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Using option pricing for valuation of mineral deposits

Key words

Valuation, mineral deposits, options, scenario analysis

Abstract

The article presents three different approaches to a valuation of a mineral deposit. The use of classical DCF methodology was tested against option pricing supplemented by scenario analysis. The option-pricing approach was applied in order to include the freedom of choice factor into consideration. The DCF methodology disregards the fact that acquiring party may alternate time scheme of investing in mining activities according to when the deposit will be really needed. The scenario analysis was employed in order to translate financial solutions into real action plans.

Introduction

While the valuation of mineral deposits shares some characteristics of other assets' pricing, it has its own very distinctive features, listed as follows:

— mineral deposits, as opposed to most other assets, cannot be created by humans[1],

— mineral deposits are exhaustible[2],

— each mineral deposit is unique in an economic sense — standardisation is impossible,

— mineral deposits, once exhausted, have no further value — on the contrary, substantial costs typically must be incurred in order to restore land from which the minerals were extracted.

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1 The validity of this statement is limited, but not fully contradicted, by the fact that some minerals can be substituted by manufactured products, e.g. mineral sulphur may readily be substituted on the market by sulphur-based chemicals obtained as by-products in various production processes (cement, oil refining, etc.).

2 For a theory on the economics of exhaustible resources (Hotelling 1931).

Due to these properties, mineral resource prices are very sensitive to market fluctuations and mining enterprise management is constantly seeking to determine the right time and price for purchase of offered deposits[3]. This issue has to be considered in the context of business cycles. Managers have to find a balance between the need to secure sources of minerals when the market asks for supply on one side, and cost and time burdens on the other. Rendering a deposit productive is time consuming, typically requiring between 6 and 24 months. Deposit prices are often positively correlated with the market value of respective minerals. For these two reasons, mining companies are forced to buy deposits when they actually do not need them and value them based on expected market developments. This is exactly like purchasing a call option, albeit a very expensive one.

Let us consider an example based on actual events[4]. A company, Silesia Mining (SM), operates four natural rock quarries (stone-pits) with quite limited resources. In view of an expected upturn in construction activities over the next three years, specifically highways and railways, SM management foresees a need to expand production capacities and it is likely that presents deposits will not be sufficient to yield the required future output. At the same time, SM has been approached by an owner of a highly attractive deposit — Stone-Hill — located on the crossroads between the four currently operated stone-pits. The asking price is 17.7 million USD[5].

1. The classical DCF approach to the purchase of the Stone-Hill deposit

SM has carried out a discounted cash flow analysis, which is unfortunately yielding a negative result — NPV below zero (Table 1). Capital expenditures represent the asking price in 2002 and necessary equipment purchases over the period 2004—2006.

Based on these calculations, the maximum price SM could accept is ca. 14.65 million USD, which is the asking price minus NPV.

There are a number of implicit, important assumptions in the calculation presented above, of which neither managers nor analysts are usually fully aware. **The first such assumption** is the **discretionary application of the same discount rate to all cash flows,** which is based on a risk evaluation of operational cash flow. There is usually no supporting argument for use of the same rate to discount investments on PPE. Here, costs do not depend mostly on minerals markets but rather on progress and competition in the construction and machinery industries[6]. One reason that this issue is commonly neglected might be a fact that, in many cases, investments occur in the project commencement period and therefore the real impact of an actual difference resulting from the use of a incorrect rate is minimal[7]. Over a long time-span of investment activities, however, such as in the Stone-Hill analyses, one must address this problem. We shall start from

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[3] Kernot 1999, pp. 231—234.

[4] Names, currencies and some details have been changed but in such a way that these departures are irrelevant for the discussion.

[5] The deposit offered was free from any legal claims and material constrains on further exploitation.

[6] Luehrman 1988, pp. 60—62.

[7] In the author's experience in FMCG and retail businesses, typically all investment outlays occur in the first year. The opposite situation is observed in R&D activities, exploration, and mineral and energy businesses.

TABLE 1

DCF evaluation of purchasing the Stone-Hill deposit

TABELA 1

Wycena złoża „Kamienna Góra" metodą zdyskontowanych przepływów gotówki

	Year of expoloitation										
Item	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Cash Flom from Operations											
Revenues	0	0	0	9 557 190	18 143 310	18 143 310	18 143 310	18 143 310	18 143 310	18 143 310	18 143 310
Operating Costs (net of deprecation)	0	(226 745)	(232 151)	(5 911 785)	(6 954 191)	(7 541 209)	(7 541 209)	(7 541 209)	(7 541 209)	(7 541 209)	(7 541 209)
Change in Net Working Capital	0			(1 625 907)	(1 184 519)	(253)	0	0	0	0	0
Income tax	0	54 419	55 716	(604 773)	(2 438 637)	(2 072 668)	(2 072 668)	(2 072 668)	(2 072 668)	(2 072 668)	(2 072 668)
Residual Value	0	0	0	0	0	0	0	0	0	0	37 018 230
Total CF from operations	**0**	**(172 326)**	**(176 435)**	**1 414 725**	**7 565 963**	**8 529 179**	**8 529 433**	**8 529 433**	**8 529 433**	**8 529 433**	**45 547 662**
Cash Flow from Investment Activities											
Capital Expenditures	(17 700 000)	0	(16 818 560)	(2 700 000)	(855 000)	0	0	0	0	0	0
Total CF from Investment Activities	**(17 700 000)**	**0**	**(16 818 560)**	**(2 700 000)**	**(855 000)**	**0**	**0**	**0**	**0**	**0**	**0**
Total CF	(17 700 000)	(172 326)	(16 994 995)	(1 285 275)	6 710 963	8 529 179	8 529 433	8 529 433	8 529 433	8 529 433	45 547 662
DCF	(17 700 000)	(147 603)	(12 468 300)	(807 654)	3 612 079	3 932 084	3 368 052	2 884 841	2 470 956	2 116 451	9 680 487
NPV	(3 058 607)										
Accumulated discaunting factor	1.00	1.17	1.36	1.59	1.86	2.17	2.53	2.96	3.45	4.03	4.71
Annual discount rate	16.75%	16.75%	16.75%	16.75%	16.75%	16.75%	16.75%	16.75%	16.75%	16.75%	16.75%
Risk free rate		5%	5%	5%	5%	5%	5%	5%	5%	5%	5%
Risk		11.75%	11.75%	11.75%	11.75%	11.75%	11.75%	11.75%	11.75%	11.75%	11.75%

the observation that the use of unjustifiably high discount rate for investment expenses overestimates a project's NPV[8]. As a consequence we observe a situation in which the high risk of the projected operational cash flow decreases the exposure to capital expenditures. This is not only counterintuitive, but also almost certainly untrue. Therefore, we need to adjust for this problem by differentiating discount rates for operational and investment activities. Luehrman suggests the use of the federal bond rate as the most prudential course of action[9]. We will use a risk-free rate because provisions for unexpected expenditures have already been included into the investment budget.

A second problem relates to the classification of residual value. Some analysts place it among cash flows from investment activities; few opt to classify it into operational cash flow. If we had considered this before the discussion of different discount rates, this question could have seemed academic. Now, however, it has a very practical implication regarding the use of a proper discount rate. To address this, we should return to the origin of residual value. Does it represent proceeds from the liquidation of assets? Or, maybe, it represents the uncertainty of cash flows in the far future and is calculated by discounting the stabilised stream of net profits through the final depletion of deposit being valued?[10] In the case of SM's valuation of Stone-Hill, the residual value calculation was based on the latter option, therefore putting residual value among operational activities and the use of the same discount rate is justified, at least to some extent[11]. If, however, residual value represents the liquidation of machinery and equipment, one should consider a different solution.

There are other commonly used and hidden assumptions in the DCF analysis presented above such as that the losses in first 3 years are tax deductible, the deposit is depreciated according to the real depletion rate and so on[12].

After consideration of the aforementioned issues, the adjusted DCF yields quite different results (Table 2).

Now, based on the adjusted calculations, the maximum price SM could accept is ca. 10.85 million USD, far below the some 14.65 million USD we obtained previously. This is extremely discouraging. One can reasonably expect to close a 15—20% gap in negotiations, but here SM's maximum acceptable price is close to 40% below the other party's asking price. Here, we shall come to the root of SM's problem. The purchase of a mineral deposit does not carry an obligation to exploit the deposit; it is rather an option to do so. In view of the factors by which SM management is guided, this transaction may be assumed as a case of hedging against the risk of

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[8] This occurs because investment expenses have a negative impact on a project's NPV while a discount rate, by transferring the time value of money to the present, reduces their absolute value.

[9] Luehrman assumes that a company would invest money in the safest financial instrument over the waiting period (Luehrman 1998, p. 62). This is very close to using a risk-free rate, which also seems justified because even if the financial means designated for the deferred investment are used for other projects, one does not incorporate unrelated risk rates into DCF calculations.

[10] Or using another, similar method.

[11] This is the solution Luehrman applied (Luehrman 1998, p. 57).

[12] The article intentionally avoids a discussion of the controversial, but appealing, Hotelling proposition, since this would require a thorough analysis beyond the scope of this paper (Brealey, Myers 1996, pp. 274—276; Hotelling 1931; Liteenberger, Rabinovitz 1995).

TABLE 2

Adjusting DCF based valuation of the Stone-Hill deposit to different risk rates

TABELA 2

Wycena złoża „Kamienna Góra" zmodyfikowaną metodą zdyskontowanych przepływów gotówki

Item	Year of expoloitation										
	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Total CF from operations	0	(172 326)	(176 435)	1 414 725	7 565 963	8 529 179	8 529 433	8 529 433	8 529 433	8 529 433	45 547 662
DCF (A)	**0**	**(147 603)**	**(129 441)**	**888 999**	**4 072 271**	**3 932 084**	**3 368 052**	**2 884 841**	**2 470 956**	**2 116 451**	**9 680 487**
NPV (A)	29 137 098										
Total CF from Investment Activities	(17 700 000)	0	(16 818 560)	(2 700 000)	(855 000)	0	0	0	0	0	0
DCF (B)	**(17 700 000)**	**0**	**(15 254 930)**	**(2 332 362)**	**(703 411)**	**0**	**0**	**0**	**0**	**0**	**0**
NPV (B)	(35 990 702)										
ANPV	(6 853 604)										
Accumulated discaunting factor (A)	1.00	1.17	1.36	1.59	1.86	2.17	2.53	2.96	3.45	4.03	4.71
Annual discount rate (A)		16.75%	16.75%	16.75%	16.75%	16.75%	16.75%	16.75%	16.75%	16.75%	16.75%
Risk free rate		5%	5%	5%	5%	5%	5%	5%	5%	5%	5%
Risk		11.75%	11.75%	11.75%	11.75%	11.75%	11.75%	11.75%	11.75%	11.75%	11.75%
Accumulated discaunting factor (B)	**1.00**	**1.05**	**1.10**	**1.16**	**1.22**	**1.28**	**1.34**	**1.41**	**1.48**	**1.55**	**1.63**
Annual discount rate (B)		5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%
Risk free rate		5%	5%	5%	5%	5%	5%	5%	5%	5%	5%
Risk		0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

TABLE 3

Decomposition of the DCF based evaluation of Stone-Hill deposit using different discount rates

TABELA 3

Wycena złoża „Kamienna Góra" zmodyfikowaną metodą zdyskontowanych przepływów gotówki — rozgraniczenie fazy I i II

Item	Year of expoloitation										
	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
1	2	3	4	5	6	7	8	9	10	11	12
Stage 1 purchase of deposit											
Cash Flom from Operations											
Revenues	0	0	0	0	0	0	0	0	0	0	0
Operating Costs (net of deprecation)	0	(226 745)	(232 151)	(232 151)	(232 151)	(232 151)	(232 151)	(232 151)	(232 151)	(232 151)	(232 151)
Change in Net Working Capital	0	0	0	0	0	0	0	0	0	0	0
Income tax	0	54 419	55 716	55 716	55 716	55 716	55 716	55 716	55 716	55 716	55 716
Total CF (1) from operations	**0**	**(172 326)**	**(176 435)**	**(176 435)**	**(176 435)**	**(176 435)**	**(176 435)**	**(176 435)**	**(176 435)**	**(176 435)**	**(176 435)**
Cash Flow from Investment Activities											
Capital Expenditures	(17 700 000)	0	0	0	0	0	0	0	0	0	0
Total CF (1) from Investment Activities	**(17 700 000)**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
DCF (1)	(17 700 000)	(147 603)	(129 441)	(110 870)	(94 963)	(81 339)	(69 670)	(59 674)	(51 113)	(43 780)	(37 499)
ANPV (1)	(18 525 950)										
Stage 2 excercising the option to exploit the deposit											
Cash Flom from Operations											
Revenues	0	0	0	9 557 190	18 143 310	18 143 310	18 143 310	18 143 310	18 143 310	18 143 310	18 143 310
Operating Costs (net of deprecation)	0	0	0	(5 679 634)	(6 722 040)	(7 309 058)	(7 309 058)	(7 309 058)	(7 309 058)	(7 309 058)	(7 309 058)
Change in Net Working Capital	0	0	0	(1 625 907)	(1 184 519)	(253)	0	0	0	0	0

cd. table 3

cont. tabela 3

1	2	3	4	5	6	7	8	9	10	11	12
Income tax	0	0	0	(660 489)	(2 494 354)	(2 128 385)	(2 128 385)	(2 128 385)	(2 128 385)	(2 128 385)	(2 128 385)
Residual Value	0	0	0	0	0	0	0	0	0	0	37 018 230
Total CF (2) from operations	**0**	**0**	**0**	**1 591 160**	**7 742 397**	**8 705 614**	**8 705 867**	**8 705 867**	**8 705 867**	**8 705 867**	**45 724 097**
DCF (2) from operations	0	0	0	999 869	4 167 234	4 013 423	3 437 722	2 944 515	2 522 069	2 160 230	9 717 985
PV (2) of CF from operations	29 963 048										
Cash Flow from Investment Activities											
Capital Expenditures	0	0	(16 818 560)	(2 700 000)	(855 000)	0	0	0	0	0	0
Total CF (2) from Investment Activities	**0**	**0**	**(16 818 560)**	**(2 700 000)**	**(855 000)**	**0**	**0**	**0**	**0**	**0**	**0**
DCF (2) from Investment Activities	0	0	(15 254 930)	(2 332 362)	(703 411)	0	0	0	0	0	0
PV (2) of CF for PPE	(18 290 702)										
ANPV (2)	11 672 346										
ANPV (1) + (2)	(6 853 604)										
Accumulated discaunting factor (A)	1.00	1.17	1.36	1.59	1.86	2.17	2.53	2.96	3.45	4.03	4.71
Annual discount rate (A)		16.75%	16.75%	16.75%	16.75%	16.75%	16.75%	16.75%	16.75%	16.75%	16.75%
Risk free rate		5%	5%	5%	5%	5%	5%	5%	5%	5%	5%
Risk		11.75%	11.75%	11.75%	11.75%	11.75%	11.75%	11.75%	11.75%	11.75%	11.75%
Accumulated discaunting factor (B)	**1.00**	**1.05**	**1.10**	**1.16**	**1.22**	**1.28**	**1.34**	**1.41**	**1.48**	**1.55**	**1.63**
Annual discount rate (B)		5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%
Risk free rate		5%	5%	5%	5%	5%	5%	5%	5%	5%	5%
Risk		0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

being unable to supply the market in an economic upturn. Therefore, we are faced with two decisions instead of one:

— Purchasing an option now to be exercised (or not) in three years by extracting resources from the Stone-Hill deposit.

— Exercising the option by investing, 3 years from now, in mineral mining.

Consequently, we can rearrange our DCF calculations in order to reflect the dual decision structure (Table 3). The only change is in the presentation of data; therefore, we still have the same result. One may observe that the negative ANPV of Stage 1 of about –18.53 million USD is only partially covered by the positive ANPV of Stage 2 of about +11.67 million USD.

At this stage, we should reconsider whether a DCF analysis is appropriate for financial evaluation of both of the above decisions. In the first case, the answer is yes. Cash flows are determined and calculated at present; a level of risk is incorporated into the discount rate. In the second instance, however, and as pointed out previously, the decision is to be made three year hence, therefore, SM still has a considerable freedom. This is an area in which DCF is of limited utility; DCF carries an underlying assumption that all cash flows are determined by present decision(s). Therefore, the PV of Stage 2, about 11.67 million USD, reflects the value of the project as seen three years before its commencement. In the language of derivative markets, the DCF approach implies that SM is making a forward transaction and it values the freedom of decision over this period at zero. This means that all knowledge and experience SM gains over the deferred period have no impact on our analysis because a forward transaction has to be executed[13]. This issue cannot be neglected. It is clear that SM needs an instrument that values freedom of choice in financial terms.

2. Valuing the purchase of the Stone-Hill deposit as an option

In this section, we address options in a financial sense. One view on the value of an option (as opposed to a future/forward transaction) is that it gives an owner freedom of choice, provided that for other (business) reasons he will have to consider buying the underlying asset at the time of option expiration. Therefore, we can start mapping variables into an option-pricing model: namely Black-Scholes[14] (see Table 4)[15]. We will use a European option as an example[16]. Mapping the first four variables does not cause much difficulty in view of the above interpretation of Stage 2. The PV of the expected return represents the present value of the future price of the Stone-Hill deposit (S), if all conditions remain the same. The present value of expected expenditures on PPE[17] represents the present value of the exercise price PV (X). An option expiration time is the expected time to the final decision, in this case $t = 3$ years. At the time of

[13] Even by mean of settling the differences (Hull 1994).

[14] Although other attempts to value options have been considered and implemented, the Black-Scholes model is predominantly used in the financial world (Hull 1994).

[15] The presentation is based on Luehrman (1998). A good presentation of applying option-pricing in deposits valuation may also be found in Torries (1998).

[16] A European option can be exercised only on the terminal date while an American option may be exercised at any date prior to the call date (Hull 1994).

[17] PPE stands for Property, Plant and Equipment.

expiration, the risk free rate (r(f)) is given as 5%. We still require an equivalent of variance of returns on underlying assets, which will represent the risk to project assets. We do, however, have a risk rate and therefore can use the Capital Asset Pricing Model (CAPM) to estimate the variance of returns. Applying this model, we estimated this variance at 1.02[18]. Consequently, we could calculate the two variables determining a relative value of an option: NPVq and Cumulative Volatility (standard deviation multiplied by the square root of time)[19]. Next, we can determine a relative option value from a table — here about 75%[20] of S.

TABLE 4

Mapping a component DCF analysis onto a call option pricing (Black-Scholes) model*

TABELA 4

Wycena złoża z wykorzystaniem modelu wyceny opcji (Blacka-Scholesa)

S — PV (2) of CF from operations	29 963 048	*Other standard deviation rates applied*			
PV(X) — PV of CF for PPE	18 290 702				
t — time the decision can be deferred	3				
r (f) — risk free rate of return	5%				
Standard deviation	1.14				
NPVq	1.64	*1.64*	*1.64*	*1.64*	*1.64*
Standard devation	1.02	*0.80*	*0.60*	*0.50*	*0.30*
St. deviation · sq. root of time	1.77	*1.39*	*1.04*	*0.87*	*0.52*
Relative option value	0.71	*0.66*	*0.54*	*0.51*	*0.42*
PV (2) = Option value	**21 273 764**	***19 775 612***	***16 180 046***	***15 281 154***	***12 584 480***
PV (1) = ANPV (1)	**(18 525 950)**	***(18 525 950)***	***(18 525 950)***	***(18 525 950)***	***(18 525 950)***
NPV of the project	**2 747 814**	***1 249 661***	***(2 345 904)***	***(3 244 796)***	***(5 941 470)***

* The is based on a methodology presented in Luehrman (1998).

Table 4 presents the final valuation of the Stone-Hill deposit based on the option approach. Stage 1 has the same value of about — 18.53 million USD, but the value of Stage 2 is now as high as about 21.27 million USD. In view of these figures, SM management can safely

[18] For other variance estimation methods (Luehrman 1998, p. 58). Here, it was assumed that a correlation coefficient between mineral prices and a market index of the country of origin was 0.48. It is worth noticing that the standard deviation cannot be lower than 0.5 provided that the rate of risk was correctly estimated.

[19] $\mathrm{NPVq} = \frac{S}{PV(X)} = \frac{29963048}{18290702};\ \delta\sqrt{t} = 1.02 \cdot \sqrt{3}$

[20] (Brealey, Myers 1996), p. AP12-AP13.

commit the asking price (17.7 million USD) or even accept a higher price up to 20.45 million USD[21].

At the first glance, the calculations presented above seem to be a slick mathematical trick designed do decrease SM shareholders value. Why should they pay more (21.27 million USD) for an option to earn less (ANPV of Stage 2 based on the standard DCF calculations is only 11.67 million USD)? The answer is as follows. Are we sure that in three years time the Stage 2 value will still be the same? Of course, not. Will it be higher or lower? We do not know. Risk, in financial terms, is not the same as in insurance[22]. Financial risk assumes that the outcome may deviate from expectations in both directions, positively or negatively. The option obtained through an early purchase of the deposit gives SM an opportunity to take the full advantage of more positive scenarios than expected and decreases the risk of taking losses.

It is also worth noting that the value of deposit, under the option approach, is positively correlated with market volatility. This is even intuitively true because an option, which in essence gives security, shall be the least valuable asset on the most secure market. Table 4 also presents values for different standard deviations: 0.3 represents the level of blue chip volatility and 0.6 is an upper limit of non-speculative investments on developed markets. The one assumption here looks very high but it is (and must be) consistent with a high rate of risk rate used in the DCF calculation. After all, what would be the point in buying a deposit three years before commencing extraction if SM management believed that it operated in a stable and safe environment? Nevertheless, one can see that even the lowest possible standard deviation yields a higher valuation of Stage 2 than the classical DCF approach (15.28 million USD as opposed to 11.67 million USD). In this case freedom of choice is still quite valuable: –3.59 million USD. Following this logic, even if someone used the standard deviation of blue chips on New York Stock Exchange: 0.3[23], then the result would be closer to 12.58 million USD, only 0.91 million but still higher than the previously calculated figure.

It is worth confronting the figures calculated above to illustrate freedom of choice against what the actual impact these values may have in terms of possible managerial decisions. The SM management acts in a certain business with its burdens and rules and it is possible to formulate likely scenarios and what sort of decisions are to be made in each case.

3. Scenario analysis

SM management, starting from the very early stage of discussions on the Stone-Hill deposit purchase must have had in mind several possible scenarios of market development. Even if they were not made explicit, we may find them implicitly in the risk assumptions for the classical DCF analysis. Below we present four key scenarios:

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21 Representing the asking price plus a positive NPV from the project's evaluation.

22 Crane 1984, pp. 3—17.

23 In such a case, however, also the previously applied discount rate would have to be reconsidered and the DCF valuation would yield a higher value for Stone-Hill (Luehrman 1998, p. 58).

A. In three years, the minerals extracted by SM become obsolete due to the unexpected invention of a chemical substitute produced as a by-product at a very low cost.

B. Low demand persist last for 20-years. The deposit is temporally useless but sooner or later present SM deposits will be depleted. At present levels of production, the Stone-Hill deposit will become necessary in 10 years.

C. A nation-wide program of road construction commences as expected and brings about expected changes on the demand-side.

D. Not only a nation-wide program of road construction commences, but also, at the same time, several major competitors exhaust their resources and will not be able to increase production. On the contrary, natural depletion ensues sooner than expected and 2—3 years plus new deposits are needed in order to return to their former capacity.

Of course, any intermediate scenario is possible but, for simplicity of argument, has been omitted. Let us consider likely SM decisions in practice, scenario by scenario:

A. Here, the average price will drop by 80%. Consequently, extracting from the Stone-Hill deposit makes no economic sense and the deposit's value will be equal to the value of the forest and farming land over it[24]. The value of the forest and farming land is estimated at PV 0.73 million USD.

B. In this case, prices will remain 20% lower than expected and SM management will delay investment and start exploiting the Stone-Hill deposit seven years later. The PV of Stage 2 will be about 6.95 million USD.

C. This is the scenario for which DCF results hold.

D. In this scenario, prices increase by 40% more than expected. SM management will decide to accelerate extraction even at increased costs and the PV of Stage 2 will be 55.83 million USD.

Is this the expected result from the standard DCF calculation? A key remaining problem that arises here is the need to establish the probability of each scenario. What we know about the relationship between scenarios is that the cumulative NPV of each scenario weighted by its probability yields ANPV in a classical DCF analysis. Otherwise we should revise our assumptions due to the fact that the result obtained is sub-optimal[25]. We have also to assume that a decision to go with the whole investment plan was made at point 0 and will not be modified. Therefore, the difference between scenarios can be restricted to a price level only. While this assumption might be questioned, it is made only for simplicity and does not have any real impact. One may use a quite complex model with many different variables provided that the principle on cumulative NPV is observed.

If we conduct a sensitivity analysis against the price level of the DCF calculation, the PV of Stage 2 for each scenario in million USD is as follows: A (–29.17), B (1.46), C (11.67), D (32.09) — Table 5. In scenario C, SM management will not modify its present decision and will proceed with extraction. If one of the other scenarios comes to fruition, SM management will modify its decision. The problem is that DCF analysis does not capture this change. In this analysis, we assume that in each scenario SM management will commence extraction at the Stone-Hill deposit as scheduled and incur resulting losses (or opportunity costs).

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[24] In the discussed case, the offer also covers land.

[25] Brealey, Myers 1996, pp. 255— 263.

TABLE 5

Scenario comparison for Stone-Hill deposit

TABELA 5

Porównanie wyników analizy scenariuszowej dla złoża „Kamienna Góra"

Stone-Hill purchase	Scenario	Probability	Optimal decisions and their outcomes				DCF implied decisions and their outcomes		
ANPV of Stage 1			PV in 3 years of Stage 2	PV at the moment 0 of Stage 2	Expected outcome	Weighted ANPV	PV at the moment 0 of Stage 2	Expected outcome	Weighted ANPV
(18 525 950)	Scenario A	10%	1 000 000	733 645	(17 792 305)	(1 779 230)	(29 167 905)	(47 693 856)	(4 769 386)
	(Prices lower by 80%)								
	Scenario B	20%	9 473 885	6 950 473	(11 575 478)	(2 315 096)	1 462 283	(17 063 667)	(3 412 733)
	(Prices lower by 20%)								
	Scenario C	40%	20 675 821	11 672 346	(6 853 604)	(2 741 442)	11 672 346	(6 853 604)	(2 741 442)
	(Prices as expected)								
	Scenario D	30%	55 829 587	40 959 122	22 433 171	6 729 951	32 092 472	13 566 521	4 069 956
	(Prices 40% higher)								
Cumulative ANPV						**(105 816)**			**(6 853 604)**

The key problem here is the need to determine the probability of each scenario. Among available techniques, Monte Carlo simulations are often applied[26]. One may still drawn interesting conclusions if this procedure is not applied, but only by analysing different outcomes. Table 5 shows results of ANPV calculations for two different assumptions: that the decision on investing in Stone-Hill exploitation is to be made in 3 years and that such a decision has already been predetermined. In this case, probabilities are determined based on expert evaluation[27]: scenario A: 10%, B: 20%, C: 40%, D: 30%.

In keeping with assumptions, the cumulative ANPV under DCF analysis equals the results from a classical DCF analysis (see: Table 2). The cumulative ANPV that takes into account possible managerial decisions is much higher, than the value obtained from the option-pricing model. Before commenting, however, let us investigate further possible results. Using linear programming methods, one can determine combinations that produce the maximum and minimum ANPV (see: Table 6). The minimum ANPV equals that obtained in classical DCF calculations. The maximum is 9.02 million USD and would imply that only scenarios A and D are possible, the latter being twice as probable as the former. If the value obtained from the option-pricing approach turns out to be correct, SM management implicitly assigns the following combinations of probabilities rounded to full percentage points: 18, 11, 30 and 41. This would imply that SM had to believe that scenario D is more likely to happen that the baseline.

Before attempting to draw a conclusion, we shall try to understand what these numbers signify in practice. The maximum can be interpreted as a borderline between reasonable and illogical solutions. Any price above the maximum ANPV, e.g. higher than 26.7 million USD cannot be accepted under any circumstance[28]. The value obtained from the option-pricing

TABLE 6

Comparison of project ANPV under various combinations of scenario probability

TABELA 6

Porównanie zmodyfikowanego NPV projektu dla wyszczególnionej kombinacji prawdopodobieństw scenariuszy

Combination	Resulted ANPV	Probability of			
		Scenario A	Scenario B	Scenario C	Scenario D
Value using options	2 747 814	17.83%	11.12%	29.85%	41.21%
Maximum	9 024 679	33.33%	0.00%	0.00%	66.67%
Minimum	(6 853 604)	0.00%	0.00%	100.00%	0.00%
Mean	1 085 537	13.28%	16.28%	35.75%	34.69%
Expert's guess	(105 816)	10.00%	20.00%	40.00%	30.00%

[26] Brealey, Myers 1996, pp. 247—265, (Howell 1998).

[27] The same experts were evaluating risk factor in a classical DCF analysis and probability of different scenarios.

[28] The question as to why such a high price be even considered immediately comes to mind. In observing deals struck in a heated atmosphere, however, especially when a buyer is emotionally involved, this upper borderline has considerable value.

approach appears high especially in light of expert estimates and the mean result. Yet, this value could be accepted if standard deviation data were strong. Here, however, we used experts' estimates, which cannot be treated more seriously than the estimations of the likelihood of each scenario. The remaining two results are within 7% of the offered prices. This suggests that the price worth paying is somewhere between 17.6 and 18.8 million USD. If the standard deviation applied in Table 4 were 0.7—0.8, we would receive similar results using an option-pricing approach. Such a standard deviation is still very high. Therefore, the range of prices based on ANPV falling between minus 0.1 and 1.1 million USD has the strongest justification for its likelihood.

4. Final conclusions

The article presented three different approaches to a valuation of the mineral deposit Stone-Hill. It was offered to Silesia Mining for 17.7 million USD. The use of classical DCF methodology yielded a valuation of 6.85 million USD below this price; this would discourage the SM management even to enter negotiations with a potential seller. Such a valuation, however, disregards the freedom of choice the SM management would have regarding investing in mining activities when the deposit is needed (in three years). In order to include this factor in valuation, the option-pricing approach was applied resulting in a much higher estimate of the Stone-Hill value (expected 20.45 million USD with a minimum of 15.28 million USD). These results have been reviewed using scenario analysis in order to get a better understanding of their practical meaning and consequences. While these results are logical, it turned out that a downward correction of the expected value to a level of 17.6—18.8 million USD was necessary.

In an attempt to draw general conclusions, the following implications of the case discussed in this paper may be observed:

— It is curious that the classical DCF approach is so often used to value properties of the type as Stone-Hill in view of its deficiency in recognising the freedom of choice value. A reason can be found in the fact that analysts simultaneously apply a uniform rate of risk to investment and operational cash flow, thereby overestimating NPV and somehow "compensating" for disregarding the option value of deferred investments.

— The usefulness of scenario analysis cannot be overlooked. While not a good tool to estimate risk and volatility factors in initial stages, it is an indispensable reviewing vehicle. It affords some opportunities that sensitivity tests do not. Using linear programming, analysts can easily identify possible solutions and understand what underlying assumptions regarding key variables have to be made in order to justify each value[29]. Moreover, results of the analyses are given in a manner friendly to non-financial managers and serve as a basis for interdisciplinary discussion.

[29] We may consider what answers will be obtained if SM management is asked the following two questions: "Do you believe it is possible that prices may be 40% higher than expected? and "Do you believe that there is a 67% percent probability that prices will be 40% higher than expected?

— Using an option-pricing approach as a stand-alone method of appraisal is risky since necessary data are not readily available[30] and assumptions must be made on variables that are very artificial for those well-acquainted with the mining business.

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WYKORZYSTANIE MODELU WYCENY OPCJI DLA OKREŚLANIA WARTOŚCI ZŁOŻA

Słowa kluczowe

Wycena, złoża minerałów, opcje, analiza scenariuszowa

Streszczenie

Artykuł prezentuje trzy różne podejścia do problemu wyceny złoża. Klasyczna postać modelu zdyskontowanych przepływów gotówki została skonfrontowana z podejściem opartym na metodologii wyceny opcji z wykorzystaniem elementów analizy scenariuszowej. Metodologia wyceny opcji została zastosowana w celu uwzględnienia w wycenie czynnika swobody wyboru. Metodologia zdyskontowanych przepływów gotówki nie bierze bowiem pod uwagę faktu, że nabywająca strona może kształtować swobodnie harmonogram zagospodarowywania i eksploatacji złoża w zależności od zapotrzebowania rynku. Analiza scenariuszowa została wykorzystania w celu pokazania praktycznych konsekwencji dla podejmowanych działań zarządczych.

[30] With a possible exception for several heavily traded minerals such as oil, gas and gold.