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BUDGETING OF THE MINING OUTPUT COSTS BY MEANS OF DUAL PROGRAMMING

BUDŻETOWANIE KOSZTÓW WYDOBYCIA KOPALNI Z WYKORZYSTANIEM PROGRAMOWANIA DUALNEGO

New conditions of market economy have caused the Polish mining to undergo comprehensive restructurization, most problems being created by financial restructurization which is expected to bring about profitable activities of mines. A modified management through budgeting of costs is introduced more frequently to achieve the positive financial result among other methods applied, especially, in black coal mines. It is the effective method of controlling of costs at separated stands, the so-called centres of the costs and helping, at the same time, to evoke initiative of the people making decisions. Budgeting requires an earlier analysis of the organization diagram and technological process to lead to the justified simplication of the operating stands of costs, assigning of budget of costs of these stands, stating precisely both of powers and responsibilities of managers as well as a possibility of motivation and control of the realization of the established budget. The paper discusses a method of the procedure of budget control by means of linear dual programming with a possibility of optimization of the realized costs. The simultaneous analysis of total costs is the essence of the solution within the framework of the budget assigned to a given stand as well as the evaluation of the internal structure of the costs of this stand. In practice, one can carry out optimization according to an arbitrary section of costs. On the other hand, an obligatory set of category costs has been used in the paper. A section calculation of costs, i.e. the exploitation section and exploitation faces that function in it, e.g. the longwalls are important in this case.

In the process of budgeting such budget parameters as the required volume of the output of the section, budget of total costs and category costs of the section are determined. Within the control of budget completion the established deviations from budget volumes are analyzed stating the causes of these deviations and methods of budget realization.

The utilization of linear dual programming in the process of budgeting depends on taking into account the differentiated conditions occuring in cost centres, i.e. the diffe-

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rentiated category structure of costs and unit costs of the output. Linear dual programming leads to the optimum distribution of the output into analyzed centres which can be longwalls in the section, sections in the mine, mines in the coal companies and companies in the whole branch of mining. In this way we achieve a possibility of realization of the established volume of the output at the lowest costs, according to the assumption, lesser costs than the assumed arbitrary budget. It will be the result of original linear programming.

In dual programming, we establish the optimum structure of category costs of the analyzed stand as well as we state the way in which every category component of costs influences a change of the optimum solution. Additionally, the practical effect of dual programming is an establishment which states which components of costs ought to be changed in the first instance since their decrease causes the greatest decrease in total costs.

Key words: mining management, costs, budgeting, optimization, dual programming.

Nowe uwarunkowania gospodarki rynkowej spowodowały, że górnictwo polskie podlega wszechstronnej restrukturyzacji przy czym najwięcej problemów stwarza restrukturyzacja finansowa, która ma doprowadzić do rentownej działalności kopalń. Spośród kilku sposobów stosowanych, zwłaszcza w kopalniach węgla kamiennego, aby uzyskać dodatni wynik finansowy coraz częściej wprowadza się zmodyfikowane zarządzanie poprzez budżetowanie kosztów. Jest to skuteczny sposób dyscyplinowania kosztów na wydzielonych stanowiskach, tak zwanych centrach kosztów, pozwalając równocześnie wyzwalać inicjatywę osób podejmujących decyzje. Budżetowanie wymaga wcześniejszego przeprowadzenia analizy schematu organizacyjnego oraz procesu technologicznego po to aby doprowadzić do uzasadnionego uproszczenia funkcjonujących stanowisk kosztów, przydzielenia tym stanowiskom budżetu kosztów, sprecyzowania uprawnień oraz odpowiedzialności kierowników a także możliwości motywacji i kontroli realizacji ustalonego budżetu. Praca omawia metodę procedury kontroli budżetu z wykorzystaniem dualnego programowania liniowego z możliwością optymalizacji realizowania kosztów. Istotą rozwiązania jest równoczesna analiza kosztów całkowitych w ramach przydzielonego na dane stanowisko budżetu jak również ocena wewnętrznej struktury kosztów tego stanowiska. W praktyce można prowadzić optymalizację według dowolnego przekroju kosztów natomiast w pracy posłużono się obowiązkowym układem kosztów rodzajowych. Podstawą jest w tym przypadku oddziałowy rachunek kosztów, czyli zasadniczym stanowiskiem kosztów jest oddział wydobywczy oraz funkcjonujące w nim przodki wydobywcze, na przykład ściany.

W procesie budżetowania wyznaczane są takie parametry budżetowe jak żądana wielkość wydobycia oddziału, budżet kosztów całkowitych oraz koszty rodzajowe oddziału. W ramach kontroli wykonania budżetu analizuje się stwierdzone odchylenia od wielkości budżetowych, stwierdzając przyczyny tych odchyleń oraz sposoby zrealizowania budżetu.

Model dualny zapisujemy następująco:

znaleźć zmienne decyzyjne $y_1, y_2, \dots, y_m$, które spełniają ograniczające warunki uboczne:

$$\left.\begin{aligned} a_{11}y_1 + a_{21}y_2 + \dots + a_{m1}y_m &\geqslant c_1 \\ a_{12}y_1 + a_{22}y_2 + \dots + a_{m2}y_m &\geqslant c_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{1n}y_1 + a_{2n}y_2 + \dots + a_{mn}y_m &\geqslant c_n \end{aligned}\right\} ;$$

ograniczające warunki brzegowe:

$$\left.\begin{array}{l} y_i \geqslant 0 \\ y_i < 0 \end{array}\right\} \text{lub} \quad (i = 1, 2, ..., m);$$

i minimalizujące funkcję celu:

$$K = b_1 y_1 + b_2 y_2 + .. + b_m y_m.$$

Wykorzystanie dualnego programowania liniowego w procesie budżetowania polega na uwzględnieniu zróżnicowanych warunków występujących w centrach kosztowych, zatem zróżnicowanej struktury rodzajowej kosztów oraz jednostkowych kosztów wydobycia. Programowanie liniowe dualne prowadzi do optymalnego rozdziału wydobycia na analizowane centra, którymi mogą być ściany w ramach oddziału, oddziały w ramach kopalni, kopalnie w ramach spółki, czy spółki w obrębie całej branży. Uzyskujemy w ten sposób możliwość zrealizowania ustalonej wielkości wydobycia przy najniższych kosztach, z założenia mniejszych niż przyjęty w sposób arbitralny budżet. Będzie to wynik programowania liniowego pierwotnego. W programowaniu dualnym ustalamy natomiast optymalną strukturę kosztów rodzajowych analizowanych stanowisk jak również stwierdzamy w jaki sposób każdy składnik rodzajowy kosztów wpływa na zmianę rozwiązania optymalnego. Praktyczny efekt programowania dualnego to także ustalenie, które składniki kosztów należy w pierwszej kolejności zmieniać bowiem na przykład ich zmniejszenie powoduje odpowiednio największe zmniejszenie kosztów całkowitych.

Słowa kluczowe: zarządzanie kopalnią, koszty, budżetowanie, optymalizacja, programowanie dualne.

1. Introduction

Market economy challenges mining with two basic requirements, namely, maintenance of both the financial liquidity in a short period of time and profitability in a longer period of time. Self — costs are the basic factor on which the realization of these requirements depends while financial accountancy help to analyze the level of these costs *ex post*.

The above market challenges make financial accountancy insufficient and it is necessary to analyze the costs *ex ante*. The analysis is realized by means of management accountancy whose element is, among others, budgeting of costs.

Budgeting of costs is the component of the current management of the mine whose essential points is (Czopek, 1998; Grzybek, 1999; RSCC S.A.).

— indication of the objectives and means indispensable for their achievement in a given period,

— establishment of criteria for the evaluation of completed tasks,

— determination of centres responsible for costs, profits, incomes and investment expenses,

— determination of budgets for separated centres of responsibility,

— control and correction of the budget completion.

The centres of costs correspond to the places of the origin of costs thus they reflect organization units both single or aggregated. It should be remembered, however, that in the case of mining activities 60% up to 80% total costs originate in the exploitation process hence the special role and significance of the choice of cost centres in this field of activity.

The established budget of costs for the whole mine is basic in the so-called controlling, i.e. in the process of the control and correction of the budget completion. The budget is distributed for the cost centres according to the assumed objectives for the realization in a given unit.

Not only the absolute value of costs in question is the basic problem but also their internal structure is equally important, i.e. the division of costs into:

— category,
— constant and variable,
— direct and indirect.

The establishment of the budget of costs for determined centres includes the final value of the budget of total costs for a given stand as well as structural budgets for every division criterion.

2. Dual linear programming

When undertaking economic decisions and their optimization two values-profit and self-costs — are applied most frequently as functions of the objective. The mathematical programming, particularly, linear, non-linear, dynamic, marginal and probabilistic programmings help to optimize the investigated economic phenomenon with the use of only one function of the objective. Thus the profit or self-cost, although taken into account separately, can be the criterion parameter. To tell the truth, optimization of the self-cost is maximization of the profit indirectly but it is not the proof of the statement that optimization of the self-cost goes explicity with maximization of the profit. There is no mathematical relation here since in both cases other output data and assumptions are taken into consideration in the process of optimization.

From the point of view of economic effectiveness there is a motivated need to optimize profit and costs simultaneously. In this way we combine essential elements of the economic process, i.e. applied technology and its correctness expressed by costs. On the other hand, we take into consideration the degree at which production subordinates to the rules of market through profit.

A detailed analysis of linear programming showed that application of the dual model of this programming helps to consider the mentioned criteria of profit and costs giving thus two optimum solutions that are closely connected together. The fact that the method helps to determine precisely the economic factors that enable us to achieve the greatest increase in profits or decrease in costs in the positive characteristics of the presented method.

Equations, linear in — equalities as well as the matrix calculus are the basis of linear programming.

Let us consider the rectangular matrix of the type $(m \times n)$:

$$A = \begin{bmatrix} a_{11}, a_{12}, \ldots, a_{1n} \\ a_{21}, a_{22}, \ldots, a_{2n} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}, a_{m2}, \ldots, a_{mn} \end{bmatrix}, \tag{1}$$

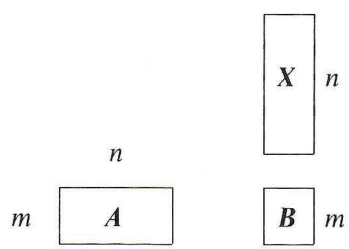
the column vector X: $$X = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix}, \tag{2}$$

and the column vector B: $$X = \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ b_m \end{bmatrix} \partial \tag{3}$$

then, in accord with the matrix calculus, the product of both the rectangular matrix A and the column vector X is the column vecor according to the formula:

$$A \cdot X = B. \tag{4}$$

Developing the above product according to the Falk diagram:



(5)

we obtain the result of the product as a set of linear equations:

$$\left.\begin{aligned}
&a_{11}x_1+a_{12}x_2+\ldots+a_{1n}x_n=b_1\\
&a_{21}x_1+a_{22}x_2+\ldots+a_{2n}x_n=b_2\\
&a_{31}x_1+a_{32}x_2+\ldots+a_{3n}x_n=b_3\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&a_{m1}x_1+a_{m2}x_2+\ldots+a_{mn}x_n=b_m
\end{aligned}\right\} \tag{6}$$

Finding of such column vector $\boldsymbol{X}$ $(x_1, x_2, \ldots, x_n)$, that optimizes the linear function called the function of the objective is the essential point of linear programming:

$$Z=c_1x_1+c_2x_2+c_2x_2+\ldots+c_nx_n=\text{optimum}\,(\text{max, min}), \tag{7}$$

with the linear limits

$$\boldsymbol{AX}=\boldsymbol{B} \tag{8}$$

Linear programming assumes non-negative solutions determined by the limit

$$x_j \geqslant 0,\ j=1,\ 2,\ 3,\ \ldots,\ n \tag{9}$$

and also the existence of the compatible set of equations that is, the condition

$$m<n. \tag{10}$$

Linear limits in linear programming can assume the form of equations or in-equalities:

$$\boldsymbol{AX}=\boldsymbol{B} \tag{11}$$

$$\boldsymbol{AX}\geqslant\boldsymbol{B} \tag{12}$$

$$\boldsymbol{AX}\leqslant\boldsymbol{B} \tag{13}$$

The function of the objective (7) is a number according to the Falk diagram, the result of the product of both the column vector $\boldsymbol{X}$ and the row vector $\boldsymbol{C}\,(c_1, c_2, \ldots, c_n)$.

$$\begin{array}{c|c|}
\cline{2-2}
 & x_1 \\
 & x_2 \\
 & \cdot \\
 & \cdot \\
 & \cdot \\
 & \cdot \\
 & x_n \\
\hline
\multicolumn{1}{|c|}{c_1\,c_2\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ c_n} & Z \\
\hline
\end{array} \tag{14}$$

Let us assume that we consider linear programming with the limits (13) present. Then we can state that for every formulated original model in the problems of linear programming find decision variables:

$$x_1, x_2, \ldots, x_n, \tag{15}$$

satisfying the indirect conditions:

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leqslant b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leqslant b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leqslant b_m \end{array}\right\}, \tag{16}$$

limiting the boundary conditons:

$$x_j \geqslant 0 \ (j = 1, 2, \ldots, n), \tag{17}$$

maximizing the function of the objective:

$$Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n. \tag{18}$$

The dual model corresponds to the original model and it reads as follows: find the decision variables:

$$y_1, y_2, \ldots, y_m, \tag{19}$$

that satisfy the limiting indirect conditions:

$$\left.\begin{array}{l} a_{11}y_1 + a_{21}y_2 + \ldots + a_{m_1}y_m \geqslant c_1 \\ a_{12}y_1 + a_{22}y_2 + \ldots + a_{m_2}y_m \geqslant c_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{1n}y_1 + a_{2n}y_2 + \ldots + a_{mn}y_m \geqslant c_n \end{array}\right\}, \tag{20}$$

limiting the boundary condition:

$$\text{or} \quad \left.\begin{array}{r} y_i \geqslant 0 \\ y_i < 0 \end{array}\right\} \quad (i = 1, 2, \ldots, m), \tag{21}$$

and minimizing the function of the objective:

$$K = b_1y_1 + b_2y_2 + \ldots + b_my_m. \tag{22}$$

In dual programming we use the matrix transposed to the A, the matrix A' of the type $(m \times n)$ and the form:

$$A = \begin{bmatrix} a_{11}, a_{12}, ..., a_{m1} \\ a_{21}, a_{22}, ..., a_{m2} \\ a_{13}, a_{23}, ..., a_{m3} \\ .... \\ a_{1n}, a_{2n}, ..., a_{mn} \end{bmatrix} . \qquad (23)$$

Hence the dual problem in the matrix calculus can be expressed in the following way: find the decision variables, components of the column vector $\boldsymbol{Y}(y_1, y_2, ..., y_m)$: that satisfy the linear limits:

$$\boldsymbol{A'Y} \geqslant \boldsymbol{C}, \qquad (24)$$

boundary limits:

$$\left.\begin{matrix} y_i \geqslant 0 \\ \text{or} \\ y_i < 0 \end{matrix}\right\} \quad i = 1, 2, ..., m. \qquad (25)$$

and minimize the function of the objective:

$$\boldsymbol{K} = \boldsymbol{BY}. \qquad (26)$$

3. Economic interpretation of dual programming

In the assumed original model a general production-economic problem has been presented. It is applied when the producer can make an arbitrary combination of several products items out of a number possible for production at his place. He, however, has limited production means: materials, money, manpower, machines that are used in production of different products in different quantities. Simultaneously, the selling prices of ready-made products (items) are also different. The problem is to produce such combination of products (items) and in such quantity as to achieve maximum profit from the sale of products at the known unit consumption of available means against the unit of a given product and unit price of the sale of products.

If the problem is interpreted in such a way, individual symbols used in the original model of linear programming denote:

n — quantity of products (items),
m — number of available means for the production of items,
a_{ij} — quantity of the i means used at the production of the unit of the item j,
b_i — maximum available quantity of the i means,

c_j — unit price of the j item,
x_j — quantity of the production of the j item.

Both in the original and dual model, the denotations a_{ij}, b_i, $\mathbf{c}_j$ and x_j remain unchanged as far as interpretation is concerned.

Their value reads as follows:

$$a_{ij} = \frac{\text{unit of the means } i}{\text{unit of the product } j},$$

$$b_i = \text{quantity of the means } i,$$

$$c_j = \frac{\text{zlotych}}{\text{unit of the product } j},$$

$$x_j = \text{quantity of the product } j.$$

Using the above denotations we can state that the value of the searched variables in the dual model reads as follows:

$$y_j = \frac{\text{zlotych}}{\text{unit of the means } i},$$

that is, as much as the consumption of the means i in the production of the items j costs.

Thus we can determine the objective of dual linear programming as minimization of total costs of the production means b_i while assuming that the total value of the production means used for the production of the item j is, at least, as great as the profit obtained due to the production of this item.

One can also use the following theorems (Gass, 1976) in the practical interpretation of dual linear programming.

— Optimum value of the decision variable y_i in the dual problem determines an increase of the maximum value of the function of the objective of the original problem $\Delta Z_{\max}$ at an increase o the limit b_i by the unit, i.e.,

$$y_i = \frac{\Delta Z_{\max}}{\Delta b_i}. \tag{27}$$

— if one of the dual problems has an optimum solution, then the optimum solution has also another solution, while the following equality appears for arbitrary optimum solutions:

$$Z_{\max} = K_{\min}. \tag{28}$$

The practical meaning of the above theorems lies in the fact that the variables y_i in the dual problem determine a degree by which the maximum value of the function of the objective of the original problem changes if the corresponding limit i changes by a unit. The greater value y_i, the greater the influence of the i means on the maximum value of the function of the objective of the original problem is. The decision variables y_i are thus the evaluations of the effectiveness of the unit of the production means i due to the assumed criterion.

The equation (28) denotes the interdependence of the optimization of two different phases of the economic process, i.e. the phase of production and phase of the sale. Thus one can optimize both phases, i.e. minimize the costs of production and maximize the profit.

Besides the mentioned profit obtained from the combined optimization of profit and self-costs, dual linear programming allows us for the following solutions which improve the economic effect:

— choice of the cheapest production means i decreasing thus the total costs and increasing the profit,

— establishment of the hierarchy of individual production means i whose use in the production of items gives the greatest increases in profit.

While making the above choices we use the following interpretation of the obtained dual solution:

— values $y_i = 0$ denote the fact that a given means i for which such a condition was obtained from calculations does not influence the change of the total profit,

— values $y_i > 0$ arranged in the order from the greatest to the least denote the sequence of production means whose consumption brings the greatest profit and their obtaining ought to increase in such a sequence,

— values $y_i > 0$ also denote how much the profit can be increased together with an increase in consumption of the production means i by a unit.

4. Budgeting in a mine by means of dual programming

The essence of budgeting is (RSCC S.A.):

— division of the mine into the centres of responsibility, the so-called cost centres headed by the people responsible for the realization of the appointed tasks,

— making of monthly budgests, i.e. limits of the outlays directed to the concrete decision sections,

— current control of carrying out the tasks included in the budget, establishment of deviations and their elimination,

— application of the responsibility calculus depending on the evaluation of the degree of realization of accepted budgets,

— assessment of workers responsible for carrying out the tasks.

In the case of the mine, the explotation sections are, by no means, included in the main centres of costs both from the point view of the leading function they satisfy and a considerable participation of the costs of the output in the total costs of the mine. Thus the exploitation section has been assumed as an example of the presented method since its organization structure (Fig. 1) reflects the essence of seperation of cost centres while taking into account the successive decision sections in the total organization structure of the mine. The diagram presented in Fig. 1 may concern a single mine, then the analysed explotation section comprises n exploitation faces. It can correspond to the coal company where the exploitation section will be an aggregated conventional centre of costs which includes exploitation sections or faces of individual mines. The above example can also be considered at the level of the whole mining of a given branch.

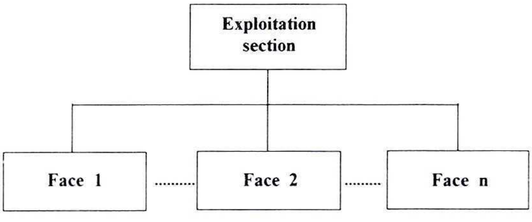


Fig. 1. Faces in the exploitation section

Let us assume then that the following budget for a given period for the stand of costs, that is, the exploitation section in the process of budgeting of the mine has been established:
X — planned volume of the output in the exploitation section, tons,
K_0 — budget of the total costs in the exploitation section, zł,
K_{0j} — budget of costs in individual faces ($j = 1, 2, ..., n$), zł,
and, according to the essence of the budget, the following condition must be satisfied:

$$\sum_{j=1}^{n} K_{0j} \leqslant K_0, \tag{29}$$

that is, the sum of the costs in the individual faces cannot be greater than the budget of the section,
k_j — unit cost of the output in the face j, zł/t.

Budgeting of the costs is not only the distribution of total costs into the established centres (stands of costs) but a necessity of realization of the established internal structure of costs, e.g., the category structure as well. It allows us to control correctly the costs at every arbitrary stage of menagement, in an arbitrary place of their origin and according to their arbitrary section.

The established budget of category costs for the exploitation section can be presented in the diagram of Table.

TABLE

Costs in the category system — exploitation section

No	Category cost	Denotation
A.	Material costs including:	
1.	Amortization	b_1
2.	Materials	b_2
3.	Power energy	b_3
4.	Repair services	b_4
5.	Transport services	b_5
6.	Others	b_6
B.	Non-material costs including:	
7.	Payments with margins	b_7
8.	Special funds	b_8
9.	Bank services	b_9
10.	Business trips	b_{10}
11.	Others	b_{11}
C.	Total costs	B

Obviously, the sum of category costs in the exploitation section has to equal to the budget of costs in this section, that is:

$$\sum_{i=1}^{m} b_i = K_0, \tag{30}$$

where:

b_i — budget i of this category cost on the section ($i = 1, 2, ..., m$)

and

$$B = \sum_{i=1}^{m} b_i \tag{31}$$

and also

$$B \leqslant K_0. \tag{32}$$

It is known that many geological — mining and organization factors whose diversification causes the exploitation cost in individual faces to vary decide about the value of the output cost. Unit costs of single faces will be varied and the category structure of costs in faces will be varied as well, most frequently being different from the established budget of category costs for the whole exploitation section at the same time.

Let us introduce the following denotations:

a_{ij} — unit cost of the i category component in the j face, zł/t,

$\sum_{j=1}^{n} a_{ij}$ — unit cost of the i category component in the exploitation section; zł/t,

and

$\sum_{i=1}^{m} a_{ij}$ — unit cost of the output in the face j, zł/t

that is:

$$k_j = \sum_{i=1}^{m} a_{ij}, \tag{33}$$

moreover:

$\sum_{i=1}^{m} a_{ij} \cdot x_{ij}$ — global costs of the output in the face j, zł,

$\sum_{j=1}^{n} a_{ij} \cdot x_j$ — sum of the costs of the i category in the exploitation section, zł,

x_j — output in the face j, tons.

Thus the in — equality must be satisfied:

$$\sum_{j=1}^{n} a_{ij} \cdot x_j \leqslant b_i, \tag{34}$$

and

$$\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} \cdot x_j \leqslant K_0. \tag{35}$$

Using the above denotations we can formulate a model of original linear programming taking into account the conditions assumed when budgeting the costs in the exploitation section. This model is interpretation in the following way.

Firstly, we want to realize the planned output X in the section:

$$\sum_{j=1}^{n} x_j = X, \tag{36}$$

with the minimum costs, i.e. the function of the objective ought to reach minimum:

$$K = x_1 \cdot k_1 + x_2 \cdot k_2 + \ldots + x_n \cdot k_n = \text{minimum}, \tag{37}$$

and, as a result, the total cost of the output in the exploitation section ought to be not greater than the assumed budget, i.e.:

$$\sum_{j=1}^{n} x_j \cdot k_j \leqslant K_0, \tag{38}$$

with the boundary conditions:

$$x_j \geqslant 0. \tag{39}$$

Secondly, we must keep the assumed budget of category costs in the exploitation section which can be expressed by the indirect conditions

$$\left.\begin{array}{l} a_{11} \cdot x_1 + a_{12} \cdot x_2 + \ldots + a_{1n} \cdot x_n \leqslant b_1 \\ a_{21} \cdot x_1 + a_{22} \cdot x_2 + \ldots + a_{2n} \cdot x_n \leqslant b_2 \\ \cdot \\ \cdot \\ a_{m1} \cdot x_1 + a_{m2} \cdot x_2 + \ldots + a_{mn} \cdot x_n \leqslant b_m \end{array}\right\} . \tag{40}$$

The above in-equalities denote that not only we have imposed the condition of non-exceeding the budget of category costs in the whole exploitation section but, at the same time, we assume a possibility of their decrease through optimization of the category structure of costs in individual exploitation faces.

To illustrate a general model of linear programming let us assume a selected exploitation section of the following data:

— assumed 24-hour output from the section, $X = 12\,000$ t,

— in the section there work three exploitation longwalls for which the analysis of costs showed:

a) unit cost of the exploitation of the longwall 1; $k_1 = 120$ zł/t,

b) unit cost of the exploitation of the longwall 2; $k_2 = 110$ zł/t,

c) unit cost of the exploitation of the longwall 3; $k_3 = 100$ zł/t,

— structure of 24 hour category costs of the exploitation section reads as follows (limited to the two basic category groups)

a) material costs — 414 000 zł/d,

b) non-material costs — 966 000 zł/d,

c) total 24 hour cost of the section — 1 380 000 zł/d,

d) mean unit cost of the section — 115 zł/t,

it means that $b_1 = 414\,000$; $b_2 = 966\,000$.

According to the assumed formula (36), we are to realize the planned output, i.e.:

$$x_1 + x_2 + x_3 = 12\,000. \tag{41}$$

Boundary conditions (39) impose:

$$x_1 \geqslant 0; \; x_2 \geqslant 0, \; x_3 \geqslant 0.$$

Moreover, we assume that in all three longwalls there is a great span of carrying one the concentration so:

$$x_1 \leqslant 12000;\ x_2 \leqslant 12000;\ x_3 \leqslant 12000 \tag{42}$$

According to the analysis of data from the section, the unit costs of the 1 st and 2 nd category components a_{ij} in individual faces are:

a) in the face 1
   unit material cost — $a_{11} = 30$ zł/t,
   unit non-material cost — $a_{21} = 90$ zł/t,

b) in the face 2
   unit material cost — $a_{12} = 32$ zł/t,
   unit non-material cost — $a_{22} = 78$ zł/t,

c) in the face 3
   unit material cost — $a_{13} = 36$ zł/t,
   unit non-material cost — $a_{23} = 64$ zł/t.

The above unit cost must confirm the formula (33), i.e.:

$$\begin{aligned} k_1 &= a_{11}+a_{21} = 30+90 = 120 \\ k_2 &= a_{12}+a_{22} = 32+78 = 110 \\ k_3 &= a_{13}+a_{23} = 36+64 = 100 \end{aligned}$$

The presented data help to record the original model of linear programming for the selected exploitation section in the following way:

We want to minimize the total costs of the output 12000 t/24 hr, i.e.

$$K = 120 \cdot x_1 + 110 \cdot x_2 + 100 \cdot x_3 = \text{minimum}.$$

The indirect conditions of the model (Formula 40), after taking into account the conditions (41 and 42), are presented in the following way:

$$\begin{array}{rrrl} 30 \cdot x_1 + & 32 \cdot x_2 + & 36 \cdot x_3 & \leqslant 414000 \\ 90 \cdot x_1 + & 78 \cdot x_2 + & 64 \cdot x_3 & \leqslant 966000 \\ x_1 + & x_2 + & x_3 & \leqslant 12000 \\ x_1 & & & \leqslant 12000 \\ & x_2 & & \leqslant 12000 \\ & & x_3 & \leqslant 12000 \end{array}$$

and boundary conditions according to the formula (39):

$$x_1 \geqslant 0;\ x_2 \geqslant 0;\ x_3 \geqslant 0.$$

Since the presented original model of linear programming minimizes the costs of the output in the faces and exploitation section, then the dual model, according to the assumption, ought to maximize a new function of the objective.

This fact follows the basic condition of original and dual linear programming expressed by the formula (28) that means that maximization of the original function is synonymus with minimization of the dual function and, obviously, *vice versa*. Thus in our case the function of the objective of dual linear programming reads as follows

$$Z = b_1 y_1 + b_2 y_2 = \text{maximum},$$

meaning that for the values b_i assumed earlier

$$Z = 414\,000\, y_1 + 966\,000\, y_2 = \text{maximum}.$$

The following in-equalities express the conditions of the dual model:

$$\begin{aligned} a_{11} \cdot y_1 + a_{21} \cdot y_2 &\leqslant k_1 \\ a_{12} \cdot y_1 + a_{22} \cdot y_2 &\leqslant k_2 \\ a_{13} \cdot y_1 + a_{23} \cdot y_2 &\leqslant k_3 \end{aligned}$$

thus, taking into account, the values of the coefficients of the presented transported matrix $\boldsymbol{A}$':

$$\begin{aligned} 30 \cdot y_1 + 90 \cdot y_2 &\leqslant 120 \\ 32 \cdot y_1 + 78 \cdot y_2 &\leqslant 110 \\ 36 \cdot y_1 + 64 \cdot y_2 &\leqslant 100 \end{aligned}$$

The values y_i, are the searched variables in the dual model thus, according to the interpretation included in the point 3 of the paper, they determine how much of the proper category cost in the course of the planned output ought to be used to obtain minimization of the total costs with the simultaneous optimization of individual category costs. A more detailed economic interpretation of the dual solution presented in the point 3, in the case of the above model, requires inclusion of the dual maximization of the function of the objective.

5. Conclusions

Budgeting is a more and more frequently applied method of management in mines, especially, management of costs. It is an effective method of disciplining the costs at the separated stands and, at the same time, helping to release an initiative of the people undertaking decisions.

The application of dual linear programming helps, moreover, to enrich budgeting due to the possibility of obtaining optimum solutions in two most important elements of effectiveness of the activity of a mine, i.e. optimization of total costs and their category structure.

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