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OPTIMISATION IN NATURAL GAS TRANSMISSION NETWORK

OPTYMALIZACJA SIECI PRZESYŁOWEJ GAZU ZIEMNEGO

The algorithm for steady-state optimisation of large gas networks, based upon. The Generalised Reduced Gradient method (GRG) is described. The networks can be of any configuration. The optimisation is treated as a non-linear problem with non-linear constraints. It is assumed that the structure of the network is known, and such a network consists of compressor stations, valves and regulators, all of which must be on.

The main goal of the described algorithms is to minimise running costs of the operating compressors. The investigation results are given and these have shown that the GRG is more effective than the Augmented Lagrangian Method. In addition, the GRG is faster and more convenient to calculate, whereas the Augmented Lagrangian Method requires a reliable initial estimate for the Lagrange multipliers. A number of recent publications are also described.

Key words: gas networks, optimisation, numerical methods, linear programming.

W artykule omówiono algorytm optymalizacji statycznej sieci gazowej wysokociśnieniowej o dowolnej konfiguracji. Wykorzystano metodę gradientu zredukowanego. Problem został sformułowany jako zadanie nieliniowej optymalizacji z nieliniowymi ograniczeniami. Przyjęto założenie, że struktura pracy sieci jest zadana, to znaczy, że wiadomo, które elementy nierurowe (stacje przetłoczne, stacje redukcyjne, pojedyncze zawory) są włączone. Jako kryterium optymalizacji przyjęto koszty eksploatacji sprężarek pracujących w tłoczniach, zakładając, że koszty eksploatacji są liniowo zależne od mocy zużywanej do przetłoczenia określonej ilości gazu przy sprężaniu od ciśnienia ssania do ciśnienia tłoczenia.

Ograniczenia równościowe uwzględniane w procesie optymalizacji to:

— I prawo Kirchhoffa,

— II prawo Kirchhoffa,

— równanie przepływu.

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Ograniczenia nierównościowe wynikają z charakterystyk statycznych sprężarek, określonych warunków pracy pozostałych elementów nierurowych, a także konieczności dotrzymania wymaganych wartości parametrów (najczęściej ciśnienia) w wybranych punktach sieci. Rozwiązaniem zadania optymalizacji są takie parametry pracy poszczególnych tłoczni (ciśnienie ssania, ciśnienie tłoczenia oraz przepływ), które gwarantują dostawę gazu każdemu odbiorcy zgodnie z kontraktem, spełniają wszystkie ograniczenia narzucone na system, a jednocześnie minimalizują sumaryczne zużycie mocy przez agregaty sprężające gaz. Aby ocenić poprawność opracowanego algorytmu, wykonano badania testowe, wykorzystując do tego celu wybrane fragmenty sieci gazowej wysokociśnieniowej w Wielkiej Brytanii. W pierwszym przypadku sieć składała się z 37 rurociągów, 30 węzłów, 2 stacji przetłocznych oraz dwóch źródeł. Optymalne parametry pracy tłoczni otrzymano po 9 iteracjach. Końcowa wartość funkcji celu w stosunku do wartości startowej zmniejszyła się w procesie optymalizacji o ponad 48%. To samo zadanie rozwiązano metodą rozszerzonego Lagrangianu, uzyskując wyniki bardzo niewiele się różniące, przy czym w tym przypadku algorytm potrzebował aż 16 iteracji.

Druga optymalizowana sieć przesyłowa składała się z 19 rurociągów, 23 węzłów, 3 stacji przetłocznych oraz 1 źródła. Algorytm optymalizacji potrzebował 19 iteracji, aby znaleźć optymalne parametry pracy poszczególnych stacji przetłocznych. Funkcja celu zmniejszyła się o ponad 56%. Rozwiązanie tego samego zadania metodą rozszerzonego Lagrangianu wymagało 27 iteracji.

Przeprowadzone badania testowe wykazały pełną poprawność działania opracowanego algorytmu. Jednocześnie badania wykazały, że algorytm optymalizacji wykorzystujący metodę gradientu zredukowanego jest szybszy w każdej sytuacji od algorytmu opartego na mnożnikach Lagrange'a, w którym występują duże trudności związane z prawidłowym doborem startowych wartości mnożników.

Słowa kluczowe: sieci gazowe, optymalizacja, metody numeryczne, liniowe programowanie.

1. Introduction

The growth of the national transmissiom system gives increasing opportunities for more efficient management. Central Control, whose main task is the overall management of the national system, because of the increasing number of compressors, has recognised the importance of efficient fuel usage.

Gas compressor stations form a major part of the operational plant on each Transmission System. Their function is to restore the gas pressure drop caused by frictional pressure losses. The compressors are driven mostly by gas turbines, which use natural gas as fuel. This gas is taken directly from the transmission pipelines.

The compressor unit comprises three main components, a gas generator, a power turbine and a centrifugal gas compressor. The maximum shaft power of the units has a range of 5.5 MSCFD (155742.235 m^3) to 20 MW, and the associated fuel consumption varies between 2.5 MSCFD (70791.925 m^3) and 5.5 MSCFD, which is equivalent respectively to 8.600 (55900 PLN) to 19.000 (123500 PLN) pounds per day of fuel cost.

The value of compressor fuel used in the UK per annum represents 80% of the total energy costs used by British Gas.

According to American Gas Association sources, the operating cost of running the compressor stations varies between 25% and 50% o the total company's operating budget.

Minimising this fuel usage is a major objective in the control of gas transmission costs. Although the fuel usage of compressors is significant and should be optimised, there are other factors, which need to be considered. Therefore, Central Control must operate the system so that gas is supplied whenever it is needed, at the appropriate pressure and volume. This is the basic problem of running the transmission system, where reliability of supply versus costs. The security needed must be judged carefully and is usually expressed as a margin of pressure above the minimum essential at any offtake.

This work is concerned with the minimisation of operating costs for high-pressure gas networks under steady-state conditions. Depending on the character of the gas flows in the system, steady and transient states need to be distinguished.

The steady-state in a gas network is given by a system of non-linear equations. In steady-state problems, since loads and supplies are not functions of time, an algorithm for optimisation determines, the structure of the network (i. e. the number of sources, compressors, valves and regulators called-units, which must be on). In addition, the algorithm must determine the optimal parameters of the operation, namely nodal pressures and flows through branches (pipes). For these reasons, the problem of optimisation is formulated in (Wilson et al., 1988) as a mixed integer problem. Each unit operates subject to a set of linear and non-linear equality and inequality constraints. By linearising the flow equation, the non-linear constraints and the objective function, the problem of steady-state optimisation can be expressed as a mixed-integer linear in the form:

$$\begin{aligned} &\min f(\boldsymbol{x}) = \boldsymbol{c}^T \cdot \boldsymbol{x} \\ &\text{subject to } \boldsymbol{A} \cdot \boldsymbol{x} \leqslant \boldsymbol{b}, \end{aligned} \tag{1}$$

where some components of the vector $\boldsymbol{x}$ can take integer values (only 0 or 1) and the rest continuous values. Alternatively, assuming that the structure is known, an algorithm for steady-state optimisation of large gas networks is described in (Osiadacz and Bell, 1988). In this case, the problem of optimisation has been treated without simplification, i. e. as a non-linear problem with non-linear constraints. In the first case, the problem has been solved using the Branch and Bound method. In the second case, the chosen method at each iteration minimises a quadratic approximation to the Lagrangian function subject to linear approximations to the constraints. A line search procedure utilising the 'watchdog technique' is used to force convergence when the initial values of the variables are far from the solution.

Described in (Luongo et al., 1989) the optimisation strategy consists of two levels. The first level involves optimising the system with suction/discharge pressures as variables. Dynamic programming is used at this level. An important part of this process is the determination of how many compressors should be switched on. The higher level optimiser is a search method with the system flow rates as independent variables. Combination of Nelder and Mead's and Simplex methods, plus a sequential exploration around a final point gives good results under all situations encountered.

CIMSA-SINTRA (Anglard and David, 1988) has developed a large control system for the trans-siberian Russian gas pipeline. The paper concentrates on the optimisation methods developed for fuel use and steady-state planification. The pipeline control as a whole is solved using a hierarchical structure.

The purpose of the steady-state optimiser described in (Percell and Ryan, 1987) is to find pressures, flows, temperatures, and compressors station configurations (i. e., choice of compressors units to be on), given fixed demands and resources for the network, which are optimal with respect to a chosen objective functions. The following objective function were used:

• minimisation of the amount of fuel which is consumed by the gas turbine drivers,
• maximisation of the total delivered flow,
• maximisation line pack.

2. Problem formulation

The goal of optimisation is to minimise the following expression

$$I = \sum_{j=1}^{M} A_j \cdot Q_j(t) \cdot \left\{ \left(\frac{p_{dj}(t)}{p_{sj}(t)} \right)^{R_j} - 1 \right\}, \tag{2}$$

where:

- M — the number of operating compressors,
- p_d — discharge pressure for j^{th} compressor,
- p_s — suction pressure for j^{th} compressor,
- Q_j — flow through j^{th} compressor,
- A_j, R_j — constants for the j^{th} compressor.

The equality constraints are the following:

$$\boldsymbol{A} \cdot \boldsymbol{Q} - \boldsymbol{K} \cdot \boldsymbol{F} - \boldsymbol{L} = \boldsymbol{0}, \tag{3}$$

$$\boldsymbol{\Delta P} + \boldsymbol{A}^T \cdot \boldsymbol{P} = \boldsymbol{0}, \tag{4}$$

$$\boldsymbol{\Delta P} = \boldsymbol{K}_f^T \cdot \boldsymbol{Q}, \tag{5}$$

where:

- $\boldsymbol{A}$ — is the nodal — branch incidence matrix, (dim $\boldsymbol{A} = n \times m$),
- n — is the number of nodes,
- m — is the number of branches,
- $\boldsymbol{P}$ — is the vector of squared nodal pressures, (dim $\boldsymbol{P} = n \times 1$),

$\boldsymbol{K}$ — is the unit — nodal incidence matrix, (dim $\boldsymbol{K} = n \times r$),
r — is the number of units,
$\boldsymbol{F}$ — is the vector of flows through units, (dim $\boldsymbol{F} = r \times 1$),
$\boldsymbol{L}$ — is the vector of loads,
$\boldsymbol{\Delta P}$ — is the vector of squared drop pressures, (dim $\boldsymbol{\Delta P} = m \times 1$),
$\boldsymbol{K}_f$ — is the vector of pipe constants, (dim $\boldsymbol{K}_f = m \times m$),
$\boldsymbol{Q}$ — is the vector of flows through pipes, (dim $\boldsymbol{Q} = m \times 1$).

The inequality constraints associated with the various units are as follows:

a) compressors

The surge line is formulated by the inequality

$$a_1 \cdot Q^2 - b_1 \cdot Q + 1 \geqslant \frac{p_d}{p_s}, \tag{6}$$

where — Q is the flow through compressor (m^3/h) and a_1, b_1 are specified coefficients. Choking line is formulated by the inequality

$$a_2 \cdot Q^2 + b_2 \cdot Q + 1 \leqslant \frac{p_d}{p_s}, \tag{7}$$

$$Q_{max} \geqslant Q, \tag{8}$$

$$\frac{p_d}{p_s} \leqslant CR_{max}, \tag{9}$$

where — CR_{max} is the maximum compression ratio,

$$(rpm)_{min} \leqslant rpm \leqslant (rpm)_{max}, \tag{10}$$

where — (rpm) is the number of revolutions per minute

$$N \leqslant N_{max}, \tag{11}$$

b) valves

$$p_{in} \geqslant p_{out}, \tag{12}$$

where:

p_{in} — is the inlet pressure,
p_{out} — is the outlet pressure.

$$Q_{max} \geqslant Q, \tag{13}$$

where — Q is the flow through valve,

c) regulators

$$p_{in} \geqslant p_{out}, \tag{14}$$

$$Q_{max} \geqslant Q, \tag{15}$$

d) sources

The outlet pressure remains constant for any flow rate.
This is stated by

$$p_0 = \text{const.} \tag{16}$$

3. Problem solution

The above problem has been formulated as a non-linear with non-linear constraints in the form

$$\min f(\boldsymbol{x}) \tag{17}$$

subject to equality and inequality constraints

$$\left.\begin{array}{ll} c_i(\boldsymbol{x}) = 0, & i = 1, 2, ..., m' \\ c_i(\boldsymbol{x}) \geqslant 0, & i = m'+1, 2, ..., m \end{array}\right\} \\ \boldsymbol{x} = (\boldsymbol{Q}, \boldsymbol{F}, \boldsymbol{P}) \tag{18}$$

and the functions $f(x)$ and $c_i(\boldsymbol{x})$; $i = 1, 2, .., m$ are real and differentiable. To solve this problem, a Generalised Reduced Gradient method is used. Generalised Reduced Gradient methods solve the following non-linear problem

$$\left.\begin{array}{ll} \underset{\boldsymbol{x}}{\text{minimise}}\, f(\boldsymbol{x}) & \\ \text{subject to:} & \\ c(\boldsymbol{x}) = 0 & \text{equality constraints} \\ l_i \leqslant x_i \leqslant u_i & \text{bounds on variables} \end{array}\right\} \tag{19}$$

where:

$c(\boldsymbol{x})$ — is a vector of constraints,

x_i — is the i-th component of the vector of variables, x.

Inequality constraints are converted to equality ones by means of slack variables. The variables, i.e. the n components of the vector $\boldsymbol{x}$ are divided into n_b basic variables, x_b, and $n-n_b$ non-basic variables, $\boldsymbol{x}_{nb}$, such that the $n_b \times n_b$ basis matrix, $\boldsymbol{S}$, whose components are $\delta c_i/\delta x_{bj}$, is non-singular at the current point, $\boldsymbol{x}^{(k)}$. Using the constraints, the n_b basic variables are expressed in terms of the non-basic ones.

Initially, the basic variables are chosen to ensure that the basis matrix is non-singular and all slack variables are chosen as basic. Any variable, which is initially close to one of its bounds, is designated as a non-basic.

The reduced gradient, $\nabla F(x_{nb})$, is calculated, at each iteration, as follows:

$$\left.\begin{array}{l}
\text{1. Define matrix } \boldsymbol{D}^{(k)} = \dfrac{\partial \boldsymbol{c}}{\partial \boldsymbol{x}_b} \dfrac{\partial \boldsymbol{c}}{\partial \boldsymbol{c}_{nb}}. \\
\text{2. Calculate objective function, } f(\boldsymbol{x}_b^{(k)}, \boldsymbol{x}_{nb}^{(k)}) \text{, at current point.} \\
\text{3. Increment } i\text{-th element of nonbasic variable vector by small amount } \eta, \\
\quad \bar{x}_{nbi}^{(k)} = x_{nbi}^{(k)} + \eta. \\
\quad \text{Increment all elements of basic variable vector as follows:} \\
\quad \bar{x}_{bj}^{(k)} = x_{bj}^{(k)} + \eta D_{ji}^{(k)} \\
\text{4. The } i\text{-th element of the reduced gradient is given by:} \\
\quad \nabla F_i = \dfrac{f(\bar{x}_{bi}, \ldots, \bar{x}_{bm}, \bar{x}_{nbi}, \ldots, \bar{x}_{nbq}) - f(\boldsymbol{x}_b, \boldsymbol{x}_{nb})}{\eta}.
\end{array}\right\} \quad (20)$$

A projected reduced gradient is then calculated as follows

$$\left.\begin{array}{l}
\nabla F_{pi} = 0 \quad \begin{cases} \text{if } x_{nbi} = u_i \text{ and } \nabla F_i < 0 \\ or \\ \text{if } x_{nbi} = l_i \text{ and } \nabla F_i > 0 \end{cases} \\
\nabla F_{pi} = \nabla F_i \text{ otherwise.}
\end{array}\right\} \quad (21)$$

The projected gradient is used in a gradient based unconstrained minimisation algorithm to determine a search direction, $\hat{\delta}^{(k)}$. An approximate search direction, $\hat{\pi}$, for the basic variables is also determined, for use in the procedure used for adjusting the basic variables to maintain feasibility, from:

$$\hat{\pi} = -\left[\frac{\partial \boldsymbol{c}}{\partial \boldsymbol{x}_b}\right]^{-1} \frac{\partial \boldsymbol{c}}{\partial \boldsymbol{x}_{nb}} \hat{\delta}. \quad (22)$$

A line search minimisation in the direction of the non-basic search direction is carried out to determine the step lenght, α, to the next point, $\boldsymbol{x}^{(k+1)}$, about which the next GRG subproblem is formed. In this algorithm, the movement from $\boldsymbol{x}^{(k)} = (\boldsymbol{x}_b, \boldsymbol{x}_{nb})^{(k)}$ to $\boldsymbol{x}^{(k+1)} = (\boldsymbol{x}_b, \boldsymbol{x}_{nb})^{(k+1)}$ is achieved as follows:

$$\left.\begin{array}{l}
x_{nbi}^{(k+1)} = \begin{cases} u_i & \text{if } x_{nbi}^{(k)} + \alpha\hat{\delta}_i \geq u_i \\ l_i & \text{if } x_{nbi}^{(k)} + \alpha\hat{\delta}_i \leq l_i \\ x_{nbi}^{(k)} + \alpha\hat{\delta} & \text{otherwise} \end{cases} \\
x_{bj}^{(k+1)} = x_{bj}^{(k)} + \alpha\hat{\pi}_i.
\end{array}\right\} \quad (23)$$

The new point is likely to be infeasible because of the non-linear constraints and, therefore, the non-basic variables are held constant while the basic ones are adjusted until feasibility is obtained for $\boldsymbol{x}^{(k+1)}$. The adjustment is equivalent to solving the constraint equations, $\boldsymbol{c}(\boldsymbol{x}) = \boldsymbol{0}$, for the basic variables, $\boldsymbol{x}_b$. The adjustment is carried out via a Newton method. After adjustment, it is possible that one of the basic variables will lie outside its bounds. In this case, linear interpolation is carried out, between the points $\boldsymbol{x}^{(k)}$ and $\boldsymbol{x}^{(k+1)}$, to determine the step length at which the nearest bound becomes active. This gives a situation with one basic variable on its bound and all the others safely inside theirs. Tests are then performed to determine whether the line search minimum lies at the bound or whether it has been bracketed, in which case a refined search locates the minimum within the bracket. If the point $x^{(k+1)}$ is feasible, the search for the minimum continues normally.

Having obtained a minimum, feasible new point, $\boldsymbol{x}^{(k+1)}$, a new GRG subproblem is formed and solved, to find the next approximation to $\boldsymbol{x}^*$, until the stopping criterion, of the L_2 norm of the projected reduced gradient, $\|\nabla \boldsymbol{F}_p\|_2$, being less than the required value, is satisfied. The formation of a new subproblem involves a re-partitioning of the variables. The basic variables, which are equal to their bounds, must be exchanged for unbounded non-basic variables, while still maintaining the non-singularity of the basis matrix. The choice of a non-basic variable to exchange with a bounded basic one is made by maximising the following expression:

$$\mathrm{Min}\{\boldsymbol{D}_{ji}|(u - x_{nbi}|\boldsymbol{D}_{ji}|(x_{nbi} - l_i)\} \tag{24}$$

$\boldsymbol{D}$ is defined in (20).

The Basic Algorithm

1. Specify any bounded basic variables as non-basic variables.
2. Calculate the reduced gradient, $\nabla \boldsymbol{F}$.
3. Form the projected reduced gradient, $\nabla \boldsymbol{F}_p$.
4. If the L_2 norm of the projected gradient is small enough then TERMINATE.
5. Determine the search direction, $\hat{\delta}$.
6. Take a step of lenght α in direction $\hat{\delta}$.
7. Adjust the basic variables using Newton method.
8. If the Newton method does NOT converge then reduce α and GOTO STEP 6.
9. If the basic variables are out of bounds then adjust them until the nearest bound becomes active. If the line search minimum is located at this bound then GOTO STEP 1. Otherwise GOTO STEP 11.
10. If the minimum is NOT bracketed then increase α and GOTO STEP 6.
11. Refine α to locate line search minimum and GOTO STEP 1.

A large amount of computation time is spent performing a Newton algorithm for the adjustment of the basic variables to maintain constraint feasibility. This time can be reduced by implementing the following modified Newton method.

$$\boldsymbol{x}_b^{(t+1)} = \boldsymbol{x}_b^{(t)} - \left[\frac{\partial \boldsymbol{c}}{\partial \boldsymbol{x}_b}\right]_0^{-1} \boldsymbol{c}\left(\boldsymbol{x}_b^{(t)}, \boldsymbol{x}_{nb}^{(k)}\right), \tag{25}$$

where:

$\left[\frac{\partial \boldsymbol{c}}{\partial \boldsymbol{x}_b}\right]_0^{-1}$ — is the inverse used for the reduced gradient calculation,

t — is the Newton iteration count.

This method avoids successive reformulations of the inverse Jacobian matrix of the constraints but, unfortunately, its convergence properties are inferior to those of the usual Newton method. Compared to other iterative algorithms, the GRG is known for its robustness and reliability. It has been shown to be successful at finding the optimal solution even when the starting approximation is far away.

(Wong, 1988) has developed algorithm for steady-state optimisation of high-pressure gas networks using Augmented Lagrangian Method.

The augmented Lagrangian is the original Lagrangian with a penalty for the violation of the constraints $c(\boldsymbol{x})$, to ensure positive curvature near to the stationary point, so that a minimum exists wrt $\boldsymbol{x}$, and so minimisation techniques can be used to find this point. A sequence of minimisations of quadratic estimates to this equation is used iteratively tending to the stationary point, updating the Lagrange multiplier estimates at each minimum. The augmented Lagrangian equation is used:

$$L_{aug}^{k}(\boldsymbol{x}^{k}, \lambda^{k}) = F(\boldsymbol{x}^{k}) - \lambda^{k^{T}} \boldsymbol{c}(\boldsymbol{x}^{k}) + \sigma^{k} \boldsymbol{c}(\boldsymbol{x}^{k})^{T} \boldsymbol{S}^{k} \boldsymbol{c}(\boldsymbol{x}^{k}), \tag{26}$$

where:

S^{k} — is positive definite diagonal to weight each constraint,
σ^{k} — is a scalar to determine the weight of the augmented term.
Then, the stationary point of the Lagrangian is given by:

$$\begin{aligned} \nabla_{x} L_{aug}(\boldsymbol{x}^{*}, \lambda^{*}) &= \boldsymbol{g}(\boldsymbol{x}^{*}) - A(\boldsymbol{x}^{*})\lambda^{*} + \sigma A(\boldsymbol{x}^{*})\boldsymbol{S}, \\ c(\boldsymbol{x}^{*}) &= 0 \\ \nabla_{\lambda} L_{aug}(\boldsymbol{x}^{*}, \lambda^{*}) &= \boldsymbol{c}(\boldsymbol{x}^{*}) = \boldsymbol{0}, \end{aligned} \tag{27}$$

so that this stationary point is the same as that of the original Lagrangian:

$$L(\boldsymbol{x}, \boldsymbol{\lambda}) = F(\boldsymbol{x}) - \boldsymbol{\lambda}^{T} \boldsymbol{c}(\boldsymbol{x}), \tag{28}$$

when $\boldsymbol{\lambda}$ has been adjusted so that $\boldsymbol{c}(\boldsymbol{x}^{*}) = \boldsymbol{0}$.
Further, the Quasi-Newton approximation to the second derivative matrix is:

$$\nabla_{xx}^{2} L_{aug}(x^{*}, \lambda^{*}) \approx \nabla_{xx}^{2} F(x) - \sum_{i=1}^{m} \nabla_{xx}^{2} c_{i}^{*} \lambda_{i}^{*} + \sigma \boldsymbol{A}^{*} \boldsymbol{S}^{*} \boldsymbol{A}^{*T}. \tag{29}$$

If $\boldsymbol{A}^{*}$ is of the full rank then $\boldsymbol{A}^{*}\boldsymbol{S}^{*}\boldsymbol{A}^{*T}$ is positive definite, and so for large enough σ, $\nabla_{xx}^{2} L_{aug}$ will be positive definite, thus ensuring the stationary point $(\boldsymbol{x}^{*}, \lambda^{*})$ is a minimum. Sequential minimisation of L_{aug} is performed by updating $(\boldsymbol{x}, \lambda)$ until the minimum is reached.

Investigations have shown that GRG is more effective then Augmented Lagrangian Method. The GRG is faster and more convenient to calculate. The Augmented Lagrangian Method needs a good initial estimate for the Lagrange multipliers.

An inner iteration minimises an Augmented Lagrangian function subject to variable bounds for fixed Lagrange multipliers. The outer iteration updates the multipliers. The active set strategy is to add constraints to the active set as soon as they become violated, but to delete constraints only when they are satisfied and their

Lagrange multipliers appear to have converged in sign and indicate that a reduction in objective function can be obtained.

4. Results of investigations

To prove the correctness of the algorithm based on GRG technique two existing networks have been solved. These will show the results for the gas networks presented in figure 1 which comprises 37 pipes, 30 nodes, 2 compressor stations and 2 sources.

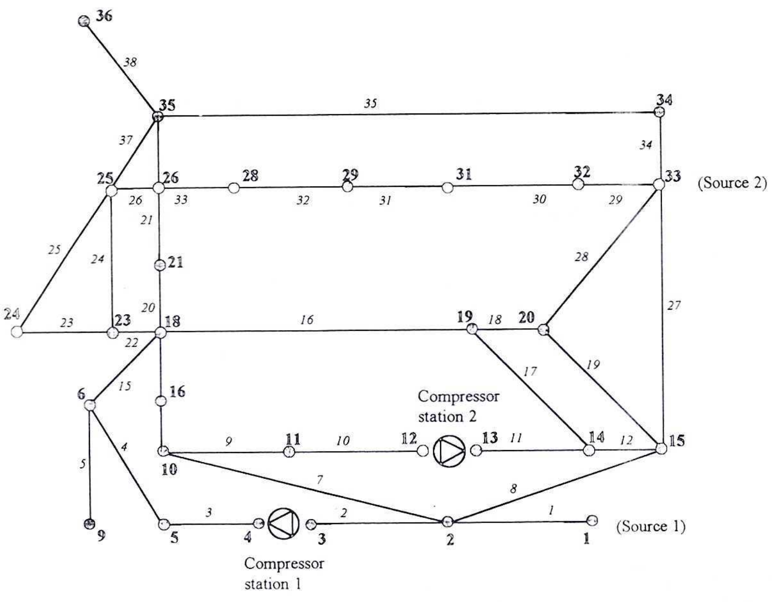

Fig. 1. Structure of the network

All the pipes have the same diameter. This is equal to 36″ (0.914 m). One of the requirements in this problem is the pressure at nodes No 9 and No 36 can not drops below 5.20 MPa and 5.40 MPa respectively. The length of the pipes are given in table 1. The nodal loads are given in table 2. Compressor station and source constraints are given in table 3 and table 4 respectively.
In this example constraints are as follows:

1. Equality constraints:

• I-st Kirchhoff's Law:

$$A \cdot Q - K \cdot F - L = 0$$

where:

A — is the nodal — branch incidence matrix, ($\dim A = 30 \times 37$),
K — is the unit — nodal incidence matrix, ($\dim K = 30 \times 4$),

TABLE 1

The length of the pipes

Pipe No	Length (m)	Pipe No	Length (m)
1	$50*10^3$	21	$75*10^3$
2	$35*10^3$	22	$55*10^3$
3	$20*10^3$	23	$65*10^3$
4	$80*10^3$	24	$80*10^3$
5	$85*10^3$	25	$50*10^3$
7	$60*10^3$	26	$35*10^3$
8	$150*10^3$	27	$80*10^3$
9	$35*10^3$	28	$60*10^3$
10	$45*10^3$	29	$40*10^3$
11	$65*10^3$	30	$50*10^3$
12	$70*10^3$	31	$60*10^3$
13	$85*10^3$	32	$55*10^3$
14	$60*10^3$	33	$60*10^3$
15	$95*10^3$	34	$80*10^3$
16	$40*10^3$	35	$50*10^3$
17	$20*10^3$	36	$90*10^3$
18	$65*10^3$	37	$75*10^3$
19	$55*10^3$	38	$80*10^3$
20	$45*10^3$		

TABLE 2

The nodal loads

Node No	Load (m^3/h)	Node No	Load (m^3/h)
2	117987	20	117987
5	,,	23	,,
6	,,	24	,,
9	,,	25	,,
10	,,	26	,,
11	,,	29	,,
14	,,	32	,,
15	,,	34	,,
18	,,	35	,,
19	,,	36	,,

TABLE 3

Compressor station constraints

Compressor Station No	Max. discharge pressure (bar)	Max. compressor ratio
Compressor station 1	60.0	1.5
Compressor station 2	"	1.5

TABLE 4

Source constraints

Source No	Max. pressure (bar)	Max. flow (m^3/h)
Source 1	60.0	2420000.0
Source 2	60.0	2420000.0

$\boldsymbol{Q}$ is the vector of flows through pipes, ($\dim \boldsymbol{Q} = 37 \times 1$),
$\boldsymbol{F}$ is the vector of flows through units, ($\dim \boldsymbol{F} = 4 \times 1$),
$\boldsymbol{L}$ is the vector of loads, ($\dim \boldsymbol{L} = 30 \times 1$).

• II-nd Kirchhoff's Law:

$$\boldsymbol{\Delta P} - \boldsymbol{K}_f^T \cdot \boldsymbol{Q} = \boldsymbol{0},$$

where:

$\boldsymbol{\Delta P}$ is the vector of squared drop pressures, ($\dim \boldsymbol{\Delta P} = 37 \times 1$),
$\boldsymbol{K}_f$ is the vector of pipe constants, ($\dim \boldsymbol{K}_f = 37 \times 37$),
$\boldsymbol{Q}$ is the vector of flows through pipes, ($\dim \boldsymbol{Q} = 37 \times 1$).

2. Inequality constraints

$$p_9 \geqslant p_{9_{\min}},$$

$$p_{36} \geqslant p_{36_{\min}}.$$

Inequality constraints were converted into equality ones by adding two new variables:

$$p_9 - p_{9_{\min}} - u_1 = 0,$$

$$p_{36} - p_{36_{\min}} - u_2 = 0,$$

$\dim \boldsymbol{h}(\boldsymbol{x}) = 69 \times 1.$

All variables were divided into basic and non-basic ones:

$x_b = [p_4 \ p_{13}]^T$

$$x_{nb} = \begin{matrix}[p_2 & p_3 & p_5 & p_6 & p_9 & p_{10} & p_{11} & p_{12} & p_{14} & p_{15} & \cdots & p_{32} & p_{34} & p_{35} \\ p_{36} & Q_{S1} & Q_{S2} & Q_1 & Q_2 & \cdots & Q_5 & Q_7 & \cdots & Q_{38} & f_1 & f_2 & u_1 & u_2]^T,\end{matrix}$$

where Q_{S1}, Q_{S2} — flows in sources, f_1, f_2 — flows through compressor stations $\dim x_b = 2 \times 1$, $\dim x_{nb} = 69 \times 1$.

$$\dim \frac{\partial f(x)}{\partial x_b} = 2 \times 1 \qquad \dim\left[\frac{\partial h(x)}{\partial x_b}\right] = 69 \times 2,$$

$$\dim\left[\frac{\partial h(x)}{\partial x_{nb}}\right] = 69 \times 69 \quad \dim\left[\frac{\partial f(x)}{\partial x_{nb}}\right] = 69 \times 1.$$

Thus $\dim g_R = 2 \times 1$, $\dim \Delta x_b = 2 \times 1$, $\dim \Delta x_{nb} = 69 \times 1$.
The results of optimisation are presented in tables 5, 6, 7, 8 and 9.

TABLE 5

The nodal pressures

Node No	Pressure (Pa)	Node No	Pressure (Pa)
2	4952279.4	20	5515743.8
3	4741864.3	21	5410402.6
5	5849845.2	23	5400502.3
6	5555732.6	24	5398839.6
9	5540327.1	25	5403114.7
10	4952286.3	26	5411346.7
11	4879920.5	28	5449142.6
12	4829617.1	29	5483560.0
14	5482098.4	31	5563985.8
15	5485629.4	32	5630129.7
16	5225366.6	34	5501397.3
18	5409836.1	35	5414898.4
19	5469052.6	36	5400022.2

TABLE 6

The pipes flow

Pipe No	Flow (m^3/h)	Pipe No	Flow (m^3/h)
1	272582.3	21	−27618.4
2	725185.8	22	112284.2
3	725185.8	23	40441.6
4	607198.8	24	−46144.3
5	117987.0	25	77545.4
7	−2077.2	26	133923.9
8	568513.3	27	532108.2
9	410901.9	28	577832.3
10	292914.9	29	464338.5
11	292914.9	30	346351.5
12	−58818.6	31	346351.5
13	530966.1	32	228364.5
14	530966.1	33	228364.5
15	371224.8	34	512878.7
16	362394.1	35	394891.7
17	233746.5	36	−51164.8
18	246634.5	37	107752.9
19	213210.7	38	117987.0
20	−27618.4		

TABLE 7

Changes in objective function during optimisation process

The number of iteration	Objective function [kW]
1	13488.417
2	12561.462
3	10656.334
4	8267.381
5	7694.291
6	7286.590
7	7026.288
8	6996.973
9	6995.364

TABLE 8

Optimal parameters of work for compressor stations

Compressor Station No	Suction pressure (bar)	Discharge pressure (bar)	Flow (m^3/h)	Power (kW)
Compressor station 1	47.41	59.48	725185.8	5623.238
Compressor station 2	48.29	55.46	292914.9	1368.127

TABLE 9

Optimal parameters of work for sources

Source No	Output pressure (bar)	Flow (m^3/h)
Source 1	50.0	272582.3
Source 2	57.2	2087157.7

Using, the Augmented Lagrangian Method, the same results were obtained in 16 iterations with 43 Quasi-Newton steps.
The second network shown in figure 2 contains 19 pipes, 23 nodes, 3 compressor stations and 1 source.

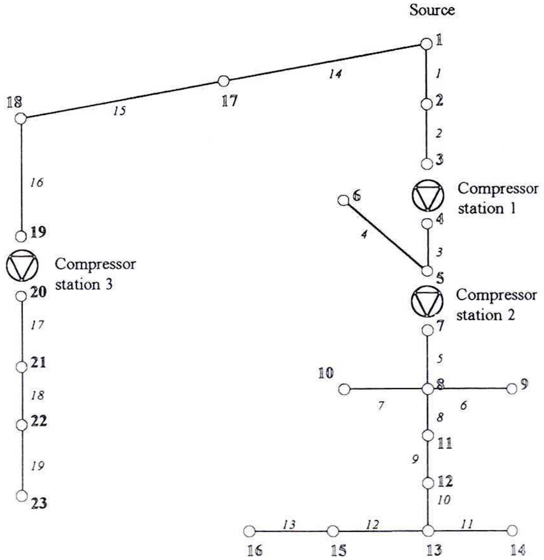

Fig. 2. Structure of the network

TABLE 10

The length and diameter of the pipes

Pipe No	Length [m]	Diameter [mm]
1	32800	900
2	34080	900
3	88160	900
4	29920	600
5	26880	900
6	13120	400
7	10400	600
8	8640	700
9	1760	600
10	1760	700
11	19680	600
12	13920	700
13	25440	700
14	64480	900
15	53280	900
16	13600	900
17	13280	900
18	28000	900
19	6400	700

TABLE 11

The nodal loads

Node No	Loads [Nm^3/h]
2	50000
6	150000
8	40000
9	55000
10	20000
13	30000
14	100000
15	100000
16	160000
18	30000
21	30000
22	110000
23	150000

The pressure at nodes No 9, No, No 13 and No 23 can not drops below 5.00 MPa, 5.00 MPa, 5.00 MPa and 5.56 MPa respectively. The length and diameter of the pipes are given in table 10. The nodal loads are given in table 11. Compressor station constraints are given in table 12.

TABLE 12

Compressor station constraints

Compressor Station No	Max. discharge pressure (bar)	Max. compressor ratio
Compressor station 1	70.0	1.5
Compressor station 2	70.00	1.5
Compressor station 3	70.00	1.5

In the example constraints are as follows:

1. Equality constraints:

- I-st Kirchhoff's Law:

$$\boldsymbol{A} \cdot \boldsymbol{Q} - \boldsymbol{K} \cdot \boldsymbol{F} - \boldsymbol{L} = \mathbf{0},$$

where:

$\boldsymbol{A}$ — is the nodal — branch incidence matrix, (dim $\boldsymbol{A} = 23 \times 19$),
$\boldsymbol{K}$ — is the unit — nodal incidence matrix, (dim $\boldsymbol{K} = 23 \times 4$),
$\boldsymbol{Q}$ — is the vector of flows through pipes, (dim $\boldsymbol{Q} = 19 \times 1$),
$\boldsymbol{F}$ — is the vector of flows through units, (dim $\boldsymbol{F} = 4 \times 1$),
$\boldsymbol{L}$ — is the vector of loads, (dim $\boldsymbol{L} = 23 \times 1$).

- II-nd Kirchhoff's Law:

$$\boldsymbol{\Delta P} - \boldsymbol{K}_f^T \cdot \boldsymbol{Q} = \mathbf{0},$$

where:

$\boldsymbol{\Delta P}$ is the vector of squared drop pressures, (dim $\boldsymbol{\Delta P} = 19 \times 1$)
$\boldsymbol{K}_f$ is the vector of pipe constants, (dim $\boldsymbol{K}_f = 19 \times 19$),
$\boldsymbol{Q}$ is the vector of flows through pipes, (dim $\boldsymbol{Q} = 19 \times 1$).

2. Inequality constraints

$$p_9 \geqslant p_{9_{\min}}$$

$$p_{10} \geqslant p_{10_{\min}}$$

$$p_{13} \geqslant p_{13_{\min}}$$

$$p_{23} \geqslant p_{23_{\min}}.$$

Inequality constraints were converted into equality constraints by adding two new variables:

$$p_9 - p_{9_{\min}} - u_1 = 0$$
$$p_{10} - p_{10_{\min}} - u_2 = 0$$
$$p_{13} - p_{13_{\min}} - u_3 = 0$$
$$p_{23} - p_{23_{\min}} - u_4 = 0$$

dim $h(x) = 46 \times 1$.
All variables were divided into basic and non-basic ones:

$$\boldsymbol{x}_b = [p_4 \; p_7 \; p_{20}]^T$$

$$\boldsymbol{x}_{nb} = \begin{matrix}[p_2 & p_3 & p_5 & p_6 & p_8 & p_9 & \cdots & p_{19} & p_{21} & p_{22} & p_{23} \\ Q_{S1} & Q_1 & Q_2 & \cdots & Q_{19} & f_1 & f_2 & f_3 & u_1 & \cdots & u_4]^T,\end{matrix}$$

where Q_{S1}, — flow in source, f_1, f_2 f_3 — flows through compressor stations, $\dim \boldsymbol{x}_b = 3 \times 1$, $\dim \boldsymbol{x}_{nb} = 46 \times 1$.

$$\dim \frac{\partial f(\boldsymbol{x})}{\partial \boldsymbol{x}_b} = 3 \times 1 \qquad \dim \left[\frac{\partial \boldsymbol{h}(\boldsymbol{x})}{\partial \boldsymbol{x}_b}\right] = 46 \times 3$$

$$\dim \left[\frac{\partial \boldsymbol{h}(\boldsymbol{x})}{\partial \boldsymbol{x}_{nb}}\right] = 46 \times 46 \quad \dim \left[\frac{\partial f(\boldsymbol{x})}{\partial \boldsymbol{x}_{nb}}\right] = 46 \times 1.$$

Thus $\dim \boldsymbol{g}_R = 3 \times 1$, $\dim \Delta \boldsymbol{x}_b = 3 \times 1$, $\dim \Delta \boldsymbol{x}_{nb} = 46 \times 1$
The results of optimisation are presented in tables 13, 14, 15 and 16.

TABLE 13

The nodal pressures

Node No	Pressure (Pa)	Node No	Pressure (Pa)
1	5000000.0	13	5000028.2
2	4800062.1	14	4977462.0
3	4611321.7	15	4932751.3
5	4315425.5	16	4899269.6
6	4230481.9	17	4910132.8
8	5094447.5	18	4834615.1
9	5059522.9	19	4818401.9
10	5093856.6	21	5033578.1
11	5036880.2	22	5007369.0
12	5011894.7	23	5000026.8

TABLE 14

The pipes flow

Pipe No	Flow (m^3/h)
1	705000.0
2	655000.0
3	655000.0
4	150000.0
5	505000.0
6	55000.0
7	20000.0
8	390000.0
9	390000.0
10	390000.0
11	100000.0
12	260000.0
13	160000.0
14	320000.0
15	320000.0
16	290000.0
17	290000.0
18	260000.0
19	150000.0

TABLE 15

Changes of objective function during optimisation process

The number of iteration	Objective function [kW]	The number of iteration	Objective function [kW]
1	10430.581	11	4832.112
2	9766.931	12	4829.287
3	8732.672	13	4619.785
4	7083.059	14	4619.673
5	6730.107	15	4592.776
6	6411.981	16	4558.683
7	6085.291	17	4546.842
8	5700.281	18	4543.142
9	5231.138	19	4542.502
10	5053.662		

TABLE 16

Optimal parameters of work for compressor stations

Compressor Station No	Suction pressure (Pa)	Discharge pressure (Pa)	Flow (Nm^3/h)	Power (kW)
Compressor station 1	4611321.7	4818500.7	655000.0	958.947
Compressor station 2	4315425.5	5178652.4	505000.0	3129.240
Compressor station 3	4818401.9	5048735.7	290000.0	451.290

Using Augmented Lagrangian Method, the same results were obtained in 27 iterations with 87 Quasi-Newton steps.

5. Conclusions

The optimiser was tested on the real systems. In each case, the optimiser improved results by minimising cost function (table 7 and table 14). Starting values for the networks were calculated using the steady-state simulator. The optimiser presented here is under constant evaluation at Warsaw University of Technology. Locally is used by the Polish Oil and Gas Industry.

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