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JERZY LITWINISZYN*

THE INFLUENCE OF THE RATE OF MINING OPERATIONS ON THE DEVELOPMENT OF SUBSIDENCE TROUGHS

UWAGA O WPŁYWIE PRĘDKOŚCI FRONTU EKSPLOATACJI NA KSZTAŁTOWANIE POLA NIECEK OSIADANIA

Eq (1) was taken as the model for subsidence trough development, where $w = w(x, z)$ is the vertical component of rock mass displacement. The solution to (1) for the imposed condition (4) has the form (5). The form of the functions $w_0 = w_0(x, t)$ in Eq (4) depends on rock properties and the velocity of moving working front (Fig). As the analytical form of this functions is not available, some properties were assumed in qualitative terms, depending on how the roof subsided over the mined-out areas. The influence of velocity v and acceleration $\frac{dv}{dt}$ of the working front motion on velocity and acceleration of vertical components of subsidence troughs was investigated.

Key words: rock mechanics, mining exploitation.

W równaniu (1) $w = w(x, t)$ oznacza, w nieruchomym w stosunku do ziemi kartezjańskim układzie współrzędnych $\{x, z\}$ o osi z skierowanej pionowo do góry, pionową składową przemieszczeń górotworu. Wielkość a w rów. (1) charakteryzuje własności górotworu. Dla $z = 0$, $-\infty < x < +\infty$ w momencie $t = 0$ zadana jest, rów. (4), wielkość $w_0 = w_0(x, t)$. Rozwiązanie rów. (1), dla warunku (4) ma postać (5), gdzie czas t jest parametrem. Rozwiązanie to opisuje w czasoprzestrzeni $\{x, z, t\}$ pole niecek w obszarze $z \geqslant 0$, $-\infty < x < +\infty$, dla $t > 0$. Warunek (4) przyjmujemy w postaci przemieszczającego się względem osi x frontu eksploatacji O z prędkością $-v$, skierowaną przeciwnie niż oś x (rys). Zadanie polega na zbadaniu wpływu prędkości v na kształtowanie pola niecek osiadania $w = (x, z, t)$. Dla rozwiązania tego zadania przyjmujemy jako układ odniesienia oś ξ, równoległą do osi x i zgodnie z nią skierowaną. Oś ξ porusza się z prędkością $-v$ względem osi x. Punkt zerowy osi ξ znajduje się w punkcie $\tilde{O}$ (rys. 1), który jest punktem przemieszczającego się razem z osią ξ frontu eksploatacji.

* INSTYTUT MECHANIKI GÓROTWORU, POLSKA AKADEMIA NAUK, 30-059 KRAKÓW, UL. REYMONTA 27

Między współrzędnymi x, t i ξ, t zachodzi związek (6) (rys. 1). Pionowa współrzędna osiadania niecki dla $z = 0$ opisana jest w układzie $\{\xi, t\}$ wielkością $\tilde{w}_0 = \tilde{w}_0(\xi, t)$. Pomiędzy nią a wielkością $w_0 = w_0(x, t)$ w układzie $\{x, t\}$ zachodzi związek (7).

Rozważmy związki, jakie zachodzą między prędkościami i przyspieszeniami pionowych składowych niecek osiadania $w_0(x, t)$ i $\tilde{w}_0(\xi, t)$. Dokonujemy tego różniczkując funkcję $\tilde{w}_0 = \tilde{w}_0[\xi(x, t), t]$ wg czasu t jako funkcję złożoną i podstawiając za $\xi = \xi(x, t)$ wielkość z rów. (6). Otrzymujemy wielkości prędkości $W_0 = W_0(x, t)$ *i* przyspieszeń $V_0 = V_0(x, t)$ rów. (10) i (12) panujące na poziomie $z = 0$ w nieruchomym w stosunku do ziemi układzie $\{x, t\}$. Dysponując tymi wielkościami za pomocą rów. (5) wyznaczamy prędkość osiadania niecek w czasoprzestrzeni $\{x, z, t\}$, $W = W(x, z, t)$ rów. (15) i podobnie przyspieszeń osiadania tych niecek $V = V(x, z, t)$.

Dla zilustrowania wyżej podanych rozważań podano przykład. W przykładzie tym funkcję $\tilde{w}_0 = \tilde{w}_0(\xi, t)$ przyjęto w postaci zgodnej z krzywą osiadania stropu nad eksploatowanym pokładem. Obrazem jest jej krzywa $\tilde{w}_0(\xi, t)$ dla $t > t_0$ (rys.). Spełnia ona warunki (16)—(20). Przedyskutowano przypadki, gdy prędkość frontu eksploatacji $v = 0$, $v = \text{const} \neq 0$ i gdy $\frac{dv}{dt} \neq 0$, tzn. gdy istnieje przyspieszenie frontu eksploatacji. Dobór tych wielkości ma znaczenie w profilaktyce szkód górniczych, a w szczególności przy rozruchu i zatrzymaniu frontów eksploatacji.

Słowa kluczowe: mechanika górotworu, wpływy eksploatacji górniczej.

This communication presents certain remarks pertaining to rock subsidence due to time-variant mining conditions. This process is discussed in detail in provided bibliography.

This problem was considered by Knothe (1953) who developed the formula describing the phenomenon. The conclusions are verified against the real life data; accordingly certain modifications are introduced.

That problem involves the impacts of the mining rate on the development of subsidence regions. In recent years more attention has been given to time-variations of working face advancement (acceleration or delays of mining operations).

Extensive information can be found in the works by Sroka (1999); and the results had some practical merits as they were the basis for the design of mining operations in built-up areas in the Ruhr Basin. This work inspired the Author of this communication to present certain remarks.

Let us then choose a model describing rock subsidence due to underground mining Litwiniszyn (1953). The two-dimensional model yields the parabolic equation:

$$\frac{\partial w(x, z)}{\partial z} = a \frac{\partial^2 w(x, z)}{\partial x^2} \tag{1}$$

This equation describes the vertical component of rock mass displacement $w = w(x, z)$. It is described in the Cartesian co-ordinate system $\{x, z\}$, immobile with respect to the Earth. The z-axis is directed upwards. The domain $z \geqslant 0$,

$-\infty < x < +\infty$ of the function $w = w(x, z)$ is considered, whereas a is the coefficient related to rock properties.

Let us consider the following problem: a function describing the vertical component of roof displacement at the level $z = 0$ is given as:

$$w(x, 0) = w_0(x) \tag{2}$$

The solution to Eq (1) satisfying the condition (2) describing the vertical component $w = w(x, z)$ of rock displacement for $z \geqslant 0$ and in particular for the Earth's surface will have the form:

$$w(x, z) = \frac{1}{2a\sqrt{\pi z}} \int_{-\infty}^{+\infty} w_0(s) e^{\frac{-(s-x)^2}{4a^2 z}} ds \tag{3}$$

Let us then consider the case when the vertical component of displacement at the level $z = 0$ should be time-variant. That corresponds to real life situations, when the working face advances with time. Let us consider the vertical component of displacement in the time-space $\{x, z, t\}$. For $z = 0$ and $-\infty < x < +\infty$; the condition (2) can be rewritten as:

$$w(x, 0, t) = w_0(x, t) \tag{4}$$

The actual form of the function $w_0 = w_0(x, t)$ depends on properties of rock above the roof and the rate of moxing of time-variant mined area boundaries.

The function $w_0 = w_0(x, t)$ at the level $z = 0$ determines the process of subsidence trough development in time-space $\{x, z, t\}$ for $z \geqslant 0$.

Analytical form of this function is not available. Its forms may vary for different geological and technological conditions. Certain properties of this function being found in qualitative terms, the plot of the function resembles the curve of roof subsidence above the mined-out area, hence we can extract certain information on how subsidence troughs would behave within the time-space.

Assuming the condition (4) for $z = 0$, the solution to Eq (1) can be written as:

$$w(x, z, t) = \frac{1}{2a\sqrt{\pi z}} \int_{-\infty}^{+\infty} w_0(s, t)\, e^{\frac{-(s-x)}{4a^2 z}} ds \tag{5}$$

where time t is the parameter. This solution does not take into account the delaying effects of rock above the mined-out area.

For further considerations, let us assume that at the moment $t = t_0$ the origin of the co-ordinate system $\{x, z\}$ immobile with respect to the Earth will be at the working face at point O (Fig. 1). The working face moves with respect to that co-ordinate system with the velocity $-v$ parallel to the x-axis and having the opposite direction. Let us consider another co-ordinate system $\{\xi, z\}$ where the ξ-axis coincides with the x-axis in the co-ordinate system $\{x, z\}$ and has the same direction.

The origin of the co-ordinate system $\{\xi, z\}$ is at the working face at the point Õ (Fig.), which moves with respect to the fixed co-ordinate system $\{x, z\}$ with the velocity $-v$, in the direction opposite to that of x-axis.

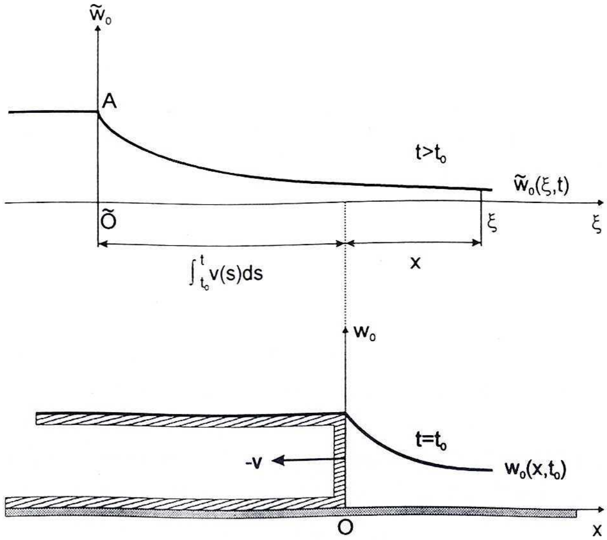

Fig. 1.

At the moment $t > t_0$ the relationship between the co-ordinates ξ and x of the systems $\{\xi, z\}$ and $\{x, z\}$ will be as follows:

$$\xi = \xi(x, t) = x + \int_{t_0}^{t} v(\tau)\, d\tau \tag{6}$$

At the moment $t = t_0$, $\xi(x_0, t_0) = x$.

Let $w_0 = w_0(x, t)$ denote the vertical component of rock displacement against $\{x, t\}$ at the level $z = 0$, whereas $w = w_0(\xi, t)$ — the displacement against the co-ordinate system $\{\xi, t\}$ moving with respects to $\{x, t\}$. It follows from Eq (6) that he relationship between the displacement $\tilde{w}_0 = \tilde{w}_0(\xi, t)$ given in the co-ordinate system $\{\xi, t\}$ and the displacement $w_0(x, t)$ given in the co-ordinate system $\{x, t\}$ will be as follows:

$$\tilde{w}_0(\xi, t) = \tilde{w}_0\left[x + \int_{t_0}^{t} v(\tau)\, d\tau, t\right] \tag{7}$$

When we assume that $v = 0$, then basing on (6) and (7) we get $\tilde{w}_0(\xi, t) = w_0(x, t)$. In this case the process of vertical rock displacement described with the function $w_0 = w_0(x, t)$ depends on rock properties exclusively.

An observer moving within the system $\{\xi, t\}$ with the velocity $v = v(t)$ (Fig.) in the direction along the ξ-axis, that is opposite to the working face advancement, will be immobile with respect to the co-ordinate system $\{x, t\}$. His co-ordinate x within this system will be time-invariant. An observer within the system $\{\xi, t\}$ will find at the moment t that the vertical displacement is the same as that observed by the stationary observer at point x of the co-ordinate system $\{x, t\}$.

Let us now consider the relations between the velocity and acceleration of displacement displacement $w_0 = (x, t)$ and $\tilde{w}(\xi, t)$ given in the co-ordinate systems $\{x, t\}$ and $\{\xi, t\}$. Accordingly, we compute the velocity of vertical displacement $\frac{d\tilde{w}_0(\xi, t)}{dt}$ at the moment t within the co-ordinate system $\{\xi, t\}$ moxing with respect to the system $\{x, t\}$ with the velocity $-v$.

Let us substitute $\xi = \xi(x, t)$ present in (6) into the formula $\tilde{w}_0 = \tilde{w}_0(\xi, t)$ describing the displacement within the co-ordinate system $\{\xi, t\}$. Taking into account (6) and the expression

$$\frac{\partial \xi(x,t)}{\partial t} = v(t) \tag{8}$$

and differentiating the function $\tilde{w}_0 = \tilde{w}_0[\xi(x, t), t]$ with respect to time (it is a compound function), we get:

$$\frac{d}{dt}\{\tilde{w}_0[\xi(x,t),t]\} = \frac{\partial \tilde{w}_0[\xi(x,t),t]}{\partial \xi} v(t) + \frac{\partial \tilde{w}_0[\xi(x,t),t]}{\partial t} \tag{9}$$

Substituting $\xi = \xi(x, t)$ from equation (6) to the right-hand side of the equation (9), we get:

$$\left[\frac{d\tilde{w}_0(\xi,t)}{dt}\right]_{\xi = \xi(x,t) = x + \int_{t_0}^{t} v(\tau)d\tau} = \frac{dw_0(x,t)}{dt} = W_0(x,t) \tag{10}$$

where $W_0 = W_0(x, t)$ describes the velocity of displacement $w_0 = w_0(x, t)$ at the level $z = 0$, within the co-ordinate system $\{x, t\}$ immobile with respect to the Earth's surface.

It follows from (9) and (10) that velocity of the displacement $W_0 = W_0(x, t)$ is the sum of local velocity $\frac{\partial \tilde{w}_0[\xi(x,t),t]}{\partial t}$ and the velocity $\frac{\partial \tilde{w}_0[\xi(x,t),t]}{\partial \xi} v(t)$ resulting from co-ordinate system $\{\xi, t\}$ motion and hence working face Õ motion with respect to the system $\{x, t\}$. To find the acceleration of the displacement $w_0 = w_0(x, t)$ we differentiating (9) with respect to time. Taking into account (8), we get:

$$\frac{d^2}{dt^2}\{\tilde{w}_0[\xi(x,t),t]\} = \frac{\partial^2 \tilde{w}_0[\xi(x,t),t]}{\partial \xi^2} v^2(t) + $$
$$+ \frac{\partial \tilde{w}_0[\xi(x,t),t]}{\partial \xi}\frac{dv(t)}{dt} + \frac{\partial^2 \tilde{w}_0[\xi(x,t),t]}{\partial t^2} \tag{11}$$

Substituting $\xi = \xi(x,t)$ from equation (6) to the right-hand side of the last equation (11), we get:

$$\left[\frac{d^2\tilde{w}_0(\xi,t)}{dt^2}\right]_{\xi=\xi(x,t)=x+\int_{t_0}^{t} v(\tau)d\tau} = \frac{d^2 w_0(x,t)}{dt^2} = V_0(x,t) \tag{12}$$

where $V_0 = V_0(x,t)$ describes the acceleration of displacement $w_0 = w_0(x,t)$ at the level $z = 0$ within the system $\{x,t\}$ stationary with respect to the Earth. The function $\tilde{w}_0 = \tilde{w}_0(\xi,t)$ given in the co-ordinate system $\{\xi,t\}$ being known, we can utilise (6) and for the specified working face velocity $v = v(t)$ find the function $w_0 = w_0(x,t)$ in the co-ordinate system $\{x,t\}$ describing the vertical displacement at the level $z = 0$ and at the moment t, that is the imposed condition in (4).

Basing on (5), this condition allows to find the subsidence troughs in the time-space $\{x,z,t\}$ for $z \geqslant 0$, particularly the subsidence trough at the Earth's surface level.

Differentiating (5) with respect to time t, being the parameter in this equation, we get:

$$\frac{dw(x,z,t)}{dt} = \frac{1}{2a\sqrt{\pi z}} \int_{-\infty}^{+\infty} \frac{dw_0(s,t)}{dt} e^{\frac{-(s-x)^2}{4a^2 z}} ds \tag{13}$$

Introducing the notation

$$\frac{dw(x,z,t)}{dt} = W(x,z,\mathrm{t}) \tag{14}$$

and taking into account (10), on the basis of (13), (14) we get:

$$W(x,z,t) = \frac{1}{2a\sqrt{\pi z}} \int_{-\infty}^{+\infty} W_0(s,t)\, e^{\frac{-(s-x)^2}{4a^2 z}} ds \tag{15}$$

$W = W(x,z,t)$ describes the velocity of vertical rock displacement within the time-space $\{x,z,t\}$ for the imposed condition $w_0 = w_0(x,t)$ in (4), at the moment and the level $z = 0$. Let us follow the procedure as before, differentiate (15) with respect to t, taking into account (11) and (12) and introducing the notation

$$\frac{dw(x,z,t)}{dt} = V(x,z,t)$$

Hence, on the basis of (12), we get:

$$V(x,z,t) = \frac{1}{2a\sqrt{\pi z}} \int_{-\infty}^{+\infty} V_0(s,t)\, e^{\frac{-(s-x)^2}{4a^2 z}} ds$$

$V = V(x, z, t)$ describes the acceleration of vertical rock displacement within the time-space $\{x, z, t\}$ for the imposed condition $w_0 = w_0(x, t)$ in (4), at the moment t and the level $z = 0$. The function $w_0 = w_0(x, t)$ given in the co-ordinate system $\{x, t\}$ can be obtained from (6) and (7) on the basis of the function $\tilde{w}_0 = \tilde{w}_0(\xi, t)$ given in the co-ordinate system $\{\xi, t\}$.

Actually we do not know the analytical form of the function $\tilde{w}_0 = \tilde{w}_0(\xi, t)$. As this function is not available, these theoretical considerations are illustrated with an example.

The function $\tilde{w}_0 = \tilde{w}_0(\xi, t)$ is represented by the curve $\tilde{w}_0(\xi, t)$ in the co-ordinate system $\{\xi, t\}$, which is shown in Fig. for the instant $t > t_0$. We assume certain properties of the function in qualitative terms, relating to the shape of roof subsidence curve. It is assumed that for $\xi \geqslant 0$, $t > 0$ the function $\tilde{w}_0 = \tilde{w}_0(\xi, t)$ will be regular and satisfy the following conditions:

$$\tilde{w}_0(\xi, t) \geqslant 0 \tag{16}$$

$$w_0(0, t) = A \tag{17}$$

$$\lim_{\xi \to \infty} \tilde{w}_0(\xi, t) = 0 \tag{18}$$

$$\frac{\partial \tilde{w}_0(\xi, t)}{\partial \xi} < 0 \tag{19}$$

$$\frac{\partial^2 \tilde{w}_0(\xi, t)}{\partial \xi^2} > 0\,* \tag{20}$$

The velocity of vertical displacement is given by (9). In accordance with (19) and for $v > 0$ (Fig.), the first term on the right hand side satisfies the inequality:

$$\frac{\partial \tilde{w}_0(\xi, t)}{\partial \xi} v(t) < 0$$

It follows therefore, that as the velocity $v = v(t)$ increases, the rate of roof subsidence over the mined-out area will increase, too. The higher the absolute value of the derivative $\left|\frac{\partial \tilde{w}_0(\xi, t)}{\partial \xi}\right|$ the greater that increase. The form of the function $\tilde{w}_0 = \tilde{w}_0(\xi, t)$ suggests that this absolute value is greatest for $\xi = 0$, that is at the working face. Within the co-ordinate system $\{x, t\}$, Eq (6), that is valid when

$$x = -\int_{t_0}^{t} v(s)\, ds.$$

* The example o this function, for $A > 0$, $\alpha > 0$, $f(\xi) = Ae^{-\alpha\xi}$;

$$\frac{df}{d\xi} = -\alpha A e^{-\alpha\xi};\quad \frac{d^2 f}{d\xi^2} = \alpha^2 A e^{-\alpha\xi}$$

When $v(t) = 0$, (7) and (9) will yield:

$$\frac{d}{dt}\{\tilde{w}_0[\xi(x,t),t]\} = \frac{\partial \tilde{w}_0[\xi(x,t),t]}{\partial t} = \frac{\partial w_0(x,t)}{\partial t}$$

The velocity of vertical displacement in this case will depend only on rock properties.

The acceleration of vertical displacement $\frac{d^2\tilde{w}_0(\xi,t)}{dt^2}$ at the level $z = 0$ is given by (11). It is the sum of three quantities. In accordance with (20), the first term on the right hand side in (11) satisfies the inequality:

$$\frac{\partial^2 \tilde{w}(\xi,t)}{\partial \xi^2} v^2 > 0$$

The sign of the second term, in accordance with (19), depends on the sign of the working face acceleration $\frac{dv(t)}{dt}$. When $\frac{dv(t)}{dt} > 0$, then $\frac{\partial \tilde{w}_0(\xi,t)}{\partial \xi}\frac{dv(t)}{dt} < 0$ when $\frac{dv(t)}{dt} < 0$ then $\frac{\partial \tilde{w}_0(\xi,t)}{\partial \xi}\frac{dv(t)}{dt} > 0$.

In the first case the first and second term on the right hand side in (11) have different signs, in the second case both are positive.

The third term on the right hand side in (11) $\frac{\partial^2 \tilde{w}(\xi,t)}{\partial t^2}$ does not depend on velocity v and the accerelation $\frac{dv}{dt}$. It depends on roof rock properties only. When the velocity of the working face $v \equiv 0$, the acceleration $\frac{\partial^2 \tilde{w}(\xi,t)}{dt^2}$ depends on these properties exclusively.

When the velocity of the working face motion is constant $v = \text{const}$, then $\frac{dv}{dt} = 0$ and in accordance with (11) the acceleration $\frac{d^2\tilde{w}(\xi,t)}{dt^2}$ depends on $\frac{\partial^2 \tilde{w}_0(\xi,t)}{\partial \xi^2} v^2(t)$ and the derivative $\frac{\partial^2 \tilde{w}_0(\xi,t)}{\partial \xi^2}$.

When the velocity of the working face motion is time-variant, the acceleration $\frac{dv}{dt} \neq 0$.

When we choose values of v and $\frac{dv}{dt}$ in this case, it is possible to obtain the required acceleration $\frac{d^2\tilde{w}(\xi,t)}{dt^2}$ throughout a certain range, in accordance with (11).

This selection is of primary importance, as it was shown in Sroka (1999), to protect against mining damage, especially while starting and stopping the mining operations.

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