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HOT WIRE ANEMOMETER WITH SINE MODULATION OF WIRE TEMPERATURE

ANEMOMETR CIEPLNY Z SINUSOIDALNĄ MODULACJĄ TEMPERATURY WŁÓKNA

The paper demonstrates a method for simultaneous measurements of flow velocity v and the temperature of the medium T_g by means of a single-wire probe operating in a CTA configuration where the overheating ratio *N(t)* is modulated sinusoidally.

Assuming that parameters a and b in the King's equation (1) are not directly related to the temperature of the flowing medium, it becomes readily apparent that variations in power losses due to the change of the overheat ing ratio do not depend on the temperature of the medium T_g (Fig 2).

When power losses are expressed in terms of instantaneous overheating ratio *N(t)* and voltage across the hot wire U_w and the voltage developed in Fourier series, the relationships between the zero order component U_0 and the first order component U_1 of the voltage across the hot wire and the flow velocity v and medium temperature T_g can be found.

Thus obtained two nonlinear algebraic equations allow for simple computation of flow velocity v and medium temperature T_g.

Key words: hot-wire anemometer, temperature compensation

W artykule podaje się metodę równoczesnego pomiaru prędkości przepływu v i temperatury medium T_g za pomocą jednowłóknowej sondy termoanemometrycznej pracującej w układzie anemometru stałotemperaturowego, którego współczynnik nagrzania *N(t)* jest modulowany sinusoidalnie.

Zakładając, że parametry a i b w prawie Kinga [równaniu (1)] nie zależą bezpośrednio od temperatury płynącego medium wykazano, że przyrosty lub straty mocy wywołane zmianą współczynnika nagrzania nie zależą od temperatury medium T_g (rys. 2).

Wyrażając traconą moc za pomocą zależnego od czasu współczynnika nagrzania włókna *N(t)* i napięcia występującego na grzanym włóknie anemometru U_w i rozkładając to napięcie w szereg Fouriera znaleziono związki jakie zachodzą pomiędzy zerową składową U_0 i pierwszą składową U_1 napięcia na grzanym włóknie a prędkością v przepływu medium i temperaturą medium T_g.

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W przypadku modulacji współczynnika nagrzania włókna po rozwinięciu furierowskim napięcia na włóknie U_w na składowe otrzymuje się dodatkowe informacje w postaci U_0 i U_1 i φ, które tu wykorzystuje się do wyliczenia prędkości przepływu v i temperatury medium T_g. Wielkości te wylicza się z równań (23), (24) i (27).

Słowa kluczowe: anemometr z grzanym włóknem, kompensacja temperatury

Nomenclature

a — coefficient in King's equation,
b — coefficient in King's equation,
c — specific heat of the wire material,
f — modulation frequency,
k — k-th component of voltage supplying the wire or phase,
m — mass of wire,
n — power exponent in King's equation,
t — time,
N_0 — mean of wire overheating ratio,
N_1 — amplitude of wire overheating ratio modulation,
$N(t)$ — average overheating ratio of the anemometer wire, which is then modulated,
R_0 — resistance of the wire at the reference temperature T_0,
R_c — resistance of the "cold" wire,
R_w — resistance of the hot wire,
T_g — temperature of the flowing medium,
T_w — temperature of the hot wire,
U_0 — average wire supplly voltage (determined from the Fourier expansion),
U_1 — first harmonic amplitude of wire supply voltage, determined from Fourier expansion,
U_w — voltage across the hot wire,
γ — temperature coefficient of wire resistance,
ω — overheat modulation frequency,
ϕ — phase shift of voltage first harmonic across the wire.

Introduction

Flow velocity measurements the conditions of changing temperature can be taken with the use of hot wire probes, providing for adequate temperature correction of compensation. These methods are described in more detail in the paper by Ligęza (Ligęza 2001). In most general terms, two measuring probes are used and one measures heat losses in the given flow conditions while the other is used for measuring the temperature of the medium. The measurement points, though, are spatially separated which in consequence may lead to measurements errors. A similar situation is encountered in flows with temperature or velocity gradients.

Attempts were made, therefore, to measure flow velocity and temperature of the medium by means of a single- wire constant-temperature anemometer in which the overheating ratio changes cyclically (Ligęza 1993, 1994); the measured parameter being the voltage supplying the bridge. In other words, spatial separation of two measuring wires is replaced by time separation while the measurements are taken with the single wire. An example of measurements of temperature and velocity fields behind then hot wire are provided in (Kiełbasa 1991). Overheat ratio modulation frequency would be 11.5 Hz and the applied modulating signal was a square wave.

The results of velocity (velocity modulus) and temperature measurements were mostly overstated because at the instant the overheating level was switched from lower to higher value, the wire would be momentarily superheated (Fig. 1) and when the overheating level was changed down, the wire would cool down naturally, that is rather slowly. That is why the average component of voltage amplitude half-period was higher when the wire was heated that when it was cooling; as the result the measured velocity values tended to be overstated.

The same approach, also utilising single-wire probes is described in (Ferreira et al. 2000). The main point is that the operating conditions of a CTA bridge are cyclically varied. This is the same idea as that patented by the Author in 1991 (Kiełbasa 1991).

The conceptual design was accordingly changed and bistable anemometer operation was replaced with sine modulations of the wire overheating ratio, which were implemented digitally. In this way the states of wire superheating were eliminated. Fig. 2 depicts the power losses from the hot anemometric wire in the function of time for sine modulations.

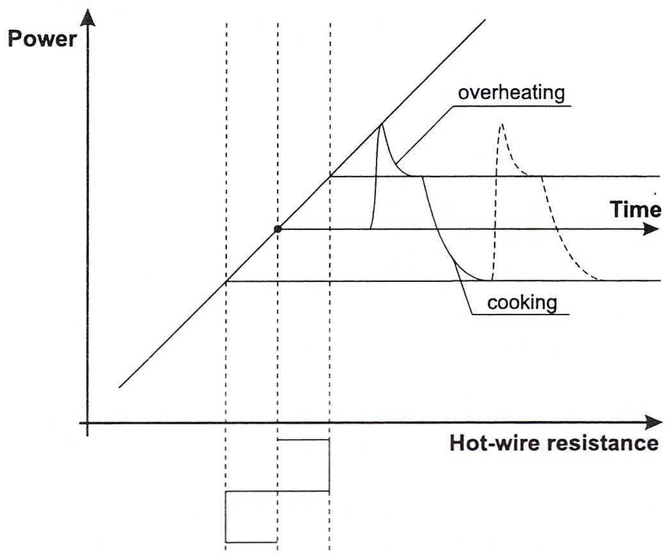

Fig. 1. Bi-stable anemometer operation

Rys. 1. Anemometr z dwoma stanami nagrzania

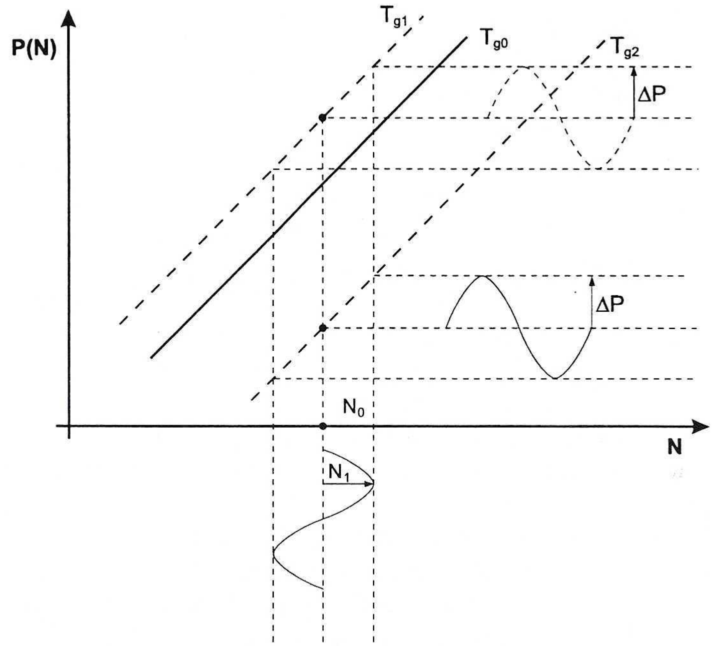

Fig. 2. Hot-wire anemometer in the function of time for sine modulations

Rys. 2. Termoanemometr z sinusoidalną modulacją współczynnika nagrzania

The main idea can be described as follows:

$$\frac{U_w^2}{R_w} = (a + bv^n)[T_w - T_g] \tag{1}$$

where:

- U_w — voltage across the hot wire,
- R_w — resistance of the hot wire,
- v — velocity of gas,
- $a,\ b,\ n$ — coefficients in the King's equation (King 1914),
- T_w — temperature of the hot wire,
- T_g — temperature of the flowing medium, taken to be independent of time.

It is readily apparent that the left-hand side of the equation provides the power supplied to the wire while the right-hand side term is linearly related to the wire temperature.

The temperature of flowing medium is the parameter in this equation. When the equation (1) is represented graphically, we get the situation presented in Fig. 2. The working point of the hot wire is determined by its temperature T_w related to the preset resistance R_w, which in turn is related to overheating ratio N_0, and to the linear function of power losses derived from (1). Any change in the temperature of the flowing medium to T_{g1} or T_{g2} will result in shifting of the straight line to the right or to the left. When the overheating ratio is modulated with an amplitude N_1, the generated power increase ΔP in the hot wire will be independent of temperature T_g. When the power increase ΔP is related to flow velocity, it will also become independent of the temperature of the medium.

2. Anemometer with sinusoidal modulations of overheating ratio

Measurements of air flow velocity and temperature can be taken with singlewire CT anemometers where the overheating ratio is modulated sinusoidally. The formula determining the heat losses from the hot wire in contact with the flowing gas and in constant-temperature configuration is given by the well-known King's law (King 1914):

$$\frac{U_w^2(t)}{R_w(t)} = (a + bv^n)[T_w(t) - T_g] + mc\frac{dT_w(t)}{dt} \tag{2}$$

where:

$U_w(t)$ — time-variant voltage across the anemometer wire,
$R_w(t)$ — time-variant resistance of the hot wire,
v — time-independent velocity of gas,
$T_w(t)$ — time-variant temperature of the hot wire,
T_g — temperature of the flowing medium taken to be time-invariant,
m — mass of the hot wire,
c — specific heat of the wire material.

Making use of the linear relationship between wire resistance and its temperature in (2)

$$R_w(t) = R_0\{1 + \gamma[T_w(t) - T_0]\} \tag{3}$$

and

$$R_g = R_0[1 + \gamma(T_g - T_0)] \tag{4}$$

we get

$$\frac{U_w^2}{R_w} = \frac{a + bv^n}{\gamma R_0}[R_w(t) - R_g] + \frac{mc}{\gamma R_0}\frac{dR_w(t)}{dt} \tag{5}$$

where:

R_0 — wire resistance at the temperature T_0 at which the linear temperature coefficient of wire resistance γ was determined,
R_g — resistance of the cold wire at the temperature of flowing medium T_g.

When an anemometer operates in a CT system, the overheat ratio $N(t)$ has to be preset before the measurements are commenced. The overheating ratio, time-variant in the present conditions, is defined as the ratio of wire resistance $R_w(t)$ to the resistance of cold wire R_c. Accordingly, we get:

$$N(t) = \frac{R_w(t)}{R_c} \tag{6}$$

Substituting (7) into (6), we obtain:

$$\frac{U_w^2}{N(t)R_c} = \frac{(a+bv^n)R_c}{\gamma R_0}\left[N(t) - \frac{R_g}{R_c}\right] + \frac{mcR_c}{\gamma R_0}\frac{dN(t)}{dt} \tag{7}$$

Using the following designations:

$$C_0 = \frac{(a+bv^n)R_c^2}{\gamma R_0} \tag{8}$$

and

$$C_1 = \frac{mcR_c^2}{\gamma R_0} \tag{9}$$

Equation (8) can be rewritten as:

$$U_w^2(t) = C_0 N(t)\left[N(t) - \frac{R_g}{R_c}\right] + C_1 N(t)\frac{dN(t)}{dt} \tag{10}$$

When the overheating ratio in a constant-temperature system is modulated in accordance with the formula:

$$N(t) = N_0 + N_1 \sin\omega t \qquad \text{and} \qquad \omega = 2\pi f \tag{11}$$

where:

N_0 — stands for the mean overheating ratio,
N_1 — modulation amplitude;
f — modulation frequency, we get the formula determining the voltage across the hot wire:

$$U_w^2(t) = C_0(N_0 + N_1\sin\omega t)\left[\left(N_0 - \frac{R_g}{R_c}\right) + N_1\sin\omega t\right] + \tag{12}$$

$$+C_1 N_1 \omega(N_0 + N_1 \sin\omega t)\cos\omega t$$

Ordering this formula, we get:

$$U_w^2(t) = C_0 N_0\left(N_0 - \frac{R_g}{R_c}\right) + C_0 N_0\left(N_0 - \frac{R_g}{R_c}\right)\sin\omega t + C_1 N_1 \omega\cos\omega t + \ldots = \tag{13}$$

$$= C_0 N_0\left(N_0 - \frac{R_g}{R_c}\right) + C_1 N_0 \sqrt{1+\omega^2\tau^2}\,\sin(\omega t + \psi)$$

where

$$\tau = \frac{C_1 N_1}{C_0\left(2N_0 - \frac{R_g}{R_c}\right)} \quad \text{and} \quad \psi = \mathrm{arc\,tg}\,\omega\tau \tag{14}$$

The unknown quantity $U_w(t)$ is sought in form of a series:

$$U_w(t) = \sum_{k=0}^{\infty} U_k \sin(k\omega t + \phi_k) \tag{15}$$

where k is the order of the harmonic component. In this case our considerations are limited to the first harmonic, given by the formula:

$$U_w(t) = U_0 + U_1 \sin(\omega t + \varphi) \tag{16}$$

In other terms:

$$U_w^2(t) \cong U_0^2 + 2U_0 U_1 \sin(\omega t + \varphi) \tag{17}$$

Thus we get the expression with the accuracy of the first order terms

$$U_0^2 = C_0 N_0\left(N_0 - \frac{R_g}{R_c}\right) \tag{18}$$

and

$$2U_0 U_1 = C_0 N_0 N_1 \sqrt{1+\omega^2\tau^2} \tag{19}$$

When the time constant τ is sufficiently small, the following approximation can be made:

$$U_1 = C_0 \frac{N_1 N_0}{2U_0} \tag{20}$$

3. Temperature compensation

Equations (18) and (19) contain two unknown parameters: C_0, R_g/R_c (it is assumed that the time constant τ is known). Solving this equations we obtain:

$$\frac{R_g}{R_c} = N_0 - \frac{N_1 U_0}{2U_1}\sqrt{1+\omega^2\tau^2} \tag{21}$$

and

$$C_0 = \frac{2U_0 U_1}{N_0 N_1 \sqrt{1+\omega^2\tau^2}} \tag{22}$$

R_g, R_c and C_0 being known, we can proceed to find the temperature of the medium at the measurement point and the flow velocity.

Utilising (21) and (8), we get:

$$\Delta T_g - T_g - T_0 = \frac{1}{\gamma}\left\{\frac{R_g}{R_c}\left[N_0 - \frac{N_1 U_0}{2N_1 U_0}\sqrt{1+\omega^2\tau^2} - 1\right]\right\} \tag{23}$$

where ΔT_g stands for temperature excess at the measurement point over the temperature T_0[1] and

$$v = \left[\frac{1}{b}\left(\frac{2U_0 U_1 \gamma R_0}{N_1 N_0 R_c^2 \sqrt{1+\omega^2\tau^2}} - a\right)\right]^{1/n} \tag{24}$$

When $R_c = R_g$ and the time constant τ are very small, the formulas (23) and (24) can be simplified yielding:

$$\Delta T_g = \frac{1}{\gamma}\left(N_0 - \frac{N_1 N_0}{2N_0 U_1} - 1\right) \tag{25}$$

and

$$v = \left[\frac{1}{b}\left(\frac{2U_0 U_1 \gamma}{N_1 N_0 R_0} - a\right)\right]^{1/n} \tag{26}$$

N_0, N_1, and $\omega = 2\pi f$ are parameters predetermined in the experiment; U_0, U_1 are obtained from measurement data by way of Fourier expansion. C_0, R_g/R_c are unknown parameters obtained from solving (22) and (23). The phase shift φ, obtained experimentally and is related to the time constant:

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[1] The Author is well aware of the fact that in the formula (23) we have τ, which is a function of velocity. It would seem, then, that (23) is a complex formula. However, the parameter τ is well known — Eq. (28).

$$tg\varphi = \omega\tau \tag{27}$$

hence:

$$\tau = \frac{tg\varphi}{\omega} \tag{28}$$

The remaining five quantities a, b, n, R_0 and R_c are obtained from calibration of the anemometric sensor, γ is obtained in measurements of the temperature coefficient of probe material resistance.

The detailed description of the experimental setup used for verification of the presented measurement method as well as discussion of results will be provided in the separate study.

REFERENCES

Ligęza P., 2001: Układy termoanemometryczne — struktura, modelowanie, przyrządy i systemy pomiarowe (Hot-wire anemometric systems — structure, modeling, instruments and measuring systems). Rozprawy i Monografie AGH nr 98, s. 135.

Ligęza P., 1993: Metoda pomiaru prędkości w nieizotermicznych przepływach pojedynczą sondą termoanemometryczną (Measurement of flow velocity with single hot-wire probe). Praca doktorska, Instytut Mechaniki Górotworu PAN, Kraków.

Ligęza P., 1994: Thermoanemometric method for measuring velocity and temperature in non-isothermal flows (Termoanemometryczna metoda pomiaru prędkości i temperatury w przepływach nieizotermicznych). Arch. Min. Sci. vol. 39, i. 3, p. 367–380.

Kiełbasa J., 1999: Measuring the velocity and temperature wake. Arch. Min. Sci, vol. 44, i. 1, p. 71–84.

Kiełbasa J., 1991: Sposób pomiaru prędkości przepływu płynu oraz anemometr skompensowany temperaturowo (Method of measurement fluid flow and temperature compensation anemometer). Patent RP nr 152363.

Ferreira R.P.C., Freire R.C.S., Deep G.S., Neto J.S.R., Oliveira A., 2000: Fluid temperature compensation in a hot wire anemometer using a single sensor. IEEE Instrumentation and Measurement Technology Conference, Baltimore, Maryland, USA, 1–4 May 2000.

King L.V., 1914: On the convection of heat from small cylinders in a stream of fluid. Phil. Trans. Roy. Soc. London A 214, p. 373–432.

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