Archives of Mining Sciences 45, 4 (2000) 555–565

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COMMENTS ON STOCHASTIC FLOWS THROUGH CHANNELS

UWAGI O PRZEPŁYWACH STOCHASTYCZNYCH PRZEZ KANAŁY

The present article contains an analysis of stochastic flows of liquids through straight channels. Calculations were carried out for a one-dimensional incompressible flow model, which is described by a linear differential equation with the application of the correlation theory of random functions.

Key words: fluid mechanics, stochastic processes, mine ventilation.

W kopalnianych sieciach wentylacyjnych przez cały czas występują nieustalone przepływy powietrza. Przeprowadzone pomiary prędkości powietrza [Trutwin i inni 1996] *, [Wasilewski 1984] pokazały, że widmo częstotliwości rośnie wraz z prędkością i dla średniej prędkości 1,8 m/s jego istotna część zawiera się w przedziale 0–1 Hz, a wariancja może dochodzić do 25% średniej wartości prędkości.

Pomiary ciśnienia absolutnego [Trutwin i inni 1996] wykazały, że jego zmiany mogą dochodzić do 75 Pa oraz że istotna część jego widma zawiera się w przedziałach 0–0,03 Hz. W tym samym czasie i wyrobisku mierzona różnica ciśnień dochodziła do 15 Pa, a jej widmo zawierało się w przedziale 0–0,001 Hz.

Przeprowadzone pomiary wykazują bardzo wolne zmiany różnicy ciśnienia między początkiem i końcem wyrobiska, a co za tym idzie i wydatku. Upoważnia to do zaniedbania zjawisk falowych. Ze względu na duże masy powietrza, wypełniające wyrobiska, nie można zaniedbać bezwładności i dlatego do rozważań przyjęto model przedstawiony równaniem (3).

Jak wiadomo, zjawiska stochastyczne można scharakteryzować pewnymi uśrednionymi parametrami. W tym celu w artykule skorzystano z korelacyjnej teorii zjawisk losowych [Łojek 1993], [Swiesznikow 1965] wyprowadzając związki dla wartości średnich lewej i prawej strony równania (3). Dla przepływów niestacjonarnych wynik tych obliczeń pokazuje wzór (12), a dla stacjonarnych (13). Wyprowadzono też związek dla kowariancji lewej i prawej strony zlinearyzowanego równania (3), co pokazano wzorem (19) dla dowolnych rozkładów wydatku. Dla przepływu niestacjonarnego kowariancje wydatku

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i różnicy ciśnień przedstawia zależność (19), a dla stacjonarnego równanie (21), które po przekształceniu Fouriera daje związki między gęstościami widmowymi (22).

Przy założeniu, że wydatek i jego pochodna tworzą dwuwymiarowy rozkład normalny wyprowadzono związki dla kowariancji równania (3) dane wzorami (28) dla przepływów niestacjonarnych i (31) dla stacjonarnych.

Przy założeniu, że przepływ jest procesem stacjonarnym o rozkładzie symetrycznym względem wartości średniej, korzystając z definicji mocy chwilowej danej wzorem (34) wyprowadzono zależność na średnią moc, potrzebną do przetłoczenia przez wyrobisko opisane modelem (3), ilości powietrza o pewnej średniej wartości i wariancji w postaci wzoru (38).

Pokazano też możliwości wyznaczenia współczynników modelu (3) ze związków dla wartości średnich i kowariancji zapisanych zależnościami (32) i (33).

Z przeprowadzonej analizy przepływów z fluktuacjami wysnuto następujące wnioski:

Po pierwsze, do opisu wartości średniej przepływu w przypadku, gdy wariancja wydatku w porównaniu z jego wartością średnią osiąga znaczące wielkości, nie można stosować modelu (3), nawet gdy $\frac{dQ(t)}{dt}$ jest małe, lecz zależność (13), w której występuje wariancja wydatku.

Po drugie: przy eksperymentalnym wyznaczaniu współczynników równania A, B i H równania (3) stosując sposób opisany w rozdziale *Mathematical heading model*, należy uwzględnić wariancję wydatku (formuła (13)) lub skorzystać z metody podanej w dalszej części (wzory (32), (33)).

Po trzecie: występowanie fluktuacji w wydatku powoduje wzrost mocy potrzebnej dla przetłaczania tej samej średniej wartości wydatku płynu w porównaniu z mocą potrzebną do przetłaczania płynu o ustalonej wartości wydatku.

Uwzględnienie fluktuacji przepływu przy wyznaczaniu współczynników modelu wymaga dokonania długiego ciągu jednoczesnych pomiarów wydatku i różnicy ciśnień. Obecnie jest to możliwe dla kanałów o małych wymiarach poprzecznych, dla kanałów wielkości wyrobisk górniczych nie opracowano takich metod.

Wysnute powyżej wnioski z teoretycznych badań modelu wymagają weryfikacji eksperymentalnej. W tym celu budowane jest odpowiednie stanowisko badawcze, umożliwiające wytworzenie przepływów o wymaganych własnościach oraz ich badanie.

Słowa kluczowe: mechanika płynów, procesy stochastyczne, wentylacja kopalń.

LIST OF DENOTATIONS:

A, B, H — coefficients of proportionality,
d — channel diameter [m],
E — average value operator,
f — frequency [1/s],
$f[Q(t_1), Q(t_2)]$ — two-dimensional flow distribution function,
g — gravitational acceleration [m/s^2],
$K_Q(\tau)$ — flow covariance for stationary process [kg^2/s^2],
$K_p(t_1, t_2)$ — differential pressure covariance function, when the process is transient [Pa^2],
$K_p(\tau)$ — stationary process pressure covariance [Pa^2],
L — channel length [m],
M_{ik} — ordinary moments of a two-dimensional random variable of the order i, k,
$M(t)$ — temporary power [W],
$\bar{M}$ — average power value [W],

$\bar{P}(t)$	— average differential pressure value [Pa],
$P(t)$	— difference in pressure between the beginning and end of the channel [Pa],
$Q(t)$	— flow (flow quality) [kg/s],
$\bar{Q}(t)$	— average flow value (following all realisations) [kg/s],
$Q'(t)$	— flow derivative with respect to time [kg/s^2],
$\bar{Q}'(t)$	— average flow value derivative [kg/s^2],
$q(t)$	— flow fluctuations in the average value environment [kg/s],
$q'(t)$	— flow fluctuations derivative [kg/s^2],
$S_Q(j\omega)$	— spectral flow density function,
$S_p(j\omega)$	— differential pressure spectral density function,
t, t_1, t_2	— time [s],
Z	— vertical co-ordinate [m],
V_Q	— stationary process flow variance [kg^2/s^2],
$V_Q(t)$	— transient process flow variance [kg^2/s^2],
ϱ	— fluid density [kg/m^3],
$\varphi(t_1, t_2)$	— characteristic function of two-dimensional random variable $Q(t_1)$, $Q(t_2)$,
λ	— resistance coefficient,
$\omega = 2\pi f$	— frequency [1/s],

1. Introduction

Transient airflows are constantly present in mine ventilation networks. We may distinguish two fundamentally different scales of this phenomenon. On a small scale, these are turbulence phenomena with dimensions of the order of transverse sections of galleries. Work [Wasilewski, 1984] *, as well as measurements carried out in one such gallery with the co-participation of the author [Trutwin et al., 1996] show that the standard deviation of velocity measured at a point may reach up to approximately 25% of average velocity, with the frequency spectrum increasing along with the velocity, and that, for a velocity of approximately 1.8 m/s, a significant part thereof is ranging between 0 – 1 Hz.

The large-scale transient states, even under conditions of normal work that is not disturbed by any special occurrences, encompass the entire mine. The reasons therefore are: transport (movement of strings of cars and hutches), the opening and closing of ventilation stoppings, and — to a certain extent — changes in atmospheric pressure or the outflows of methane during the mining of coal.

Measurements executed [Trutwin et al., 1996] reveal that momentary (lasting no more than a few seconds) pressure changes in headings may equal even 75 Pa. A significant part of the pressure spectrum is located within the range 0 – 0.03 Hz. Differences in pressure measured in the same heading between the beginning and end of the gallery were of the order of about 15 Pa, while a significant part of the spectrum was situated in the range 0 – 0.001 Hz. We may expect the flow spectrum to be of the same order as the differential pressure spectrum.

* The relevant items of cited literature have been given in square brackets.

2. Mathematical heading model

One dimensional flow in conduits is described by well known equation of motion [Pawiński et al., 1995]:

$$\frac{\partial w}{\partial t}+w\frac{\partial w}{\partial s}+\frac{1}{\varrho}\cdot\frac{\partial p}{\partial s}+g\frac{dz(s)}{ds}+\lambda\frac{w^2}{2g}=0 \tag{1}$$

derived from the momentum equation, in which the term $\lambda\frac{w^2}{2g}$ represents dissipation of energy.

For small pressure and temperature fluctuations compressibility of air may be neglected without risk of making a large error. This assumption together with continuity equation, for a conduit of a constant cross section area leads to conclusion that $\frac{\partial \mathrm{w}}{\partial \mathrm{s}}=0$. Therefore equation (1) may be integrated along s and transormed to a form:

$$L\frac{dw}{dt}+g(z_1-z_2)+L\lambda\frac{w^2}{2g}=-\frac{1}{\varrho}(p_2-p_1). \tag{2}$$

In terms of a mass flow quantity we obtain:

$$A\frac{dQ(t)}{dt}+BQ^2(t)+H=P(t). \tag{3}$$

For straight, round conduits coefficients in (3) can be expressed as:

$$A=\frac{4L}{\pi d^2};\quad B=\frac{8\lambda}{\varrho\pi^2d^5}L;\quad H=\varrho g(z_1-z_2). \tag{4}$$

Of course, such a model may be applied within a somewhat limited frequency range and only when the flow proceeds in one direction.

It would seem that if the period of the greatest harmonic component is many times longer (10 times, for example) that the passage of the wave through the channel, the following will be of decisive importance for the type and magnitude of occurring phenomena: mass of the liquid in the channel, losses being the result of movement, and also the differences in altitude and pressure between the beginning and end of the channel. In this situation the wave phenomena may be neglected and model (3) shall sufficiently describe the flow through the channel.

Real mine headings have different transverse sections, roughness and temperatures of walls along their entire length, and, what is more, their course is frequently non-linear. These factors force us to carry out the experimental determination of factors of proportionality in model (3).

Normally, A and H are determined on the basis of averaging geometric measurements of the heading and air density. Coefficient B is determined on the basis of an flow quality measurement and differential pressure for a predetermined H, assuming that $dQ/dt = 0$. Such a procedure may lead to significant errors, which fact may be easily observed by calculating the average value for the left and right sides of expression (3).

3. Relations for average values

Averaging [Cierniak et al., 1977] the left and right sides of formula (3), we receive:

$$E\left\{A\frac{d}{dt}Q(t)+BQ^2(t)+H\right\} = E[P(t)] \tag{5}$$

and next

$$AE\left\{\frac{d}{dt}Q(t)+BE[Q^2(t)]+H\right\} = E[P(t)]. \tag{6}$$

Substituting

$$Q(t) = Q(t)-E[Q(t)+E[Q(t)]$$

we receive:

$$\begin{aligned} E[Q^2(t)] &= E[Q(t)-E[Q(t)+E[Q(t)]]^2 = E[Q(t)-E[Q(t)]]^2+ \\ &+E\{2E[Q(t)][Q(t)-E[Q(t)]]\}+\{E[Q(t)]\}^2 = \{E[Q(t)]\}^2+E\{Q(t)-E[Q(t)]\}^2. \end{aligned} \tag{7}$$

From definition $E\{Q(t)-E[Q(t)]\}^2$ is a variance $Q(t)$.
Introducing denotations to be used below:

$$E[Q(t)] = \bar{Q}(t), \tag{8}$$

$$E[p(t)] = \bar{P}(t) \tag{9}$$

we may write:

$$E\{Q(t)-E[Q(t)]\}^2 = V_Q(t), \tag{10}$$

$$E[Q^2(t)] = \{E[Q(t)]\}^2+V_Q(t) = \bar{Q}^2(t)+V_Q(t). \tag{11}$$

Applying the theorem on changing the order of averaging and differentiating, expression (6) may be presented as follows:

$$A\frac{d}{dt}\bar{Q}(t)+B[\bar{Q}^2(t)+V_Q(t)]+H = \bar{P}(t). \tag{12}$$

When the process is stationary, then $\bar{P}(t) = \text{const.}$ and $\bar{Q}(t) = \text{const.}$, while equation (12) may be reduced to the following equation:

$$B[\bar{Q}^2+V_Q]+H = \bar{P}. \tag{13}$$

Two facts follow from dependence (13). First, the increase in flow variance leads to an increase in the average differential pressure value necessary to maintain unchanged average flow value, while second — the omission of flow variances when determining coefficient B results in an error, even when the process in stationary. Of course, to determine B it is required to have H, which may be determined from (4).

For the non-stationary process we may, applying dependence (12), determine all of the coefficients on the basis of flow and differential pressure measurements only, since for various time periods it is possible to arrange a system of equations with a non-zero matrix determinant, like from dependence (3). It would appear, however, that the derivative of the average flow value may be determined with greater ease and precision than the temporary value derivative.

Yet another possibility consists in applying the covariance of the left and right sides of dependence (3).

4. Relations for the covariance

Because of the presence of flow in the square, the derivation [Cierniak et al., 1977] of analytical dependencies for the covariance of the left and right sides is possible only for specific distributions of the two-dimensional variable $Q(t_1)$, $Q(t_2)$. This will next be derived for a normal distribution. Due to the fact that there are no such requirements concerning a distribution in the case of linear differential equations, dependencies for the linearized form of equation (3) will be derived first.

If we assume that flow $Q(t)$ fluctations are small, we may proceed to linearise dependence (3) and thus obtain the following equation:

$$A\frac{dq(t)}{dt}+A\frac{d}{dt}\bar{Q}(t)+2Bq(t)+B\bar{Q}^2(t)+H = P(t), \tag{14}$$

where:

$$Q(t) = \bar{Q}(t)+q(t). \tag{15}$$

The notation of the covariance for both sides of the equation (14) upon introduction of the convention:

$$\frac{d}{dt}Q(t) = Q'(t); \quad \frac{d}{dt}q(t) = q'(t)$$

has the form:

$$\begin{aligned} &E\{[Aq'(t_1)+A\bar{Q}'(t_1)+2Bq(t_1)+B\bar{Q}^2(t_1)+H-A\bar{Q}'(t_1)-B\bar{Q}^2(t_1)-H]\cdot \\ &\cdot[Aq'(t_2)+A\bar{Q}'(t_2)+2Bq(t_2)+B\bar{Q}^2(t_2)+H-A\bar{Q}'(t_2)-B\bar{Q}^2(t_2)-H]\} = \\ &= E[P(t_1)-\bar{P}(t_1)]\cdot[P(t_2)-\bar{P}(t_2)] \end{aligned} \tag{16}$$

and then, following reductions:

$$E\{[Aq'(t_1)+2Bq(t_1)]\cdot[Aq'(t_2)+2Bq(t_2)]\} = K_p(t_1, t_2). \tag{17}$$

Upon multiplying, it takes the form:

$$A^2E[q'(t_1)q'(t_2)]+2ABE[q'(t_1)q(t_2)]+2ABE[q(t_1)q'(t_2)]+ \\ +B^2E[q(t_1)q(t_2)] = K_p(t_1, t_2). \tag{18}$$

If we change the order of differentiating with averaging, the above expression may be transformed into the following form:

$$A^2\frac{\partial^2}{\partial t_1 \partial t_2}K_Q(t_1, t_2)+2AB\frac{\partial}{\partial t_1}K_Q(t_1, t_2)+2AB\frac{\partial}{\partial t_2}K_Q(t_1, t_2)+ \\ +4B^2K_Q(t_1, t_2) = K_Q(t_1, t_2). \tag{19}$$

In the event of a stationary process:

$$K(t_1, t_2) = K(t_2-t_1) = K(\tau) \tag{20}$$

and equation (19) receives a shorter form:

$$-A^2\frac{\partial}{\partial\tau^2}K_Q(\tau)+4B^2K_Q(\tau) = K_p(\tau). \tag{21}$$

If we subject the last expression to Fourier transformation, we shall receive the relation:

$$(A^2\omega^2+4B^2)S_Q(j\omega) = S_p(j\omega) \tag{22}$$

connecting the functions of spectral output density and differential pressure, which may be easily calculated applying Fast Fourier Tranformation.

Seeking covariance relations in the event of non-linearity is considerably more difficult. The issue is simplified if the non-linearities are polinomials and the form of the distribution function is known. This is the case of equation (3) if we assume that the two-dimensional distribution function $Q(t_1)$, $Q(t_2)$ is known.

As was mentioned previously, hereonforward it is assumed that the distribution is normal and two-dimensional.

Applying dependence (11), the covariance of process (3) may be presented by dependence.

$$E\{[A\frac{d}{dt_1}Q(t_1)+BQ^2(t_1)+H-A\frac{d}{dt_1}\bar{Q}(t_1)-\bar{B}Q^2(t_1)-BV_Q(t_1)-H]\cdot \\ \cdot[A\frac{d}{dt_2}Q(t_2)+BQ^2(t_2)+H-A\frac{d}{dt_2}\bar{Q}(t_2)-B\bar{Q}^2(t_2)-BV_Q(t_2)-H]\} = \\ = E\{[P(t_1)-\bar{P}(t_1)][P(t_2)-\bar{P}(t_2)]\} = K_p(t_1, t_2). \tag{23}$$

Applying the theorem on changing the order of averaging and differentiating and having executed the appropriate multiplications and ordered the formulae, we receive:

$$\begin{aligned}
& A^2 \frac{\partial}{\partial t_1 \partial t_2} K_Q(t_1, t_2) + \\
& \quad + AB \frac{\partial}{\partial t_1} \{E\{[Q(t_1) - \bar{Q}(t_1)][Q(t_2) - \bar{Q}(t_2)]^2\} + 2\bar{Q}(t_2) K_Q(t_1, t_2)\} + \\
& \quad + AB \frac{\partial}{\partial t_2} \{E\{[Q(t_2) - \bar{Q}(t_2)][Q(t_1) - \bar{Q}(t_1)]^2\} + 2\bar{Q}(t_1) K_Q(t_1, t_2)\} + \\
& \qquad + B^2 E\{[Q(t_1) - \bar{Q}(t_1)]^2 [Q(t_2) - \bar{Q}(t_2)]^2\} + \\
& \qquad + 2B^2 \bar{Q}(t_2) E\{[Q(t_1) - \bar{Q}(t_1)]^2 [Q(t_2) - \bar{Q}(t_2)]\} + \\
& \qquad + 2B^2 \bar{Q}(t_1) E\{[Q(t_1) - \bar{Q}(t_1)][Q(t_2) - \bar{Q}(t_2)]^2\} + \\
& \qquad\quad + 4B^2 \bar{Q}(t_1)\bar{Q}(t_2) K_Q(t_1, t_2) - B^2 V_Q(t_1) V_Q(t_2) = K_p(t_1, t_2).
\end{aligned} \tag{24}$$

If we make no assumptions regarding the form of the distribution density function $f[Q(t_1), Q(t_2)]$, then formula (24) is the final form.

On the left side of the above formula there appear average flow values, the covariances thereof, derivatives of covariances and terms situated beneath the sign of the averaging operator E, which are the central moments of different grades. It is most convenient to calculate [Łojek 1993], [Smiesznikow 1965] these moments from the characteristic function that for the two-dimensional random variable has the following form:

$$\begin{aligned}
\varphi(Z_1, Z_2) &= E\{\exp[jZ_1 Q(t_1), jZ_2 Q(t_2)]\} = \\
&= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f[Q(t_1), Q(t_2)] \exp[jZ_1 Q(t_1) + jZ_2 Q(t_2)] dQ(t_1) dQ(t_2),
\end{aligned} \tag{25}$$

while moments are calculated applying the formula:

$$M_{i,k} = \frac{1}{j^{i+k}} \frac{\partial^{i+k}}{\partial^i Z_1 \partial^k Z_2} \varphi(Z_1, Z_2)_{|Z_1 = Z_2 = 0|}. \tag{26}$$

In practice, normal distribution is most common. The authors of several works have voiced the opinion that output in mine headings is characterized by exactly this type of distribution. An experiment carried out with the co-participation of the author does not contradict this. Thus, further calculations will be made for two-dimensional normal distribution, the function of which is characteristic of the average value, this equal to zero (and such is the case in expressions on the left side of eq. (24)), has the form:

$$\varphi(Z_1, Z_2) = \exp\left[-\frac{1}{2} Z_1^2 V_Q(t_1) - Z_1 Z_2 K_Q(t_1, t_2) - \frac{1}{2} Z_2^2 V_Q(t_2)\right]. \tag{27}$$

Applying dependencies (25), (26) for dependences (24), we receive:

$$A^2 \frac{\partial^2}{\partial t_1 \partial t_2} K_Q(t_1, t_2) + 2AB \frac{\partial}{\partial t_1} [\bar{Q}(t_1, t_2) K_Q(t_1, t_2)] + 2AB \frac{\partial}{\partial t_2} [\bar{Q}(t_2) K_Q(t_1, t_2] +$$

$$+ B^2 K_Q^2(t_1, t_2) + 4B^2 \bar{Q}(t_1) \bar{Q}(t_2) K_Q(t_1, t_2) = K_p(t_1, t_2). \quad (28)$$

If the process is stationary, then

$$\bar{Q}(t_1) = \bar{Q}(t_2) = \bar{Q}$$

and:

$$K_Q(t_1, t_2) = K_Q(t_2 - t_1) = K_Q(\tau) \quad (29)$$

and equation (28) obtains the following form:

$$-A^2 \frac{\partial^2}{\partial \tau^2} K_Q(\tau) - 2AB \frac{\partial}{\partial \tau} K_Q(\tau) + 2AB\bar{Q} \frac{\partial}{\partial \tau} K_Q(\tau) + B^2 K_Q^2(\tau) + 4B^2 \bar{Q}^2 K_Q(\tau) = K_p(\tau) \quad (30)$$

and, further,

$$-A^2 \frac{\partial^2}{\partial \tau^2} K_Q(\tau) + 4B^2 \bar{Q}^2 K_Q(\tau) + B^2 K_Q^2(\tau) = K_p(\tau). \quad (31)$$

Relations for covariances given by formulae (19), (21), (28) and (31) or relation (22) for spectral density make it possible to obtain additional equations which are required in order to determine the entirety of coefficients of model (3).

Dependencies (21), (22), (31) are of the form:

$$af(x) + bg(x) = h(x), \quad (32)$$

where: $x = \tau$ or $x = \omega$.
This allows us to arrange N equations for $x = x_i$ $i = 1 \ldots N$ and determine a and b from dependence:

$$\min_{a,b} \left\{ \sum_{i=1}^{N} [af(x_i) + bg(x_i) - h(x_i)]^2 \right\} \quad (33)$$

and this considerably reduces the uncertainty of determining a and b.
Since it is assumed that a and $b > 0$ then $A = \sqrt{a}$ and $B = \sqrt{b}$.

5. Average power consumption

The result of the calculation of the average value (13) for both the left and right side of equation (3) implies an increase in the average power requirement, needed in order to force through the liquid for an flow of a stochastic nature as compared with a stationary flow. The average power consumption value will be calculated on the assumption that process $Q(t)$ is stationary and with a symmetric distribution with respect to the average value (for example, normal distribution).

The temporary power required in order to force a flow with an flow $Q(t)$ with a differential pressure $P(t)$ is expressed by the dependence:

$$M(t) = Q(t)P(t). \tag{34}$$

For the adopted model we may thus write:

$$M(t) = AQ(t)\frac{d}{dt}Q(t) + BQ^3(t) + HQ(t). \tag{35}$$

Following the introduction of the replacement:

$$Q(t) = \bar{Q} + q(t)$$

the average power value is expressed by formula:

$$\bar{M} = AE\left[\bar{Q} + q(t)\frac{d}{dt}q(t)\right] + BE\left[\bar{Q}^3 + 3\bar{Q}^2 q(t) + 3\bar{Q}q^2(t) + q^3(t)\right] + \\ + H\bar{Q} + HE[q(t)] \tag{36}$$

and, further:

$$\bar{M} = AE\left[q(t)\frac{d}{dt}q(t)\right] + B\bar{Q}^3 \ldots + 3B\bar{Q}E[q^2(t)] + BE[q^3(t)] + H\bar{Q} \tag{37}$$

since the average value $q(t)$ is by assumption equal to zero, and also the expression:

$$E\left[q(t)\frac{d}{dt}q(t)\right] = \frac{1}{2}\frac{d}{dt}E\left[q^2(t)\right] = \frac{1}{2}\frac{d}{dt}K_q(t)_{t=0} = 0$$

since the covariance has a maximum for $t = 0$.

The expression

$$E[q^3(t)] = \int_{-\infty}^{\infty} q^3 f(q)dq = 0$$

since function $f(q)$ is by assumption a symmetric function and when multiplied by the unsymmetrical function $q^3(t)$ produces an unsymmetrical function, the integral of which in the range $+\infty$, $-\infty$ is equal to zero.

Thus, expression (36) may be transformed to form:

$$\bar{M} = B\bar{Q}^2 + 3B\bar{Q}V_Q + H\bar{Q}. \tag{38}$$

If follows from the above dependence that the increase in power necessary in order to force through liquid with an average flow value $\bar{Q}$ is proportional to the average value of the flow and the variance thereof.

6. Conclusions

Several conclusions may be drawn from the analyses of flows with fluctuations.

First of all, to describe the average flow value when the flow variance attains considerable values in comparison with its average value we cannot apply model (3) — even if $\frac{dQ(t)}{dt}$ is small — but dependence (13), in which there is present an flow variance.

Secondly: when experimentally determining the equation coefficients A, B and H using the method described in the chapter *Mathematical heading model*, it is necessary to take into consideration the flow variance (formula (13)) or otherwise use the method set forward subsequently (formulae (32), (33)).

Thirdly: the occurrence of fluctuations in flow brings about an increase in the power which is required to force through the same average value of fluid flow quantity as compared with the power necessary in order to force through fluid with a stationary flow value.

Taking into consideration flow fluctuations when determing the coefficients of the model requires making a lengthy series of simultaneous measurements of flow quantity and differential pressure. At present, this is possible only for channels which have small transverse dimensions; no such methods have been elaborated for channels of the size of mine headings.

The conclusions presented above, reached on the basis of theoretical model research, need to be verified experimentally. To this end, an appropriate test bed is being constructed, and this will make it possible to generate and research flows that have that required properties.

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Received: 06 June 2000.