Archives of Mining Sciences 45, 1 (2000) 3—22

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ROCK BURSTS IN THE LIGHT OF THE CATASTROPHE THEORY

ZJAWISKO TĄPNIĘCIA W ŚWIETLE TEORII KATASTROF

Rock burst phenomenon because of its jump like character may be considered as a process of the loss of stability and using the catastrophe theory it is possible to define conditions of such instability. Analysis of some geomechanical models shows two mechanisms of rock burst: rock burst as the catastrophic jump or rock burst as a loss of bearing capacity of a seam. Conditions of rock burst existence were obtained for three and uni-axial stress states and additionally for a case when roof and floor rocks were considered as the rheological medium.

Key words: geomechanics, rheology, rock burst, coal bump.

W polskim górnictwie węgla kamiennego oraz rud miedzi tąpania są głównym zagrożeniem naturalnym, a wiedzę o warunkach ich powstania trudno uznawać za kompletną; odnosi się to zarówno do problemu genezy tego zjawiska, jak też zagadnień prognozowania i zwalczania. Z geomechanicznego punktu widzenia zasadniczą cechą tąpnięcia jest nagłe ⇒ skokowe przejście z jednego stanu równowagi do drugiego, co oznacza, że tąpnięcie można utożsamiać z procesem utraty stateczności skał otaczających wyrobisko, a jako kryterium przyjmować warunki niestateczności rozwiązań opisujących zachowanie się odpowiednich modeli geomechanicznych. Metodą pozwalającą na analizowanie warunków powstawania skokowych zmian stanu równowagi jest teoria katastrof. Ponieważ metoda energetyczna może być wykorzystywana do opisu zmian zachodzących w ośrodku odkształcalnym, toteż akumulowaną w odpowiednich układach geomechanicznych energię przyjęto do analizy istnienia przemian katastroficznych.

Analizę fizycznej strony procesu oparto o możliwie proste geomechaniczne modele układu zbudowanego z połączonych szeregowo elementów liniowo odkształcalnych (w tym także cechujących się właściwościami reologicznymi), które charakteryzowały warstwy stropowe i spągowe oraz z nieliniowo odkształcalnej calizny. Na podstawie analizy przemian energetycznych zdefinowane zostały warunki wystąpienia niestateczności (czyli tąpnięcia), a mianowicie: tąpnięcie wskutek przeskoku lub tąpnięcie wskutek utraty nośności. Na podstawie otrzymanych warunków wykazano, że:

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1. Zjawisko utraty stateczności utożsamiane z procesem tąpnięcia (dla obu jego mechanizmów) nie jest wyłącznie cechą jednoosiowego stanu naprężenia, lecz również może zachodzić dla stanów trójosiowych.

2. Z punktu widzenia możliwości wystąpienia tąpnięcia najbardziej niekorzystnym jest jednoosiowy stan naprężenia, co wynika z dwóch przyczyn:

— niezależnie od mechanizmu stan jednoosiowy jest najmniej chłonny energetycznie,

— dla jednoosiowego stanu naprężenia maksymalne wartości modułów pozniszczeniowych decydujących o wystąpieniu niestateczności są większe niż dla stanów trójosiowych, a zatem istnieje większe prawdopodobieństwo wystąpienia przeskoku.

3. Występowanie skokowych zmian stanu równowagi jest cechą układu, a konkretnie istnieniem określonych relacji między własnościami odkształceniowymi warstw stropowych i spągowych oraz pozniszczeniowymi własnościami pokładu (ściśle rzecz biorąc elementu ulegającego zniszczeniu). Z tej przyczyny tąpiącym nie może być węgiel, czy (w przypadku górnictwa rud miedzi) określona skała; zatem pojęcie naturalnej skłonności skały (węgla) do tąpań, które wynika ze stanu wiedzy sprzed ponad ćwierć wieku, nie mając żadnego fizykalnego uzasadnienia jest aktualnie anachronizmem.

Z punktu widzenia minimalizacji wielkości zagrożenia tąpaniami ze wszech miar korzystne są działania wykonywane w utworach stropowych lub spągowych prowadzące do uaktywniania się w nich procesów dyssypacyjnych. Dzięki temu zmniejsza się zdolność tych utworów do oddawania energii i w efekcie może dojść do transformacji układu z tąpiącego w nietąpiący.

Słowa kluczowe: geomechanika, reologia, tąpania.

1. Introduction

The first works which attempted to define the conditions for potential rock burst occurrence were phenomenological rock burst theories (Parysiewicz, 1967) providing the fundamental relations based mainly on the observations made in mines. These phenomenological theories were marked with certain verbalism — i. e. the absence of quantitative criteria, hence they were of little practical use in mining where quantitative prognoses are required. On the other hand they may be used as the basis for evaluating the theories which give the quantitative criteria for rock burst occurrence. Following the assumption that one consequence of rock bursts (unlike the explosions within the strata) is destruction of mining workings, the phenomenological theories (Filcek et al., 1984, Parysiewicz, 1967) allow to distinguish:

a) Rock bursts due to stress concentration, being the consequence of:

— appearance of the fracture zone in the part of seam adjacent to the workings,

— ability of floor, roof and virgin rock to accumulate the sufficient amount of elastic energy,

— specific relations between the stress — strain characteristics and the geometry of rock surrounding the excavations.

b) Rock bursts due to dynamic loading, which may occur in certain conditions:

— appearance of the fracture zone in the part of seam adjacent to the workings,

— ability of floor, roof and virgin rock to accumulate the sufficient amount of elastic energy,

— there are specific relations between the stress-strain characteristics and geometrical parameters of rock round the excavations,

— occurrence of mining tremor of high seismic energy,

— the source of mining tremor should be near the excavations since the kinetic energy reaching the excavation depends on its distance from the source.

In the light of geomechanics, the distinctive feature of rock burst is a sudden, violent transition from one state of equilibrium to another. According to the definition (Filcek at al., 1984) — which is our starting point — a rock burst is "a physical explosion within the strata round the excavation leading to its failure", and "the physical explosion is the process of rapid change of the state of equilibrium requiring mechanical work and accompanied by acoustic effects" (Leksykon, 1984). Accordingly, rock bursts are associated with the loss of stability of rocks surrounding the excavation (Salamon, 1970, Zorychta, Kłeczek, 1998, Zorychta, 1984, Zorychta, 1988), and the conditions for instability of solutions describing the geomechanical models can be used as the rock bursts criteria.

The method allowing to analyse the conditions for violent changes of system equilibrium (it follows from the definition that a rock burst is such an violent change) is the catastrophe theory (Awrejcewicz, 1996, Geresz, 1980, Poston, Steward, 1978, Thompson, 1982), which claims that for the function $\Phi(x_1, x_2, ..., x_n)$ describing the physical system there exists a critical point $u_k(x_{1_k}, ..., x_{n_k})$ if

$$\left.\frac{\partial \Phi}{\partial x_1}\right|_{u_k} = \left.\frac{\partial \Phi}{\partial x_2}\right|_{u_k} = ... = \left.\frac{\partial \Phi}{\partial x_n}\right|_{u_k} = 0$$

and if additionally

$$\det\left[\left.\frac{\partial^2 \Phi}{\partial x_i \partial x_j}\right|_{u_k}\right] = 0,$$

then it is named the degenerated critical point or the catastrophe point. For the function of one variable $\Phi(x)$ the condition for the existence of a catastrophe point (meaning the possibility of violent transitions from one state of equilibrium to another) is given by the system of equations:

$$\frac{d\Phi(x)}{dx} = 0 \quad \text{and} \quad \frac{d^2\Phi(x)}{dx^2} = 0.$$

The mechanics of deformable bodies offers a number of methods to describe the changes due to outside interactions. One of these is the energy- based method, where the state of the system is defined by the amount of accumulated energy, which means that energy may be the function analysed to find whether a catastrophe point is possible.

2. Geomechanical model of rock burst systems

The analysis of the physical aspects of the process is confined to the simplest model of the system (Fig. 1) made of linearly deformable roof and floor strata (S) and non-linearly deformable unmined coal seam (N),
where the vertical stress — strain relations is given by the function $\sigma_n = f(\varepsilon_n, \eta_n)$ (Fig. 2).

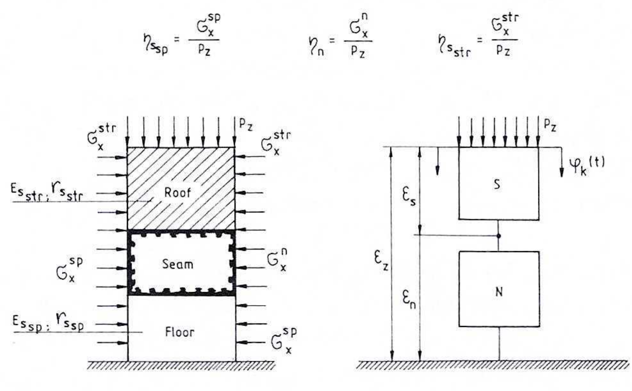

Fig. 1. The scheme of the geomechanical model for three-axial state of stress

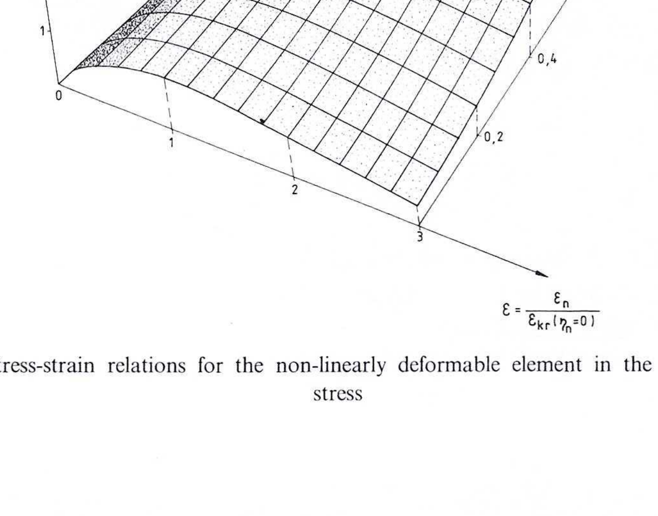

Fig. 2. Vertical stress-strain relations for the non-linearly deformable element in the three-axial state of stress

The results of laboratory tests (Zorychra, 1988) indicate that for the function $f(\varepsilon_n, \eta_n)$ we can assume

$$\text{— for } 0 \leqslant \varepsilon_n \leqslant \varepsilon_{kr}(\eta_n)\colon \frac{\partial f(\varepsilon_n, \eta_n)}{\partial \varepsilon_n} \geqslant 0 \tag{1}$$

$$\text{— for } \varepsilon_{kr}(\eta_n) < \varepsilon_n < \infty\colon \frac{\partial f(\varepsilon_n, \eta_n)}{\partial \varepsilon_n} < 0 \tag{2}$$

$$\text{— for } 0 \leqslant \varepsilon_n < \infty\colon \frac{\partial f(\varepsilon_n, \eta_n)}{\partial \eta_n} > 0, \tag{3}$$

where:

$\varepsilon_{kr}(\eta_n)$ — strain value at which failing begins — corresponding to the state of stress described with the parameter η_n.

When these two elements are connected in series, the following relations will be valid:

$$\sigma_z = \sigma_s = \sigma_n$$
$$\frac{\partial^{(i)}\sigma_z}{\partial t^{(i)}} = \frac{\partial^{(i)}\sigma_s}{\partial t^{(i)}} = \frac{\partial^{(i)}\sigma_n}{\partial t^{(i)}} \tag{4}$$

$$\varepsilon_z = \varepsilon_s + \varepsilon_n$$
$$\frac{\partial^{(i)}\varepsilon_z}{\partial t^{(i)}} = \frac{\partial^{(i)}\varepsilon_s}{\partial t^{(i)}} + \frac{\partial^{(i)}\varepsilon_n}{\partial t^{(i)}}. \tag{5}$$

Energy accumulated in the system considered here is equal to:

$$A(\varepsilon_n, \eta_n) = \frac{[f(\varepsilon_n, \eta_n) - p_z]^2}{2E_s} + \int_{\varepsilon_{n0}}^{\varepsilon_n} [f(\xi, \eta_n) - p_z]\, d\xi, \tag{6}$$

where:

$$E_s = \frac{E_{s_{str}} E_{s_{sp}}}{(1 - 2v_{s_{sp}}\eta_{s_{sp}}) E_{s_{str}} + (1 - 2v_{s_{sp}}\eta_{s_{str}}) E_{s_{sp}}},$$

$E_{s_{str}}$, $E_{s_{sp}}$ — modulus of elasticity for the roof and floor, respectively,

$v_{s_{str}}$, $v_{s_{sp}}$ — Poisson's ratio for the roof and floor, respectively.

The co-ordinate $\{\varepsilon_{n_k}, \eta_{n_k}\}$ of the catastrophe point is derived from the conditions:

$$\frac{\partial A(\varepsilon_{n_k}, \eta_{n_k})}{\partial \varepsilon_n} = 0,$$
$$\frac{\partial A(\varepsilon_{n_k}, \eta_{n_k})}{\partial \eta_n} = 0, \tag{7}$$
$$\Delta(\varepsilon_{n_k}, \eta_{n_k}) = 0,$$

where:

$$\Delta(\varepsilon_n,\eta_n)=\frac{\partial^2 A(\varepsilon_n,\eta_n)}{\partial\varepsilon_n^2}\cdot\frac{\partial^2 A(\varepsilon_n,\eta_n)}{\partial\eta_n^2}-\left[\frac{\partial^2 A(\varepsilon_n,\eta_n)}{\partial\varepsilon_n\partial\eta_n}\right]^2,$$

$$\frac{\partial A(\varepsilon_n,\eta_n)}{\partial\varepsilon_n}=[f(\varepsilon_n,\eta_n)-p_z]\cdot\left[\frac{1}{E_s}\frac{\partial f(\varepsilon_n,\eta_n)}{\partial\varepsilon_n}+1\right],$$

$$\frac{\partial A(\varepsilon_n,\eta_n)}{\partial\eta_n}=[f(\varepsilon_n,\eta_n)-p_z]\cdot\frac{1}{E_s}\frac{\partial f(\varepsilon_n,\eta_n)}{\partial\eta_n}+\int\limits_{\varepsilon_{n0}}^{\varepsilon_n}\frac{\partial f(\xi,\eta_n)}{\partial\eta_n}d\xi,$$

$$\frac{\partial^2 A(\varepsilon_n,\eta_n)}{\partial\varepsilon_n^2}=\frac{\partial f(\varepsilon_n,\eta_n)}{\partial\varepsilon_n}\cdot\left[\frac{1}{E_s}\frac{\partial f(\varepsilon_n,\eta_n)}{\partial\varepsilon_n}+1\right]+[f(\varepsilon_n,\eta_n)-p_z]\cdot\frac{1}{E_s}\cdot\frac{\partial^2 f(\varepsilon_n,\eta_n)}{\partial\varepsilon_n^2},$$

$$\frac{\partial^2 A(\varepsilon_n,\eta_n)}{\partial\eta_n^2}=\frac{1}{E_s}\cdot\left[\frac{\partial f(\varepsilon_n,\eta_n)}{\partial\eta_n}\right]^2+[f(\varepsilon_n,\eta_n)-p_z]\cdot\frac{1}{E_s}\cdot\frac{\partial^2 f(\varepsilon_n,\eta_n)}{\partial\eta_n^2}+\int\limits_{\varepsilon_{n0}}^{\varepsilon_n}\frac{\partial^2 f(\xi,\eta_n)}{\partial\eta_n^2}d\xi,$$

$$\frac{\partial^2 A(\varepsilon_n,\eta_n)}{\partial\varepsilon_n\partial\eta_n}=\frac{\partial f(\varepsilon_n,\eta_n)}{\partial\eta_n}\cdot\left[\frac{1}{E_s}\cdot\frac{\partial f(\varepsilon_n,\eta_n)}{\partial\varepsilon_n}+1\right]+[f(\varepsilon_n,\eta_n)-p_z]\cdot\frac{1}{E_s}\cdot\frac{\partial^2 f(\varepsilon_n,\eta_n)}{\partial\varepsilon_n\partial\eta_n}.$$

Taking into account

$$\int\limits_{\varepsilon_{n0}}^{\varepsilon_n}\frac{\partial^2 f(\xi,\eta_n)}{\partial\eta_n^2}d\xi=\frac{\partial}{\partial\eta_n}\int\limits_{\varepsilon_{n0}}^{\varepsilon_n}\frac{\partial f(\xi,\eta_n)}{\partial\eta_n}d\xi$$

and the conditions for the critical point occurrence

$$[f(\varepsilon_n,\eta_n)-p_z]\cdot\left[\frac{1}{E_s}\frac{\partial f(\varepsilon_n,\eta_n)}{\partial\varepsilon_n}+1\right]=0, \tag{8}$$

$$[f(\varepsilon_n,\eta_n)-p_z]\cdot\frac{1}{E_s}\frac{\partial f(\varepsilon_n,\eta_n)}{\partial\eta_n}+\int\limits_{\varepsilon_{n0}}^{\varepsilon_n}\frac{\partial f(\xi,\eta_n)}{\partial\eta_n}d\xi=0, \tag{9}$$

we get:

$$\int\limits_{\varepsilon_{n0}}^{\varepsilon_n}\frac{\partial^2 f(\xi,\eta_n)}{\partial^2\eta_n}d\xi=-\frac{1}{E_s}\left[\frac{\partial f(\varepsilon_n,\eta_n)}{\partial\eta_n}\right]^2-[f(\varepsilon_n,\eta_n)-p_z]\cdot\frac{1}{E_s}\cdot\frac{\partial^2 f(\varepsilon_n,\eta_n)}{\partial\eta_n^2}.$$

As the result:

$$\Delta=-\left\{\left[\frac{\partial f(\varepsilon_n,\eta_n)}{\partial\eta_n}\right]^2\cdot\left[\frac{1}{E_s}\cdot\frac{\partial f(\varepsilon_n,\eta_n)}{\partial\varepsilon_n}+1\right]^2+[f(\varepsilon_n,\eta_n)-p_z]^2\cdot\frac{1}{E_s^2}\cdot\left[\frac{\partial^2 f(\varepsilon_n,\eta_n)}{\partial\varepsilon_n\partial\eta_n}\right]^2\right\}. \tag{10}$$

Considering the form of the conditions (8) and (10) we have to analyse three situations:

1. When we assume that for $\varepsilon_{n0}(\eta_n) < \varepsilon_n < \infty$ the following inequality is satisfied:

$$[f(\varepsilon_n, \eta_n) - p_z] > 0, \tag{11}$$

the co-ordinate $\{\varepsilon_{n_k}, \eta_{n_k}\}$ of the catastrophe point can be obtained from the system of equations:

$$\begin{aligned} \frac{1}{E_{s_k}} \cdot \frac{\partial f(\varepsilon_{n_k} \cdot \eta_{n_k})}{\partial \varepsilon_n} + 1 = 0, \\ \frac{\partial^2 f(\varepsilon_{n_k}, \eta_{n_k})}{\partial \varepsilon_n \partial \eta_n} = 0. \end{aligned} \tag{12}$$

The very existence of the catastrophe point $\{\varepsilon_{n_k}, \eta_{n_k}\}$ indicates that there might be sudden, violent changes of the state of equilibrium of the system, furthermore, those changes will appear in the post-failure regime of the stress-strain characteristics $\varepsilon_{n_k} > \varepsilon_{kr}(\eta_{n_k})$. Such instability is called a catastrophic jump, and the violent changes of the state of equilibrium will proceed for $0 \leqslant \eta_n < \eta_{n_k}$; it may also occur for three-axial stress states. We have to bear in mind that the occurrence of a catastrophic jump depends on the properties of the system $\{f(\varepsilon_n, \eta_n), E_s\}$. Accordingly, a whole system may be prone to bursts, while separate elements of the system never have that property.

2. If we assume that for $\varepsilon_{n0}(\eta_n) \leqslant \varepsilon_n < \infty$

$$\frac{1}{E_s} \frac{\partial f(\varepsilon_n, \eta_n)}{\partial \varepsilon_n} + 1 > 0, \tag{13}$$

hence for the critical point:

$$\begin{aligned} f(\varepsilon_{n_n}, \eta_{n0}) - p_z = 0, \\ \Delta = -\left[\frac{\partial f(\varepsilon_{n_n}, \eta_{n_n})}{\partial \eta_n}\right]^2 \cdot \left[\frac{1}{E_s} \cdot \frac{\partial f(\varepsilon_{n_n}, \eta_{n_n})}{\partial \varepsilon_n} + 1\right]^2, \end{aligned} \tag{14}$$

which means that:

$$\Delta(\varepsilon_{n_n}, \eta_{n_n}) < 0. \tag{15}$$

In this case the critical point $\{\varepsilon_{n_n}, \eta_{n_n}\}$ is not the catastrophe point, therefore the state of equilibrium of the system may not undergo and rapid, violent changes. However, as:

$$\begin{aligned} \frac{\partial A(\varepsilon_{n_n}, \eta_{n_n})}{\partial \varepsilon_n} = 0, \\ \Delta(\varepsilon_{n_n}, \eta_{n_n}) < 0, \end{aligned} \tag{16}$$

the energy, which the system can accumulate, reaches its peak value at the critical point $\varepsilon_{n_n} > \varepsilon_{kr}(\eta_{n_n})$. As a result, for the strains $\varepsilon_n > \varepsilon_{n_n}$ there is loss of stability due to the fact that the external energy supplied to the system is greater than the system can absorb. This type of instability will be called the loss of a bearing capacity of the system.

3. If we assume that for $\varepsilon_{n0}(\eta_n) < \varepsilon_n < \infty$ the following inequalities are satisfied:

$$f(\varepsilon_n, \eta_n) - p_z > 0, \tag{17}$$

$$\frac{1}{E_s}\frac{\partial f(\varepsilon_n, \eta_n)}{\partial \varepsilon_n} + 1 > 0, \tag{18}$$

then

$$\frac{\partial A(\varepsilon_{n_n}, \eta_{n_n})}{\partial \varepsilon_n} > 0. \tag{19}$$

That means that a critical point (and the catastrophe point) will not exist in this case. In other words, regardless of the value of the supplied energy no loss of stability will occur because the energy absorbed by the system is the increasing function of strain. In this case no rock burst will occur.

As in the catastrophe theory the conditions for instabilities — the necessary condition for rock bursts are defined, we can clearly see two mechanisms of this process:

— rock bursts due to catastrophic jumps,

— rock bursts due to the loss of bearing capacity of the seam.

The uni-axial case (Fig. 3) where $\eta_n = 0$ (the element non-linearly deformable, characterising the properties of the seam will be referred to as the pseudo-elastic element) will be thoroughly analysed, especially in view of the physical aspects.

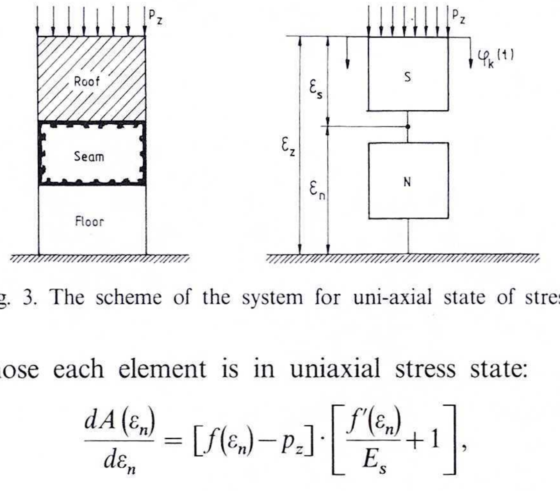

Fig. 3. The scheme of the system for uni-axial state of stress

For the system whose each element is in uniaxial stress state:

$$\frac{dA(\varepsilon_n)}{d\varepsilon_n} = [f(\varepsilon_n) - p_z] \cdot \left[\frac{f'(\varepsilon_n)}{E_s} + 1\right], \tag{20}$$

$$\frac{d^2A(\varepsilon_n)}{d\varepsilon_n^2} = f'(\varepsilon_n) \cdot \left[\frac{f'(\varepsilon_n)}{E_s} + 1\right] + [f(\varepsilon_n) - p_z] \cdot \frac{f''(\varepsilon_n)}{E_s}, \tag{21}$$

where:

$$f'(\varepsilon_n) = \frac{df(\varepsilon_n)}{d\varepsilon_n},$$

— for $0 \leqslant \varepsilon_n \leqslant \varepsilon_{kr}$: $\dfrac{df(\varepsilon_n)}{d\varepsilon_n} \geqslant 0,$

— for $\varepsilon_{kr} < \varepsilon_n < \infty$: $\dfrac{df(\varepsilon_n)}{d\varepsilon_n} < 0.$

As the conditions defining the ocurrence of the catastrophe point have the form:

$$\frac{dA(\varepsilon_n)}{d\varepsilon_n} = 0 \quad \text{and} \quad \frac{d^2A(\varepsilon_n)}{d\varepsilon_n^2} = 0. \tag{22}$$

Accordingly, at that point:

$$\begin{gathered} [f(\varepsilon_n)-p_z]\cdot\left[\frac{f'(\varepsilon_n)}{E_s}+1\right] = 0, \\ f'(\varepsilon_n)\cdot\left[\frac{f'(\varepsilon_n)}{E_s}+1\right]+[f(\varepsilon_n)-p_z)\frac{f''(\varepsilon_n)}{E_s} = 0. \end{gathered} \tag{23}$$

Three cases will be considered here, as in previous sections.

1. If we assume that for $\varepsilon_{no} \leqslant \varepsilon_n < \infty$ the inequality is satisfied (Fig. 4 a):

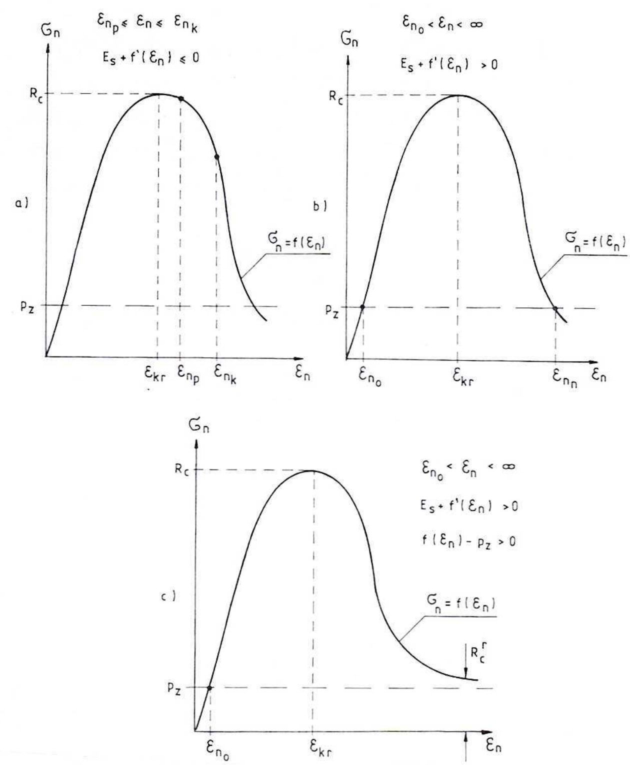

Fig. 4. Vertical stress-strain relation for uni-axial state of stress

$$f(\varepsilon_n) - p_z > 0, \tag{24}$$

then the system of equations allowing to determine the co-ordinates of the catastrophe point $\{\varepsilon_{n_k}, E_{s_k}\}$ have the form:

$$\frac{f'(\varepsilon_{n_k})}{E_{s_k}} + 1 = 0, \qquad f''(\varepsilon_{n_k}) = 0. \tag{25}$$

It follows from the first equation that the inequality $f'(\varepsilon_{n_k}) < 0$ must be satisfied for the catastrophe point, hence the catastrophic jump may occur only in post-failure regime. It can be demonstrated that for a catastrophe system the possibility for catastrophic jump occurrence is not related to the catastrophe point only, but also to a certain range of strain values: $\varepsilon_{kr} < \varepsilon_{n_p} \leqslant \varepsilon_{n_k}$, on the condition that:

$$\frac{f'(\varepsilon_{n_p})}{E_{s_p}} + 1 = 0, \qquad E_{s_p} \leqslant E_{s_k}. \tag{26}$$

The changes of strain ε_n in the non-linear term present in the function of total strain are presented in Fig. 5 while the stress variations σ_n are given in Fig. 6.
The processes taking place within the rock medium result from rock displacement due to mining activities, accordingly we get the problem of kinematic disturbances [14], where the total strain is a function of time $\varepsilon_z = \varphi_k(t)$. Since these relations are valid:

$$\frac{d\varepsilon_z}{d\varepsilon_n} = \frac{f'(\varepsilon_n)}{E_s} + 1,$$

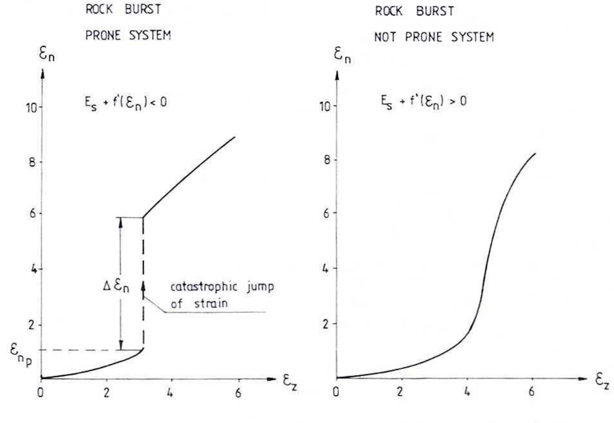

Fig. 5. Vertical strain in the non-linear element as the function of the external strain

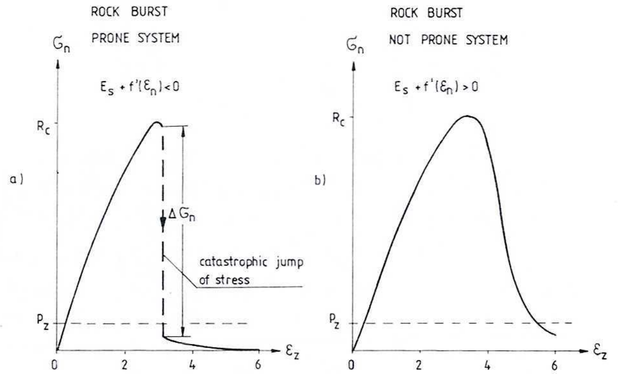

Fig. 6. Vertical stress in the non-linear element as the function of the external strain

$$\frac{d\varepsilon_n}{dt} = \frac{d\varepsilon_n}{d\varepsilon_z}\frac{d\varepsilon_z}{dt},$$

$$\frac{d\varepsilon_z}{dt} = \frac{d\varphi_k(t)}{dt}$$

and additionally for the moment the catastrophic jump occurs the equation (26) is satisfied, therefore we get:

$$\lim_{t \to t_p} \frac{d\varepsilon_n(t)}{dt} = \infty. \tag{27}$$

Accordingly, at the moment of the catastrophic jump the strain rate in the pseudo-elastic element tends to infinity, the rate of total strain changes being still finite. We have to emphasise that this condition (27) for a rock burst occurrence due to the catastrophic jump may be used for analysing more complex geomechanical systems, such as three-axial stress systems and systems involving rheological elements.

2. When we assume that for $\varepsilon_{n_0} \leqslant \varepsilon_n < \infty$ the following inequality (Fig. 4 b) is satisfied:

$$\frac{f'(\varepsilon_n)}{E_s} + 1 > 0, \tag{28}$$

then at the point $\varepsilon_{kr} < \varepsilon_n < \varepsilon_{n_n}$:

$$f(\varepsilon_{n_n}) - p_z = 0 \tag{29}$$

and additionally:

$$f'(\varepsilon_{n_n}) \cdot \left[\frac{f'(\varepsilon_{n_n})}{E_s} + 1\right] < 0. \tag{30}$$

As the result:

$$\frac{dA(\varepsilon_{n_n})}{d\varepsilon_n} = 0 \quad \text{and} \quad \frac{d^2 A(\varepsilon_{n_n})}{d\varepsilon_n^2} < 0. \tag{31}$$

Hence the point $\varepsilon_n = \varepsilon_{n_n}$ is not a catastrophe point, in this case no catastrophic jump may occur. However, at the point $\varepsilon_n = \varepsilon_{n_n}$ the energy absorbed by the system reaches its maximum value, so for the strains $\varepsilon_n > \varepsilon_{n_n}$ there will be a loss of stability as the absorbing ability of the system is thus exceeded. We have to emphasise that in this case also the loss of stability occurs in the post-failure regime because $\varepsilon_{n_n} > \varepsilon_{kr}$.

3. When we assume that for $\varepsilon_{n0} < \varepsilon_n < \infty$ the following inequality (Fig. 4 c) is satisfied:

$$f(\varepsilon_n) - p_z > 0,$$
$$\frac{1}{E_s}\frac{df(\varepsilon_n)}{d\varepsilon_n} + 1 > 0, \tag{32}$$

hence:

$$\frac{dA(\varepsilon_n)}{d\varepsilon_n} > 0, \tag{33}$$

then there will be no critical point (nor the catastrophe point). That means that regardless of the actual amount of energy supplied to the system there will be no loss of stability as the energy, wich the system can absorb, is the incresing function of strain.

Thus derived formulas define the necessary conditions for a rock burst explained by the given mechanism. In the light of the sufficient condition for a burst, the external energy supplied to the system must exceed the maximal energy the system can absorb for the given mechanism, that is:

$$A_z > A_{n_i}^{\max}\{i = p, n\}, \tag{34}$$

while the expressions defining the maximal energy have the form:

— for the catastrophic jump

$$A_{n_p}^{\max} = \frac{[f(\varepsilon_{n_p}, \eta_{n_p}) - p_z]^2}{2E_s} + \int_{\varepsilon_{n0}}^{\varepsilon_{n_p}} [f(\xi, \eta_{n_p}) - p_z]\, d\xi, \tag{35}$$

— for the loss of a bearing capacity

$$A_{n_n}^{\max} = \frac{[f(\varepsilon_{n_n}, \eta_{n_n}) - p_z]^2}{2E_s} + \int_{\varepsilon_{n0}}^{\varepsilon_{n_n}} [f(\xi, \eta_{n_n}) - p_z]\, d\xi. \tag{36}$$

As the rocks possess certain properties of rheological media, let us consider the system made of a pseudo-elastic element modelling the seam (N) and a rheological elements (R) simulating the work of roof and floor strata — assuming that each elements is in uni-axial state of stress.

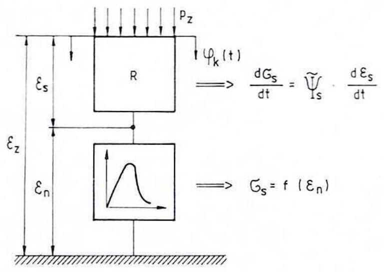

Fig. 7. The scheme of a system including a rheological element

The equation of state for the rheological element can be written as (Derski, Ziemba, 1968, Kłeczek, 1994, Rabotnow, 1977):

$$\sigma_s(t) = \int_0^t \tilde{\Psi}_s(\zeta) \frac{d\varepsilon_s}{d\zeta} d\zeta \tag{37}$$

That means:

$$\frac{d\sigma_s}{dt} = \tilde{\Psi}_s(t) \frac{d\varepsilon_s}{dt}$$

where:

$\tilde{\Psi}_s(t)$ — the function of stress relaxation

$$\tilde{\Psi}_s(t) > 0$$

After the transformations arising from the conditions (4), (5), we obtain:

$$\frac{dA}{d\varepsilon_n} = [f(\varepsilon_n) - p_z] \cdot \left[\frac{f'(\varepsilon_n)}{\tilde{\Psi}_s(t)} + 1\right] \tag{38}$$

$$\frac{d^2 A}{d\varepsilon_n^2} = f'(\varepsilon_n) \cdot \left[\frac{f'(\varepsilon_n)}{\tilde{\Psi}_s(t)} + 1\right] + [f(\varepsilon_n) - p_z] \cdot \frac{f''(\varepsilon_n)}{\tilde{\Psi}_s(t)} \tag{39}$$

As the system of equations determining the existence of the catastrophe point has the form:

$$\begin{gathered} [f(\varepsilon_n) - p_z] \cdot \left[\frac{f'(\varepsilon_n)}{\tilde{\Psi}_s(t)} + 1\right] = 0 \\ f'(\varepsilon_n) \cdot \left[\frac{f'(\varepsilon_n)}{\tilde{\Psi}_s(t)} + 1\right] + [f(\varepsilon_n) - p_z] \cdot \frac{f''(\varepsilon_n)}{\tilde{\Psi}_s(t)} = 0 \end{gathered} \tag{40}$$

then, like before, we will consider two cases determining the possibility of a catastrophic jump or the loss of bearing capacity.

1. When we assume that for $\varepsilon_{n_0} \leqslant \varepsilon_n < \infty$ the following inequality is satisfied:

$$f(\varepsilon_n) - p_z > 0 \tag{41}$$

then the system of equations allowing to obtain the co-ordinate of the catastrophe point $\{\varepsilon_{n_k}, \tilde{\Psi}_{s_k}(t)\}$ has the form:

$$\begin{aligned} &\frac{f'(\varepsilon_{n_k})}{\tilde{\Psi}_{s_k}(t)} + 1 = 0 \\ &f''(\varepsilon_{n_k}) = 0 \end{aligned} \tag{42}$$

This case defines the possibility of a catastrophic jump occurrence. In the case of a system including a rheological element we can demonstrate that a catastrophe jump will occur when:

$$\lim_{t \to t_p} \frac{d\varepsilon_n(t)}{dt} = \infty$$

2. When we assume that for $\varepsilon_{n_0} \leqslant \varepsilon_n < \infty$ the following inequality is satisfied:

$$\frac{f'(\varepsilon_n)}{\tilde{\Psi}_s(t)} + 1 > 0 \tag{43}$$

therefore the existence of a catastrophe point $\varepsilon_{kr} < \varepsilon_n = \varepsilon_{n_n}$ is determined by the equation:

$$f(\varepsilon_{n_n}) - p_z = 0 \tag{44}$$

By virtue of:

$$f'(\varepsilon_{n_n}) \cdot \left[\frac{f'(\varepsilon_{n_n})}{\tilde{\Psi}_s(t)} + 1\right] < 0$$

we obtain

$$\frac{dA(\varepsilon_{n_n})}{d\varepsilon_n} = 0 \quad \text{and} \quad \frac{d^2 A(\varepsilon_{n_n})}{d\varepsilon_n^2} < 0 \tag{45}$$

which means that the energy absorbed by the system at the point $\varepsilon_n = \varepsilon_{n_n}$ reaches its peak value, so the loss of stability due to the loss of bearing capacity is quite possible.

Four models (Zorychta, 1988) are considered now to illustrate the possibility of a catastrophic jump occurrence in systems including rheological elements.

— model consisting of the pseudo-elastic element and the elastic element connected in series (Fig. 8 a)

the seam: $\sigma_n = f(\varepsilon_n)$

the roof: $\sigma_s = E_s \varepsilon_s$

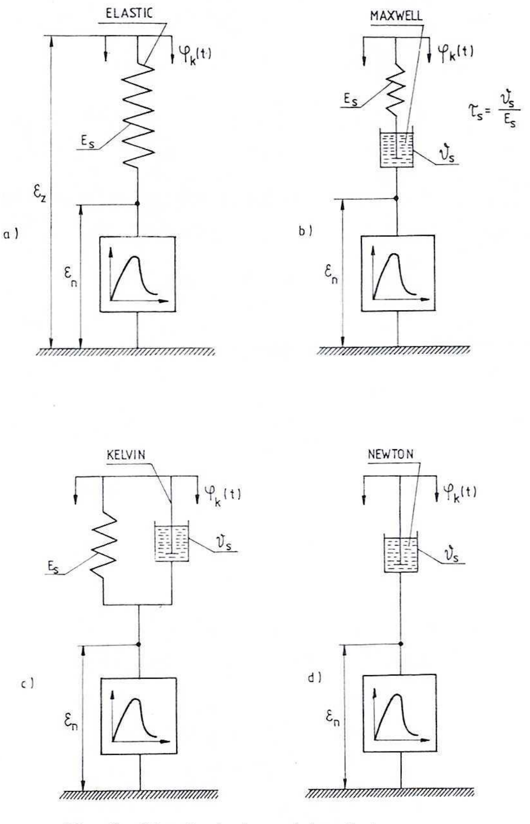

Fig. 8. Rheological models of the system

$$\text{the system: } \left[\frac{f'(\varepsilon_n)}{E_s}+1\right]\frac{d\varepsilon_n}{dt}=\frac{d\varphi_k}{dt}$$

— model consisting of the pseudo-elastic element and the Maxwell's element connected in series (Fig. 8 b)

$$\text{the seam: } \sigma_n=f(\varepsilon_n)$$

$$\text{the roof: } \sigma_s+\tau_s\frac{d\sigma_s}{dt}=\vartheta_s\frac{d\varepsilon_s}{dt}$$

$$\text{the system: } \left[\tau_s+\frac{(\tau_s)^2 f'(\varepsilon_n)}{\vartheta_s}\right]\frac{d^2\varepsilon_n}{dt^2}+\frac{(\tau_s)^2 f''(\varepsilon_n)}{\vartheta_s}\left(\frac{d\varepsilon_n}{dt}\right)^2+\left[\frac{\tau_s f'(\varepsilon_n)}{\vartheta_s}+1\right]\frac{d\varepsilon_n}{dt}=\frac{d\varphi_k}{dt}+\tau_s\frac{d^2\varphi_k}{dt^2}$$

— model consisting of the pseudo-elastic element and the Kelvin's element connected in series (Fig. 8 c)

$$\text{the seam: } \sigma_n=f(\varepsilon_n)$$

$$\text{the roof: } \sigma_s = E_s \varepsilon_s + \vartheta_s \frac{d\varepsilon_s}{dt}$$

$$\text{the system: } \frac{\vartheta_s d^2 \varepsilon_n}{E^2 dt^2} + \left[\frac{f'(\varepsilon_n)}{E_s} + 1\right] \frac{d\varepsilon_n}{dt} = \frac{d\varphi_k}{dt} + \frac{\vartheta_s}{E_s} \frac{d^2\varphi_k}{dt^2}$$

— model consisting of the pseudo-elastic element and the Newton's element connected in series (Fig. 8 d)

$$\text{the seam: } \sigma_n = f(\varepsilon_n)$$

$$\text{the roof: } \sigma_s = \vartheta_s \frac{d\varepsilon_s}{dt}$$

$$\text{the system: } \frac{d\varepsilon_n}{dt} + \frac{f(\varepsilon_n)}{\vartheta_s} = \frac{d\varphi_k}{dt}$$

where:

E_s — modulus of elasticity,
ϑ_s — viscosity,
τ_s — relaxation time.

In numerical calculations the function is used which in qualitative terms represents the results of laboratory tests (Zorychta, Kłeczek, 1998):

$$f(\varepsilon_n) = R_c \frac{\varepsilon_n}{\varepsilon_{kr}} \cdot \exp\left(1 - \frac{\varepsilon_n}{\varepsilon_{kr}}\right)$$

where:

R_c — compressive strength,
$\varepsilon_n \leqslant \varepsilon_{kr}$ — pre-failure regime,
$\varepsilon_n > \varepsilon_{kr}$ — post-failure regime.

Several additional assumptions were made:

— system loading depends on time-variant total strain $\varepsilon_z(t)$, while: $\frac{d\varepsilon_z}{dt} = \frac{d\varphi_k}{dt} = \text{const.}$

— the initial loading: $p_z = 0$,

— modulus of elasticity E_s was chosen such that a catastrophic jump should occur, in accordance with (26),

— to compare the results of these calculations it was assumed (Derski, Ziemba, 1968) that $\tau_s = \frac{\vartheta_s}{E_s}$ for the Maxwell's model.

We have to bear in mind, however, that the results of numerical simulation shown in Fig. 9 and Fig. 10 should be considered only in qualitative terms, since the calculations were done for the given form of the function $f(\varepsilon_n)$ and for subjectively chosen rheological parameters (Kłeczek, 1994). For practical reasons it is impossible to present the exact quantitative relations as the geological structure of the rock is very variable and the main aim was to present the phenomenon in qualitative terms.

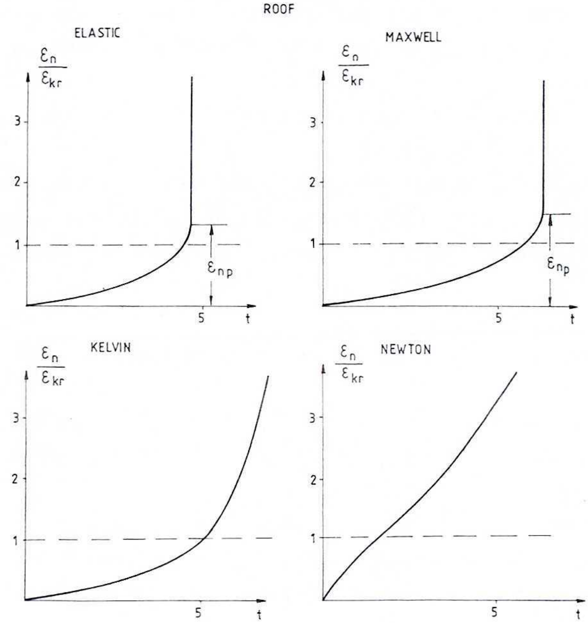

Fig. 9. Vertical strain in the non-linear element as the function of time for various rheological models

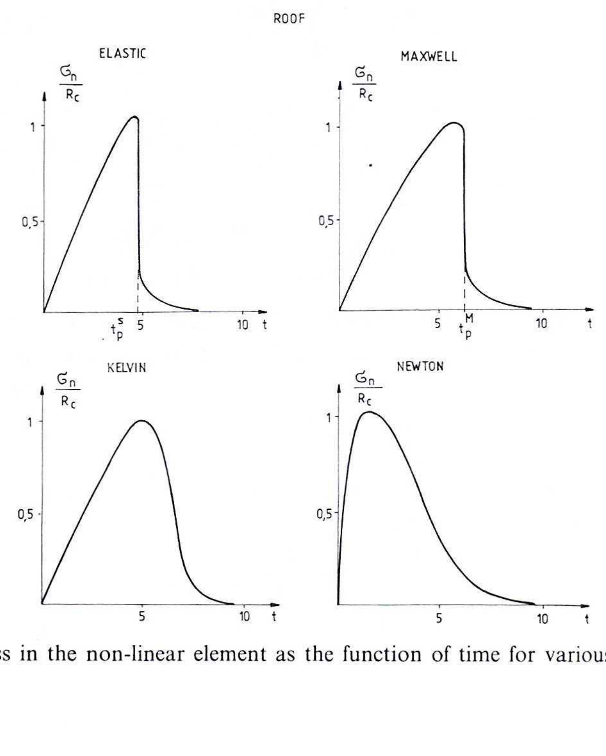

Fig. 10. Vertical stress in the non-linear element as the function of time for various rheological models

Analysing the results of numerical simulation we notice that:

— a catastrophic jump manifested by a violent change in strain (involving a violent reduction of stress in the pseudo-elastic element) may occur not only when the roof and floor are modelled as elastic elements, but also when those strata are modelled as the Maxwell's element,

— the occurrence of a catastrophic jump depends on whether the roof and floor strata can immediately impart the energy they accumulated — such effects are not present in Kelvin's and Newton's models as they do not have such property.

3. The influence of geomechanical factors on possibility of rock burst occurrence

Analysing the conditions for loss of stability determining two mechanisms of rock bursts: rock bursts due to catastrophic jump and to the loss of bearing capacity we have to consider certain problems:

1. The loss of stability identified as a rock burst (for both mechanisms) is not a feature of the uni-axial stress state only; it may also occur for three-axial states while the endangered stress states regime is given by the inequality:

$$0 \leqslant \eta_n < \eta_{n_i} \qquad \{i = p, n\}$$

2. In the light of the possibility of rock burst occurrence the uni-axial stress state seems most hazardous, for two reasons:

— regardless of the mechanism for the uni-axial state (for the same external strain ε_z) energy absorption ability is smallest:

$$A_{n_i}^{\max}\Big|_{\eta_n=0} < A_{n_i}^{\max}\Big|_{\eta_n>0} \qquad \{i = p, n\}$$

— for the uni-axial stress state the maximal values of post-failure moduli are greater than those for three-axial states; hence the probability of a catastrophic jump occurrence is greater, too:

$$\left|\frac{\partial f(\varepsilon_n, \eta_n = 0)}{\partial \varepsilon_n}\right|_{\max} > \left|\frac{\partial f(\varepsilon_n, \eta_n > 0)}{\partial \varepsilon_n}\right|_{\max}$$

3. Violent changes of the state of equilibrium (i. e. a rock burst due to a catastrophic jump) is a property of the system — a definite relation between the deformability of roof and floor strata and post-failure characteristics of the seam (the fracturing element, to be more specific). Catastrophic jumps occur when floor and roof strata impart the previously accumulated energy. Furthermore, we have to discuss the term: "natural propensity of rocks (e. g. coal) to bursts" and resulting indices, such as W_{et}. It must be pointed out that:

— the assumption that rock has a natural tendency to burst means there will always exist a catastrophe point (or rather a catastrophe regime) on the stress-strain characteristics, while the results of experiments performed on sedimentary rocks using rigid testers reveal that such a point does not exist.

— burst propensity indices for rock (or coal) are determined for the raising part of the stress-strain characteristics, hence they are not related to post-failure properties which determine rock burst occurrence.

4. A rock burst due to the loss of bearing capacity is not a natural property of rock because the loss of stability is unavoidable for each rock subjected to loading such that $p_z > R_c^r(\eta_n)$, provided the energy supplied to the system is more than can be absorbed.

5. The condition for the existence of a catastrophe point allows to propose the index of rock burst hazard due to catastrophic jump Ω_p:

$$\Omega_p = \frac{\left|f'(\varepsilon_n)\right|_{\varepsilon_n > \varepsilon_{kr}}^{\max}}{E_s}$$

On that basis two cases can be distinguished:

$\Omega_p \leqslant 1$ — systems not prone to catastrophic jump $\Rightarrow$ burst-free systems,

$\Omega_p > 1$ — systems prone to catastrophic jumps $\Rightarrow$ burst-prone systems.

The index value can be determined by way of laboratory tests, where we have to obtain:

— post-failure properties of coal or the virgin ore rocks in copper mining; such tests should be run using rigid testing machines,

— elasticity modulus of the roof and floor rock during the unloading phase.

6. With an eye to minimise the burst hazard, all activities performed in roof and floor strata aimed to activate the dissipation processes are strongly recommended. As the result, roof and floor strata capacity to impart energy is reduced, so a burst-prone system may become burst-free. For example, when the elasticity modulus for the unloading phase is increased (for instance when the structure of roof and floor rock is destroyed), $E_s \to \infty$):

$$\lim_{E_s \to \infty} \left[\frac{f'(\varepsilon_n)}{E_s} + 1 \right] = 1$$

That means that a rock burst due to a catastrophic jump will not occur; yet the rock burst due to the loss of bearing capacity is still possible. Theoretically speaking, similar effects are produced when rheological processes within the floor and roof rocks are intensified (for instance when viscosity is increased).

4. Conclusions

Rock bursts are considered to be the major natural hazard in coal and copper mining in Poland, while the knowledge of rock burst conditions is far from complete: that refers both to the causes of burst processes as well as forecasting methods and the ways to mitigation their impacts. These considerations (the paper was confined to relatively simple geomechanical models) reveal that even for more complex geomechanical systems, which better simulate the actual mining conditions, it is still possible to determine the criteria for the occurrence of rock bursts treated as the loss of stability. That may lead to new methods to minimise the hazard.

One more thing calls for an explanation. As it was shown, a rock burst is the result of energy transitions within the system made of the seam and the surrounding rock; so the burst (because of the possibility of catastrophic jump) must involve the whole system. Accordingly, there will be no burst of coal or a given rock in copper mining — hence it will be necessary to update the regulations where we can find the term "tendency of coal (rock) to bursts". Let us conclude, then: the concept of 'natural tendency of a given rock (coal) to bursts' was introduced more than twenty five years ago. It is not justified on the grounds of physics, so at present it becomes an outdated term.

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Received: 12 May 1999.