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MEASUREMENT UNCERTAINTY OF STATIC DIFFERENTIAL PRESSURE MEASUREMENT IN MINE VENTILATION NETWORKS USING THE BAROMETRIC METHOD

NIEPEWNOŚĆ POMIARU RÓŻNICY CIŚNIEŃ STATYCZNYCH METODĄ BAROMETRYCZNĄ W KOPALNIANEJ SIECI WENTYLACYJNEJ

Measurements of the static differential pressure in mine ventilation networks by means of barometric methods are often performed to determine pressure losses in ventilation side-branches and to calculate their aerodynamic resistance coefficient. As experience shows, the results of these measurements are distorted by a significant level of measurement error (Biernacki and Gumiński, 1999). According to guidelines published in "Wyrażanie Niepewności Pomiaru, Przewodnik" [Guide to the Presentation of Measuring Uncertainty] (GUM 1999), the accuracy of measurement is expressed by the so-called *measuring uncertainty*. In Part 2, concepts of standard uncertainty and composite standard uncertainty are discussed (1). In Part 3, three methods of measuring differential pressures are outlined:

- with the use of two different instruments at a certain time interval (Section 3.1);
- with the use of one instrument at different times (Section 3.2);
- with the use of two instruments at the same time (Section 3.3);

and for each of these methods the measurement modelling function is determined (5), (14), (16).

Partial derivatives of the measurement modelling function are calculated for all input values (both measured and not measured). Furthermore, sensitivity coefficients and formulas expressing their composite variances are presented for those values. In addition, variances and covariances of coefficients of the measuring instruments' static characteristics are calculated by means of the minimum chi-square method in the process of calibrating instruments. Using these relationships of the sensitivity coefficients and the variances of input values, two sample calculations are presented in Part 4, performed with the help of a measurement database prepared by KWK "Anna" Coal Mine and the results of recording pressure values measured in a cross heading of this mine. Data for calculating the uncertainty of coefficients of measuring instruments' static characteristics were obtained from a collection of calibration certificates of anemometers and pressure gauges issued by the Institute of Strata Mechanics, Polish Academy of Sciences (IMG PAN). The results of calculations of composite variances for differential pressures are presented in tabular form to demonstrate the impact of uncertainty of individual input values on the measuring uncertainty of the differential pressure. Conclusions from the analysis of data listed in the tables are presented in Part 5.

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Pomiar różnicy ciśnień statycznych w kopalnianej sieci wentylacyjnej metodą barometryczną jest często wykonywany w celu wyznaczenia straty naporu na bocznicy wentylacyjnej i obliczenia współczynnika oporu aerodynamicznego bocznicy. Jak pokazuje praktyka, wynik tego pomiaru obarczony jest dużym błędem (Biernacki i Gumiński 1999). Miarą dokładności pomiaru, zgodnie z wytycznymi podanymi w publikacji „Wyrażanie Niepewności Pomiaru, Przewodnik" (GUM 1999), jest niepewność pomiaru. Przewodnik wprowadza pojęcie niepewności standardowej i złożonej niepewności standardowej (1). Ponieważ estymaty wielkości wejściowych funkcji modelującej pomiar nie są wyznaczane z powtarzanych obserwacji, to estymaty wariancji tych wielkości określa się metodą typu B. W tym przypadku estymatę wielkości wejściowej lub jej niepewność standardową określa się metodą analizy naukowej, opartej na wszystkich dostępnych informacjach o możliwej zmienności wielkości wejściowej.

W artykule omówiono niepewność pomiaru różnicy ciśnień metodą barometryczną dla trzech przypadków:

- z niejednoczesnym pomiarem ciśnień statycznych dwoma przyrządami,
- z niejednoczesnym pomiarem jednym przyrządem,
- z jednoczesnym pomiarem tych ciśnień dwoma przyrządami.

Za wyjątkiem ostatniego przypadku, należy uwzględnić zmianę ciśnienia barometrycznego w czasie dzielącym pomiary. Realizuje się to wprowadzając tzw. ciśnienie zredukowane (2), (3). Do pomiaru ciśnień najczęściej używa się aneroidy (np. baroluks), a ostatnio również mikroprocesorowego przyrządu μBAR. Przyjęto, że charakterystyki tych przyrządów można wyrazić zależnością (4).

Jeżeli pomiary ciśnień wykonano dwoma różnymi przyrządami w pewnym odstępie czasu, to różnica ciśnień wyrazi się wzorem (5), a wariancja różnicy ciśnień — wzorem (6). Jeżeli współczynniki charakterystyki (4) przyrządów są skorelowane, to do zależności (6) dodaje się składnik (7). W przypadku gdy te współczynniki są wyznaczane metodą najmniejszych kwadratów, równanie aproksymujące dane jest wzorem (8), a wariancje i kowariancje współczynników charakterystyki przyrządu otrzymuje się z macierzy kowariancji (9). Kowariancję współczynników można wyeliminować, przyjmując w równaniu aproksymującym (8) warunek (10). Korzystając z danych ze świadectw wzorcowania można obliczyć wariancje eksperymentalne (12) i kowariancję (13) współczynników charakterystyki dla danego typu przyrządu.

W przypadku pomiaru jednym przyrządem różnica ciśnień wyrazi się wzorem (14) i wariancja tej różnicy — wzorem (15), a przy pomiarze dwoma przyrządami w tym samym czasie różnica ciśnień wyrazi się wzorem (16) i wariancja różnicy ciśnień — wzorem (17).

Korzystając z wyników pomiarów kopalnianych parametrów wentylacyjnych zamieszczonych w literaturze oraz z wyników badań prowadzonych w Pracowni Wentylacji Kopalń IMG, podjęto próbę oszacowania niepewności pomiaru różnicy ciśnień metodą barometryczną. W przykładach obliczania niepewności wykorzystano wartości z bazy danych pomiarowych KWK Anna. Niepewność wyznaczenia współczynników charakterystyki przyrządów do pomiaru ciśnień obliczono na podstawie zbioru świadectw wzorcowania przyrządów typu baroluks oraz typu μBAR, wydanych przez IMG PAN w latach 1996—2001. Z tego zbioru wybrano po jednym świadectwie wzorcowania dla każdego przyrządu. Następnie obliczono współczynniki a_r i b_r równania aproksymującego (8) i ciśnienie odniesienia p_{ro} z warunku (10), wariancje eksperymentalne $s^2(a_r)$ i $s^2(b_r)$ współczynników oraz wariancję eksperymentalną całkowitą s^2 jako miarę niepewności aproksymacji. Dane te przedstawiono w tablicy 1 — dla baroluksu i w tablicy 2 — dla μBARa.

Jeżeli wyniki pomiarów ciśnień nie są korygowane poprawkami wyliczanymi z zależności (9), to dla tego przypadku wyznaczono wariancje i kowariancje eksperymentalne współczynników a i b, obliczone z zależności (11). Wartości tych współczynników podano w tablicach 3 i 4.

Niepewności pomiarów ciśnień na początku i na końcu bocznicy, ze względu na zmienność w czasie wartości tych wielkości, określono na podstawie wyników pomiarów wykonanych w KWK Anna (Cierniak i in. 1996). Jeden z tych pomiarów oraz jego histogram pokazano na rysunku 1. Czasy trwania poszczególnych rejestracji ciśnień i wariancje eksperymentalne tych ciśnień oraz ich różnicy $\Delta p_o = p_{o1} - p_{o2}$ zestawiono w tablicy 5. Na podstawie tych danych określono wariancje składowe wariancji różnicy ciśnień $u^2(\Delta p)$, obliczanej z zależności (6) w przypadku pomiaru ciśnień na początku i na końcu bocznicy dwoma przyrządami w różnym czasie, z zależności (15) w przypadku pomiaru ciśnień jednym przyrządem w różnym czasie i z zależności (17) w przypadku pomiaru ciśnień dwoma przyrządami w tym samym czasie.

Przedstawiono następujące przypadki:

- ciśnienia mierzono przyrządami typu baroluks ze świadectwami wzorcowania i wyniki pomiarów korygowano poprawkami,
- ciśnienia mierzono przyrządami typu baroluks bez świadectw wzorcowania i bez korekcji wyników pomiarów,
- ciśnienia mierzono przyrządami typu μBAR ze świadectwami wzorcowania i wyniki pomiarów korygowano poprawkami,
- ciśnienia mierzono przyrządami typu μBAR bez świadectw wzorcowania i bez korekcji wyników pomiarów.

Uwzględniono następujące źródła niepewności:

- niepewność współczynników a i b równania (8) aproksymującego krzywą błędów przyrządów użytych do pomiaru ciśnień p_1 i p_2,
- niepewność pomiaru ciśnienia p_b na zrębie szybu,
- niepewność z powodu zmienności w czasie ciśnień p_1 i p_2.

Obliczone dla wymienionych przypadków wariancje i niepewności złożone różnicy ciśnień, wariancje składowe dla różnych źródeł niepewności i udziały wariancji składowych w wariancji złożonej dla wymienionych wyżej przypadków, dla przykładu 1 zestawiono w tablicach 6, 7, 8 i dla przykładu 2 w tabliach 9, 10, 11. Zestawienie niepewności standardowych i niepewności standardowych względnych pomiaru różnicy ciśnień zamieszczono w tablicach 12 i 13.

Analiza przykładów obliczeniowych wyznaczania niepewności pomiaru różnicy ciśnień metodą barometryczną prowadzi do następujących wniosków:

- Niepewność pomiaru różnicy zależy od metody pomiaru i typu przyrządu. Najmniejszą niepewnością cechuje się pomiar dwoma przyrządami typu μBAR w tym samym czasie i z korekcją wyników pomiaru na podstawie świadectw wzorcowania. Około trzykrotnie większą niepewność ma pomiar różnicy ciśnień, wykonany jednym przyrządem typu baroluks lub μBAR w różnym czasie, a największą niepewność ma pomiar wykonany dwoma baroluksami w różnym czasie i bez korekcji wyników pomiaru.
- Największe wartości mają wariancje związane z fluktuacjami ciśnienia przy niejednoczesnym pomiarze ciśnień oraz wariancje związane z charakterystyką statyczną przyrządu, gdzie decydujące znaczenie ma wariancja współczynnika przesunięcia charakterystyki.
- Bezwładności związane z układem mechanicznym przyrządu powodują, że jego charakterystyka dynamiczna powinna być zbliżona do charakterystyki filtru dolnoprzepustowego, co powoduje zmniejszenie amplitudy fluktuacji wskazań przyrządu i tym samym zmniejszenie związanej z tymi fluktuacjami wariancji składowej w złożonej wariancji pomiaru różnicy ciśnień. W przypadku pomiaru przyrządami typu μBAR znaczące zmniejszenie niepewności pomiaru różnicy ciśnień można osiągnąć rejestrując cyfrowe sygnały wyjściowe tych przyrządów w określonym odcinku czasu i wyznaczając średnie wartości mierzonych ciśnień w tym okresie. Jednak, jeżeli pomiary ciśnień na początku i na końcu bocznicy wykonane zostaną w pewnym odstępie czasu, to obliczone średnie wartości ciśnień będą zmiennymi losowymi, ponieważ wpływ na te wartości mają losowo zmieniające się warunki przewietrzania w kopalni.

Słowa kluczowe: wentylacja kopalń, pomiar ciśnienia, niepewność pomiaru

1. Introduction

The measurement of static differential pressure is often performed in mines to determine pressure losses in a side-branch, a dam or other ventilation elements, and to calculate the aerodynamic resistance coefficient of a given element. The static differential pressure can be measured directly with the use of a differential pressure gauge connected to with static pressure probes with impulse cables; the distance between measuring points being is limited by the length of the cables. For measurement of the differential pressure in remote points of a ventilation network (e.g. at the beginning and at the end of a long ventilation side-branch), this differential pressure is most commonly measured by means of the barometric method as a difference of absolute pressures measured at those points (Roszczynialski, Trutwin, Wacławik 1992). However, as experience shows, the differential pressure calculated on the basis of these measurements is distorted by a significant measuring error. After comparing the accuracy of measurements of pressure losses in side-branches using different methods, Biernacki and Gumiński (1999) indicated a low measurement-accuracy of measured losses in a single side-branch with the use of the barometric method (which is most often used in practice), and pointed out that it resulted from high measurement errors of the differential pressure.

2. Measuring uncertainty

The results of measurement may be presented with a certain level of reliability, which is expressed as *measuring uncertainty*. Rules of calculating and presenting measuring uncertainty are outlined in "Wyrażanie Niepewności Pomiaru, Przewodnik" (1999) published by the General Measurement Agency (GUM). The theory of uncertainty and the theory of error are discussed in more detail in the appendix to the aforementioned publication by J.M. Jaworski and in his article (1997). According to the definition presented in this publication, the measuring uncertainty is a parameter related to the measurement result characterised by a dispersion of values, which can be reasonably attributed to the measured value. This publication introduces the concept of a standard uncertainty expressed as a standard deviation of a measurement result and the concept of a composite measuring uncertainty for intermediate measurements, where the measurement result is calculated on the basis of a certain number of other values. The composite measuring uncertainty is a positive square root of the composite variance equal to the following sum:

$$u_c^2(y)=\sum_{i=1}^{N} u_i^2(y)=\sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i)+2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}u(x_i,x_j) \tag{1}$$

where f is the measurement modelling function, x_i and x_j are estimates of input values, $u^2(x_i)$ is the estimate of variance of the value x_i; and $u(x_i, x_j)$ is the estimate of covariance related to x_i and x_j, when these values are correlated. Partial derivatives in this formula are often called *sensitivity coefficients*.

Because the estimates of input values are not calculated on the basis of repeated observations, the estimates of variances of these values are determined by means of the B-type method. According to "Przewodnik do wyrażania niepewności pomiaru" (GUM 1999), in this case the variance estimate of the input value or its standard uncertainty is determined by means of a scientific analysis based on all available information about possible variability of the input value, such as:

- previous measurement data,
- experience and general knowledge of phenomena and properties of measuring instruments,
- technical specifications prepared by manufacturers of measuring instruments,
- data obtained in the process of calibration and certification,
- uncertainties attributed to data from various publications.

3. Modelling of the measurement of differential pressure by means of the barometric method

Because the pressure-gauge method is rarely used in practice, the measurement uncertainty of differential pressure by means of the barometric method is discussed below for three situations:

- measurement of differential pressure with two instruments at different times,
- measurement with one instrument at different times,
- simultaneous measurement of these pressures with two instruments.

When measurements of pressures at the beginning and at the end of the side-branch are not performed with the use of two instruments at the same time, the change in barometric pressure during the interval between measurements should be taken into account. This correction is made by introducing so-called *reduced pressures*:

$$p_{z1} = p_1 - (p_{b1} - p_{b0}) \qquad p_{z2} = p_2 - (p_{b2} - p_{b0}) \tag{2}$$

hence:

$$\Delta p = p_{z1} - p_{z2} = p_1 - p_2 - (p_{b1} - p_{b2}) \tag{3}$$

where p_{b0} is the barometric pressure at the outset measured at the moment t_0, which is the conventional beginning of measurements; and p_{b1} and p_{b2} are values of barometric pressures at moments when pressures p_1 and p_2 are measured.

The measuring uncertainty of static differential pressure depends on whether measurements of pressures at the beginning and at the end are made with two different instruments, or with the same instrument after a certain time interval, or with two different instruments at the same time. Most frequently, different aneroid barometers are used for measuring these pressures, such as Baroluks or, increasingly often in recent times, a microprocessor-based μBAR instrument with a pressure converter manufactured by the SETRA Company. It was assumed that characteristics of these instruments could be described by the following formula:

$$p - p_o = a + b(p_o - p_r) \tag{4}$$

where a and b are shift and inclination coefficients; p_o is the instrument's reading; p_r is a constant reference pressure of the value within the measurement range of the instrument.

Results of the recording of absolute pressures in selected remote points of a ventilation branch (Cierniak at al. 1996) show that this value can be presented as a sum of two components: a component changing slowly in time (called a *usable component*) and a random component of expected value zero. Random components may lead to a significant measuring uncertainty.

3.1. Measurement of pressures with two different instruments at different times

If measurements of pressures are performed using two different instruments at a certain time interval, the differential pressure is as follows:

$$\Delta p = p_{o1} - p_{o2} + a_1 - a_2 + b_1(p_{o1} - p_{r2}) - \tag{5}$$

$$-b_2(p_{o2} - p_{r2}) - (1 + b_b)(p_{b1} - p_{b2})$$

where b_b is the coefficient of inclination of the barometric pressure measuring instrument's characteristic at the outset; and p_{b1} and p_{b2} are readings of this instrument at moments when pressures p_1 and p_2 are measured. Given that $b_1 << 1$, $b_2 << 1$ and $b_b << 1$ and assuming that $u^2(p_{b1}) = u^2(p_{b2}) = u^2(p_b)$, the variance of differential pressure in this case will be:

$$u_c^2(\Delta p) = u^2(a_1) + u^2(a_2) + (p_{o1} - p_{r1})^2 u^2(b_1) + (p_{o2} - p_{r2})^2 u^2(b_2) + \tag{6}$$

$$+ u^2(p_{o1}) + u^2(p_{o2}) + 2u^2(p_b) + (p_{b2} - p_{b2})^2 u^2(b_b)$$

If coefficients a and b of the characteristics of these instruments are determined in the same calibration process, they may be correlated and the above relationship should be supplemented with a component including the co-variances of these coefficients:

$$2(p_{o1} - p_{r1})u(a_1, b_1) + 2(p_{o2} - p_{r2})u(a_2, b_2) \tag{7}$$

If coefficients a and b are calculated with the use of the minimum chi-square method, the approximating equation has the following form:

$$y = a + bx, \qquad \text{where:} \quad y = p - p_o \text{ and } x = p_o - p_r \tag{8}$$

The matrix of covariances for N pairs of measurement results (p — set pressure, p_o — read pressure) is as follows:

$$\mathbf{cov}(a,b)=\frac{\sum_{n=1}^{N}[y_n-a-bx_n]^2}{(N-2)\left[N\sum_{n=1}^{N}x_n^2-\left(\sum_{n=1}^{N}x_n\right)^2\right]}\begin{bmatrix}\sum_{n=1}^{N}x_n^2 & -\sum_{n=1}^{N}x_n\\ -\sum_{n=1}^{N}x_n & N\end{bmatrix} \tag{9}$$

Hence, variances $u^2(a)$, $u^2(b)$ and covariance $u(a, b)$ can be calculated. The covariance can be eliminated, assuming in the approximating equation that (8):

$$x=p_o-p_{ro} \qquad \text{where:} \quad p_{ro}=\frac{1}{N}\sum_{n=1}^{N}p_{on} \tag{10}$$

New coefficients a_r and b_r are related to coefficients of the equation (8) by the following relationships:

$$a-p_r b=a_r-p_{ro}b_r, \qquad b=b_r \tag{11}$$

If the coefficients of the instrument's characteristics are disregarded in the calculation of the differential pressure in equation (5), variances of these coefficients can be calculated on the basis of their likelihood density distributions, using available data, such as the set of calibration certificates for a given type of instrument. Using the set of N values of coefficients for various instruments of the same type, experimental variables can be calculated:

$$s^2(a)=\frac{\sum_{n=1}^{N}(a_n-\bar{a})^2}{N-1}, \qquad s^2(b)=\frac{\sum_{n=1}^{N}(b_n-\bar{b})^2}{N-1} \tag{12}$$

and co-variance:

$$s(a,b)=\frac{\sum_{n=1}^{N}(a_n-\bar{a})(b_n-\bar{b})}{N-1} \tag{13}$$

where:

$$\bar{a}=\frac{\sum_{n=1}^{N}a_n}{N}, \qquad \bar{b}=\frac{\sum_{n=1}^{N}b_n}{N}$$

3.2. Measurement of pressures with one instrument at different times

For measurement with one instrument, the differential pressure is as follows:

$$\Delta p = (1+b)(p_{o1} - p_{o2}) - (1+b_b) + (p_{b1} - p_{b2}) \tag{14}$$

The variability calculated from the above formula for $b << 1$ and $b_b << 1$ is equal to:

$$u_c^2(\Delta p) = u^2(p_{o1}) + u^2(p_{o2}) + (p_{o1} - p_{o2})^2 u^2(b) + \tag{15}$$

$$+2u^2(p_b)(p_{b1} - p_{b2})^2 u^2(b_b)$$

For the calculation of the variability of the instrument's characteristic inclination coefficient b see Section 3.1.

3.3. Measurement with two instruments at the same time

For measurement by means of two instruments at the same time the differential pressure is as follows:

$$\Delta p = p_{o1} - p_{o2} + a_1 - a_2 + b_1(p_{o1} - p_{r1}) - b_2(p_{o2} - p_{r2}) \tag{16}$$

Taking into account that $b_1 << 1$ and $b_2 << 1$, the variability in this case is equal to:

$$u_c^2(\Delta p) = u^2(a_1) + u^2(a_2) + (p_{o1} - p_{r1})^2 u^2(b_1) + \tag{17}$$

$$+ (p_{o2} - p_{r2})^2 u^2(b_2) + u^2(Dp_o)$$

Where $\Delta p_o = p_{o1} - p_{o2}$. The comments in Section 3.1 regarding the calculation of variabilities of an instrument's characteristic coefficients and any correlation are also valid in this case.

4. Calculation of the measuring uncertainty of the differential pressure

Using the results of measurements of mine ventilation parameters presented in publications and the results of research conducted by the Mine Ventilation Institute, Institute of Strata Mechanics, Polish Academy of Sciences (IMG PAN) under its mission statement and grants, an attempt was made to estimate the uncertainty. In calculating examples of this uncertainty, values from the measurement database produced by the KWK "Anna" Coal Mine were used.

Example 1

- pressure at the branch's beginning and end [mbar]
  p_1 = 1035.02 p_2 = 1034.94
- pressure at the outset [mbar] p_{b1} = 995.39 p_{b2} = 995.37
- average air speed in the branch's [m/s] v = 1.68
- area of the branch's lay-by's section [m^2] A = 7.5
- the branch's lay-by's length [m] L = 30
- depth from the surface of the branch's beginning and end [m]
  z_1 = –207.9 z_2 = –207.7
- average air density [kg/m^3] ρ = 1.23
- hydrostatic pressure [Pa] $g\rho(z_1 - z_2)$ = –2.4
- reduced differential pressure [Pa] Δp = 6

Example 2

- pressure at the branch's beginning and end [mbar]
  p_1 = 1096.02 p_2 = 1103.02
- pressure at the outset [mbar] p_{b1} = 995.03 p_{b2} = 995.15
- average air speed in the branch [m/s] v = 0.96
- area of the branch's section [m^2] A = 9
- branch's length [m] L = 600
- depth from surface of the branch's beginning and end [m]
  z_1 = –599.02 z_2 = –655.39
- average air density [kg/m^3] ρ = 1.274
- hydrostatic pressure [Pa] $g\rho(z_1 - z_2)$ = 704.5
- reduced differential pressure [Pa] Δp = –688

The uncertainty of coefficients of pressure-measuring instruments' characteristics was calculated on the basis of the set of calibration certificates of Baroluks and μBAR instruments issued by the Institute of Strata Mechanics, Polish Academy of Sciences (IMG PAN) between 1996 and 2001. From this set one calibration certificate was selected for each instrument. Afterwards, coefficients a_r and b_r of the approximating equation (8) and reference pressure p_{ro} from the condition (10), experimental variables $s^2(a_r)$ and $s^2(b_r)$ of coefficients and the total experimental variability s^2 as a measure of approximation uncertainty were calculated. These data are presented in the tables below.

The analysis of the above data shows that 75% of all Baroluks instruments have variance s^2 in a range between 0 and 0.2 $mbar^2$ (after rejecting the instrument serial number H9228, for which the variance s^2 markedly exceeded the variations of the other instruments). From this group of instruments three instruments of variances s^2, which were closest to the average variance (H9321, H9908 and H9350), were chosen for the sample calculation. Three μBAR instruments (nos. 13, 18 and 19) were selected by the same method. It can be noted that variances s^2 for these instruments are many times lower than for the Baroluks instruments.

TABLE 1

Instrument type Baroluks

TABLICA 1

Typ przyrządu baroluks

Instrument's serial number	P_{ro} [mbar]	a_r [mbar]	$s^2(a_r) \cdot 10^2$ [mbar^2]	$b_r \cdot 10^3$	$s^2(b_r) \cdot 10^6$	s^2 [mbar^2]
H9002	1009.5455	−12.74546	0.2560741	−0.692535	0.1609170	0.0563363
H9228	1020.0000	−25.10000	8.291629	−104.4381	5.150080	1.824158
H9321	1000.0000	2.276190	0.8902694	−43.55195	0.6070019	0.1869566
H9345	990.0000	−4.136355	3.247751	−41.93105	2.017237	0.7145053
H9350	999.5238	−5.157148	0.3456942	5.022787	0.2387271	0.0725958
H9354	1000.0000	−6.942859	2.853654	2.474044	1.945674	0.5992674
H9362	1000.0000	−1.744761	0.0424769	−5.727186	0.0289615	0.0089201
H9902	1000.0000	−1.345237	0.2353723	−1.129844	0.1604811	0.0494282
H9908	1000.0000	−5.199995	0.6805822	−1.675272	0.4640333	0.1429223

TABLE 2

Instrument type μBAR

TABLICA 2

Typ przyrządu μBAR

Instrument's serial number	p_{ro} [mbar]	a_r [mbar]	$s^2(a_r) \cdot 10^5$ [mbar^2]	$b_r \cdot 10^3$	$s^2(b_r) \cdot 10^8$	$s^2 \cdot 10^4$ [mbar^2]
10	985.08547	−0.393655	8.723931	0.9609395	8.690865	9.596324
11	985.30273	−0.610918	6.831328	0.7795360	6.802968	7.514461
12	985.89454	−0.523621	6.170140	2.261813	6.161622	6.787154
13	985.25545	−0.563638	6.884079	0.7981868	6.855756	7.572487
14	985.87274	−0.501826	4.472466	2.115764	4.464989	4.919712
15	985.90908	−0.538164	6.165849	2.207300	6.156667	6.782434
16	985.96636	−0.595442	4.877291	2.243360	4.870377	5.365020
17	985.29636	−0.604548	8.189970	0.8157518	8.156561	9.008967
18	985.27819	−0.5863758	8.053438	0.7887578	8.020153	8.858782
19	985.91090	−0.539984	8.049127	2.152455	8.036262	8.854040

TABLE 3

Instrument type Baroluks

TABLICA 3

Typ przyrządu baroluks

Instrument's serial number	a [mbar]	$b \cdot 10^3$
H9002	−12.73885	−0.69254
H9321	2.27619	−43.55195
H9345	−4.55567	−41.93105
H9350	−5.15476	5.02279
H9354	−6.94286	2.47404
H9362	−1.74476	−5.72719
H9902	−1.34524	−1.12984
H9908	−5.20000	−1.67527

Variances: $s^2(a) = 19.74$ mbar^2; $s^2(b) = 3.9614 \cdot 10^{-4}$.
Covariance: $s(a, b) = -4.4015 \cdot 10^{-2}$ mbar.

TABLE 4

Instrument type μBAR

TABLICA 4

Typ przyrządu μBAR

Instrument's serial number	a [mbar]	$b \cdot 10^3$
10	−0.39374	0.96094
11	−0.61115	0.77954
12	−0.52564	2.26181
13	−0.56384	0.79819
14	−0.50367	2.11576
15	−0.54017	2.20730
16	−0.59761	2.24336
17	−0.60479	0.81575
18	−0.58660	0.78876
19	−0.54194	2.15246

Variances: $s^2(a) = 4.196 \cdot 10^{-3}$ mbar^2; $s^2(b) = 0.524 \cdot 10^{-6}$.
Covariance: $s(a, b) = 6.286 \cdot 10^{-6}$ mbar.

If the results of measurements of pressures are not corrected according to formula (9), in this case experimental variances and covariances of coefficients a and b were calculated on the basis of the formula (11), assuming the reference pressure p_r = 1000 mbar for Baroluks instruments and p_r = 985 mbar for μBAR instruments. Values of these coefficients are presented in the tables below.

Because of the variability of values in time, measurement uncertainties of pressures at the branch's beginning and end were calculated on the basis of results of measurements performed in the KWK "Anna" Coal Mine (Cierniak at al. 1996). Measurements

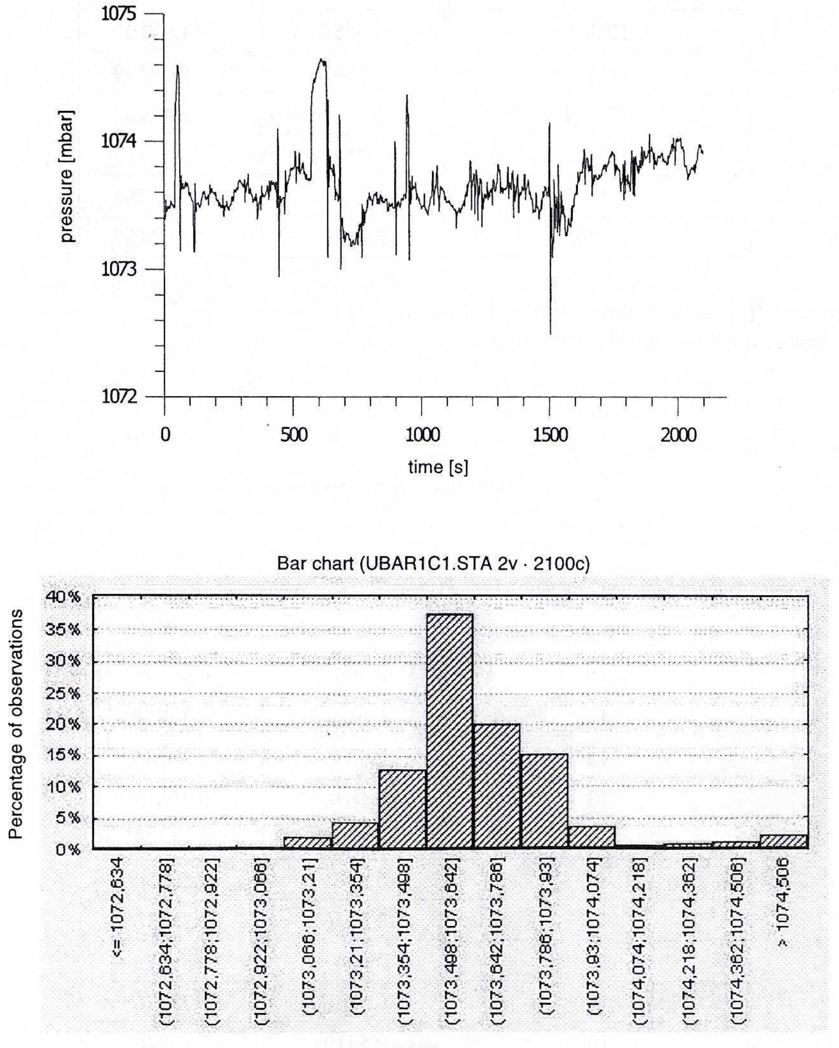

Fig. 1. Sample distribution of barometric pressure fluctuations in a mine heading and its bar chart

Rys. 1. Przykładowy przebieg zmian ciśnienia barometrycznego w wyrobisku kopalni i jego histogram were conducted in a cross heading at level 800. μBAR pressure gauges were located in the close proximity of ventilation dams at a distance of approximately 200 metres from each other. Measurements were carried out every second. Time sections, where pressure fluctuations caused by a change in the position of a ventilation dam were present, were removed from the records. Therefore, four successive registrations of pressures for each of the two instruments were obtained. One of these records and its bar chart are presented in Fig. 1.

In the table below, times of individual pressure registrations and experimental variances of these pressures and the differential pressure $\Delta p_o = p_{o1} - p_{o2}$ are presented.

TABLE 5

Registration times and experimental variations of pressure and differential pressure

TABLICA 5

Czasy rejestracji i wariancje eksperymentalne ciśnień oraz ich różnicy

Registration number	T [sec]	$s^2(p_{o1})$ [mbar^2]	$s^2(p_{o2})$ [mbar^2]	$s^2(\Delta p_o)$ [mbar^2]
1	2100	0.0618	0.0583	0.0088
2	2200	0.0109	0.0098	0.0012
3	1600	0.0138	0.0177	0.0015
4	1000	0.0259	0.0261	0.0051

In further calculations the weighted average variances were used according to the following formula:

$$s^2(p_x) = \frac{\sum_{n=1}^{4} T_n s_n^2(p_{ox})}{\sum_{n=1}^{4} T_n} \tag{18}$$

The calculated values are as follows (mbar^2): $s^2(p_{o1}) = 0.0292$, $s^2(p_{o2}) = 0.0288$, $s^2(\Delta p_o) = 0.0041$.

On the basis of the above-mentioned data, component variances of the differential pressure variability $u^2(\Delta p)$ were calculated according to formula (6) for measurement of pressures at the branch's beginning and end by means of two instruments at different times; according to formula (15), for measurement of pressures by means of one instrument at different times; and according to formula (17) for measurement of pressures by means of two instruments simultaneously.

1. Variances related to the variability of pressures measured in time.

$$u^2(p_1) = 292 \text{ Pa}^2, \qquad u^2(p_2) = 288 \text{ Pa}^2, \qquad u^2(\Delta p) = 41 \text{ Pa}^2,$$

2. Variance related to the pressure gauge's reading uncertainty at the outset.
For the Baroluks instrument, the a resolution equal to 0.2 of the scale interval was chosen, while for the μBAR instrument with digital reading the resolution was equal to the lowest significant digit:
Baroluks instrument $u^2(p_b) = 1.33 \text{ Pa}^2$,
μBAR instrument $u^2(p_b) = 0.08 \text{ Pa}^2$.

3. Variances related to the uncertainty of the instrument's static characteristic:

Case 1

Pressures were measured by Baroluks instruments with calibration certificates and results were corrected.

- Measurement of pressure p_1: instrument no. H9321,
  correction factors $a_1 = 227.6$ Pa, $b_1 = -4.355 \cdot 10^{-2}$, for $p_{r1} = 10^5$ Pa,
  coefficient variances $u^2(a_1) = 89.027 \text{ Pa}^2$, $u^2(b_1) = 0.6070 \cdot 10^{-6}$,
- Measurement of pressure p_2: instrument no. H9908,
  correction factors $a_2 = -520$ Pa, $b_2 = -1.675 \cdot 10^{-3}$, for $p_{r2} = 10^5$ Pa,
  coefficient variances $u^2(a_2) = 68.058 \text{ Pa}^2$, $u^2(b_2) = 0.4640 \cdot 10^{-6}$,
- Measurement of pressure p_b: instrument no. H9350,
  correction factors $a_b = -515.7$ Pa, $b_b = -5.023 \cdot 10^{-3}$, for $p_{rb} = 99952.4$ Pa,
  coefficient variances $u^2(a_b) = 34.569 \text{ Pa}^2$, $u^2(b_b) = 0.2387 \cdot 10^{-6}$.

Case 2

Pressures were measured by Baroluks instruments without calibration certificates and without correction of measurement results.

- Measurement of pressure p_1, p_2 and p_b: $p_r = 10^5$ Pa,
  coefficient variances $u^2(a_1) = u^2(a_2) = u^2(a_b) = u^2(a) = 1.974 \cdot 10^5 \text{ Pa}^2$,
  $u^2(b_1) = u^2(b_2) = u^2(b_b) = u^2(b) = 3.96 \cdot 10^{-4}$,
  $u(a,b) = -4.4$ Pa.

Case 3

Pressures were measured by μBAR instruments with calibration certificates and results were corrected.

- Measurement of pressure p_1: instrument no. 13,
  correction factors $a_1 = -56.4$ Pa, $b_1 = 7.982 \cdot 10^{-4}$, for $p_{r1} = 98525.5$ Pa,
  coefficient variances $u^2(a_1) = 0.688 \text{ Pa}^2$, $u^2(b_1) = 6.8557 \cdot 10^{-8}$,
- Measurement of pressure p_2: instrument no. 18,
  correction factors $a_2 = -58.6$ Pa, $b_2 = 7.888 \cdot 10^{-4}$, for $p_{r2} = 98527.8$ Pa,
  coefficient variances $u^2(a_2) = 0.805 \text{ Pa}^2$, $u^2(b_2) = 8.0202 \cdot 10^{-8}$,
- Measurement of pressure p_b: instrument no. 19,
  correction factors $a_b = -54.0$ Pa, $b_b = -2.152 \cdot 10^{-3}$, for $p_{rb} = 98591.1$ Pa,
  coefficient variances $u^2(a_b) = 0.805 \text{ Pa}^2$, $u^2(b_b) = 8.0363 \cdot 10^{-8}$.

Case 4

Pressures were measured by μBAR instruments without calibration certificates and without correction of measurement results.

- Measurement of pressure p_1, p_2 and p_b: $p_r = 98500$ Pa,
  coefficient variances $u^2(a_1) = u^2(a_2) = u^2(a_b) = 41.96$ Pa^2,
  $u^2(b_1) = u^2(b_2) = u^2(b_b) = 5.24 \cdot 10^{-7}$,
  $u(a, b) = 6.29 \cdot 10^{-6}$ Pa.

The tables on the following pages present variances and composite uncertainties of differential pressure, component variances for different sources of uncertainty and proportions of component variances in the composite variance for the aforementioned examples and cases and for various methods of measuring pressures at the branch's beginning and end.

Sources of uncertainty:

A. Uncertainty of coefficients a and b in the equation (8) that approximates the curve of errors of instruments used for measuring pressures p_1 and p_2.

B. Measuring uncertainty of pressure p_b at the outset.

C. Uncertainty caused by the fluctuation of pressures p_1 and p_2 in time.

Measuring methods:

1. First measuring method: pressures p_1 and p_2 were measured by two instruments at different times.

Variances for individual sources of uncertainty were calculated on the basis of the formula (6), after grouping components as follows:

- measurement with corrections of results

$$u_A^2 = u^2(a_1) + u^2(a_2) + (p_{o1} - p_{r1})^2 u^2(b_1) + (p_{o2} - p_{r2})^2 u^2(b_2)$$

$$u_B^2 = 2u^2(p_b) + (p_{b1} - p_{b2})^2 u^2(b_b)$$

- measurement without correction of results

$$u_A^2 = 2u^2(a) + [(p_{o1} - p_r)^2 + (p_{o2} - p_r)^2] u^2(b) + 2(p_{o1} + p_{o2} - 2p_r) u(a,b) \tag{19}$$

$$u_B^2 = 2u^2(p_b) + (p_{b1} - p_{b2})^2 u^2(b)$$

$$u_C^2 = u^2(p_{o1}) + u^2(p_{o2})$$

2. Second measuring method: pressures p_1 and p_2 were measured by one instrument at different times.

Variances for individual sources of uncertainty were calculated according to the formula (15), after grouping components as follows:

$$u_A^2 = (p_{o1} - p_{o2})^2 u^2(b) \tag{20}$$

$$u_B^2 = 2u^2(p_b) + (p_{b1} - p_{b2})^2 u^2(b_b)$$

$$u_C^2 = u^2(p_{o1}) + u^2(p_{o2})$$

3. Third measuring method: pressures p_1 and p_2 were measured by two instruments simultaneously.

Variances for individual sources of uncertainty, calculated according to formula (17), are expressed by the following relationships:

$$u_A^2 \text{ — as for the first measuring method,} \quad u_B^2 = 0, \quad u_C^2 = u^2(\Delta p_o) \tag{21}$$

Example 1

TABLE 6

First measuring method: pressures p_1 and p_2 were measured by two instruments at different times

TABLICA 6

Pierwszy sposób pomiaru: ciśnienia p_1 i p_2 mierzono dwoma przyrządami w różnym czasie

Source of uncertainty	Measuring instruments							
	Baroluks with correction		Baroluks without correction		μBAR with correction		μBAR without correction	
	u_x^2 [Pa^2]	u_x^2/u^2	u_x^2 [Pa^2]	u_x^2/u^2	u_x^2 [Pa^2]	u_x^2/u^2	u_x^2 [Pa^2]	u_x^2/u^2
A	170.19	0.2261	$3.429 \cdot 10^5$	0.9983	5.17	0.0088	68.19	0.1052
B	2.66	0.0035	2.66	10^{-6}	0.16	0.0003	0.16	0.0002
C	580	0.7704	580	0.0017	580	0.9909	580	0.8946
$u^2(\Delta p)$ [Pa^2]	753		$3.435 \cdot 10^5$		585		648	
$u(\Delta p)$ [Pa]	27.4		586		24.2		25.5	

TABLE 7

Second measuring method: pressures p_1 and p_2 measured by one instrument at different times (Baroluks no. H9321, μBAR no. 13)

TABLICA 7

Drugi sposób pomiaru: ciśnienia p_1 i p_2 mierzono jednym przyrządem w różnym czasie, (barolux nr H9321, μBAR nr 13)

Source of uncertainty	Measuring instruments							
	Baroluks with correction		Baroluks without correction		μBAR with correction		μBAR without correction	
	u_x^2 [Pa^2]	u_x^2/u^2	u_x^2 [Pa^2]	u_x^2/u^2	u_x^2 [Pa^2]	u_x^2/u^2	u_x^2 [Pa^2]	u_x^2/u^2
A	$3.885 \cdot 10^{-5}$	$6.67 \cdot 10^{-8}$	0.0254	$4.36 \cdot 10^{-5}$	$4.39 \cdot 10^{-6}$	$7.57 \cdot 10^{-9}$	$3.35 \cdot 10^{-5}$	$5.77 \cdot 10^{-8}$
B	2.66	0.0046	2.66	0.0046	0.16	0.0003	0.16	0.0003
C	580	0.9954	580	0.9964	580	0.9997	580	0.9997
$u^2(\Delta p)$ [Pa^2]	583		583		580		580	
$u(\Delta p)$ [Pa]	24.1		24.1		24.1		24.1	

TABLE 8

Third measuring method: pressures p_1 and p_2 were measured by two instruments simultaneously

TABLICA 8

Trzeci sposób pomiaru: ciśnienia p_1 i p_2 mierzono dwoma przyrządami w tym samym czasie

Source of uncertainty	Measuring instruments							
	Baroluks with correction		Baroluks without correction		μBAR with correction		μBAR without correction	
	u_x^2 [Pa^2]	u_x^2/u^2	u_x^2 [Pa^2]	u_x^2/u^2	u_x^2 [Pa^2]	u_x^2/u^2	u_x^2 [Pa^2]	u_x^2/u^2
A	170.19	0.8059	$3.429 \cdot 10^5$	0.9999	5.17	0.1120	68.19	0.6245
B	0	0	0	0	0	0	0	0
C	41	0.1941	41	0.0001	41	0.8880	41	0.3755
$u^2(\Delta p)$ [Pa^2]	211		$3.429 \cdot 10^5$		46.2		109.2	
$u(\Delta p)$ [Pa]	14.5		586		6.8		10.4	

Example 2

TABLE 9

First measuring method: pressures p_1 and p_2 were measured by two instruments at different times

TABLICA 9

Pierwszy sposób pomiaru: ciśnienia p_1 i p_2 mierzono dwoma przyrządami w różnym czasie

Source of uncertainty	Measuring instruments							
	Baroluks with correction		Baroluks without correction		μBAR with correction		μBAR without correction	
	u_x^2 [Pa^2]	u_x^2/u^2	u_x^2 [Pa^2]	u_x^2/u^2	u_x^2 [Pa^2]	u_x^2/u^2	u_x^2 [Pa^2]	u_x^2/u^2
A	262.29	0.3104	$4.725 \cdot 10^5$	0.9988	21.03	0.0350	220.90	0.2758
B	2.66	0.0032	2.66	$5.6 \cdot 10^{-6}$	0.16	0.0003	0.16	0.0002
C	580	0.6864	580	0.0012	580	0.9647	580	0.7240
$u^2(\Delta p)$ [Pa^2]	845		$4.731 \cdot 10^5$		601		801	
$u(\Delta p)$ [Pa]	29.1		688		24.5		28.3	

TABLE 10

Second measuring method: pressures p_1 and p_2 were measured by one instrument at different times (Baroluks no. H9321, μBAR no. 13)

TABLICA 10

Drugi sposób pomiaru: ciśnienia p_1 i p_2 mierzono jednym przyrządem w różnym czasie (baroluks nr H9321, μBAR nr 13)

Source of uncertainty	Measuring instruments							
	Baroluks with correction		Baroluks without correction		μBAR with correction		μBAR without correction	
	u_x^2 [Pa^2]	u_x^2/u^2	u_x^2 [Pa^2]	u_x^2/u^2	u_x^2 [Pa^2]	u_x^2/u^2	u_x^2 [Pa^2]	u_x^2/u^2
A	0.30	0.0005	194.12	0.2499	0.03	$5.2 \cdot 10^{-5}$	0.26	0.0004
B	2.66	0.0046	2.66	0.0034	0.16	0.0003	0.16	0.0003
C	580	0.9949	580	0.7467	580	0.9997	580	0.9993
$u^2(\Delta p)$ [Pa^2]	583		777		580		580	
$U(\Delta p)$ [Pa]	24.1		27.9		24.1		24.1	

TABLE 11

Third measuring method: pressures p_1 and p_2 were measured by two instruments simultaneously

TABLICA 11

Trzeci sposób pomiaru: ciśnienia p_1 i p_2 mierzono dwoma przyrządami w tym samym czasie

Source of uncertainty	Measuring instruments							
	Baroluks with correction		Baroluks without correction		μBAR with correction		μBAR without correction	
	u_x^2 [Pa^2]	u_x^2/u^2	u_x^2 [Pa^2]	u_x^2/u^2	u_x^2 [Pa^2]	u_x^2/u^2	u_x^2 [Pa^2]	u_x^2/u^2
A	262.29	0.8648	$4.725 \cdot 10^5$	0.9999	21.03	0.3390	220.90	0.8435
B	0	0	0	0	0	0	0	0
C	41	0.1352	41	0.0001	41	0.6610	41	0.1565
$u^2(\Delta p)$ [Pa^2]	303		$4.725 \cdot 10^5$		62		262	
$u(\Delta p)$ [Pa]	17.4		687		7.9		16.2	

Standard uncertainties and relative standard uncertainties of the calculation of reduced differential pressures are presented below.

TABLE 12

Standard uncertainties of reduced differential pressures, example 1

TABLICA 12

Niepewności standardowe różnicy ciśnień zredukowanuch, przykład 1

Example 1, $\Delta p = 6$ Pa								
Measuring method	Baroluks with correction		Baroluks without correction		μBAR with correction		μBAR without correction	
	$u(\Delta p)$ [Pa]	$u(\Delta p)/\Delta p$ [%]	$u(\Delta p)$ [Pa]	$u(\Delta p)/\Delta p$ [%]	$u(\Delta p)$ [Pa]	$u(\Delta p)/\Delta p$ [%]	$u(\Delta p)$ [Pa]	$u(\Delta p)/\Delta p$ [%]
1	27.4	457	586	9767	24.2	403	25.5	425
2	24.1	402	24.1	402	24.1	402	24.1	402
3	14.5	242	586	9767	6.8	113	10.4	173

TABLE 13

Standard uncertainties of reduced differential pressures, example 2

TABLICA 13

Niepewności standardowe różnicy ciśnień zredukowanuch, przykład 2

Example 2, $\Delta p = -688$ Pa								
Measuring method	Baroluks with correction		Baroluks without correction		μBAR with correction		μBAR without correction	
	$u(\Delta p)$ [Pa]	$u(\Delta p)/\Delta p$ [%]	$u(\Delta p)$ [Pa]	$u(\Delta p)/\Delta p$ [%]	$u(\Delta p)$ [Pa]	$u(\Delta p)/\Delta p$ [%]	$u(\Delta p)$ [Pa]	$u(\Delta p)/\Delta p$ [%]
1	29.1	4.23	688	100	24.5	3.56	28.3	4.11
2	24.1	3.50	27.9	4.1	24.1	3.50	24.1	3.50
3	17.4	2.53	687	99.9	7.9	1.15	16.2	2.35

Conclusions

As shown in Tables 12 and 13, the measuring uncertainty of the differential pressure depends on the measuring method and the type of a measuring instrument. The lowest uncertainty is observed for simultaneous measurement using two μBAR instruments and with correction of measurement results on the basis of calibration certificates. An uncertainty approximately three times higher is noted for measurement of the differential pressure by one Baroluks or μBAR instrument at different times, while the highest uncertainty occurs for measurement using two Baroluks instruments at different times and without correction of measurement results.

These observations can be explained by taking into account component variances of the differential pressure variance presented in Tables 6 to 11. The highest values are recorded for variances related to pressure fluctuations with measurement of pressures not made at the same time, and variances related to the instrument's static characteristic, where the variance of the characteristic's shift coefficient is of the greatest importance. While measuring pressures by one instrument and subsequently calculating the differential pressure, the characteristic's shift coefficient is eliminated and, consequently, its variance disappears. Only a significantly lower variance related to the inclination coefficient and the dominant component variance related to pressure fluctuations remains. Therefore, for this measuring method, the uncertainties have nearly identical values.

The variance related to the instrument's static characteristic has the lowest value when pressures are measured by μBAR instruments with correction of measured results, whereas the highest value occurs in the case of Baroluks instruments without correction of results, and the variance related to pressure fluctuations has a significantly lower value when pressures are measured at the same time compared to other measuring methods. This is the cause of the aforementioned highest and lowest measuring uncertainties of the differential pressure. It can be noted that the variance related to the μBAR instrument's static characteristic without correction of measurement results has a lower value than the variance for the Baroluks instrument with correction, which indicates reflects the much better metrological properties of the former.

For a small pressure differential (see Example 1), very high values of relative standard uncertainties were recorded, particularly when pressures were measured by Baroluks instruments. In fact, these values may be lower because the dynamic properties of these instruments were not taken into account in these calculations.

Inertia related to the instrument's mechanical structure results in its dynamic characteristic being similar to that of a low-pass filter, which leads to a reduced amplitude of the instrument's reading and, consequently, to a reduced component variance related to these fluctuations in the composite variance of measurement of the differential pressure. For measurement by μBAR instruments, a significant reduction in the measurement uncertainty of the differential pressure may be achieved by recording digital output signals of these instruments at a specific time interval and by calculating average values of measured pressures during this period. However, if measurements of pressures at the beginning and at the end of a branch are conducted at a certain time interval, the calculated average values of pressures will be random variables because they are affected by randomly changing mine ventilation conditions.

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