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PROCESS OF COLMATAGE IN POROUS MEDIUM WITH CLOSED CIRCULATION OF SUSPENSION

PRZEBIEG ZJAWISKA KOLMATACJI W OŚRODKU POROWATYM PRZY ZAMKNIĘTYM OBIEGU ZAWIESINY

In this paper a theoretical description of the phenomenon of colmatage observed in a porous medium with a finite length L and a closed circulation has been presented. Such circulation allows one to use the same suspension many times because after flowing through the medium, it can be forced again into it.

Theoretical considerations have been presented on the basis of a system of balance-transport equation (1) and those of the kinetics of the colmatage process (2) with initial-boundary conditions (3), (4), (5).

Functions obtained determine the distribution of the concentration of flowing suspension $N\ (x, t)$ (17) and the porosity $\varepsilon(x, t)$ (18) in space x and time t of the proceeding phenomenon.

Based on equation of motion (20) the distribution of pressure in porous medium were determined (23).

Key words: the flow with mass and momentum exchange, colmatage.

W niniejszej publikacji został przedstawiony teoretyczny opis zjawiska kolmatacji obserwowanego w ośrodku porowatym o skończonej długości L, w zamkniętym obiegu zawiesiny. Taki obieg pozwala wielokrotnie wykorzystać tę samą zawiesinę, którą po przepływie przez ośrodek powtórnie zatłacza się do tego ośrodka.

Rozważania teoretyczne zostały przedstawione w oparciu o układ równań bilansu-transportu (1) i kinetyki procesu kolmatacji (2) z warunkami początkowo-brzegowymi (3), (4), (5).

Uzyskane funkcje określają rozkład — w przestrzeni x i czasie t trwającego zjawiska — koncentracji przepływającej zawiesiny $N(x, t)$ (17) i porowatości $\varepsilon(x, t)$ (18).

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Opierając się na równaniu ruchu postaci (20) wyznaczono rozkład ciśnienia w ośrodku porowatym (23).

Słowa kluczowe: przepływy z wymianą masy i pędu, kolmatacja.

1. Introduction

A team making research into the process of colmatage should satisfy a series of basic conditions. These conditions refer both to a porous medium and to suspension forced into it. In the papers so far presented (Litwiniszyn, 1961, 1963, Trzaska, 1972, 1983, Szechtman, 1961), we have assumed that the concentration of suspension forced into a porous medium has constant value during the whole experimental period. In this situation, since the concentration of suspension flowing through the medium is traditionally denoted by the letter N, we have: $N(0, t) = n = \text{const}$.

During their experimental investigation the authors of this paper have many times observed that verification of the theory needed long periods of time for the proceeding of the phenomenon. It was connected with making large quantities of suspension measured in hundreds of litres or, even, in cubic metres.

For economy's sake we decided to carry out such experiments during which suspension flowing out of the porous medium could be used again and be forced back into this medium. The present theoretical description of the phenomenon of colmatage takes into consideration this fact of the suspension return. Thus, it concerns the flows in which $N(0,t) \neq \text{const}$.

2. Formulation of the problem

Suspension having the concentration n is forced into a porous medium of the length L and the initial porosity ε_0. When suspension reaches point $x = L$, its flow is cut off, and what flows out at this point is forced into point $x = 0$ of the medium. In this way a closed circulation of suspension is formed.

Sedimentation of solid particles composing this suspension occurs in the porous medium; thus the phenomenon of colmatage takes place. The aim of this paper is to determine functions of the medium porosity $\varepsilon(x,t)$ and the volume concentration of solid particles in suspension $N(x,t)$, depending on position and time.

The process in question can be described by a system of balance-transport partial differential equations

$$\frac{\partial \varepsilon(x,t)}{\partial t} = \frac{\partial N(x,t)\varepsilon(x,t)}{\partial t} + q(t)\frac{\partial N(x,t)}{\partial x} \tag{1}$$

and those of the colmatage process kinetics

$$\frac{\partial \varepsilon(x,t)}{\partial t} = -\alpha\, q(t) N(x,t) \tag{2}$$

where α is a certain constant, and $q(t)$ a unitary flow discharge.

Because of complicated and non-typical calculations equation (1) is replaced by a simplified equation (1′)

$$\frac{\partial \varepsilon(x,t)}{\partial t} = \varepsilon_0 \frac{\partial N(x,t)}{\partial t} + q(t)\frac{\partial N(x,t)}{\partial x} \tag{1′}$$

Its form can be accepted when the phenomenon of colmatage can be regarded as shallow, single-layer, and proceeds in proportion to a short period of time. Then it can accepted that $\varepsilon(x,t) \approx \varepsilon_0$.

Let us denote that $Q(t) = \int_0^t q(t)dt$. Let t_1 denote the time in which the wave front of flowing suspension reaches point $x = L$. It results from simple dependence that $Q(t_1) = \varepsilon_0 L$. Initial boundary conditions can be written in the form

$$N(0,t) = n \quad \text{when} \quad 0 \le t < t_1 \tag{3}$$

$$N(0,t) = N(L,t) \quad \text{when} \quad t \ge t_1 \tag{4}$$

$$\varepsilon(x,t) = \varepsilon_0 \quad \text{when} \quad Q(t)/\varepsilon_0 \le x \le L;\ 0 \le t < t_1 \tag{5}$$

3. Determination of the functions of suspension concentration and medium porosity

Let us notice that introduction of a new variable $U \ge 0$:

$$U = Q(t)/\varepsilon_0 - x \tag{6}$$

connected with time t and path x, helps to reduce the system of equations (1′), (2) to the form (7), (8):

$$\frac{\partial \varepsilon(x,U)}{\partial U} = \varepsilon_0 \frac{\partial N(x,U)}{\partial x} \tag{7}$$

$$\frac{\partial \varepsilon(x,U)}{\partial U} = -\alpha\, \varepsilon_0 N(x,U) \tag{8}$$

It should be also noticed that for $x = 0$ and at time $t = t_1$ relation $U = Q(t_1)/\varepsilon_0 = L$ occurs, i.e. the wave front of the colmatant has reached the end of the medium. After introducing the new variable, boundary-initial conditions, take the form:

$$N(0,U) = n \quad \text{for} \quad 0 \le U < L \tag{9}$$

$$N(0,U) = N(L, U-L) \quad \text{for} \quad U \ge L \tag{10}$$

$$\varepsilon(x,0) = \varepsilon_0 \quad \text{for} \quad 0 \le x \le L \tag{11}$$

After eliminating function $\varepsilon(x, U)$ from the system (7), (8) we obtain an equation

$$\frac{\partial N(x,U)}{\partial x} + \alpha N(x,U) = 0$$

Function given below is its solution

$$N(x,U) = C(U)e^{-\alpha x} \tag{12}$$

Using conditions (9) for $0 \le U < L$ function $C(U)$ can be determined. Its form for $0 \le U < L$ is as follows: $C(U) = n$.
Hence

$$N(x\,,U) = n\,e^{-\alpha x}, \quad \text{for} \quad 0 \le U < L, \quad 0 \le x \le L \tag{13}$$

Let us notice that for $x = L$ the following takes place

$$N(L,U) = n\,e^{-\alpha L}, \quad \text{for} \quad 0 \le U < L \tag{14}$$

Now function $C(U)$ for $U \ge L$ will be determined. Let the situation when $L \le U < 2L$ be considered. Because $0 \le U - L < L$, we can use condition (10) and formula (14). We obtain

$$N(0,U) = N(L,U-L) = n\,e^{-\alpha L}, \quad \text{for} \quad L \le U < 2L$$

The above being taken into consideration in formula (12), we can calculate now

$$C(U) = n\,e^{-\alpha L}, \quad \text{when} \quad L \le U < 2L$$

and

$$N(x,U) = n\,e^{-\alpha L}e^{-\alpha x}, \quad \text{when} \quad L \le U < 2L, \quad 0 \le x \le L$$

Similar procedure is applied for $2L \le U < 3L$. Making use of the last formula for $x = \mathrm{L}$ and condition (10), we obtain an initial condition as shown below

$$N(0,U) = N(L,U-L) = n\,e^{-2\alpha L}, \quad \text{when} \quad 2L \le U < 3L$$

Hence, basing on (12)

$$C(U) = n\,e^{-2\alpha L}, \quad \text{when} \quad 2L \le U < 3L$$

and

$$N(x,U) = n\,e^{-2\alpha L}e^{-\alpha x}, \quad \text{when} \quad 2L \le U < 3L, \quad 0 \le x \le L$$

Continuing the same technique we come to the conclusion that function $N(x, U)$ can be expressed by

$$N(x,U) = n\,e^{-m\alpha L}e^{-\alpha x}, \quad \text{when} \quad mL \le U < (m+1)L$$

$$m = 0,\ 1,\ 2,... \tag{15}$$

$$0 \le x \le L$$

or else by

$$N(x, U) = n\,e^{-\alpha L E\left[\frac{U}{L}\right]} e^{-\alpha x}, \quad \text{when} \quad 0 \leq x \leq L \tag{15'}$$

where $E\left[\frac{U}{L}\right]$ denotes entire part of the number U/L.

The form of the function second unknown in thus proceeding process of colmatage, i.e. $\varepsilon(x, t)$ can be obtained by introducing formula (15) to equation (8).

Integrating equation (8) in relation to U and taking condition (5) into account we obtain

$$\varepsilon(x, t) = \varepsilon_0 - \alpha\varepsilon_0 \int_0^U N(x, U)\,dU$$

Function $N(x, U)$ can be expressed by various formulae within intervals of the form $[kL, (k+1)L)$ where $k = 0, 1, 2, \ldots$ Therefore, if $E[U/L] = \text{m.}$, the last formula will be written as

$$\varepsilon(x, U) = \varepsilon_0\left[1 - \alpha\left(\int_0^L N(x, U)\,dU + \int_L^{2L} N(x, U)\,dU + \ldots + \int_{mL}^U N(x, U)\,dU\right)\right]$$

Now expression (15) is introduced to the above formula. We have

$$\varepsilon(x, U) = \varepsilon_0\left[1 - \alpha n\left(\int_0^L e^{-\alpha x}dU + \int_L^{2L} e^{-\alpha L} e^{-\alpha x}dU + \ldots + \int_{mL}^U e^{-\alpha m L} e^{-\alpha x}dU\right)\right]$$

or

$$\varepsilon(x, U) = \varepsilon_0\{1 - \alpha n\,e^{-\alpha x}[L(1 + e^{-\alpha L} + e^{-2\alpha L} + \ldots + e^{-\alpha(m-1)L}) + e^{-m\alpha L}(U - mL)]\}$$

In the above formula a sum of expressions of the geometrical sequence appears in round brackets. Using an appropriate formula we can write

$$\varepsilon(x, U) = \varepsilon_0\left\{1 - n\alpha\,e^{-\alpha x}\left[L\frac{1 - e^{-m\alpha L}}{1 - e^{-\alpha L}} + e^{-m\alpha L}(U - mL)\right]\right\}; \quad \text{for} \quad 0 \leq x \leq L$$
$$mL \leq U < (m+1)L \tag{16}$$

or else

$$\varepsilon(x, U) = \varepsilon_0\left\{1 - n\alpha\,e^{-\alpha x}\left[L\frac{1 - e^{-\alpha L E\left[\frac{U}{L}\right]}}{1 - e^{-\alpha L}} + e^{-\alpha L E\left[\frac{U}{L}\right]}\left(U - LE\left[\frac{U}{L}\right]\right)\right]\right\}; \quad \text{for } 0 \leq x \leq L$$
$$U \geq 0 \tag{16'}$$

Formula (6) enables the transition from the variables (x, U) to (x, t), introducing this formula in (15′) and (16′) we finally obtain

$$N(x,t) = n\,e^{-\alpha x} e^{-\alpha L E\left[\frac{Q(t)-\varepsilon_0 x}{\varepsilon_0}\right]}; \quad \text{when} \quad 0 \le x \le \frac{1}{\varepsilon_0} Q(t) \quad \text{and} \quad 0 \le t \le t_1$$
$$\text{or} \quad 0 \le x \le L \quad \text{and} \quad t > t_1 \tag{17}$$

and

$$\varepsilon(x,t) = \begin{cases} \varepsilon_0; \quad \text{when} \quad \frac{1}{\varepsilon_0} Q(t) < x \le L \quad \text{i} \quad 0 \le t \le t_1 \\ \varepsilon_0 \left\{ 1 - n\alpha e^{-\alpha x} \left[L \dfrac{1 - e^{-\alpha L E\left[\frac{Q(t)-\varepsilon_0 x}{\varepsilon_0}\right]}}{1 - e^{-\alpha L}} + e^{-\alpha L E\left[\frac{Q(t)-\varepsilon_0 x}{\varepsilon_0}\right]} \left(\dfrac{1}{\varepsilon_0} Q(t) - x - L E\left[\dfrac{Q(t)-\varepsilon_0 x}{\varepsilon_0 L} \right] \right) \right] \right\}; \\ \quad \text{when} \quad 0 \le x \le \frac{1}{\varepsilon_0} Q(t) \quad \text{and} \quad 0 \le t \le t_1 \\ \quad \text{or} \quad 0 \le x \le L \quad \text{and} \quad t > t_1 \end{cases} \tag{18}$$

If $E\left[\dfrac{Q(t)-\varepsilon_0 x}{\varepsilon_0 L}\right] = m+1$, for $m = 0, 1, 2, \ldots$ then the formula (18) is presented in the form (19)

$$\varepsilon(x,t) = \begin{cases} \varepsilon_0 \left\{ 1 - n\alpha e^{-\alpha x} \left[L \dfrac{1 - e^{-\alpha(m+1)L}}{1 - e^{-\alpha L}} + e^{-\alpha(m+1)L} \left(\dfrac{1}{\varepsilon_0} Q(t) - x - (m+1)L \right) \right] \right\} \\ \quad \text{for} \quad 0 \le x \le \frac{1}{\varepsilon_0} Q(t) - (m+1)L \\ \varepsilon_0 \left\{ 1 - n\alpha e^{-\alpha x} \left[L \dfrac{1 - e^{-\alpha m L}}{1 - e^{-\alpha L}} + e^{-\alpha m L} \left(\dfrac{1}{\varepsilon_0} Q(t) - x - mL \right) \right] \right\} \\ \quad \text{for} \quad \frac{1}{\varepsilon_0} Q(t) - (m+1)L < x \le L \end{cases} \tag{19}$$

Now the distribution of pressure in porous media will be determined. To this end we use equation of motion (Trzaska, 1983) in the form:

$$\frac{\partial h(x,t)}{\partial x} = -\frac{a\,q(t)}{\left[\varepsilon(x,t)\right]^3} \tag{20}$$

where a is a certain constant.

Assuming that well-known is pressure under which the liquid flows into the porous medium in point $x = 0$.

$$h(0,t) = h_0 \tag{21}$$

We integrate equation (20) taking condition (21). We have (22)

$$h(x,t) = h_0 - \int_0^x \frac{aq(t)dx}{[\varepsilon(x,t)]^3} \tag{22}$$

Introducing formula (19) into the equation (22) we obtain the function of the distribution of pressure in the porous medium. We have

$$h(x,t) = \begin{cases} h_0 - \displaystyle\int_0^x \frac{aq(t)dx}{\varepsilon_0^3\left\{1 - n\alpha e^{-\alpha x}\left[L\dfrac{1-e^{-\alpha(m+1)L}}{1-e^{-\alpha L}} + e^{-\alpha(m+1)L}\left(\dfrac{1}{\varepsilon_0}Q(t) - x - (m+1)L\right)\right]\right\}^3} \\ \qquad \text{for} \quad 0 \le x \le \dfrac{1}{\varepsilon_0}Q(t) - (m+1)L \\ h_0 - \displaystyle\int_0^{\frac{1}{\varepsilon_0}Q(t)-(m+1)L} \frac{aq(t)dx}{\varepsilon_0^3\left\{1 - n\alpha e^{-\alpha x}\left[L\dfrac{1-e^{-\alpha(m+1)L}}{1-e^{-\alpha L}} + e^{-\alpha(m+1)L}\left(\dfrac{1}{\varepsilon_0}Q(t) - x - (m+1)L\right)\right]\right\}^3} + \\ + \displaystyle\int_{\frac{1}{\varepsilon_0}Q(t)-(m+1)L}^{0} \frac{aq(t)dx}{\varepsilon_0^3\left\{1 - n\alpha e^{-\alpha x}\left[L\dfrac{1-e^{-\alpha mL}}{1-e^{-\alpha L}} + e^{-\alpha mL}\left(\dfrac{1}{\varepsilon_0}Q(t) - x - mL\right)\right]\right\}^3} \\ \qquad \text{for} \quad \dfrac{1}{\varepsilon_0}Q(t) - (m+1)L < x \le L \end{cases} \tag{23}$$

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