Archives of Mining Sciences **47**, 1 (2002) 51–67

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CONCEPTS OF THE EVALUATION OF MULTI-PRODUCT SEPARATION PROCESSES OF MINERAL ENGINEERING

KONCEPCJE OCENY WIELOPRODUKTOWYCH PROCESÓW ROZDZIELCZYCH INŻYNIERII MINERALNEJ

The utilisation of mineral materials requires a detailed control and evaluation of both the products properties and the course and a result of processes occurring in the consecutive utilisation stages. These tasks are of special significance in case of processing of materials of complex properties, used in a compound way. The properties of mineral raw materials gave them a position of a classic object of complex processing. The first technological stage of utilisation of mineral raw materials is constituted by a set of operations of mineral engineering (processing of mined minerals and other mineral raw materials). This stage comprises the tasks discussed for the first time and those occurring in the most developed way. The separation operations constitute the basic part of unitary technological operations of mineral engineering. There are numerous principles and detailed methods of description and evaluation of such operations. The applied methods concern usually only single-useful component raw materials (having one property) and single technological operations. There are no methods considering the specific conditions of evaluation of complex processes, and especially the processing of materials of the complex system of useful components (properties). The paper presents the concept of joint evaluation of effectiveness of these separation processes in which more than two products are obtained from a multi-component feed. For sake of an example, the most popular technological criteria of evaluation were assumed. Also, the method of quantitative evaluation of separation results, proposed earlier by the author, in separation processes according to the feed segregation rate, purposefully obtained during such processes. This method has been adapted to the results of processing on heterogeneous raw materials. It has been also indicated that that the evaluations of the process according to the analysis of the segregation rate corresponds the conditions, which are necessary for evaluation of the multi-product process. The system provided for mineral engineering — of the most complex conditions resulting from the primary properties of the processed materials — can be adapted to the description and evaluation of any similar processing processes in various branches of industry.

Key words: mineral engineering, separation processes, multi-product processes, evaluation of effectiveness, segregation

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Wykorzystywanie materiałów pochodzenia mineralnego wymaga szczegółowej kontroli i oceny zarówno właściwości produktów, jak i przebiegu i wyników procesów występujących w kolejnych etapach utylizacji. Szczególne znaczenie mają te zadania w przypadku przetwarzania materiałów o złożonych właściwościach, wykorzystywanych kompleksowo. Właściwości surowców pochodzenia mineralnego dały im pozycję klasycznego obiektu przetwarzania kompleksowego. Pierwszym etapem technologicznym użytkowania surowców mineralnych jest zespół operacji inżynierii mineralnej (przeróbki kopalin i innych surowców mineralnych), w nim więc omawiane zadania występują po raz pierwszy, a także najbardziej rozwinięty. Podstawowa część jednostkowych operacji technologicznych inżynierii mineralnej to operacje rozdzielcze. Znane są liczne zasady i metody szczegółowe opisu i oceny takich operacji. Stosowane metody dotyczą zwykle surowców o jednym składniku (jednej właściwości) użytecznym oraz pojedynczych operacji technologicznych. Brak metod uwzględniających specyficzne warunki oceny procesów złożonych, a zwłaszcza przetwarzania materiałów o złożonym układzie składników (właściwości) użytecznych. Treścią opracowania jest przedstawienie koncepcji dokonywania łącznej oceny skuteczności tych procesów rozdzielczych, w których z nadawy wieloskładnikowej otrzymuje się więcej niż dwa produkty. Do przedstawienia przykładu przyjęto najbardziej rozpowszechnione technologiczne kryteria oceny. Powołano również zaproponowany wcześniej przez autora sposób oceny ilościowej wyników rozdziału w procesach rozdzielczych na podstawie stopnia segregacji nadawy, osiąganej celowo w toku takich procesów. Sposób ten został dostosowany do oceny wyników przerabiania surowców niejednorodnych. Wskazano, że ocena procesu na podstawie analizy stopnia segregacji odpowiada warunkom niezbędnym do oceny procesu wieloproduktowego. System przewidziany dla inżynierii mineralnej o najbardziej złożonych warunkach, wynikających z pierwotnych właściwości przetwarzanych surowców, może być adaptowany do opisu i oceny dowolnych, podobnych procesów przetwórczych, występujących w różnych gałęziach gospodarki.

Słowa kluczowe: inżynieria mineralna, procesy rozdzielcze, procesy wieloproduktowe, ocena skuteczności, segregacja

1. Introduction

The economic use of mineral materials, such as minerals and products of their processing, together with secondary and waste materials originating in the course of processing and utilization of raw materials, requires a detailed control and evaluation, both of the properties of products and of the course and results of processes occurring in consecutive stages of utilization. These tasks are special importance in case of processing of materials of compound properties which are utilized in a complex way. It is the mineral raw materials which have such compound properties and the limitations as well as unreproducibility of their primary sources, together with the economic necessity of utilization of such raw materials, contributed to their position of a classical object of complex processing, more and more commonly perceived in the world. Most generally, it consists in the application of a possibly large number of components and properties, most often clearly differentiated (Sztaba 1970b). The first technological stage of utilization of mineral raw materials is represented by the set of mineral engineering operations, i.e. the processing of minerals and mineral raw materials (Sztaba 2000d); the discussed tasks occur chronologically for the first time and are well developed. The subject of processing is constituted by the materials of the original primary composition, as a rule very heterogeneous and complex. The present practice includes numerous principles and detailed methods of description and evaluation of the mentioned operations (Barskij, Plaksin 1967; Barskij, Rubinštejn 1970; Sztaba 1970–2001, 2000a, 2000c; and others). Apart from individual methods of evaluation, generally applied in mineral engineering, there is no general and coherent system. The applied methods usually concern the raw materials of a single useful component (single property) and single technological operations. There are no methods which take into account the specific conditions of evaluation of complex processes and, first of all, the processing materials of the complex system of useful components (or properties). This work discusses the assumptions of the coherent system of evaluation of the course and results of processing of materials of complex properties, carried out in order to separate the useful components, contained in them. The system is provided for mineral engineering of the most complex conditions, resulting from primary conditions of processed raw materials, can be adapted to the description and evaluation of any similar processing processes, occurring in different branches of economy.

2. Systematic of evaluation methods of results of separation operations

Separation operations constitute the most important and numerous group of the technological operations of mineral engineering. The results of separation of the feed into required products are subjected to various evaluations out of which this paper deals only with these concerning the efficiency (effectiveness) of separation. It is understood as a numerically expressed (most often in percent values) relation of the obtained result to the expected (or required) one. In the literature and industrial or laboratory practice several groups of effectiveness efficiency are found (Barskij, Plaksin 1967; Barskij, Rubinštejn 1970; Sztaba 1970a, 1970–2001, 2000a). These are the evaluations (main groups) such as:

- technological,
- statistical,
- thermodynamic,
- power,
- economic,

which are called in such a way depending on the type of the assumed basic criteria. In the industrial conditions, for the current and result estimation of the condition of the technological process, the technological and economic evaluation (respectively) are most important while in the research (performed from the laboratory to industrial scales) the technological and statistic ones play the leading role. A numerous group of technological evaluations can be divided, due to the evaluation priority, into the qualitative, quantitative and general ones (considering both the qualitative and quantitative aspect).

In order to illustrate the idea of creating the discussed evaluations of process effectiveness the author presents an example of applying the most generally known and used technological methods for evaluating the results of a certain separation operation of the material containing one separated ("useful") component — ***a***. Entering the operation, the feed containing the separated product a_0 is introduced with efficiency Q_0 (kg/h) (practically, usually the contents a are given in percent, here for the sake of convenience fractions were used, similarly as for further estimations and descriptions) and two products are obtained (1. and 2.) of the yields and contents of the $\boldsymbol{a}$, respectively Q_1 and Q_2 and a_1 and a_2. Moreover, it is assumed that the distinguished (separated) component will be concentrated in product 1. (concentrate) and others ("useless") components in product 2. (waste). It should be noticed that always $1 \leq a_{max} \leq a_{(0,1,2)} \leq$ $\leq a_{min} \leq 0$ and $a_1 > a_0 > a_2$. The basic quantitative description of products is given by their yields: $\gamma_1 = Q_1/Q_0$, $\gamma_2 = Q_2/Q_0$ (hence $\gamma_1 + \gamma_2 = 1$), not counted as evaluations of effectiveness. Practically, in majority of cases the yields are determined indirectly, e.g. by means of the formula:

$$\gamma_1 = \frac{a_0 - a_2}{a_1 - a_2} \tag{1}$$

The following technological qualitative criteria can be considered most important (Sztaba 1970–2001) (E — conventional numbering of respective formulas):

$$E'_5 = a_1 - a_2 \quad \text{— when selectivity of separation is evaluated} \tag{2}$$

$$E'_9 = \frac{a_1 - a_0}{a_{max} - a_0} \quad \text{— when the quality of product 1. is evaluated} \tag{3}$$

$$E'_{10} = \frac{a_0 - a_2}{a_0 - a_{min}} \quad \text{— when the quality of product 2. is evaluated} \tag{4}$$

Among the quantitative criteria the recovery is most important ($\varepsilon \equiv E''$ — relation of the amount of the component separated in its concentrate, here in product 1., to the amount introduced together with the feed — always $0 \leq \varepsilon \leq 1$). This is practically the only commonly applied index of the quantity group.

$$\varepsilon = E''_1 = \frac{a_1 \cdot (a_0 - a_2)}{a_0 \cdot (a_1 - a_2)} = \gamma_1 \cdot \frac{a_1}{a_0} \tag{5}$$

Similarly, in the group of general technological indices of the effectiveness evaluation (E), only the following formula is of practical significance (Hancock 1918 in: Barskij, Plaksin 1967; Barskij, Rubinštejn 1970):

$$E_1 = \frac{(a_0 - a_2)\cdot(a_1 - a_0)}{a_0 \cdot (a_1 - a_2)\cdot(a_{max} - a_0)} \tag{6}$$

When $a_{max} = 1$ (100%) and all values of a are in percent, formula (6) obtains a better known form:

$$E_1 = 10\,000 \cdot \frac{(a_0 - a_2)\cdot(a_1 - a_0)}{a_0 \cdot (a_1 - a_2)\cdot(100 - a_0)}\ \% \tag{6.1}$$

An example of calculation results (Sztaba 1970–2001) for a certain operation of the flow classification:

$$a_0 = 52.4\%,\ a_1 = 96.3\%,\ a_2 = 3.7\%,\ a_{max} = 100\%,\ a_{min} = 0;$$

$$E'_5 = 73.1\%,\ E'_9 = 92.2\%,\ E'_{10} = 55.7\%,\ E''_1 = 73.4\%,\ E_1 = 70.3\%.$$

In the majority of enrichment operations the analyses of the contents of the feed and products are performed to achieve the content of the pure component (e.g. metal). In these cases the values a_{max} and a_{min} are assumed to be equal to, respectively, the content of the component marked in its mineral (average in minerals) separated into the concentrate and the content of metal dispersed in waste minerals (so called background values).

3. The evaluation of effectiveness of multi-product operations

Only very few evaluation methods among the ones known at present correspond to the implementation of a postulate of complex utilization of raw materials and treating them as multi-product.

The evaluation of effectiveness of multi-product operations is not, of course, limited to the case of applying estimations belonging to only one group mentioned before and also presented in the further part of this paper, the estimations, which are based on the segregation rate. Therefore it is necessary to determine a general, principle (or principles) of formulating such estimations.

Because of this fact, this problem in an introductory approach is treated somehow autonomously in this work, without the direct relation with concrete evaluation methods.

The presentation of a general concept of construction of such evaluations, however, requires for its illustration an exemplary selection of a certain initial method. For this sake the author chose the technological evaluation (mentioned before) from the quantitative subgroup, limited practically to determining the recovery. There are no formal obstacles to perform the same considerations for every other group of basic evaluations.

The discussion of the principles of construction of evaluation criteria for the processes of complex utilization of the raw material requires to assume at first certain limitations:

1) considering exclusively the separation operations (and processes — operation systems),

2) assuming the process of multi-product processing of the multi-component raw materials while it must be possible to determine the properties of its feed and all products, in particular their yields and contents of chosen components,

3) presenting not so much final solutions but the methods of searching them and, especially, mastering, therefore there are, among others, two variants of presented propositions, taking into account various degrees of accessibility of numerical data about the feed and process products.

The separation process is being considered the feed of which (of the yield from the assumption $\gamma_0 = 1$) contains the components of symbols "i" (in the most general case these symbols are consecutive natural numbers: $i = 1, 2, \ldots, n$) the contents of which are a_i; $\sum_i a_i = 1$. As a result of the performed process the feed is subjected to separation into the products of symbols "j"; $j = 1, 2, \ldots, m$.

For each j^{th} product it is possible to indicate univocally one or more i^{th} components which are assumed to be concentrated in this product; they are conventionally called "proper" components, i_j. In the relation to the same product the remaining components are "improper" (polluted) — i_{-j}. The set $\{i_j\}$ of "proper" components determines the symbol of the product $j = \{i_j\}$. Each j^{th} product characterizes its yield in relation to the feed — γ_j (1) and the contents of the i^{th} components — a_{ij} — $\sum_i a_{ij} = 1$.

The recovery of the i^{th} component in the j^{th} product is assumed to be the basic elementary notion

$$\varepsilon_{ij} = \gamma_j \cdot \frac{a_{ij}}{a_i} \tag{7}$$

It should be observed that the recovery could be formally interpreted also as a product of the yield of the product and a rate of enrichment of the i^{th} component (s_{ij}) in it:

$$s_{ij} = \frac{a_{ij}}{a_i} \tag{8}$$

For the j^{th} product the author introduces a notion of a generalized effectiveness of its separation as a difference of the obtained recovery of the proper component (components) and the losses[1] of components appropriate for other products. The multi-component ability of the considered materials makes it necessary to accept the way of considering the mutual relations between the components. Here there are several possibilities out of which two are presented as starting points for the performed considerations.

1. If only there are data about the contents of components, without any possibility of grading in a more rational way, than it is better to prefer the components most easily transfered into a given product when establishing the weights. The measure of easy separation, i.e. the weight, can be the already mentioned enrichment rate s_{ij} (8).

2. When there is some economic information, especially the unitary values of the i^{th} components (W_i — value of a unit of measure of the i^{th} component) and the cost of obtaining a unit of measure of respective components of the j^{th} product (K_{ij}), the unitary "relevant value" of the i^{th} component, contained in the j^{th} product — $w_{ij} = W_i/K_{ij}$, is to be assumed the weight. It should be observed K_{ij} should be a joint gaining cost,

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[1] The losses are in the fact the recovery of components in the products inappropriate for them, especially in waste (Polish Standard 1999).

starting from the feed of the processing process till the commercial product, containing the i^{th} component.

Assuming the weight of the first variant, we obtain a generalized index of separation of the j^{th} product in the form:

$$\varepsilon_j = \frac{\sum\limits_{i=i_j} s_{ij} \cdot \varepsilon_{ij}}{\sum\limits_{i=i_j} s_{ij}} \cdot \frac{\sum\limits_{i=i_{-j}} s_{ij} \cdot \varepsilon_{ij}}{\sum\limits_{i=i_{-j}} s_{ij}} \tag{9}$$

and at γ_j = const:

$$\varepsilon_j = \gamma_j \cdot \left[\frac{\sum\limits_{i=i_j} s_{ij}^2}{\sum\limits_{i=i_j} s_{ij}} - \frac{\sum\limits_{i=i_{-j}} s_{ij}^2}{\sum\limits_{i=i_{-j}} s_{ij}} \right] \tag{9.1}$$

If the weight is assumed according to the second variant, then analogically

$$\varepsilon_j = \frac{\sum\limits_{i=i_j} w_{ij} \cdot \varepsilon_{ij}}{\sum\limits_{i=i_j} w_{ij}} - \frac{\sum\limits_{i=i_{-j}} w_{ij} \cdot \varepsilon_{ij}}{\sum\limits_{i=i_{-j}} w_{ij}} \tag{10}$$

or

$$\varepsilon_j = \gamma_j \cdot \left[\frac{\sum\limits_{i=i_j} w_{ij} \cdot s_{ij}}{\sum\limits_{i=i_j} w_{ij}} - \frac{\sum\limits_{i=i_{-j}} w_{ij} \cdot s_{ij}}{\sum\limits_{i=i_{-j}} w_{ij}} \right] \tag{10.1}$$

The values ε_j should be calculated for all the products obtained in a given process. They constitute the base for calculating a generalised index of separation effectiveness for the entire process. It should consider the values ε_j for respective products with appropriate weights. The analysis of possible variants of assuming such weights leads to the conclusion that it would be most rational to assume them in the same way as in the calculation of the ε_j values but in this case the weights would be constituted by the entire values of numerators of the first terms (grouping the influences of "appropriate" components) of expressions (9) or (10). Such an assumption would be a certain form of preference of positive effects of separation of respective products whereas the secondary effects of polluting these products with "inappropriate" components are already taken into consideration in values ε_j. The formulas of generalised effectiveness of separation depending on the variant of weight determination when calculating ε_j are of the form:

variant 1:

$$\varepsilon = \frac{\sum_j \gamma_j \cdot \left(\sum_{i=i_j} s_{ij}^2 \right) \cdot \varepsilon_j}{\sum_j \gamma_j \cdot \left(\sum_{i=i_j} s_{ij}^2 \right)} \tag{11}$$

variant 2:

$$\varepsilon = \frac{\sum_j \gamma_j \cdot \left(\sum_{i=i_j} w_{ij} \cdot s_{ij} \right) \cdot \varepsilon_j}{\sum_j \gamma_j \cdot \left(\sum_{i=i_j} w_{ij} \cdot s_{ij} \right)} \tag{12}$$

(in variant 1. there are, in fact, weights of values w_{ij} = const = 1).

The discussed evaluations of technological effectiveness of processing processes can also be used to analyse many other separation processes, they can also be the base of selecting the criteria of their optimization.

The idea of the presented propositions is an attempt of creating the basis of evaluation of multi-product processes of processing of multi-component raw materials.

The example concerns the enrichment of Zn-Pb ore from which in the past also marcasite was separated. The components to be marked are: Zn, Pb and Fe. Their main carriers are, respectively: sphalerite (ZnS), galenite (PbS) and marcasite (FeS_2). Therefore: $a_{max,Zn}$ = 67.2%, $a_{max,Pb}$ = 86.7%, $a_{max,Fe}$ = 46.7%. Another useful component is also sulfur for which $a_{max,S}$ = 53.3% (in marcasite). The remaining minerals are considered to be waste rock of negligible metals content (a_{min} = 0). The introductory data, according to (Stępiński 1964) and the results of calculations — formulas (9.1) and 11 — are listed in Table 1.

TABLE 1

Results of calculations of generalized separation effectiveness

TABLICA 1

Wyniki obliczeń uogólnionej skuteczności rozdziału

j	Product	γ_j [%]	a_{Zn} [%]	a_{Pb} [%]	a_{Fe} [%]	a_{waste} [%]	ε_j [%]	ε [%]
0	Feed	100.00	16.15	3.90	9.70	50.70	—	79.70
1	Zn concentrate	25.00	55.00	1.00	3.00	10.58	78.28	
2	Pb concentrate	5.00	3.00	66.00	5.00	8.70	82.44	
3	Fe concentrate	20.00	5.00	1.00	36.00	14.32	68.50	
4	Waste	50.00	2.50	0.30	3.00	89.51	76.93	

It can be observed, among others, that Pb concentrate is separated most effectively, next Zn concentrate, then waste (as the "concentrate" of waste minerals) and marcasite concentrate.

As it was mentioned, the same principle can be used for constructing the evaluations of effectiveness of multi-product operations (processes), substituting ε and s_{ij} respectively, which are the technological estimations, with the estimations of another selected group analogical as far as the function is concerned.

4. Segregation

The essence of every separation process consists in separation of the feed grains according to the differences of their properties and features, mainly physical. In natural conditions, the phenomenon of segregation reveals an identical mechanism.

Segregation consists in a spontaneous, occurring in certain conditions (Sztaba 1993), concentration of grains of heterogeneous grained material of differentiated values of certain physical features — mass and conditioning it: size, density, shape, rarely others — in the separated parts of the area occupied by this material (tank, heap, transport stream, etc.).

Segregation is an ultimately undesired phenomenon in case of the material to be processed and to be sold since it decreases the homogeneity of its properties. It is also very much undesired in many technological operations of mineral processing. This concerns mainly the averaging operations and also the controlling which comprise the separation of a representative part of the material in the form of a sample to be investigated. The concurrence of segregation is one of main factors hindering the averaging of the material. It can be said that segregation is a process opposite to averaging.

The method of evaluation of the segregation rate, proposed by the author (Sztaba 1993), is based on the analysis of distribution of any measurable material features in the area occupied by this material. As a result of broad considerations on the choice of the way of evaluation of the segregation rate (S) and, agreement with the previous remarks about its relation with averaging, the averaging rate (H), it was assumed that the simplest possibility enabling the estimation of these two values in relation to the chosen feature (a) of the material is:

$$S = \frac{1}{H} = \frac{\sigma}{\sigma'} \tag{13}$$

where:

- σ — standard deviation of feature a in the tested area occupied by the given material,
- σ' — boundary value of deviation σ which would occur if the material was ideally segregated into differentiable components:

$$\sigma'^2 = \frac{(a_{\max} - a_m)(a_m - a_{\min})^2 + (a_m - a_{\min})(a_{\max} - a_m)^2}{a_{\max} - a_{\min}} \qquad (14)$$

where:

a_{nnn} — value of feature a: minimum — $a_{\min}$, maximum — $a_{\max}$ and mean in the entire material — a_m (a_{mean}).

There is always a relation $\sigma \leq \sigma'$. Therefore always $S \leq 1$ while $H \geq 1$ and with the increase of the averaging rate $H \to \infty$, and $S \to 0$. The first two out of the values a_{nnn} ($a_{\min}$ and $a_{\max}$) are evaluated according to the knowledge of material characteristics and the investigated feature. The average content (a_m) is calculated from the analysis of the material composition (granulometric, chemical, etc., depending on the choice of the feature a). In case of the metal ore the minimum value is the content of metal in pure grains of waste rock (usually the so called background value), the maximum one in the pure grains of the ore mineral, while the average one is obtained from the chemical analysis of the representative sample of the entire material. In the order to obtain the value σ, depending also on the configuration of the system of grains of different values of feature a in the tested area, an appropriate sampling of the material in this area should be carried out (Sztaba 1993, 1999–2000).

5. The separation process as controlled segregation

It should be additionally made more precise that the discussed separation operations (processes) are technological procedures the aim of which is to achieve the separation of the feed (initial material) of the technological processes into at least two products differing from each other by the selected feature (e.g. content of components, physical properties, etc.). They comprise the most important main operations of mineral engineering (including mineral processing). These are most often: enrichment — when they aim at grouping the selected component (components) of the feed in its proper concentrate or classification — when an analogical goal concerns grain classes. Dewatering is also a separation operation.

It is easy to observe that all separating technological operations of processing give in fact the results which are qualitatively convergent with the results of segregation, occurring, however, not in the conditions of a spontaneous uncontrolled course but in strictly determined conditions, forcing obtaining the desired technological effects. Therefore the results of separation operations can be treated in the same way as the results of spontaneous segregation and treated with the methods used for the evaluation of segregation rate for their quantitative evaluation. In this case this rate determines the obtained discrepancy between the characteristics of products of the separation operation (state of material after segregation) and its feed (primary state of material mixed "ideally"). Here the author presents a developed method of quantitative evaluation of results of processing heterogeneous raw materials in separation operations. Its idea consists in treating the products obtained from separation processes as subareas (in this case incompatible) of the entire area i.e. "occupied" by the material of the feed. In the products the results of forced segregation were revealed and they are evaluated quantitatively by the method presented before. The properties of the method of segregation evaluation make it possible, for separation operations, to estimate jointly the effectiveness of separation according to an arbitrary number of mutually additive properties, for example according to the contents of a larger number of useful components. It is also possible to compare the evaluations obtained for the operations different from one another.

The range of problems of the title of the paper was discussed in the works quoted before, especially in Refs (Sztaba 1993, 1999–2000) in the fullest form.

The data about the distribution of feature ***a*** in the area comprising the process products, obtained in any way, must make it possible to calculate the parameters of this distribution. For instance, in case of assuming a regular sampling grid (x,y), the values of the chosen feature: $a\,(x,y)$ in the samples taken in all the points of the grid (in number N — usually large) are used for calculating, in a known way, its average value (a_m) and standard deviation (σ):

$$a_m = \frac{\sum_{j=1}^{N} a(x,y)_j}{N}, \qquad \sigma = \sqrt{\frac{\sum_{j=1}^{N} (a(x,y)_j - a_m)^2}{N}} \tag{15}$$

where:

$j = 1, 2, 3, \dots, N$ — successive numeration of sampling points of (x,y) coordinates.

Using the standard deviation the discussed purposes requires usually the assumption of normality of distribution of the tested phenomena. It is practically always possible to be accepted in case of investigating the random mass phenomena, which are constituted by any objects (materials) and operations of processing of mineral raw materials.

The regular arrangement of sampling points enables us also the testing of the phenomenon of segregation as such to be performed. It consists in finding whether in the given area there is a tendency of regular changes of the local material properties, which is the principal attribute of segregation (Sztaba 1993). Finding the occurrence of segregation and its quantitative estimation are based on testing the gradient of distribution of selected features (components) of the estimated material in given conditions in the area under investigation (Sztaba 1993). These problems are not investigated in this paper.

Any number of features, corresponding to certain fairly general conditions, can be evaluated. In case of a few groups of features evaluated jointly the use of hierarchising rates (weights w) contributes to obtaining the relativity of taking such groups into consideration. The modification of the method enables also the relative quality of obtained products to be evaluated which allows us to treat this quality as a pat of the task of effectiveness evaluation of segregation in the given process. These problems are not developed in this paper.

Having the values σ and the σ' calculated for mutually additive features of the given material (for instance, the content of mutually excluding components): $a_i - i = 1, 2, 3, \ldots, n$, it is possible to evaluate the group value of the $S - S$ indicator for this group of features. Attributing, for example, the average contents of the i^{th} components in the material to the respective features of the investigated weight group — w_i, it can be calculated:

$$s^2 = \frac{\sum_{i=1}^{n} w_i \sigma_i^2}{\sum_{i=1}^{n} w_i}, \qquad s'^2 = \frac{\sum_{i=1}^{n} w_i \sigma'^2_i}{\sum_{i=1}^{n} w_i}, \qquad \mathfrak{S} = \frac{s}{s'} = \ldots = \sqrt{\frac{\sum_{i=1}^{n} w_i \sigma_i^2}{\sum_{i=1}^{n} w_i \sigma'^2_i}} \tag{16}$$

where:

s, s' — group standard deviation, real and boundary.

The group value of the indicator of segregation rate — $\mathfrak{S}$ — is proposed to be used for the evaluation of effectiveness of separation processes.

6. Segregation rate as the evaluation of effectiveness of multi-product operations

The application of the segregation rate for evaluating of separation processes requires as assumption that the process products are separated in mutually separates (as it has already been mentioned) parts of the area occupied by the feed material. Any possible segregation, occurring inside each part is not significant for the estimation of the entire process which enables them to be called internally homogeneous and described exclusively by proper quantity quantifiers (average values of selected features and masses, yields, etc.).

Under such an assumption one should choose the evaluated features of the separated material (a_i) and determine their average values for particular k^{th} products of the process (a_{ki}) $- k = 1, 2, 3, \ldots, m$. Either particular components of the material and product processes or their groups can be assumed to be the features (components) and products. The rational evaluation of the process can be performed only when analyzing such features which are really separated. In case of metal ores they will not be the metals as such but ore minerals containing them in certain amounts. The calculations are carried out applying formulas (15) and (14) respectively, for each component separately whereas the values in them are interpreted as:

$a(x,y) = a_{ki}$ — average contents of the i^{th} component in the k^{th} product,
$a_m = a_i$ — average content of the i^{th} component in the process feed,
$\sigma, \sigma' = \sigma_i, \sigma'_i$ — standard deviation, real and boundary, of separation of the i^{th} component,
$w_i = w_k$ — yield of the k^{th} product.

Next, formulas (16) are applied for the entire material.

It should be observed that the values of indicator $\mathbb{S}$, calculated in the described way, allow us to evaluate the approximation of the obtained separation rate to ideal separation. They do not take into consideration technological and technical possibilities, consumers' needs and economic legitimacy of aiming at such an ideal state. Therefore the knowledge of their values can be less useful in industrial conditions but they have undisputable advantages of comparative evaluations, bases of choosing the most profitable process conditions, estimation of process progress, comparison of enrichment ability of different types of the same mineral, etc.

In practice, nobody expects obtaining an ideal separation but leading the content of components (a_i) to certain values, at least a_{konc} in the concentrate and utmost a_{odp} in the waste (in separation operations different than enrichment this concerns, respectively, the permissible — or desired — extreme values of other values which are separation features[2]). Substituting in formula (14) the values: $a_{\max}$ by a_{konc} and $a_{\min}$ by a_{odp} and performing calculations according to the described procedure, we obtain the estimation ($\mathbb{S}$) of the approximation of the obtained separation result to the expected result and not to the ideal one.

It should be noticed that the application of the segregation rate to the estimation of process results causes that the use of the notions taken from statistics becomes purely conventional and the method assumes a formal character. Therefore we are not entitled to formulate statistical conclusions on the basis of the results of the application of the above method and, especially, to formulate generalizations, aiming at constructing model descriptions of the investigated operations. In this case general remarks and restrictions are obligatory which concern the relations of phenomenological models (which comprise also such descriptions) with real basic conditionings of course respective operations.

The described application of methods of evaluation of the segregation rate for the estimation of process results indicates a possibility of adapting the coefficients of segregation estimation to the description of technological separation processes. It proposes instantly an evaluation method suitable for separation processes of any materials, neglecting a procedure of passing from "one-component" materials to "multi--component" ones. This procedure therefore concerns also the latter materials and the calculations for "one-component" materials are carried out assuming n (max i) = 1 which simplifies greatly all formulas, making redundant all summing operations by i.

This procedure constitutes also a repeatable element of the system of the multi-stage (multi-operational) separation process in which it should be adapted by sequence, passing through consecutive separation operations, as they appear in the process. The effects of performing the intersecting (in particular places of the process) non-separating operations are taken into account by means of introducing appropriate changes in feed characteristics of consecutive separation processes. If there is no non-separating operation between two operations, the characteristics of input (feed) of the second one,

[2] Properties of material grains according to which separation is performed.

respectively, is the same as the characteristics of output of the preceding operation, constituting this feed.

The group segregation rate ($\mathbb{S}$), introduced in Chapter 5 as a criterion of evaluation of the multi-product operation, fulfills the requirements of the discussed criteria. The intermediate values s_i, obtained in the course of calculations for respective i^{th} components, are the evaluating criteria for the effectiveness of separation of concentrates of these components. Therefore the presented method does not require any additional adapting to the needs of evaluation of multi-product processes. It is enough to define the components separated and considered in calculations according to the characteristics of separated products, i.e. the concentrates of respective components. It should be reminded that waste is also a separated component, treated as a "concentrate" of waste rock.

Taking into account the remarks on the relevance of the relative understanding of the effectiveness evaluation in reference to the expected and not ideal process result, a calculation example (10) can be presented for an exemplary calculation of the technological effectiveness of the multi-product process. The example is based on the same data, which were used in Chapter 3.

The initially enriched Zn-Pb ore contained (Stępiński 1964) minerals to be subject to flotation:

- 26.73% ZnS, which corresponds to 17.96% Zn,
- 4.99% PbS, which corresponds to 4.33% Pb,
- 29.51% FeS_2, which corresponds to 13.78% Fe,
- 38.77% of waste minerals ("rock").

The following products were obtained after the multi-product flotation[3] enrichment (Sztaba 2000b):

- Zn concentrate of 25% yield and composition: 88.54% ZnS, 1.18% PbS, 6.59% FeS_2 and 3.69% "rock"[4],
- Pb concentrate of 5% yield and composition: 4.55% ZnS, 82.58% PbS, 10.80% FeS_2 and 2.12% of "rock",
- Fe[5] concentrate of 20% yield and composition: 7.31% ZnS, 1.13% PbS, 84.19% FeS_2 and 7.37% of "rock",
- 50% of waste ("rock" concentrate) of composition: 5.82% ZnS, 0.68% PbS, 20.96% FeS_2 and 72.54% of "rock".

The coefficients of separation effectiveness ($\mathbb{S}$) were calculated according to the method of Chapter 3, assuming two variants of the reference level:

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[3] System of unitary operations of the process is not taken into account.

[4] Completing to 100 % of the sum of content of the remaining components, separated into separate concentrates.

[5] More precisely, the industrial concentrate of marcasite is usually the source of sulphur and not iron, for the calculation the author assumed the iron content to avoid the necessity of taking into account the sulphur contained in two other concentrates (ZnS and PbS), sulphur is treated as a part (component) of "rock", it can be obviously separated as one more "proper" component, for instance for all sulphide metal concentrates and "improper" for waste, or treating in this way only the sulphur contained in marcasite, etc.

- variant 1: "ideal" separation — separating the entire amount of components into appropriate products, corresponding to the contents of metals in these products at a 100% content of the quoted minerals, respectively 67.2% Zn, 86.7% Pb, 46.7% Fe in the concentrates and 0% of metals (100% of "rock") in waste,
- variant 2: permitting lower contents of metals[6] in the concentrates, respectively 65% Zn, 84% Pb, 45.2% Fe and in waste 1% Zn, 0.1% Pb and 4% Fe.

The results of calculations are presented in Table 2.

TABLE 2

Effectiveness of separation in relation to differentiated reference level, calculated according the segregation rate

TABLICA 2

Skuteczność rozdziału względem zróżnicowanych poziomów odniesienia, obliczona na podstawie stopnia segregacji

i	Component	w_i [%]	σ_i	Variant 1		Variant 2	
				σ'_{i1}	s_{i1}	σ'_{i2}	s_{i2}
1	ZnS	26.73	35.69	44.26	0.8064	49.03	0.7279
2	PbS	4.99	17.18	21.77	0.7892	21.16	0.8121
3	FeS2	29.51	27.99	45.61	0.6137	35.57	0.7869
4	"rock"	38.77	33.80	48.72	0.6938	43.26	0.7814
				$\mathfrak{S}_1 =$	0.7050	$\mathfrak{S}_2 =$	0.7703

As it could be expected, the higher value of the effectiveness indicator ($\mathfrak{S}_2$) corresponds to referring the real results to the level determined in a more realistic way (variant 2) than in the case of referring to the results of the unobtainable "ideal" separation ($\mathfrak{S}_1$). The same concerns in principle the effectiveness indicators calculated individually for the respective i^{th} products (s_{i2} and s_{i1}, respectively).

7. Final remarks

The basis of the presented evaluation of the processes is constituted on one hand on the well-known and applied methods, especially the technological ones, and on the other, on the methods based on the application of the segregation rate, formally belonging to statistical estimations. All of them use directly or indirectly the data about

[6] The values freely assumed.

the technological properties of the process and its products. Due to this, all of them can be applied in production industrial conditions. It is not always plausible in case of the basic group of statistical methods, based on the idea of distribution numbers, i.e. probabilities of passing of the feed grains of certain properties to respective products of the process, most often used in research tasks. To calculate the values of evaluations according to the segregation rate it is necessary to have the same values of technological characteristics of the feed and process products which in case of evaluations of the technological groups, supplemented only by basic data concerning the feed, i.e. a_{min} and a_{max}, which are invariable for a given deposit of the certain mineral composition (value a_m is always one of the currently controlled technological characteristics of the process). Therefore there are no obstacles to introduce them into industrial practice.

It should be observed, however, that the existing system of technological evaluations is the base of production computations, technological and economic, of the estimation of the rate (effectiveness) of the use of input material (especially the minerals) in the economic aspect, including the current quantitative estimation of production results, presented in reports (shift, day, etc.), determining the general economic evaluation of the process. This function is not, at the present stage of development of the method based on the segregation evaluation, accessible according to this method which results in the described possibilities of the complex evaluation of the process, with the indication of the most required directions of its modification, i.e. perfecting.

Therefore it seems that the current evaluation of the process in industrial conditions should consists of two parallel procedures:

- calculating technological evaluations for tasks of reporting,
- calculating the estimations applying the segregation rate, indicating the directions of the current modification of the process conditions.

These two procedures require the use of the same current values of technological indicators (content of components, product yields, value of processing, etc.) together with supplementing only by the mentioned values, necessary to calculate the segregation rate (a_{min} and a_{max}).

The present work has been prepared as a result of implementation of the research project of Department of Mineral Processing, Environment Protection and Waste Utilization of Mining Faculty at the University of Mining and Metallurgy in Cracow, contract no 10.10.100.458 (years 1999–2000) (Sztaba 1999–2000) and the first stage (2000–2001) of the research project no 9 T12A 032 19 (2000–2003), financed by the Committee of Scientific Research.

The Polish version of the present work was presented at the 6th International Conference of Non-Ferrous Metal Ores Processing — 6th ICNOP' 2001 in Szklarska Poręba, May 16–18th, 2001, and published in the conference materials (Institute of Non-Ferrous Metals in Gliwice).

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REVIEW BY: PROF. DR HAB. INŻ. JAKUB BODZIONY, KRAKÓW

Received: 05 November 2001