is that the researchers [1–16] used the properties and characteristics of the phase change materials such as latent heat of fusion, thermal conductivity for solid state (and liquid state), specific heat for solid state (and liquid state), density for solid state (and liquid state), cost, maintenance and operational costs, technological complexities, compatibility, flammability, risk levels, etc. for selection of a best PCM from amongst the available PCMs. Using the available data related to the properties and characteristics a large number of PCMs, the researchers used MADM methods and conducted simulation studies to choose a best PCM for the given application. After choosing a particular PCM, the researchers had then suggested that particular PCM for use in the given application. However, *real experimentation* was not conducted by the researchers [1–16] on the alternative PCMs to decide the selection of right PCM. It was because of the difficulty of experimenting on a large number of PCMs which is a costly and time-consuming activity.

Only limited number of research works are available on the real experimentation conducted on the PCMs for the purpose of selecting a best PCM out of the available ones. However, the number of PCMs experimented in such works is very less, because of the difficulty of experimenting on a large number of

PCMs. Oró et al. [18] studied a thermal energy storage system using PCM for low temperature applications such as commercial freezers. A set of PCM formulations based on ammonium chloride-water binary system were tested and analyzed to provide information useful for the selection of PCM with regard to their melting range, latent heat, stability under cycling, and cost. Yu et al. [19] conducted testing of GH-33 and GH-37 PCMs in boards mounted on internal surfaces of the main sun-facing walls of buildings for heating and curing of construction elements made of precast concrete. The theoretical and experimental study results suggested that the use of 50 mm thick board made of GH-37 composite PCM on the internal surface of the main sun-facing wall of the curing building provided the best thermal performance. Prieto et al. [20] tested thermal energy storage systems containing PCMs such as LiOH-LiBr and LiOH-KOH for direct steam generation concentrating solar power plants. After a deep characterization process, the LiOH-KOH was selected. Thus, it can be understood that the selection of best PCM for a given application was carried out by most of the researchers using MADM methods and simulations only.

Even though the above-mentioned MADM methods are useful for selection of right PCM for a given application, they also have drawbacks. For example, the TOPSIS approach necessitates extensive computations that become more difficult as the number of alternatives and attributes increases. The ranks of alternatives provided by the TOPSIS method can vary depending on the different normalization techniques applied to standardize the data. In the case of VIKOR method, there is additional processing needed. The method could lead to different outcomes for the same attribute weights in different ranking lists depending upon the weight allotted to "the majority of attributes". The other MADM techniques have drawbacks of their own and require a significant amount of processing [21,22].

The weights of the PCM selection attributes decided by the decision-maker are called the subjective weights. The AHP method [2] generates a large number of comparison matrices by comparing attributes and alternatives on a scale from 1 to 9. The issue of contradictory judgements occasionally comes up. Furthermore, the way the weights are determined (arithmetic mean, geometric mean, etc.) can affect the choice results. The BWM strategy [6] outperforms the AHP method in terms of judgment consistency, but it also requires a significant amount of computational work due to the increase in pairwise comparisons between the worst, best, and other criteria.

The weights of the PCM selection attributes can also be determined using objective approaches by utilizing methods like the entropy method [3], CRITIC [7], MEREC [11], etc. These weights are called the objective weights since the decision-maker has no control over how they are determined. It should be highlighted, nevertheless, that the decision-maker has no role in the objective weights, which are determined by the given numerical values of the attributes. These objective weights may be (most probably) entirely different from the decision-maker's opined subjective weights. The opinions of the decision-makers who actually deal with the practical values of the attributes in a given decision-making situation are therefore not taken into consideration, which makes the evaluation and ranking of the alternatives using such objective attribute weights potentially meaningless. Recently, a few studies have begun using composite weights in PCM selection, which combine the objective and subjective weights [16]. These compromise weights might not be utilized at all in actual decision-making scenarios and simply remain as an academic exercise.

The research questions (RQs) related to selection of a right alternative PCM using MADM methods are:

1. RQ1: Is there a simple and effective MADM method to weigh the PCM selection attributes logically and evaluate the performance of alternative PCMs used in different energy storage units?
2. RQ2: Can such chosen MADM method handle both qualitative and quantitative PCM selection attributes?
3. RQ3: If such simple and effective MADM method exists, will it be easy to comprehend and practical to use for selection of best PCM for different energy storage applications?
4. RQ4: Will the objective weights obtained from the performance data of the PCM selection attributes really meaningful?
5. RQ5: Is it feasible to have an appropriate MADM method that is both reliable and resistant to changes in the PCM selection attributes' weights? Can such kind of MADM method regarded as best?

The main objective of this research paper is to answer to the above-mentioned RQs. Hence, an attempt is made in this paper to develop an improved MADM method based on simple ranking procedure. The proposed decision-making method addresses the above research questions. The proposed method is a simple, systematic, logical, and effective MADM method to process the performance data of the alternative PCMs corresponding to different PCM selection attributes (both quantitative and qualitative), to logically decide the weights of importance of the PCM selection attributes, and to rank the alternative PCMs based on their total performance. The proposed method is applied for PCM selection in three different thermal energy storage applications.

The proposed decision-making methodology, named as, BHARAT-II, is explained in detail in the next section.

2. Proposed decision-making methodology for PCM selection

The following is a description of the steps of the proposed methodology for PCM selection.

Step 1: Determine the PCM selection attributes A_i (i = 1, 2,..., m), and the alternative PCMs B_j (for j = 1, 2,..., n). The PCM selection attributes are both non-beneficial (i.e., lower values are desired) and beneficial (i.e., higher values are desired).

Step 2: Decide the order of importance of the PCM selection attributes to obtain the weights w_i (for i = 1, 2,..., m). The order of importance is in terms of 1, 2, 3, 4, and so on, based on how significant they are in relation to each other. An average rank will be given if two or more attributes are thought to be equally important.

For example, let there are four PCM selection attributes − W, X, Y, and Z − and the ranks of 1, 2, 3, and 4 are assigned to them. Matrix A1 shows the rank relations:

$$A1 = \begin{array}{c} \\ W \\ X \\ Y \\ Z \end{array} \begin{array}{c} \begin{array}{cccc} W & X & Y & Z \end{array} \\ \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1/2 & 1 & 3/2 & 4/2 \\ 1/3 & 2/3 & 1 & 4/3 \\ 1/4 & 2/4 & 3/4 & 1 \end{bmatrix} \end{array}.$$

It may be noted that in matrix A1, the diagonal elements are 1 (i.e., $r_{WW} = 1$, $r_{XX} = 1$, $r_{YY} = 1$, and $r_{ZZ} = 1$) and the elements below the diagonal are the reciprocals of the rank relations of the selection attributes given above the diagonal (i.e. $r_{XW} = 1/r_{WX}$, $r_{YW} = 1/r_{WY}$, $r_{ZW} = 1/r_{WZ}$, $r_{YX} = 1/r_{XY}$, $r_{ZX} = 1/r_{XZ}$, $r_{ZY} = 1/r_{YZ}$).

The arithmetic means of each row of the A1 matrix are calculated and these are 2.5 (i.e., 10/4), 1.25 (i.e., 5/4), 0.833333 (i.e., 3.33333/4) and 0.625 (i.e., 2.5/4), respectively. The grand summation of these row sums is equal to 5.208333 (i.e., 2.5 + 1.25 + 0.83333 + 0.625). Now dividing each row sum with the grand sum of 5.208333 gives the A2 matrix, which corresponds to the weights of the four selection attributes considered:

$$A2 = \begin{bmatrix} 0.48 \\ 0.24 \\ 0.16 \\ 0.12 \end{bmatrix}.$$

Similar to the AHP and BWM approaches, if the consistency check is performed to check for consistency of rank relations provided in matrix A1, the matrix A3 is computed as A1*A2:

$$A3 = A1*A2 = \begin{bmatrix} 1.92 \\ 0.96 \\ 0.64 \\ 0.48 \end{bmatrix}.$$

Now A4 matrix is computed as A3/A2:

$$A4 = A3/A2 = \begin{bmatrix} 1.92/0.48 \\ 0.96/0.24 \\ 0.64/0.16 \\ 0.48/0.12 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 4 \\ 4 \end{bmatrix}.$$

Now the maximum eigen value (λ_{max}) is computed:

$$\lambda_{max} = \text{average of A4} = (4+4+4+4)/4 = 4.$$

Consistency index (CI) = (λ_{max}–m)/(m–1) = (4–4)/(4–1) = 0; the no. of attributes = size of A1 matrix = 4.

The CI value of 0 indicates that the rank relations provided in A1 matrix are *absolutely consistent* and there is no error present in the judgements of rank relations. As a result, weights of 0.48, 0.24, 0.16, and 0.12 can be assigned to the attributes W, X, Y, and Z respectively. *By expanding this method to any number of attributes and giving each one a rank, the attributes' weights may be found.* It may be stated here that techniques such as AHP and BWM *seldom* provide absolute consistency in the assessments of relative importance. Thus, the proposed method is more dependable.

Step 3: For every alternative PCM, obtain the performance data corresponding to the PCM selection attributes. The performances may be in qualitative or quantitative terms. Transform the qualitative attribute data (expressed in descriptive language) into quantitative data by applying a straightforward scale and avoiding the use of fuzzy logic. Rao [21–23] proved that there is no need of using fuzzy scales and simple ordinary scales will serve the same purpose. Simple ordinary scales can simply replace the fuzzy scales provided by different researchers to deal with linguistic or qualitative attributes using different membership functions. Table 1, for example, shows the transformation of a qualitative or linguistic attribute into a quantitative attribute on 11-point scale.

Table 1. Transformation of a qualitative attribute into a quantitative attribute using a 11-point scale.

Linguistic expression	Fuzzy scale value for a beneficial attribute [23]	Fuzzy scale value for a non-beneficial attribute [23]	Simple scale value for a beneficial attribute	Simple scale value for a non-beneficial attribute
Exceptionally low	0.0455	0.9545	0.0	1.0
Extremely low	0.1364	0.8636	0.1	0.9
Very low	0.2273	0.7727	0.2	0.8
Low	0.3182	0.6818	0.3	0.7
Below average	0.4091	0.5909	0.4	0.6
Average	0.5	0.5	0.5	0.5
Above average	0.5909	0.4091	0.6	0.4
High	0.6818	0.3182	0.7	0.3
Very high	0.7727	0.2273	0.8	0.2
Extremely high	0.8636	0.1364	0.9	0.1
Exceptionally high	0.9545	0.0455	1.0	0

Step 4: Normalize the data for a PCM selection attribute by comparing it to the attribute's "best" value for various alternative PCMs. To obtain the normalized data, repeat this normalization process for each attribute. When referring to a beneficial attribute, the term "best" denotes the highest value that is available, and when referring to a non-beneficial attribute, the lowest value that is available. Normalization is required for the performance measurements of alternative PCMs. For a beneficial attribute, the normalized value $(x_{ji})_{norm}$ is $x_{ji}/x_{i\text{-}best}$; and for a non-beneficial attribute, it is $x_{i\text{-}best}/x_{ji}$. The *i*-th attribute's best value is represented by $x_{i\text{-}best}$. The standing positions of the alternative PCMs in relation to the "best" values of the attributes are clearly displayed by this kind of normalization of the data with reference to the "best" values.

Step 5: Total score of an alternative PCM is $\sum w_i*(x_{ji})_{norm}$ and it is the result of multiplying the selection attributes' weights with the corresponding normalized data of the attributes for the alternatives.

Step 6: Arrange the alternative PCMs in decreasing order, based on their total scores. The alternative PCM that receives the highest total score is considered best for the particular PCM selection problem investigated.

The flowchart of the proposed decision-making method is shown in Fig. 1.

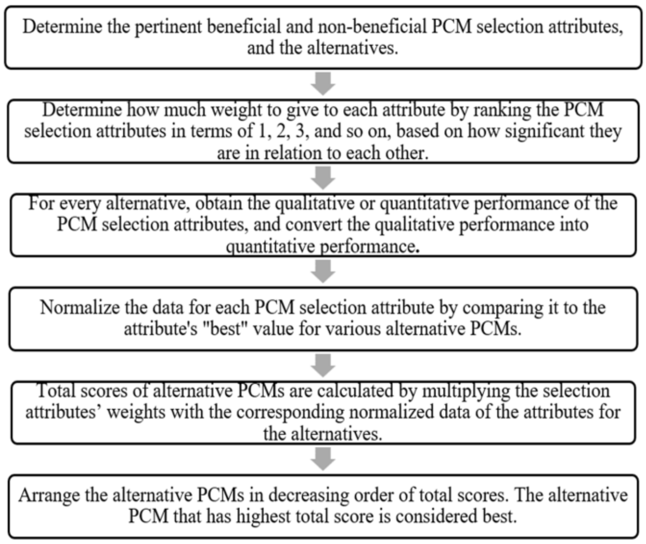

Fig. 1. Flowchart of the proposed decision-making method.

3. Applications of proposed decision-making method to the case studies of phase change material selection for energy storage

3.1. Case study 1: PCM selection for a domestic water heating system using LH storage unit

Gadhave et al. [16] conducted a case study to decide the right PCM for a domestic water heating system containing a PCM-based LH storage system, a storage tank with water, and additional accessories such as a circulation pump, flowmeter, valve, etc. The decision-making problem considered 15 alternative PCMs, designated from M1 to M15, analyzed under 5 selection attributes. Figure 2 shows the goal, selection attributes, and the alternative PCMs for case study 1.

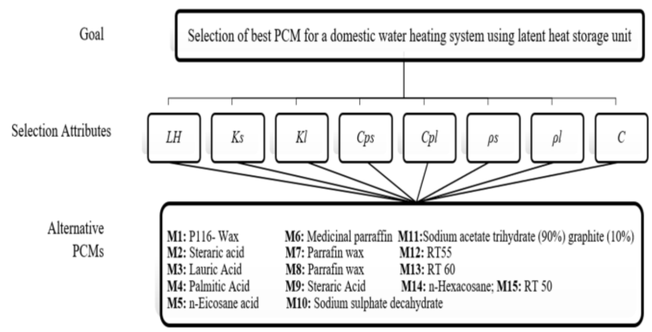

Fig. 2. Goal, selection attributes, and the alternative PCMs of case study 1.

Now to select a best PCM out of 15 PCMs, the steps of the proposed decision-making method are carried out as described below.

Step 1: Table 2 shows the PCM selection attributes and the alternative PCMs considered by Gadhave et al. [16]. The selection attributes are the material properties: latent heat of fusion (*LH*), thermal conductivity for solid state (*Ks*), thermal conductivity for liquid state (*Kl*), specific heat for solid state (*Cps*), specific heat for liquid state (*Cpl*), density for solid state (ρs), density for liquid state (ρl), and cost (*C*). The attributes *LH*, *Ks*, *Kl*, *Cps*, *Cpl*, and ρs, and ρl are beneficial and *C* is non-beneficial. The cost (*C*) is expressed linguistically and the corresponding quantitative values on a simple ordinary scale are assigned using Table 1 and shown in parentheses.

Table 2. Data of the 8 attributes and 15 alternative PCMs of case study 1.

PCM	*LH* kJ/kg	*Ks* W/(m K)	*Kl* W/(m K)	*Cps* kJ/(kg K)	*Cpl* kJ/(kg K)	*ρs* kg/m³	*ρl* kg/m³	*C*
P116-Wax (M1)	209	0.14	0.277	2.89	**2.89**	786	786	H (0.3)
Stearic acid (M2)	211.6	1.6	0.3	1.76	2.27	940	940	VH (0.2)
Lauric acid (M3)	178	1.6	0.147	1.6	2.27	870	870	A (0.5)
Palmitic acid (M4)	201	0.29	0.21	2	2.37	942	862	A (0.5)
n-Eicosane (M5)	248	0.426	0.146	1.926	2.4	910	769	L (0.7)
Medicinal paraffin (M6)	146	0.3	2.1	2.25	2.2	830	830	L (0.7)
Paraffin wax (M7)	210	0.24	0.15	**2.9**	2.1	860	780	L (0.7)
Paraffin wax (M8)	190	0.24	0.22	2	2.15	910	790	L (0.7)
Stearic acid (M9)	169	0.29	0.29	1.59	1.59	965	847	A (0.5)
Sodium sulphate decahydrate (M10)	180	0.15	0.3	2	2	**1460**	**1458**	H (0.3)
Sodium acetate trihydrate (90%)+graphite (10%) (M11)	190	**2.5**	**2.5**	2.5	2.5	1350	1350	H (0.3)
RT55 (M12)	172	0.2	0.2	2	2	880	770	L (0.7)
RT60 (M13)	167.6	0.2	0.2	2	2	880	770	**VL (0.8)**
n-Hexacosane (M14)	**256**	0.21	0.21	2	2	778.3	770	A (0.5)
RT50 (M15)	160	0.2	0.2	2	2	880	760	L (0.7)

VH: Very High; H: High; A: Average; L: Low; VL: Very Low

Step 2: To determine the weights of the 8 PCM selection attributes, ranks are assigned. The rank 1 is assigned to LH as it is considered much more important for the given application. In this case study, the attributes ρs and ρl are considered equally significant. Hence, an average rank of 2.5 (i.e., (2+3)/2) is allocated. Comparably, *Ks* and *Kl* have an average rank of 4.5 (i.e., (4+5)/2); *Cps* and *Cpl* have an average rank of 6.5 (i.e., (6+7)/2); and *C* has a rank of 8. The attributes have been assigned the same ranks as those shown in Table 3. The rank relationships and weights of the 8 attributes are shown in Table 3.

Table 3. Rank relationships of the 8 attributes in case study 1.

Attributes	Attributes									
	LH	*Ks*	*Kl*	*Cps*	*Cpl*	*ρs*	*ρl*	*C*	Means of rows	Weights of attributes
LH	1	4.5	4.5	6.5	6.5	2.5	2.5	8	36	0.37353
Ks	1/4.5	1	1	6.5/4.5	6.5/4.5	2.5/4.5	2.5/4.5	8/4.5	8	0.083
Kl	1/4.5	1	1	6.5/4.5	6.5/4.5	2.5/4.5	2.5/4.5	8/4.5	8	0.083
Cps	1/6.5	4.5/6.5	4.5/6.5	1	1	2.5/6.5	2.5/6.5	8/6.5	5.53843	0.05746
Cpl	1/6.5	4.5/6.5	4.5/6.5	1	1	2.5/6.5	2.5/6.5	8/6.5	5.53843	0.05746
ρs	1/2.5	4.5/2.5	4.5/2.5	6.5/2.5	6.5/2.5	1	1	8/2.5	14.4	0.14941
ρl	1/2.5	4.5/2.5	4.5/2.5	6.5/2.5	6.5/2.5	1	1	8/2.5	14.4	0.14941
C	1/8	4.5/8	4.5/8	6.5/8	6.5/8	2.5/8	2.5/8	1	4.5	0.04669
								Total =	96.3768	1.000

The CR value for the rank relations matrix containing 8 PCM selection attributes is 0. Thus, there exists absolute consistency in the judgments of rank relations. The last column of Table 3 gives the weights of the 8 PCM selection attributes.

Step 3: The linguistic expressions of the attribute *C* are transformed to quantitative values using Table 1 without the need of using fuzzy logic. These values are shown in Table 2 in parentheses. The values are assigned to non-beneficial *C* based on Table 1. After assigning like this, the assigned values for C can be considered beneficial for the sake of normalization.

Step 4: The data is normalized based on the "best" PCM for each attribute. The best values of the attributes are shown in bold inside Table 2. Table 4 shows the normalized values. For example, the normalized value of 0.816406 for *LH* corresponding to M1 is obtained by (209/256); the value of 0.056 for *Ks* corresponding to M1 is obtained by (0.14/2.5).

Table 4. Normalized values for case study 1.

PCM	Normalized values							
	LH	*Ks*	*Kl*	*Cps*	*Cpl*	*ρs*	*ρl*	*C*
M1	0.816406	0.056	0.1108	0.996552	1	0.538356	0.539095	0.375
M2	0.826563	0.64	0.12	0.606897	0.785467	0.643836	0.644719	0.25
M3	0.695313	0.64	0.0588	0.551724	0.785467	0.59589	0.596708	0.625
M4	0.785156	0.116	0.084	0.689655	0.820069	0.645205	0.591221	0.625
M5	0.96875	0.1704	0.0584	0.664138	0.83045	0.623288	0.527435	0.875
M6	0.570313	0.12	0.84	0.775862	0.761246	0.568493	0.569273	0.875
M7	0.820313	0.096	0.06	1	0.726644	0.589041	0.534979	0.875
M8	0.742188	0.096	0.088	0.689655	0.743945	0.623288	0.541838	0.875
M9	0.660156	0.116	0.116	0.548276	0.550173	0.660959	0.580933	0.625
M10	0.703125	0.06	0.12	0.689655	0.692042	1	1	0.375
M11	0.742188	1	1	0.862069	0.865052	0.924658	0.925926	0.375
M12	0.671875	0.08	0.08	0.689655	0.692042	0.60274	0.528121	0.875
M13	0.654688	0.08	0.08	0.689655	0.692042	0.60274	0.528121	1
M14	1	0.084	0.084	0.689655	0.692042	0.533082	0.528121	0.625
M15	0.625	0.08	0.08	0.689655	0.692042	0.60274	0.521262	0.875

Step 5: Total scores of alternative PCMs are calculated by multiplying the selection attributes' weights with the corresponding normalized data of the attributes for the alternatives. For example, the total score of PCM designated as M1 is computed as:

Total score (M1) = 0.37353*0.816406 + 0.083 + 0.056 + 0.083*0.1108 + 0.05746*0.996552 + 0.05746*1 + 0.14941*0.538356 + 0.14941*0.539095 + 0.04669*0.375 = 0.612029.

The total scores of the PCMs are given in Table 5.

Table 5. Total scores of 15 PCMs of case study 1.

PCM No.	PCM	Total score	PCM No.	PCM	Total score
M1	P116-Wax	0.612029	M9	Stearic acid	0.543704
M2	Stearic acid	0.65604	M10	Sodium sulphate decahydrate	0.673313
M3	Lauric acid	0.601936	M11	Sodium acetate trihydrate (90%) + graphite (10%)	**0.836491**
M4	Palmitic acid	0.610559			
M5	n-Eicosane	0.679525	M12	RT55	0.553467
M6	Medicinal paraffin	0.591894	M13	RT60	0.552883
M7	Paraffin wax	0.627383	M14	n-Hexacosane	0.654616
M8	Paraffin wax	0.589825	M15	RT50	0.534933

Step 6: The alternative PCMs are arranged in decreasing order of their total scores. The PCMs are ranked from highest to lowest total scores as follows: M11-M5-M10-M2-M14-M7-M1-M4-M3-M6-M8-M12-M13-M9-M15.

With the highest total score, the PCM identified as M11 can be considered as the best choice for the given application of domestic water heating system. Gadhave et al. [16] used entropy and AHP methods for obtaining the weights of the attributes and finally combined those weights to get the compromised weights of 0.4208, 0.0853, 0.0805, 0.0353, 0.0354, 0.1624, 0.1616, and 0.0187 for *LH*, *Ks*, *Kl*, *Cps*, *Cpl*, *ρs*, *ρl*, and *C*, respectively. The compromise weights were then used by Gadhave et al. [16] in TOPSIS, VIKOR, and EXPROM2 methods to calculate the scores and then ranked the PCMs. The PCMs were ranked from highest to lowest total scores as follows:

- TOPSIS [16]: M11-M2-M14-M5-M10-M6-M3-M7-M1-M4-M8-M9-M12-M13-M15,
- VIKOR [16]: M11-M5-M14-M2-M1-M7-M10-M4-M8-M3-M12-M9-M13-M15-M6,
- EXPROM2 [16]: M11-M2-M5-M10-M14-M7-M1-M4-M3-M8-M9-M6-M12-M13-M15.

These methods also suggested M11 as the best choice. However, it may be noted that the compromise weights were obtained by combining the objective weights obtained by entropy method and the subjective weights obtained by AHP method. In fact, the objective weights obtained and the subjective weights obtained by Gadhave et al. [16] were completely different. The objective and subjective weights were then combined to form the compromise weights. The compromise weights might not be utilized at all in actual decision-making scenarios and it simply remains as an academic exercise. However, for fair comparison, if the compromise weights used by Gadhave et al. [16] in TOPSIS, VIKOR, and EXPROM2 methods are used in the proposed decision-making method, then the PCMs can be arranged in the following order.

- Proposed method (using the compromise weights): M11-M10-M5-M2-M14-M7-M4-M1-M3-M8-M6-M9-M12-M13-M15.

Using the same compromise weights as those used in VIKOR, TOPSIS, and EXPROM2, the proposed decision-making method also suggested M11 as the best choice. The last choice is M15. It may be noted once again that the proposed decision-making method is involved in simple normalization procedure and the calculation of total scores of PCMs compared to the computationally intensive TOPSIS, VIKOR, and EXPROM2 methods. The ranks assignment procedure and the subsequent determinations of the weights of the PCM selection attributes by the decision maker are more logical compared to the compromise or combined weights used by Gadhave et al. [16]. The proposed method makes it easy to convert qualitative attributes into quantitative, does not require the use of fuzzy scale as that used by Gadhave et al. [16].

3.2. Case study 2: PCM selection for a triple tube heat exchanger unit

Yang et al. [15] presented the results of simulation conducted to investigate the impact of PCMs' thermophysical characteristics on a triple tube heat exchanger (TTHX) unit's heat storage ratio. The weights assigned to the attributes were obtained using range analysis at various time scales. Lastly, the ranking and selection process was carried out using the TOPSIS method. Figure 3 shows the goal, selection attributes, and the alternative PCMs for case study 2.

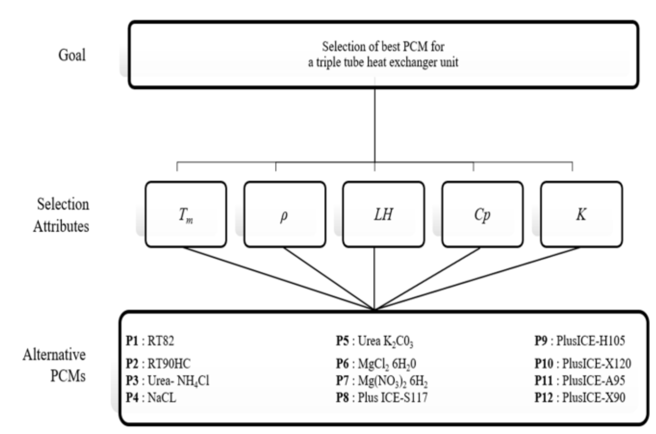

Fig. 3. Goal, selection attributes, and the alternative PCMs of case study 2.

The findings showed that the heat storage rate was significantly influenced by the thermophysical characteristics. The PCM PlusICE-S117 was found to be the ideal PCM when the melting time was 20 minutes, since the PCM density and thermal conductivity attributes were given greater weights. But when the melting time was 150 or 250 min, the most important attributes to take into account were the PCM density and melting enthalpy, and the alternative urea-NaCl was considered as the best one.

The decision-making problem considered 12 alternative PCMs, designated from P1 to P12, analyzed under 5 selection attributes. Now to select a best PCM out of the 15 PCMs, the steps of the proposed decision-making method are carried out as described below.

Step 1: Table 6 shows the PCM selection attributes and the alternative PCMs considered by Yang et al. [15]. The selection attributes are the material properties: melting temperature (T_m), density (ρ), latent heat of fusion (*LH*), specific heat (*Cp*), and thermal conductivity (*K*).

Table 6. Data of the 5 attributes and 12 alternative PCMs of case study 2.

PCM No.	PCM	T_m °C	P kg/m³	LH kJ/kg	Cp kJ/(kgK)	K W/(mK)
P1	RT82	82	770	170	2000	0.2
P2	RT90HC	90	850	170	2000	0.2
P3	Urea-NH_4Cl	102	1348	214	2090	0.58
P4	Urea-NaCl	112	1372	**230**	2020	0.6
P5	Urea-K_2CO_3	102	1415	206	2020	0.58
P6	$MgCl_2 \cdot 6H_2O$	116	1450	167	**2610**	0.57
P7	$Mg(NO_3)_2 \cdot 6H_2$	90	1550	163	2480	0.49
P8	PlusICE-S117	117	1450	160	**2610**	**0.7**
P9	PlusICE-H105	105	**1700**	125	1500	0.5
P10	PlusICE-X120	**120**	1245	180	1500	0.36
P11	PlusICE-A95	95	900	205	2200	0.22
P12	PlusICE-X90	90	1200	135	1510	0.36

Yang et al. [15] ignored the thickness of the tube walls and assumed the thermophysical properties of the PCM as equal in the liquid and solid phases. All these 5 selection attributes are beneficial.

Step 2: For sub-case 1 of Yang et al. [15], when the heat storage rate was considered for the first 20 minutes, density ρ was considered much more important followed by thermal conductivity K. Hence, rank 1 is given to ρ, rank 2 is given to K, rank 3 is given to LH, rank 4 is given to T_m, and rank 5 is given to Cp. Table 7 displays the rankings assigned to the 5 attributes along with the weights derived (similar to the procedure explained in section 2).

Table 7. Rank relationships of the 5 attributes of case study 2 and the corresponding weights (sub-case 1 of considering heat storage rate when the melting process continues up to 20 minutes).

Attributes	T_m	ρ	LH	Cp	K	Means of rows	Weights of attributes
T_m	1	1/4	3/4	5/4	2/4	3.75/5=0.75	0.75/6.85=0.109489
ρ	4	1	3	5	2	15/5=3	3/6.85=0.437956
L	4/3	1/3	1	5/3	2/3	(15/3)/5=1	1/6.85=0.145985
Cp	4/5	1/5	3/5	1	2/5	(15/5)/5=0.60	0.6/6.85=0.087591
K	2	1/2	3/2	5/2	1	7.5/5=1.5	1.5/6.85=0.218978
Total =						6.85	1.000000

It may be noted that the CR value for the rank relations matrix containing 5 PCM selection attributes is 0. Thus, there exists absolute consistency in the judgments of rank relations. The last column of Table 7 gives the weights of the 5 PCM selection attributes.

Step 3: The values shown in Table 6 are already quantitative in nature and there is no need of transformation. All 5 attributes are of beneficial type.

Step 4: The data shown in Table 6 is normalized based on the "best" PCM for each attribute. The best values of the attributes are shown in bold in Table 6. The normalized values are shown in Table 8. For example, the normalized value of 0.683333 for T_m corresponding to P1 is obtained by (82/120). Similarly, the other data is normalized and given in Table 8.

Table 8. Normalized values for sub-case 1 of case study 2.

PCM	T_m	ρ	L	Cp	K
P1	0.683333	0.452941	0.73913	0.766284	0.285714
P2	0.75	0.5	0.73913	0.766284	0.285714
P3	0.85	0.792941	0.930435	0.800766	0.828571
P4	0.933333	0.807059	1	0.773946	0.857143
P5	0.85	0.832353	0.895652	0.773946	0.828571
P6	0.966667	0.852941	0.726087	1	0.814286
P7	0.75	0.911765	0.708696	0.950192	0.7
P8	0.975	0.852941	0.695652	1	1
P9	0.875	1	0.543478	0.574713	0.714286
P10	1	0.732353	0.782609	0.574713	0.514286
P11	0.791667	0.529412	0.891304	0.842912	0.314286
P12	0.75	0.705882	0.586957	0.578544	0.514286

Step 5: Total scores of alternative PCMs are calculated by multiplying the selection attributes' weights with the corresponding normalized data of the attributes for the alternatives. For example, the total score of PCM designated as P1 is computed as:

Total score (P1) = 0.109489*0.683333 + 0.437956*0.452941 + 0.145985*0.73913 + 0.087591*0.766284 + 0.218978*0.285714 = 0510773.

The total scores of the PCMs are given in Table 9.

Table 9. Total scores of 15 PCMs of sub-case 1 of case study 2.

PCM No.	PCM	Total Score
P1	RT82	0.510773
P2	RT90HC	0.538682
P3	Urea-NH_4Cl	0.827748
P4	Urea-NaCl	0.857118
P5	Urea-K_2CO_3	0.837582
P6	$MgCl_2·6H_2O$	0.85129
P7	$Mg(NO_3)_2·6H_2$	0.821402
P8	PlusICE-S117	0.888427
P9	PlusICE-H105	0.819852
P10	PlusICE-X120	0.707434
P11	PlusICE-A95	0.591309
P12	PlusICE-X90	0.640242

Step 6: The alternative PCMs are arranged in decreasing order of their total scores. The PCMs are ranked from highest to lowest total scores as follows:

P8-P4-P6-P5-P3-P7-P9-P10-P12-P11-P2-P1.

With the highest total score, the PCM identified as P8 (i.e. PlusICE-S117) can be considered as the best choice for the given application of PCM selection for a triple tube heat exchanger unit (for the sub-case-1 of case study-2).

For sub-case 2 of Yang et al. [15], when the heat storage rate was considered for the first 150 minutes, density ρ was considered much more important followed by latent heat L. Hence, rank 1 is given to ρ, rank 2 is given to LH, rank 3 is given to T_m, rank 4 is given to K, and rank 5 is given to Cp. Table 10 displays the rankings assigned to the 5 attributes along with the weights derived.

Table 10. Rank relationships of the 5 attributes of sub-case 2 of case study 2 and the corresponding weights (considering heat storage rate when the melting process continues up to 150 minutes).

Attributes	T_m	ρ	L	Cp	K	Means of rows	Weights of attributes
T_m	1	1/3	2/3	5/3	4/3	5/5=1	1/6.85=0.145985
ρ	3	1	2	5	4	15/5=3	3/6.85=0.437956
L	3/2	1/2	1	5/2	2	7.5/5=1.5	1.5/6.85=0.218978
Cp	3/5	1/5	2/5	1	4/5	3/5=0.6	0.6/6.85=0.087591
K	3/4	1/4	2/4	5/4	1	3.75/5=0.75	0.75/6.85=0.109489
Total=						6.85	1.000000

For sub-case 3 of Yang et al. [15], when the heat storage rate was considered for the first 250 minutes, density ρ was considered much more important followed by latent heat LH, K, T_m, and Cp. Hence, rank 1 is given to ρ, rank 2 is given to LH, rank 3 is given to K, rank 4 is given to T_m, and rank 5 is given to Cp. Table 11 displays the rankings assigned to the 5 attributes along with the weights derived (similar to the procedure explained in section 2).

Table 11. Rank relationships of the 5 attributes of sub-case 3 of case study 2 and the corresponding weights (considering heat storage rate when the melting process continues up to 250 minutes).

Attrib-utes	T_m	ρ	L	Cp	K	Means of rows	Weights of attributes
T_m	1	1/4	2/4	5/4	3/4	3.75/5=0.75	0.75/6.85=0.109489
ρ	4	1	2	5	3	15/5=3	3/6.85=0.437956
L	2	1/2	1	5/2	3/2	7.5/5=1.5	1.5/6.85=0.218978
Cp	4/5	1/5	2/5	1	3/5	3/5=0.6	0.6/6.85=0.087591
K	4/3	1/3	2/3	5/3	1	5/5=1	1/6.85=0.145985
Total=						6.85	1.000000

Table 12 shows the total scores of 12 PCMs corresponding to melting process timings. The alternative PCMs can be arranged in the descending order of the total scores for all the three sub-cases of case study 2

- proposed method (for $t_{melting}$=250 minutes): P4-P8-P6-P5-P3-P7-P9-P10-P12-P11-P2-P1.
- proposed method (for $t_{melting}$=150 minutes): P4-P8-P6-P5-P3-P7-P9-P10-P12-P11-P2-P1.
- proposed method (for $t_{melting}$=20 minutes): P8-P4-P6-P5-P3-P7-P9-P10-P12-P11-P2-P1.

Table 12. Total scores of 12 PCMs corresponding to melting process continued up to 20 minutes (sub-case 1), 150 minutes (sub-case 2) and 250 minutes (sub-case 3).

PCM No.	PCM	Total score ($t_{melting}$=20 min)	Total score ($t_{melting}$=150 min)	Total score ($t_{melting}$=250 min)
P1	RT82	0.510773	0.558381	0.543869
P2	RT90HC	0.538682	0.588723	0.571778
P3	Urea-NH_4Cl	0.827748	0.835966	0.835183
P4	Urea-NaCl	0.857118	0.870326	0.867546
P5	Urea-K_2CO_3	0.837582	0.84326	0.842478
P6	$MgCl_2{\cdot}6H_2O$	0.85129	0.850414	0.844853
P7	$Mg(NO_3)_2{\cdot}6H_2$	0.821402	0.823862	0.822037
P8	PlusICE-S117	0.888427	0.8653	0.866212
P9	PlusICE-H105	0.819852	0.81325	0.807384
P10	PlusICE-X120	0.707434	0.744747	0.72702
P11	PlusICE-A95	0.591309	0.65085	0.633427
P12	PlusICE-X90	0.640242	0.654149	0.645547

The alternative PCM P4 is considered as the best PCM if the melting time of 150 minutes as well for up to 250 minutes. However, the alternative PCM P8 is found as the best alternative PCM for melting time of 20 minutes.

It may be mentioned here that Yang et al. [15] used different weights of the attributes for the three sub-cases of case study 2 and used these weights in TOPSIS method to evaluate the alternative PCMs.

The weights used by Yang et al. [15] for sub-case 1 were: 0.092, 0.404, 0.214, 0.036 and 0.254 for T_m, ρ, LH, Cp and K, respectively.

The weights used by Yang et al. [15] for sub-case 2 were: 0.112, 0.405, 0.320, 0.056 and 0.107 for T_m, ρ, LH, Cp, and K, respectively. The weights used by Yang et al. [15] for sub-case 3 were: 0.094, 0.408, 0.340, 0.053 and 0.105 for T_m, ρ, L, Cp and K, respectively. The ranking of PCMs for different melting times were as given below:

- TOPSIS [15] (for $t_{melting}$=250 minutes): P4-P5-P3-P7-P6-P8-P9-P10-P11-P12-P2-P1.
- TOPSIS [15] (for $t_{melting}$=150 minutes): P4-P5-P3-P7-P6-P8-P9-P10-P12-P11-P2-P1.
- TOPSIS [15] (for $t_{melting}$=20 minutes): P8-P4-P5-P6-P3-P9-P7-P10-P12-P11-P2-P1.

For fair comparison, the same three sub-cases with the same weights of attributes, as considered by Yang et al. [15], are attempted using the proposed decision-making method and the following rankings are obtained

- proposed method (for $t_{melting}$=250 minutes) for the same weights used in TOPSIS [15]: P4-P5-P3-P8-P6-P7-P9-P10-P11-P12-P2-P1.
- proposed method (for $t_{melting}$=150 minutes) for the same weights used in TOPSIS [15]: P4-P5-P3-P8-P6-P7-P9-P10-P11-P12-P2-P1.
- proposed method (for $t_{melting}$=20 minutes) for the same weights used in TOPSIS [15]: P4-P8-P5-P3-P6-P9-P7-P10-P12-P11-P2-P1.

It is clear that using the same weights as those used in TOPSIS [15], the proposed decision-making method also suggested P4 as the first choice and P5 as the second choice for the sub-cases of $t_{melting}$=250 minutes and $t_{melting}$=150 minutes. It may be noted that the TOPSIS method used by Yang et al. [15] involves too lengthy calculations for normalization, calculating the objective weights using range analysis method, and then using those objective weights in the remaining computationally intensive steps. However, the procedure suggested by the proposed method is straightforward and simple to comprehend, in contrast to the TOPSIS method. As seen in this case study, the proposed method allows the use of weights of attributes calculated by other methods or as decided by the decision-maker based on intuition or experience.

3.3. Case study 3: PCM selection for an optimal Trombe wall performance

Thermal energy storage in buildings considerably lowers the energy demand of the building by releasing its stored energy when the need arises. Buildings use Trombe walls to store and distribute thermal energy, which controls the ambient temperature in each space. PCMs have been investigated extensively for latent heat thermal energy storage within the walls of Trombe to enhance performance. Oulah [5] investigated 11 alternative PCMs for selection of a suitable PCM for optimal Trombe wall performance using TOPSIS method. The heat of fusion, thermal conductivity, density, and cost were the four attributes taken into account. Except the cost attribute, the remaining attributes are of beneficial type.

Figure 4 shows the schematic of the PCM selection problem and Table 13 shows the data of the 4 attributes and 11 alternative PCMs. The best values of the attributes are shown in bold in Table 13.

Following the steps of the proposed decision-making method, the data given in Table 13 is normalized and is shown in Table 14. Table 15 displays the rankings assigned to the 4 attributes along with the weights derived.

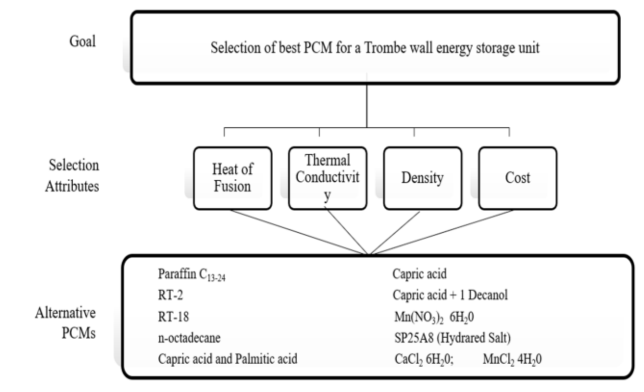

Fig. 4. Goal, selection attributes, and the alternative PCMs of case study 3.

Table 13. Data of the attributes for case study 3.

PCM	Heat of fusion kJ/kg	Thermal conductivity W/(m K)	Density kg/m³	Cost $/kg
Paraffin C13-24 (P1)	**189**	0.21	760	2
RT-27 (P2)	179	0.2	800	3.6
RT-18 (P3)	134	0.2	756	3.6
n-Octadecane (P4)	179	0.2	750	5
Capric acid and palmitic acid (P5)	177	**2.2**	784	1.78
Capric acid (P6)	142.7	0.2	815	**1.5**
Capric acid + 1 decanol (P7)	126.9	0.2	817	1.6
$Mn(NO_3)_2 \bullet 6H_2O$ (P8)	125.9	0.6	1700	2
SP25A8 Hydrated salt (P9)	180	0.6	1380	1.8
$CaCl_2 \bullet 6H_2O$ (P10)	187	0.53	**1710**	1.8
$MnCl_2 \bullet 4H_2O$ (P11)	175	1	1490	2

Table 14. Normalized data of the attributes for case study 3.

PCM	Heat of fusion kJ/kg	Thermal conductivity W/(m K)	Density kg/m³	Cost $/kg
Paraffin C13-24 (P1)	1	0.095455	0.444444	0.75
RT-27 (P2)	0.94709	0.090909	0.467836	0.416667
RT-18 (P3)	0.708995	0.090909	0.442105	0.416667
n-Octadecane (P4)	0.94709	0.090909	0.438596	0.3
Capric acid and palmitic acid (P5)	0.936508	1	0.45848	0.842697
Capric acid (P6)	0.755026	0.090909	0.476608	1
Capric acid + 1 decanol (P7)	0.671429	0.090909	0.477778	0.9375
$Mn(NO_3)_2 \bullet 6H_2O$ (P8)	0.666138	0.272727	0.994152	0.75
SP25A8 Hydrated salt (P9)	0.952381	0.272727	0.807018	0.833333
$CaCl_2 \bullet 6H_2O$ (P10)	0.989418	0.240909	1	0.833333
$MnCl_2 \bullet 4H_2O$ (P11)	0.925926	0.454545	0.871345	0.75

It may be noted that the CR value for the rank relations matrix containing 4 PCM selection attributes is 0. Thus, there exists absolute consistency in the judgments of rank relations. The last column of Table 15 gives the weights of the 4 PCM selection attributes.

Now using the normalized data of the attributes given in Table 14 and the weights of the attributes given in the last column of Table 15, the total scores of the 11 alternative PCMs are calculated and are given in Table 16.

Table 15. Rank relationships of the 4 attributes of case study 3 and the corresponding weights.

Attributes	Heat of fusion kJ/kg	Thermal conductivity W/(mK)	Density kg/m³	Cost $/kg	Means of rows	Weights of attributes
Heat of fusion	1	1/4	3/4	2/4	2.5/4=0.625	0.625/5.20833 = 0.12
Thermal conductivity	4	1	3	2	10/4=2.5	2.5/5.20833 = 0.48
Density	4/3	1/3	1	2/3	(10/3)/4 = 0.83333	0.83333/5.20833 = 0.16
Cost	2	1/2	3/2	1	5/4=1.25	1.25/5.20833 = 0.24
Total=					5.20833	1.00000

Table 16. Total scores of 11 PCMs of case study 3.

PCM	Total Scores of PCMs
Paraffin C13-24 (P1)	0.416929
RT-27 (P2)	0.332141
RT-18 (P3)	0.299453
n-Octadecane (P4)	0.299463
Capric acid and palmitic acid (P5)	0.867985
Capric acid (P6)	0.450497
Capric acid + 1 decanol (P7)	0.425652
$Mn(NO_3)_2 \bullet 6H_2O$ (P8)	0.54991
SP25A8 Hydrated salt (P9)	0.574318
$CaCl_2 \bullet 6H_2O$ (P10)	0.594367
$MnCl_2 \bullet 4H_2O$ (P11)	0.648708

The PCMs are now arranged in the descending order of the total scores:

P5-P11-P10-P9-P8-P6-P7-P1-P2-P4-P3.

From the total scores of 11 PCMs, it can be understood that PCM designated as P5 (i.e., Capric acid and palmitic acid) is the first choice and PCM designated as P11 (i.e. $MnCl_2 \bullet 4H_2O$) is the second choice for the Trombe wall.

It may be noted that Oulah et al. [5] used the objective weights of the attributes obtained by the entropy method (the weights were: 0.020109, 0.7127, 0.1124 and 0.1549 for the heat of fusion, thermal conductivity, density, and cost respectively). In fact, the objective weights are not much meaningful as they do not take into account the decision-maker's preferences. Using the objective weights, Oulah et al. [5] obtained the following rankings using the TOPSIS method:

- TOPSIS [5]: P5-P11-P8-P9-P10-P6-P7-P1-P2-P3-P4.

It can be seen that the entropy method suggests a bigger weightage of 0.7127 for thermal conductivity. Obviously, the alternative PCM which is the best with reference to the thermal conductivity emerges as the first choice (in this case, the PCM designated as P5) and the ranking of other PCMs will also be affected by such a higher weightage assigned to thermal conductivity. That was why Rao [21] opined that attributes weights should be decided by the decision-maker only as he/she is going to face the advantage or disadvantage of his/her decision. However, for fair comparison, if the same objective weights are used in the proposed method, the alternative PCMs can be arranged as shown below

- proposed method (using the objective weights of Oulah et al. [5]): P5-P11-P8-P9-P10-P6-P7-P1-P2-P3-P4.

The ranking is exactly same as that given by Oulah et al. [5] due to the reasons explained above.

4. Discussion

The research questions (RQs) related to selection of a right alternative PCM are reproduced below and discussion is made:

1. RQ1: *Is there a simple and effective MADM method to weigh the PCM selection attributes logically and evaluate the performance of alternative PCMs used in different energy storage units*?
   Yes, it is possible to develop such a simple and effective MADM method for PCM selection as proved in three different case studies of energy storage units.
2. RQ2: *Can such chosen MADM method handle both qualitative and quantitative PCM selection attributes*?
   Yes, the proposed decision-making method can handle both qualitative and quantitative attributes. This has been clearly demonstrated in the first case study. The suggested method can transform the qualitative (i.e. linguistically expressed) attributes into quantitative ones with the aid of simple linear scales.
3. RQ3: *If such simple and effective MADM method exists, will it be easy to comprehend and practical to use for selection of best PCM for different energy storage applications*?
   Yes, it is already shown in case studies 1, 2, and 3 that the proposed method can easily deal with information at hand. Even to deal with the qualitative information of the attributes, simple ordinary linear scales can be used instead of fuzzy logic-based scales. The proposed method is easy to comprehend and practical to use for selection of right alternative PCM for different energy storage applications.
4. RQ4: *Will the objective weights obtained from the performance data of the PCM selection attributes really meaningful*?
   No, not meaningful. The objective weights may be (most probably) entirely different from the decision-maker's opined subjective weights. The opinions of the decision-makers who actually deal with the practical values of the attributes in a given decision-making situation are therefore not taken into consideration, which makes the evaluation and ranking of the alternatives using such objective attribute weights potentially meaningless. Recently, a few studies have begun using composite weights in PCM selection, which combine the objective and subjective weights [3,7,16]. These compromise weights might not be utilized at all in actual decision-making scenarios and simply remain as an academic exercise.
5. RQ5: *Is it feasible to have an appropriate MADM method that is both reliable and resistant to changes in the PCM selection attributes' weights? Can such kind of MADM method regarded as best*?
   There is no doubt that an MADM method must be reliable. However, an MADM method need not be resistant to changes in the attributes' weights. How is it justified that the MADM method suggests the same ranking of alternatives even if the attributes' weights changed? It is unjustified and meaningless. It can be seen from the three sub-cases of case study. When the ranks and the weights importance of the PCM selection attributes are changed, the rankings are changed. Weights of importance are changed means the priorities of the decision-maker are changed. Then how an MADM method should remain insensitive? Rao [21] opined that the researchers may suggest that a particular MADM method indicates a particular alternative as the first choice within certain percentage of variation in each attribute's weight. One need not look upon at an MADM method which is resistant to any changes in the attributes' weights of importance.

The five research questions (RQs) answered above will make the readers more informed about the assignment of weights to the attributes and the application of the proposed MADM method for right PCM selection for a given application.

The three case studies have amply demonstrated the potential of the proposed method as a multi-attribute decision-making method. It is important to observe that the ranking remains the same even if the fuzzy scales given in Table 1—rather than the simple linear scales—are used to translate the linguistic expressions used by the decision-maker. This is quite helpful when making decisions in real-world situations.

5. Conclusions

Thermal energy storage is an emerging field of study that links the production and consumption of energy. Particularly important in latent heat storage units are phase change materials with high energy storage density and isothermal working characteristics. Selecting the right PCM is crucial to the effective and efficient heat storage of the thermal energy storage system. Selecting the right PCM to satisfy certain requirements typically necessitates finding a compromise between opposing attributes. A large number of researchers select PCMs according to cost, availability, and experience. However, PCMs in the present work are chosen using a variety of attributes. The research work reported in this paper tackled the PCM selection problem by using a simple and effective decision-making method, named as BHARAT-II.

Three case studies of PCM selection are presented to illustrate the potential of the proposed methodology. The first case study addressed the issue of choosing the best PCM selection for a domestic water heating latent heat storage system by considering 15 different PCMs and 8 selection attributes; the second case study addressed the problem of selecting the best PCM for a triple tube heat exchanger unit by considering 12 different PCMs and 5 selection attributes with three sub-cases; the third case study addressed the problem of best PCM selection for latent heat thermal energy storage within the walls of Trombe to enhance performance considering 11 PCMs and 4 selection attributes.

It may be noted that in the case of widely used AHP method for determining the weights of importance of the selection attributes, the decision-maker must indicate the relative relevance of each attribute compared to all other attributes. However, a novel feature of the proposed method is that, it simply ranks all of the attributes (1 to n) according to their priority as per the understanding of the decision-maker. A relative importance matrix is then created using these ranks to further establish the weights. Even though the idea seems to be simple, it has the advantage of ensuring consistency while prioritizing one attribute over another. The consistency index is always 0 (i.e. fully consistent). In AHP or BWM, this is not feasible, particularly for the decision-making problems containing a large number of attributes. Furthermore, the proposed method clearly explained that the objective weights obtained from the performance data of the PCM selection attributes are *not* really meaningful, and the composite weights which combine the objective and subjective weights remain as an academic exercise and actually not used in the real decision-making scenarios.

The second novel feature of the proposed method is that it can include any number of alternative PCMs and any number of quantitative and qualitative PCM selection attributes simultaneously and aids in calculating the total score values that assess the alternative PCMs for the selection problem under consideration. The third novel feature of the proposed method is that it does not require the use of fuzzy scales to transform qualitative attributes into quantitative attributes. Using the simple linear scales that the method suggests, decision-makers may find it simpler to assign numerical values to the qualitative attributes. This fact is explained in the first case study presented. The proposed method tackles the PCM selection problem holistically, and is easy for decision-makers to put into practice.

The proposed methodology offers a general procedure that may be used to address a range of selection problems that emerge in the disciplines of energy and thermal engineering that involve ambiguity, multiple attributes, and alternatives.

Acknowledgements

The research was supported by the Science and Engineering Research Board (SERB) of the Department of Science and Technology (DST), Government of India, under the Mathematical Research Impact Centric Scheme (MATRICS) with the project number MTR/2023/000071.

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