Co-published by
Institute of Fluid-Flow Machinery
Polish Academy of Sciences
Committee on Thermodynamics and Combustion
Polish Academy of Sciences

http://www.imp.gda.pl/archives-of-thermodynamics/

ARCHIVES OF THERMODYNAMICS

Effect of variable thermal conductivity in semiconducting medium underlying an elastic half-space

Praveen Ailawalia[a*], Priyanka[b]

[a]Department of Mathematics, University Institute of Sciences, Chandigarh University, Gharuan, Mohali, Punjab, India
[b]I.G.N College, Ladwa, Haryana, India
*Corresponding author email: praveen_2117@rediffmail.com

Received: 16.07.2023; revised: 06.03.2024; accepted: 9.05.2024

Abstract

The aim of the present work is to discuss the effect of varying thermal conductivity in a semiconducting medium under photothermal theory. An infinite elastic half-space is overlying the infinite semiconducting medium, and a constant mechanical force is applied along the interface. The normal mode analysis method is applied to find the analytic components of displacement, stress, carrier density and temperature distribution. It was found that all physical quantities are affected by variable thermal conductivity. The novelty of the paper lies in the fact that no such a problem of variable thermal conductivity has been discussed by any researcher so far.

Keywords: Semiconducting medium; Thermal conductivity; Temperature distribution; Normal mode analysis; Carrier density

Vol. 45(2024), No. 3, 159–166; doi: 10.24425/ather.2024.151226

Cite this manuscript as: Ailawalia.P., & Priyanka (2024). Effect of variable thermal conductivity in semiconducting medium underlying an elastic half-space. *Archives of Thermodynamics*, 45(3), 159–166.

1. Introduction

Semiconductor materials are being considered important in current years due to their wide utilization in various fields of science and engineering. The plasma waves get generated due to the excitation of electrons under exposure to a beam of laser or sunlight. As a result, the interaction between the thermal wave, elastic wave and plasma wave occurs. Gordon et al. [1] and Kreuzer [2] made remarkable contributions in developing the photothermal theory. Mandelis et al. [3] studied the coupling of thermoelastic and electronic waves under photothermal theory. Todorovic [4] investigated plasmaelastic and thermoelastic waves in the semiconducting medium. Song et al. [5] worked on a reflection problem in a semiconducting medium for finding reflection coefficient ratios. Othman et al. [6] explained the wave propagation problem in a semiconducting medium in the context of Lord-Shulman (LS) theory. Ailawalia et al. [7] analysed the influence of mechanical force at the interface of semiconducting half-space and thermoelastic micropolar cubic crystal. Alzahrani and Abbas [8] studied a two-dimensional semiconducting medium under thermoelastic theory with one relaxation time. Hobiny [9] explored wave propagation in a semiconducting medium under a hyperbolic two-temperature model without energy dissipation. Saeed et al. [10] introduced a novel model for studying photothermal interaction in a rotating microstretch semiconductor medium subjected to initial stress.

Nomenclature

a – wave number in the x-direction
C_d – coefficient related to diffusion of carriers
C_s – specific heat, J/(kg K)
E_s – gap in energy of valence and conduction band of the semiconductor, J
e_{ij} – strain tensor
F – mechanical force
K_0^s – arbitrary constant
K_1 – physical parameter
K^s – thermal conductivity, W/(m K)
N – carrier density, kg/m^3
N_0 – carrier concentration at temperature T in equilibrium
$\vec{r}$ – position vector
T – temperature, K
T_0 – reference temperature, K
t – time, s
$\vec{u}$ – displacement vector
u, w – x, z components of velocity, m/s
u_i – components of velocity (i = 1, 2, 3), m/s
x, y, z – Cartesian coordinates, m
x_i, – Cartesian coordinates (i = 1, 2, 3), m

Greek symbols

α – thermal expansion coefficient, 1/K
β – coefficient of electronic deformation
γ – $\gamma = (3\lambda^s + 2\mu^s)\alpha$
δ – $\delta = (3\lambda^s + 2\mu^s)\beta$
δ_{ij} – Kronecker delta
κ – coupling parameter for the thermal activation, $= \left(\frac{\partial N_0}{\partial t}\right)\left(\frac{1}{\tau}\right)$
λ, μ – Lame's constants
ρ – density, kg/m^3
τ – photogenerated carrier lifetime
ω – complex time constant

Subscripts and Superscripts

e – elastic medium
s – semiconducting medium
$u_{i,j}$ – differentiation of u_i with respect to x_i
$(\cdot)^*$ – amplitude of the variables

Hilal [11] demonstrated photothermal interaction in a micro-elongated semiconducting medium under the influence of gravity. Kaur et al. [12] explained photo-thermo-elastic interactions in an infinite semiconducting rotating solid cylinder subjected to the magnetic field and hall current. Lotfy et al. [13] put forward a novel model for investigating non-local semiconductor medium. Azhar et al. [14] studied the reflection problem in a non-local semiconducting medium under the effect of hall current and magnetic field.

Under exposure to high temperature, the thermal conductivity of an elastic material is found experimentally to be varied with temperature. Therefore, the thermal conductivity cannot be treated as constant. El-Bary [15] introduced a mathematical model for studying layered thin plate subjected to variable thermal conductivity. Ezzat and Youssef [16] studied thermoelastic medium subjected to variable electrical and thermal conductivity under the theory of one relaxation time. Sherief and Abd El-Latief [17] explored the impact of variable thermal conductivity in an elastic half-space with respect to the theory of fractional thermoelasticity. Zenkour and Abbas [18] applied finite element method for obtaining thermal stress for a hollow cylinder in a temperature dependent thermoelastic medium. Yasein et al. [19] demonstrated the effect of varying thermal conductivity in a semiconducting medium subjected to thermal ramp type in the context of dual-phase-lag (DPL) and L-S model of thermoelasticity. Abbas et al. [20] analysed the behaviour of semiconducting medium with cylindrical cavities subjected to variable thermal conductivity. Alzahrani et al. [21] discussed eigen value problem for variable thermal conductivity in a porous medium. Marin et al. [22] explored porothermoelastic materials subjected to fractional time derivatives by applying the finite element method. Lotfy and El-Bary [23] proposed an elastic-thermodiffusion model for studying photothermal interactions in a semiconductor subjected to mechanical ramp type and variable thermal conductivity. Hobiny and Abbas [24] utilised the finite element method for describing thermoelastic interaction in an orthotropic material with spherical cavities under variable thermal conductivity. Kumar et al. [25] explained thermodynamical interactions for a thermodiffusive medium subjected to rotation and gravitational effect. El-Sapa et al. [26] demonstrated the influence of variable thermal conductivity on wave propagation in a non-local semiconducting medium.

The present research work deals with investigating the effect of varying thermal conductivity in a semiconducting medium. The problem has been modelled as an elastic half-space overlying the semiconducting half-space. The analytic components of displacement, stress, carrier density and temperature distribution are obtained by applying the normal mode technique. It was observed that all considered physical quantities depend on variable thermal conductivity.

2. Governing equations of the problem

Let us take a semiconducting half-space with an overlying elastic half-space. A mechanical load of magnitude F is acting on the boundary separating the two half-spaces. Further, we consider coordinate system (x, y, z) in which z-axis is taken in vertically downward direction. The region $z \geq 0$ is occupied by the semiconducting half-space (medium I) and the region $z < 0$ by elastic-half space (medium II) as represented in Fig. 1.

The fundamental equations for a semiconducting medium after neglecting body forces are given by Lotfy [27]:

$$\mu^s \nabla^2 \vec{u}(\vec{r}, t) + (\lambda^s + \mu^s)\nabla\big(\nabla \cdot \vec{u}(\vec{r}, t)\big) + -\gamma \nabla T - \delta \nabla N = \rho \frac{\partial^2 \vec{u}(\vec{r}, t)}{\partial t^2}. \tag{1}$$

The coupled equations for the semiconducting medium are given by Song et al. [28]:

$$K^s \nabla^2 T - \frac{E_s}{\tau} N + \gamma T_0 \nabla \cdot \frac{\partial \vec{u}}{\partial t} - \rho C_s \frac{\partial T}{\partial t} = 0. \tag{2}$$

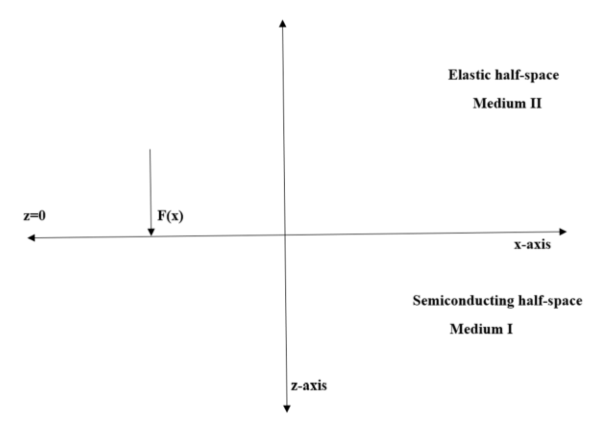

Fig. 1. Geometry of the problem.

$$C_d\nabla^2 N-\frac{1}{\tau}N+\kappa T-\frac{\partial N}{\partial t}=0. \quad (3)$$

Further the stress-strain relations for the considered medium are given by Lotfy [27]:

$$\sigma_{ij}=2\mu^s e_{ij}+\left(\lambda^s u_{k,k}-\delta N-\gamma T\right)\delta_{ij}, \quad (4)$$

where

$$e_{ij}=\frac{1}{2}\left(u_{j,i}+u_{i,j}\right).$$

The fundamental relations for elastic half-space are given by Ewing et al. [29]:

$$\mu^e\nabla^2\vec{u}^e(\vec{r},t)+(\lambda^e+\mu^e)\nabla\left(\nabla\cdot\vec{u}^e(\vec{r},t)\right)=\rho^e\frac{\partial^2\vec{u}^e(\vec{r},t)}{\partial t^2}, \quad (5)$$

$$\sigma_{lk}^e=\lambda^e u_{r,r}^e\delta_{lk}+\mu^e\left(u_{l,k}+u_{k,l}\right), \quad r,l,k=1,2,3. \quad (6)$$

3. Formulation of the problem

We assume that the waves are propagating in x-z plane. Hence the displacement vector in the semiconducting medium is considered as $\vec{u}=(u,0,w)$, where $u=u(x,z,t)$, $w=w(x,z,t)$. Equations (1)–(4) in two dimensions can be expressed as:

$$(\lambda^s+2\mu^s)\frac{\partial^2 u}{\partial x^2}+(\lambda^s+\mu^s)\frac{\partial^2 w}{\partial x\partial z}+$$
$$+\mu^s\frac{\partial^2 u}{\partial z^2}-\gamma\frac{\partial T}{\partial x}-\delta\frac{\partial N}{\partial x}=\rho\frac{\partial^2 u}{\partial t^2}, \quad (7)$$

$$\mu^s\frac{\partial^2 w}{\partial x^2}+(\lambda^s+\mu^s)\frac{\partial^2 u}{\partial x\partial z}+(\lambda^s+2\mu^s)\frac{\partial^2 w}{\partial z^2}+$$
$$-\gamma\frac{\partial T}{\partial z}-\delta\frac{\partial N}{\partial z}=\rho\frac{\partial^2 w}{\partial t^2}, \quad (8)$$

$$K^s\Delta T-\frac{E_s}{\tau}N+\gamma T_0\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)-\rho C_s\frac{\partial T}{\partial t}=0, \quad (9)$$

$$C_d\Delta N-\frac{1}{\tau}N+\kappa T-\frac{\partial N}{\partial t}=0, \quad (10)$$

$$\sigma_{xx}=(\lambda^s+2\mu^s)\frac{\partial u}{\partial x}+\lambda^s\frac{\partial w}{\partial z}-(\gamma T+\delta N), \quad (11)$$

$$\sigma_{zz}=(\lambda^s+2\mu^s)\frac{\partial w}{\partial z}+\lambda^s\frac{\partial u}{\partial x}-(\gamma T+\delta N), \quad (12)$$

$$\sigma_{zx}=\mu^s\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right), \quad (13)$$

where: $\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}$.

Further for the elastic half space, Eqs. (5) and (6) in two-dimensions can be reduced as:

$$(\lambda^e+2\mu^e)\frac{\partial^2 u^e}{\partial x^2}+(\lambda^e+\mu^e)\frac{\partial^2 w^e}{\partial x\partial z}+\mu^e\frac{\partial^2 u^e}{\partial z^2}=\rho^e\frac{\partial^2 u^e}{\partial t^2}, \quad (14)$$

$$\mu^e\frac{\partial^2 w^e}{\partial x^2}+(\lambda^e+2\mu^e)\frac{\partial^2 w^e}{\partial z^2}+(\lambda^e+\mu^e)\frac{\partial^2 u^e}{\partial x\partial z}=\rho^e\frac{\partial^2 w^e}{\partial t^2}, \quad (15)$$

$$\sigma_{xx}^e=(\lambda^e+2\mu^e)\frac{\partial u^e}{\partial x}+\lambda^e\frac{\partial w^e}{\partial z}, \quad (16)$$

$$\sigma_{zz}^e=(\lambda^e+2\mu^e)\frac{\partial w^e}{\partial z}+\lambda^e\frac{\partial u^e}{\partial x}, \quad (17)$$

$$\sigma_{zx}^e=\mu^e\left(\frac{\partial u^e}{\partial z}+\frac{\partial w^e}{\partial x}\right). \quad (18)$$

Under exposure to high temperature, heat conductivity K^s depends on the temperature of the medium, therefore it must be taken as variable as given by Lotfy [30]:

$$K^s(T)=K_0^s(1+K_1T). \quad (19)$$

We can use the Kirchhoff transformation for conversion of thermal conduction equation into linear form by the following relation given by Lotfy [30]:

$$\hat{T}=\frac{1}{K_0^s}\int_0^T K^s(\xi)d\xi. \quad (20)$$

Differentiating (20) with respect to x_i we get

$$K_0^s\widehat{T_{,\iota}}=K^s(T)T_{,i}. \quad (21)$$

Differentiating the above equation again with respect to x_i we get

$$K_0^s\widehat{T_{,\iota\iota}}=\left(K^s(T)T_{,i}\right)_{,i}. \quad (22)$$

Above equation has the following linear form after removing non-linear terms:

$$K_0^s\widehat{T_{,\iota\iota}}=K^s(T)T_{,ii}. \quad (23)$$

In $\widehat{T_{,\iota\iota}}$ the subscript should be treated as *ii* in Eqs. (22) and (23), though it appears as *ll* under $\hat{T}$ due to some software issue.

Differentiating (20) with respect to t we get

$$K_0^s\widehat{T_{,t}}=K^s(T)T_{,t}. \quad (24)$$

Equations (7) to (13) can be rewritten using (19)–(24) as:

$$(\lambda^s+2\mu^s)\frac{\partial^2 u}{\partial x^2}+(\lambda^s+\mu^s)\frac{\partial^2 w}{\partial x\partial z}+$$
$$+\mu^s\frac{\partial^2 u}{\partial z^2}-\gamma\frac{\partial\hat{T}}{\partial x}-\delta\frac{\partial N}{\partial x}=\rho\frac{\partial^2 u}{\partial t^2}, \quad (25)$$

$$\mu^s\frac{\partial^2 w}{\partial x^2}+(\lambda^s+\mu^s)\frac{\partial^2 u}{\partial x\partial z}+(\lambda^s+2\mu^s)\frac{\partial^2 w}{\partial z^2}+$$
$$-\gamma\frac{\partial\hat{T}}{\partial z}-\delta\frac{\partial N}{\partial z}=\rho\frac{\partial^2 w}{\partial t^2}, \quad (26)$$

$$K_0^s \Delta\hat{T} - \frac{E_s}{\tau}N + \gamma T_0 \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) - \rho C_s \frac{\partial \hat{T}}{\partial t} = 0, \quad (27)$$

$$C_d \Delta N - \frac{1}{\tau}N + \kappa\hat{T} - \frac{\partial N}{\partial t} = 0, \quad (28)$$

$$\sigma_{xx} = (\lambda^s + 2\mu^s)\frac{\partial u}{\partial x} + \lambda^s \frac{\partial w}{\partial z} - \left(\gamma\hat{T} + \delta N\right), \quad (29)$$

$$\sigma_{zz} = (\lambda^s + 2\mu^s)\frac{\partial w}{\partial z} + \lambda^s \frac{\partial u}{\partial x} - \left(\gamma\hat{T} + \delta N\right), \quad (30)$$

$$\sigma_{zx} = \mu^s\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right). \quad (31)$$

Further, for simplicity we use dimensionless quantities written below:

$$x' = \frac{1}{c_l t^*}x, \; z' = \frac{1}{c_l t^*}z, \; u' = \frac{1}{c_l t^*}u, \; w' = \frac{1}{c_l t^*}w,$$

$$t' = \frac{t}{t^*}, \; \sigma'_{ij} = \frac{\sigma_{ij}}{\mu^s}, \hat{T}' = \frac{\gamma\hat{T}}{(\lambda^s + 2\mu^s)}, \; N' = \frac{\delta N}{(\lambda^s + 2\mu^s)},$$

$$u^{e'} = \frac{1}{c_l t^*}u^e, \; w^{e'} = \frac{1}{c_l t^*}w^e, \; \sigma^{e'}_{ij} = \frac{\sigma_{ij}{}^e}{\mu^e}. \quad (32)$$

where:

$$c_l^2 = \frac{(\lambda^s + 2\mu^s)}{\rho}, \quad t^* = \frac{K_0^s}{C_s c_l^2}.$$

Using dimensionless variables (32) in Eqs. (25) to (31) we get equations as:

$$\frac{\partial^2 u}{\partial x^2} + b_{11}\frac{\partial^2 u}{\partial z^2} + b_{11}\frac{\partial^2 w}{\partial x \partial z} - \frac{\partial \hat{T}}{\partial x} - \frac{\partial N}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \quad (33)$$

$$b_{12}\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial z^2} + b_{11}\frac{\partial^2 w}{\partial x^2} - \frac{\partial \hat{T}}{\partial z} - \frac{\partial N}{\partial z} = \frac{\partial^2 w}{\partial t^2}, \quad (34)$$

$$a_{15}\Delta\hat{T} + a_{16}\frac{\partial \hat{T}}{\partial t} + a_{17}N + a_{18}\left(\frac{\partial^2 u}{\partial x \partial t} + \frac{\partial^2 w}{\partial z \partial t}\right) = 0, \quad (35)$$

$$a_{11}\Delta N + a_{12}N + a_{13}\frac{\partial N}{\partial t} + a_{14}\hat{T} = 0, \quad (36)$$

$$\sigma_{xx} = b_{13}\frac{\partial u}{\partial x} + b_{14}\frac{\partial w}{\partial z} - b_{13}\hat{T} - b_{13}N, \quad (37)$$

$$\sigma_{zz} = b_{14}\frac{\partial u}{\partial x} + b_{13}\frac{\partial w}{\partial z} - b_{13}\hat{T} - b_{13}N, \quad (38)$$

$$\sigma_{zx} = \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right). \quad (39)$$

Further for the elastic half space, Eqs. (14)–(18) in dimensionless form can be reduced as:

$$c_{11}\frac{\partial^2 u^e}{\partial x^2} + c_{12}\frac{\partial^2 w^e}{\partial x \partial z} + c_{13}\frac{\partial^2 u^e}{\partial z^2} = \frac{\partial^2 u^e}{\partial t^2}, \quad (40)$$

$$c_{13}\frac{\partial^2 w^e}{\partial x^2} + c_{11}\frac{\partial^2 w^e}{\partial z^2} + c_{12}\frac{\partial^2 u^e}{\partial x \partial z} = \frac{\partial^2 w^e}{\partial t^2}, \quad (41)$$

$$\sigma^e_{xx} = c_{14}\frac{\partial u^e}{\partial x} + c_{15}\frac{\partial w^e}{\partial z}, \quad (42)$$

$$\sigma^e_{zz} = c_{14}\frac{\partial w^e}{\partial z} + c_{15}\frac{\partial u^e}{\partial x}, \quad (43)$$

$$\sigma^e_{zx} = \left(\frac{\partial u^e}{\partial z} + \frac{\partial w^e}{\partial x}\right). \quad (44)$$

Here:

$$a_{11} = \frac{C_d}{\delta c_l^2 t^{*2}}, \; a_{12} = -\frac{1}{\tau\delta}, \; a_{13} = -\frac{1}{t^*\delta}, \; a_{14} = \frac{\kappa}{\gamma},$$

$$a_{15} = \frac{K_0^s}{\gamma t^* C_s}, \; a_{16} = -\frac{(\lambda^s + 2\mu^s)}{\gamma}, \; a_{17} = \frac{E_s(\lambda^s + 2\mu^s)t^*}{\tau\delta\rho C_s}, \; a_{18} = \frac{\gamma T_0}{\rho C_s},$$

$$b_{11} = \frac{\mu^s}{(\lambda^s + 2\mu^s)}, \; b_{12} = \frac{\lambda^s + \mu^s}{\lambda^s + 2\mu^s}, \; b_{13} = \frac{\lambda^s + 2\mu^s}{\mu^s}, \; b_{14} = \frac{\lambda^s}{\mu^s},$$

$$b_{14} = \frac{\lambda^s}{\mu^s}, \; c_{11} = \frac{\lambda^e + 2\mu^e}{\rho^e C_l^2}, \; c_{12} = \frac{\lambda^e + \mu^e}{\rho^e C_l^2}, \; c_{13} = \frac{\mu^e}{\rho^e C_l^2},$$

$$c_{14} = \frac{\lambda^e + 2\mu^e}{\mu^e}, \; c_{15} = \frac{\lambda^e}{\mu^e}.$$

4. Normal mode analysis

We use the following normal mode analysis for obtaining the solution for above considered physical variables:

$$\left[u, w, \hat{T}, N, \mathrm{u}^e, \mathrm{w}^e\right] = \left[u^*, w^*, \widehat{T^*}, N^*, \mathrm{u}^{e*}, \mathrm{w}^{e*}\right](z)e^{\omega t + iax}. \quad (45)$$

Using the solutions given by (45) in (33)–(36), we obtain the following equations in coupled form as:

$$(b_{11}D^2 + e_{15})u^* + iab_{12}Dw^* - ia\widehat{T^*} - iaN^* = 0, \quad (46)$$

$$iab_{12}Du^* + (D^2 + e_{16})w^* - D\widehat{T^*} - DN^* = 0, \quad (47)$$

$$e_{13}u^* + e_{14}Dw^* + (a_{15}D^2 + e_{12})\widehat{T^*} + a_{17}N^* = 0, \quad (48)$$

$$a_{14}\hat{T}^* + (a_{11}D^2 + e_{11})N^* = 0. \quad (49)$$

On solving these coupled equations (46) to (49), we get the following eighth-order differential equation:

$$[g_8 D^8 + g_9 D^6 + g_{10}D^4 + g_{11}D^2 + g_{12}]\left(u^*, w^*, \widehat{T^*}, N^*\right) = 0. \quad (50)$$

Similarly, using the solutions given by (45) in (40)–(41), we obtain the following equations in coupled form:

$$(c_{13}D^2 + e_{17})u^{e*} + iac_{12}Dw^{e*} = 0, \quad (51)$$

$$(c_{11}D^2 + e_{18})w^{e*} + iac_{12}Du^{e*} = 0. \quad (52)$$

On solving these coupled Eqs. (51) and (52), we get the following fourth-order differential equation:

$$[g_{13}D^4 + g_{14}D^2 + g_{15}](u^{e*}, w^{e*}) = 0. \quad (53)$$

Here:

$$e_{11} = -a_{11}a^2 + a_{12} + a_{13}\omega, \; e_{12} = -a_{15}a^2 + a_{16}\omega,$$

$$e_{13} = ia\omega a_{18}, \; e_{14} = \omega a_{18}, \; e_{15} = -(a^2 + \omega^2),$$

$$e_{16} = -(b_{11}a^2 + \omega^2), \; e_{17} = -(c_{11}a^2 + \omega^2),$$

$$e_{18} = -(c_{13}a^2 + \omega^2),$$

$$f_1 = iaa_{15}b_{12}, \; f_2 = ia(e_{12}b_{12} + e_{14}), \; f_3 = -e_{14}b_{11},$$

$$f_4 = iae_{13}b_{12} - e_{14}e_{15}, \; f_5 = ia(a_{17}b_{12} + e_{14}),$$

$$f_6 = b_{11}, \; f_7 = e_{15} + e_{16}b_{11+}a^2 b_{12}{}^2, \; f_8 = e_{15}e_{16},$$

$$f_9 = ia(b_{12} - 1),\ f_{10} = -iae_{16},$$

$$g_1 = f_1 f_6,\ g_2 = f_2 f_6 + f_1 f_7 - f_3 f_9,$$

$$g_3 = f_2 f_7 + f_1 f_8 - f_3 f_{10} - f_4 f_9,\ g_4 = f_2 f_8 - f_4 f_{10},$$

$$g_5 = f_5 f_6 - f_3 f_9,\ g_6 = f_5 f_7 - f_4 f_9 - f_3 f_{10},$$

$$g_7 = f_5 f_8 - f_4 f_{10},\ g_8 = a_{11} g_1,\ g_9 = e_{11} g_1 + a_{11} g_2,$$

$$g_{10} = e_{11} g_2 + a_{11} g_3 - a_{14} g_5,$$

$$g_{11} = e_{11} g_3 + a_{11} g_4 - a_{14} g_6,$$

$$g_{12} = e_{11} g_4 - a_{14} g_7,\ g_{13} = c_{11} c_{13},$$

$$g_{14} = c_{11} e_{17} + c_{13} e_{18} + a^2 {c_{12}}^2,\ g_{15} = -e_{18} e_{17}.$$

Using radiation conditions u^*, $w^*, \widehat{T^*}$, $N^* \to 0$ as $z \to \infty$, the solution of Eq.(50) can be written as:

$$\widehat{T^*} = \textstyle\sum_{j=1}^{4} C_j\, e^{-k_j z}, \tag{54}$$

$$u^* = \textstyle\sum_{j=1}^{4} A_j\, e^{-k_j z}, \tag{55}$$

$$w^* = \textstyle\sum_{j=1}^{4} B_j\, e^{-k_j z}, \tag{56}$$

$$N^* = \textstyle\sum_{j=1}^{4} \mathrm{E}_j\, e^{-k_j z}, \tag{57}$$

where k_j^2 $(j = 1,\ 2,\ 3,\ 4)$ are roots of Eq. (50). Also using Eqs. (46)–(49), the coupling constants A_j, B_j, E_j are given by $A_j = L_j C_j$, $B_j = M_j C_j$, and $E_j = N_j C_j$, where:

$$L_j = \frac{h_1 k_j^4 + h_2 k_j^2 + h_3}{h_4 k_j^4 + h_5 k_j^2 + h_6},$$

$$M_j = \frac{h_7 k_j^6 + h_8 k_j^4 + h_9 k_j^2 + h_{10}}{h_{11} k_j^5 + h_{12} k_j^3 + h_{13} k_j},$$

$$N_j = -\frac{a_{14}}{a_{11} k_j^2 + e_{11}}$$

and

$$h_1 = -f_1 a_{11},\ h_2 = -f_2 a_{11} - f_1 e_{11},\ h_3 = f_5 a_{14} - f_2 e_{11},$$

$$h_4 = f_3 a_{11},\ h_5 = f_4 a_{11} + f_3 e_{11},\ h_6 = f_4 e_{11},$$

$$h_7 = -a_{15} f_3 a_{11},$$

$$h_8 = e_{13} f_1 a_{11} - a_{15} f_3 e_{11} - a_{11} f_3 e_{12} - a_{15} f_4 a_{11},$$

$$h_9 = a_{17} f_3 a_{14} + e_{13} f_2 a_{11} + e_{13} f_1 e_{11} - e_{12} f_3 e_{11} +$$

$$-e_{12} f_4 a_{11} - a_{15} f_4 e_{11},$$

$$h_{10} = a_{17} f_4 a_{14} - e_{13} f_5 a_{14} + e_{13} f_2 e_{11} - e_{12} f_4 e_{11},$$

$$h_{11} = a_{11} f_3 e_{14},$$

$$h_{12} = a_{11} f_4 e_{14} + e_{14} f_3 e_{11}, h_{13} = e_{11} f_4 e_{14}.$$

Using Eqs. (54)–(57), we obtain the expressions for all physical quantities as:

$$\hat{T} = \textstyle\sum_{j=1}^{4} C_j\, e^{-k_j z} e^{\omega t + iax}, \tag{58}$$

$$u = \textstyle\sum_{j=1}^{4} C_j\, L_j e^{-k_j z} e^{\omega t + iax}, \tag{59}$$

$$w = \textstyle\sum_{j=1}^{4} C_j\, M_j e^{-k_j z} e^{\omega t + iax}, \tag{60}$$

$$N = \textstyle\sum_{j=1}^{4} C_j\, N_j e^{-k_j z} e^{\omega t + iax}, \tag{61}$$

$$\sigma_{xx} = \textstyle\sum_{j=1}^{4} \left[iab_{13} L_j - b_{14} k_j M_j - b_{13} N_j - b_{13}\right] \times \times C_j e^{-k_j z} e^{\omega t + iax}, \tag{62}$$

$$\sigma_{zz} = \textstyle\sum_{j=1}^{4} \left[-b_{13} k_j M_j + iab_{14} L_j - b_{13} N_j - b_{13}\right] \times \times C_j e^{-k_j z} e^{\omega t + iax}, \tag{63}$$

$$\sigma_{zx} = \textstyle\sum_{j=1}^{4} \left[-k_j L_j + iaM_j\right] C_j\, e^{-k_j z} e^{\omega t + iax}. \tag{64}$$

Using radiation conditions u^{e*}, $w^{e*} \to 0$ as $z \to \infty$, the solution of Eq. (53) can be written as:

$$u^{e*} = \textstyle\sum_{p=5}^{6} F_p\, e^{k_p z}, \tag{65}$$

$$w^{e*} = \textstyle\sum_{p=5}^{6} G_p\, e^{k_p z}, \tag{66}$$

where k_p^2 $(j = 5, 6)$ are roots of Eq. (53). Also using Eqs. (51)–(52), the coupling constant G_p is given by $G_p = O_p F_p$, where:

$$O_p = -\frac{c_{13} k_p^2 + e_{17}}{iac_{12} k_p}.$$

Using Eqs. (65)–(66), we obtain the expressions for all physical quantities in elastic half-space as:

$$u^e = \textstyle\sum_{p=5}^{6} F_p\, e^{k_p z} e^{\omega t + iax}, \tag{67}$$

$$w^e = \textstyle\sum_{p=5}^{6} O_p F_p\, e^{k_p z} e^{\omega t + iax}, \tag{68}$$

$$\sigma_{xx}^e = \textstyle\sum_{p=5}^{6} \left[iac_{14} + c_{15} k_p O_p\right] F_p\, e^{k_p z} e^{\omega t + iax}, \tag{69}$$

$$\sigma_{zz}^e = \textstyle\sum_{p=5}^{6} \left[iac_{15} + c_{14} k_p O_p\right] F_p\, e^{k_p z} e^{\omega t + iax}, \tag{70}$$

$$\sigma_{zx}^e = \textstyle\sum_{p=5}^{6} \left[iaO_p + k_p\right] F_p\, e^{k_p z} e^{\omega t + iax}. \tag{71}$$

5. Boundary conditions

For evaluating the constants C_j $(j = 1, 2, 3, 4)$ and F_p $(p = 5, 6)$, the following boundary conditions are applied:

1) A mechanical force $F e^{\omega t + iax}$ is applied at $z = 0$ along the normal direction:

$$\sigma_{zz} = \sigma_{zz}^e - F e^{\omega t + iax}. \tag{72}$$

2) The tangential stress vanishes at $z = 0$:

$$\sigma_{zx} = \sigma_{zx}^e. \tag{73}$$

3) The surface $z = 0$ is thermally insulated:

$$\frac{\partial T}{\partial z} = 0. \tag{74}$$

4) The carrier density at $z = 0$ is

$$C_d \frac{\partial N}{\partial z} = \kappa N. \tag{75}$$

5) The tangential displacement is continuous at $z = 0$:

$$u = u^e. \quad (76)$$

6) The normal displacement is continuous at $z = 0$:

$$w = w^e. \quad (77)$$

Using Eqs. (58)–(64) and (67)–(71) in Eqs. (72)–(77), we get the following non-homogenous system of four equations:

$$a_1^{**}C_1 + a_2^{**}C_2 + a_3^{**}C_3 + a_4^{**}C_4 + a_5^{**}F_5 + a_6^{**}F_6 = -F, \quad (78)$$

$$b_1^{**}C_1 + b_2^{**}C_2 + b_3^{**}C_3 + b_4^{**}C_4 + b_5^{**}F_5 + b_6^{**}F_6 = 0, \quad (79)$$

$$k_1C_1 + k_2C_2 + k_3C_3 + k_4C_4 = 0, \quad (80)$$

$$e_1^{**}C_1 + e_2^{**}C_2 + e_3^{**}C_3 + e_4^{**}C_4 = 0, \quad (81)$$

$$L_1C_1 + L_2C_2 + L_3C_3 + L_4C_4 - F_5 - F_6 = 0, \quad (82)$$

$$M_1C_1 + M_2C_2 + M_3C_3 + M_4C_4 - O_5F_5 - O_6F_6 = 0. \quad (83)$$

where:

$$a_j^{**} = -b_{13}k_jM_j + iab_{14}L_j - b_{13}N_j - b_{13},$$

$$a_p^{**} = -[c_{14}k_pO_p + iac_{15}],$$

$$b_j^{**} = iaM_j - k_jL_j, b_p^{**} = -[k_p - iaO_p],$$

$$e_j^{**} = \left(\frac{\kappa}{C_d} + k_j\right)N_j, \ \ j = 1,2,3,4, \ \ p = 5,6.$$

For evaluating the values of constants C_j $(j = 1, 2, 3, 4)$ and F_p $(p = 5, 6)$, the system of Eqs. (78)–(83) are solved using MATLAB. After evaluating the values of these constants, the expressions for temperature distribution, components of displacement, carrier density and components of stress can be obtained by the expressions (58)–(64).

Using Eqs. (19) and (20), the relation between the temperature T and the operator $\widehat{T}$ can be written as

$$T = \frac{1}{K_1}\left[\sqrt{1 + 2K_1\widehat{T}} - 1\right] = \frac{1}{K_1}\left[\sqrt{1 + 2K_1\widehat{T^*}e^{\omega t + iax}} - 1\right]. \quad (84)$$

The temperature, components of displacement, stresses and carrier density may also be expressed in terms of K_1 as:

$$T = \frac{1}{K_1}\left[\sqrt{1 + 2K_1\sum_{j=1}^{4} C_j\, e^{-k_jz}e^{\omega t + iax}} - 1\right], \quad (85)$$

$$u = \frac{L^{**}}{2}\left[K_1\left(T + \frac{1}{K_1}\right)^2 - \frac{1}{K_1}\right]e^{\omega t + iax}, \quad (86)$$

$$w = \frac{M^{**}}{2}\left[K_1\left(T + \frac{1}{K_1}\right)^2 - \frac{1}{K_1}\right]e^{\omega t + iax}, \quad (87)$$

$$\sigma_{xx} = \frac{1}{2}\left[K_1\left(T + \frac{1}{K_1}\right)^2 - \frac{1}{K_1}\right][iab_{13}L^{**} - b_{13} - b_{13}N^{**}]e^{\omega t + iax} + - b_{14}M^{**}\sum_{j=1}^{4} k_j\, C_je^{-k_jz}e^{\omega t + iax}, \quad (88)$$

$$\sigma_{zz} = \frac{1}{2}\left[K_1\left(T + \frac{1}{K_1}\right)^2 - \frac{1}{K_1}\right][iab_{14}L^{**} - b_{13} - b_{13}N^{**}]e^{\omega t + iax} + - b_{13}M^{**}\sum_{j=1}^{4} k_j\, C_je^{-k_jz}e^{\omega t + iax}, \quad (89)$$

$$\sigma_{zx} = \left\{\begin{matrix} -L^{**}\sum_{j=1}^{4} k_j\, C_je^{-k_jz} + \\ + \frac{iaM^{**}}{2}\left[K_1\left(T + \frac{1}{K_1}\right)^2 - \frac{1}{K_1}\right]\end{matrix}\right\}e^{\omega t + iax}, \quad (90)$$

$$N = \frac{N^{**}}{2}\left[K_1\left(T + \frac{1}{K_1}\right)^2 - \frac{1}{K_1}\right]e^{\omega t + iax}, \quad (91)$$

where:

$$L^{**} = \frac{h_1r^4 + h_2r^2 + h_3}{h_4r^4 + h_5r^2 + h_6},$$

$$M^{**} = \frac{h_7r^6 + h_8r^4 + h_9r^2 + h_{10}}{h_{11}r^5 + h_{12}r^3 + h_{13}r},$$

$$N^{**} = -\frac{a_{14}}{a_{11}r^2 + e_{11}}.$$

6. Numerical results

For the numerical justification of the analytical results obtained, we take example of silicon (Si) as semiconducting medium for which related values of constants are given by Song et al. [28] as:

$$\lambda^s = 3.64\times10^{10}\ \text{N/m}^2,\ \mu^s = 5.46\times10^{10}\ \text{N/m}^2,\ \rho = 2330\ \text{kg/m}^3,$$

$$T_0 = 800\ \text{K},\ \tau = 5\times10^{-5}\ \text{s},\ C_d = 2.5\times10^{-3}\ \text{m}^2/\text{s},\ C_s = 695\ \text{J/(kg K)},$$

$$E_s = 1.11\text{eV},\ \alpha = 4.14\times10^{-6}/\text{K},\ K_0^s = 150\text{W/(m K)},$$

$$\beta = -9\times10^{-31}\text{m}^3,\ F = 1.0,\ \kappa = 2.0.$$

Further the physical constants for elastic medium – granite – are given by Bullen [31]:

$$\rho^e = 2.65\times10^3\text{kg/m}^3,\ \lambda^e = 2.238\times10^3\text{N/m}^2,\ \mu^e = 2.238\times10^3\ \text{N/m}^2.$$

All calculations have been done at the surface $x = 1,\ t = 1$. Further $\omega = \omega_0 + i\xi$, where $\omega_0 = -0.03,\ \xi = 0.01$. The graphs are obtained for constant and variable thermal conductivity by taking three values of K_1, namely 0, –2 and –5, respectively.

6.1. Results and discussion

Figure 2 shows the variation of normal displacement against horizontal distance. Parameter K_1 represents thermal conductivity of the medium. $K_1 = 0$, when the variables are independent of thermal conductivity, and when K_1 is not equal to zero ($K_1 = -2, -5$), then the variables are dependent on thermal conductivity. The value of normal displacement decreases with an increase in horizontal distance for both the cases. In the case of variable thermal conductivity, the magnitude of normal displacement decreases with increase in value of K_1.

Figure 3 shows variation of normal stress with horizontal distance. The value of normal stress increases as the horizontal distance increases in the case of constant thermal conductivity. The similar behaviour is observed for $K_1 = -5$. But for

$K_1 = -2$, the magnitude of normal stress first increases to attain the maximum value, then follows a sharp decrease.

Figure 4 shows variation of carrier density against horizontal distance. The value of carrier density increases sharply for constant thermal conductivity following a sharp decrease. But for variable thermal conductivity, the magnitude of carrier density varies inversely with value of K_1.

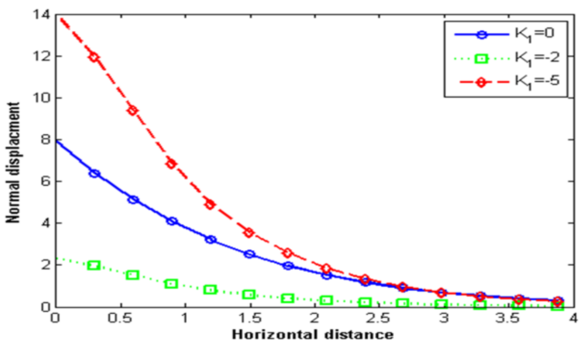

Fig. 2. Variation of normal displacement against horizontal distance.

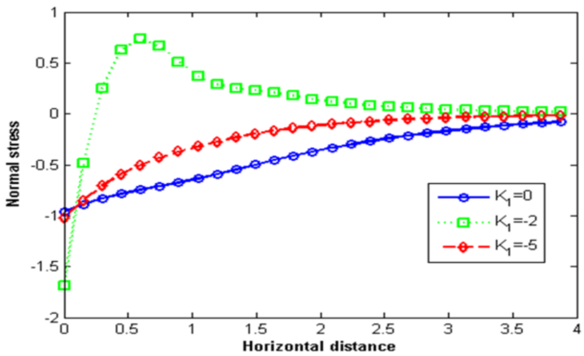

Fig. 3. Variation of normal stress against horizontal distance.

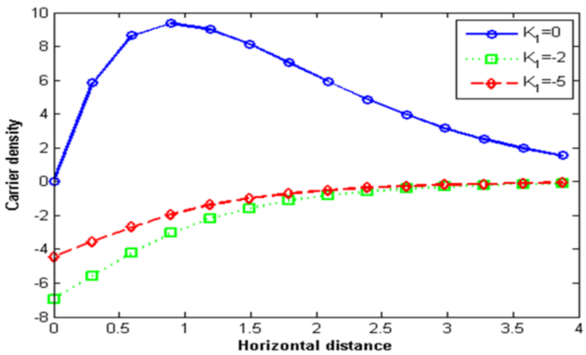

Fig. 4. Variation of carrier density against horizontal distance.

Figure 5 shows variation of temperature against horizontal distance. The magnitude of temperature increases exponentially, then becomes constant for all values of K_1. Further for variable thermal conductivity, the temperature varies inversely with parameter K_1.

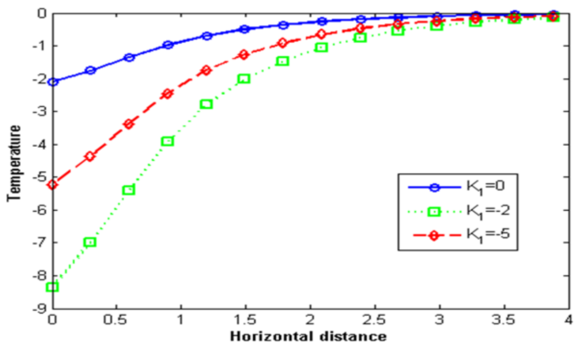

Fig. 5. Variation of temperature against horizontal distance.

7. Conclusions

The following conclusions can be drawn from the performed study on the effect of variable thermal conductivity in semiconducting medium underlying an elastic half-space:

1) The variable thermal conductivity has a considerable effect on all the physical quantities.
2) The maximum variation is obtained for carrier density in context of constant and variable thermal conductivity.
3) All the physical quantities are inversely proportional to the value of parameter K_1 , except normal stress.
4) This research work finds its application in different fields of engineering to calculate displacement, stress and carrier density in semiconductors subjected to variable thermal conductivity.
5) This problem has its importance due to the fact that physical properties of a material change drastically when there is a change in temperature.

References

[1] Gordon, J.P., Leite, R.C.C., Moore, R.S., Porto, S.P.S., & Whinnery, J.R. (1965). Long-transient effects in lasers with inserted liquid samples. *Journal of Applied Physics*, 36(1), 3–8. doi: 10.1063/1.1713919

[2] Kreuzer, L.B. (1971). Ultralow gas concentration infrared absorption spectroscopy. *Journal of Applied Physics*, 42(7), 2934–2943. doi: 10.1063/1.1660651

[3] Mandelis, A. (1997). Thermoelectronic-wave coupling in laser photothermal theory of semiconductors at elevated temperatures. *Optical Engineering (Redondo Beach, Calif.)*, 36(2), 459–468. doi: 10.1117/1.601217

[4] Todorović, D.M. (2005). Plasmaelastic and thermoelastic waves in semiconductors. *Journal de Physique IV (Proceedings)*, 125, 551–555. doi: 10.1051/jp4:2005125127

[5] Song, Y.Q., Bai, J.T., & Ren, Z.Y. (2012). Study on the reflection of photothermal waves in a semiconducting medium under generalized thermoelastic theory. *Acta Mechanica*, 223, 1545–1557. doi: 10.1007/s00707-012-0677-1

[6] Othman, M.I., Tantawi, R.S., & Eraki, E.E. (2016). Propagation of the photothermal waves in a semiconducting medium under L-S theory. *Journal of Thermal Stresses*, 39(11), 1419–1427. doi: 10.1080/01495739.2016.1216063

[7] Ailawalia, P., Sachdeva, S. K., Singh Pathania, D.S., & Hong Wu, Y. (2017). Effect of mechanical force along the interface of

semi-infinite semiconducting medium and thermoelastic micropolar cubic crystal. *Cogent Mathematics*, 4(1), 1347991. doi: 10.1080/23311835.2017.1347991

[8] Alzahrani, F.S., & Abbas, I.A. (2018). Photo-thermoelastic interactions in a 2D semiconducting medium. *European Physical Journal Plus*, 133, 505. doi: 10.1140/epjp/i2018-12285-5

[9] Hobiny, A. (2020). Effect of the hyperbolic two-temperature model without energy dissipation on photo-thermal interaction in a semi-conducting medium. *Results in Physics*, 18, 103167. doi: 10.1016/j.rinp.2020.103167

[10] Saeed, A.M., Lotfy, K., El-Bary, A., & Ahmed, M.H. (2021). Thermoelastic with photogenerated model of rotating microstretch semiconductor medium under the influence of initial stress. *Results in Physics*, 31, 104967. doi: 10.1016/j.rinp.2021.104967

[11] Hilal, M.I.M. (2022). Photothermal excitation and Thomson impact in a semiconductor microelongated thermoelastic medium with microtemperatures in the gravity. *Zeitschrift Fur Angewandte Mathematik Und Mechanik*, 102(12), e202200175. doi: 10.1002/zamm.202200175

[12] Kaur, I., Singh, K., & Craciun, E.M. (2022). A mathematical study of a semiconducting thermoelastic rotating solid cylinder with modified Moore–Gibson–Thompson heat transfer under the Hall effect. *Mathematics*, 10(14), 2386. doi: 10.3390/math10142386

[13] Lotfy, K., Ahmed, A., El-Bary, A., & Tantawi, R.S. (2023). A novel stochastic model of the photo-thermoelasticity theory of the non-local excited semiconductor medium. *Silicon*, 15, 437–450. doi: 10.1007/s12633-022-02021-x

[14] Azhar, E., Ali, H., Jahangir, A., & Anya, A.I. (2023). Effect of Hall current on reflection phenomenon of magneto-thermoelastic waves in a non-local semiconducting solid. *Waves in Random and Complex Media*, 1–18. doi: 10.1080/17455030.2023.2182146

[15] Youssef, H.M., & El-Bary, A.A. (2006). Thermal shock problem of a generalized thermoelastic layered composite material with variable thermal conductivity. *Mathematical Problems in Engineering*, 2006, 87940, 1–14. doi: 10.1155/mpe/2006/87940

[16] Ezzat, M.A., & Youssef, H.M. (2009). State space approach for conducting magneto-thermoelastic medium with variable electrical and thermal conductivity subjected to ramp-type heating. *Journal of Thermal Stresses*, 32(4), 414–427. doi: 10.1080/01495730802637233

[17] Sherief, H., & Abd El-Latief, A.M. (2013). Effect of variable thermal conductivity on a half-space under the fractional order theory of thermoelasticity. *International Journal of Mechanical Sciences*, 74, 185–189. doi: 10.1016/j.ijmecsci.2013.05.016

[18] Zenkour, A.M., & Abbas, I.A. (2014). Nonlinear transient thermal stress analysis of temperature-dependent hollow cylinders using a finite element model. *International Journal of Structural Stability and Dynamics*, 14(7), 1450025. doi: 10.1142/S0219455414500254

[19] Yasein, M., Mabrouk, N., Lotfy, K., & EL-Bary, A.A. (2019). The influence of variable thermal conductivity of semiconductor elastic medium during photothermal excitation subjected to thermal ramp type. *Results in Physics*, 15, 102766. doi: 10.1016/j.rinp.2019.102766

[20] Abbas, I., Hobiny, A., & Marin, M. (2020). Photo-thermal interactions in a semi-conductor material with cylindrical cavities and variable thermal conductivity. *Journal of Taibah University for Science*, 14(1), 1369–1376. doi: 10.1080/16583655.2020.1824465

[21] Alzahrani, F., Hobiny, A., Abbas, I., & Marin, M. (2020). An eigenvalues approach for a two-dimensional porous medium based upon weak, normal and strong thermal conductivities. *Symmetry*, 12(5), 848. doi: 10.3390/sym12050848

[22] Marin, M., Hobiny, A., & Abbas, I. (2021). The effects of fractional time derivatives in porothermoelastic materials using finite element method. *Mathematics*, 9(14), 1606. doi: 10.3390/math9141606

[23] Lotfy, K., & El-Bary, A. (2022). Elastic-thermal-diffusion model with a mechanical ramp type and variable thermal conductivity of electrons–holes semiconductor interaction. *Waves in Random and Complex Media*, 1–20. doi: 10.1080/17455030.2022.2078521

[24] Hobiny, A., & Abbas, I. (2023). Generalized thermoelastic interaction in orthotropic media under variable thermal conductivity using the finite element method. *Mathematics*, 11(4), 955. doi: 10.3390/math11040955

[25] Kumar, S., Kadian, A., & Kalkal, K.K. (2023). Thermoelastic interactions in a rotational medium having variable thermal conductivity and diffusivity with gravitational effect. *Waves in Random and Complex Media*, 1–22. doi: 10.1080/17455030.2023.2205964

[26] El-Sapa, S., El-Bary, A.A., & Lotfy, K. (2023). Effect of an excited non-local microelongated semiconductor with variable thermal conductivity on the propagation of photo-thermoelastic waves. *Optical and Quantum Electronics*, 55(6), 569. doi: 10.1007/s11082-023-04836-3

[27] Lotfy, K. (2017). Photothermal waves for two temperature with a semiconducting medium under using a dual-phase-lag model and hydrostatic initial stress. *Waves in Random and Complex Media*, 27(3), 482–501. doi: 10.1080/17455030.2016.1267416

[28] Song, Y., Bai, J., & Ren, Z. (2012). Reflection of plane waves in a semiconducting medium under photothermal theory. *International Journal of Thermophysics*, 33, 1270–1287. doi: 10.1007/s10765-012-1239-4

[29] Ewing, W.M., Jardetzky, W.S., Press, F., & Beiser, A. (1957). *Elastic Waves in Layered Media*. McGraw-Hill.

[30] Lotfy, K. (2019). Effect of variable thermal conductivity during the photothermal diffusion process of semiconductor medium. *Silicon*, 11, 1863–1873. doi: 10.1007/s12633-018-0005-z

[31] Bullen, K.E. (1963). *An Introduction of the Theory of Seismology*. Cambridge University Press.