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Mathematical models in geomechanics

Key words

Mathematical modelling, geomechanics

Abstract

The mathematical models of narrow enough class will be considered in the frames of present statement: model of geological surroundings; geometrical model of object; model of distribution of stresses and shifts around working (main working, development working, coal face); model of distribution of extension and transpherences around the system of holes; model of interaction of scanning signals (waves) with the structural and power heterogeneities in rock mass.

Introduction

The main aim of geomechanics, as the applied science, is prediction of conduction the rock mass under the formation in it the artificial cavities. An engineer who is already on the stage of designing the underground object needs to have knowledge what loading will be on the timbering of mine development and what quantity of displacement of contour weathering of the rock exposure will be expected, what probability is as for the gas dynamic display of the rock pressure, whether the sides of opencast and the dump slopes will be stable and about many other geo mechanical indices which would allow afterwards to master terrestrial entrails in a safe and economic way. This knowledge is obtained in different ways, from which the most informative ways are the construction, mathematical describing and analyses of the correspondent models.

Modeling underlies the human activity. We, not to notice, apprehend the surrounding world as a complex of models in everyday life, such as: physical, economic, psychological, philosophic and

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other models which are just the approximate reflection of the objectively existing reality in human consciousness the dep. More detailed knowledge about the surrounding world is, the closer models are to reality are. However, there is the obvious fact that the complete compliance of models to the real objects is unattainable as a matter of principle. Many vast researches were accomplished, hundreds of books and articles were written on this occasion.

The following definition of the model is known: "The object M is the model of the object A relatively to (concerning) some system S of characteristics (properties), if M is built (or is chosen) for imitation of A according to these characteristics" [1]. Under it any material body, action, situation is implied (meant) as the object A. The model may be the research one, for study of indicated characteristics, or the working one, for immediate usage, for instance, automatic pilot, toy, money and so on. Only research models are considered in geo mechanics.

The geomechanical model may be physical or mathematical one depending on setting aim and the character of research object. The mathematical models of narrow enough class will be considered in the frames of present statement. Thus, it is meant that the definite physical conceptions are always laid in their basis. There it is necessary to underline that the formation of model is very responsible and important activity. The results of next researches are directly

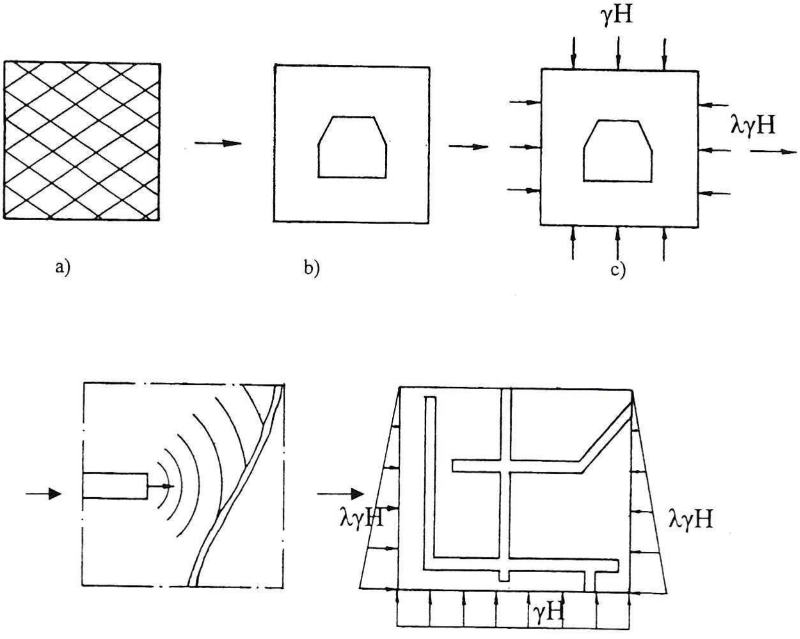

Fig. 1. The main objects of modeling in geo mechanics
a) model of geological surroundings; b) geometrical model of object; c) model deflected mode for single excavation; d) model of the dissemination (spreading) of waves in complex structural surroundings; e) model deflected mode for underground object

Rys. 1. Główne obiekty modelowane w geomechanice
a) model otoczenia geologicznego; b) geometryczny model obiektu; c) model odchylenia dla pojedynczego wyrobiska; d) model rozchodzenia się fal w otoczeniu złożonym; e) model odchylenia dla obiektu podziemnego

depended from the fact whether the initial mode was so simple and at the same time adequate to real object. The formation of models is both the science and at the same time the art, the level of which is defined in great volume by the personality and experience of the researcher.

The following principle objects of modeling are distinguished during study of mechanical phenomena occurred in the mountain-mass in consequence of formation the artificial cavities (Fig. 1).

— model of geological surroundings;

— geometrical model of object;

— model of distribution of stresses and shifts around working (main working, development working, coal face);

— model of distribution of extension and transpherences around the system of holes;

— model of interaction of scanning signals (waves) with the structural and power heterogeneities in rock mass.

Let us examine these models in above-mentioned order.

2. Model of destruction the rock mass

The rock mass, as the surroundings, containing the mine developments, is the priority object of researches in geomechanics. The structural complexity of these surroundings is that its conductivity in rock exposure still difficult to forecast.

The characteristic peculiarity of outcrops, which are located on large depth, is the presence of the zone of destructed rocks, which is formed between resiliently deformed part of rock mass and timbering (Fig. 2). This zone, as a rule, includes the contour of development and plays the role

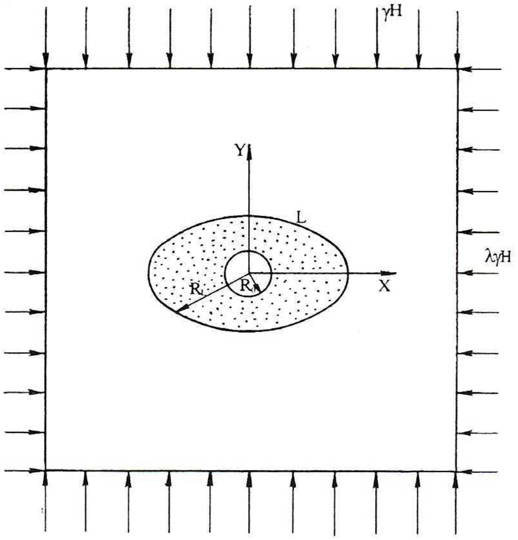

Fig. 2. Zone of inelastic deformations around single horizontal underground outcrop

Rys. 2. Obszar niedlastycznych deformacji wokół pojedynczego poziomego wyrobiska podziemnego of peculiar damper, reducing the extension level in precontour space. The peculiarities of manifestation of rock pressure in large degree depend from course of process of destruction the mass in the outcrop environment.

The rocks, if they are examined as a material, environment in which the outcrop is passed, in absolute majority may be regarded to the solid bodies.

The problem of durability, and indissolubly connected with it the phenomenon of the demolition of solid bodies, exists since when building any construction, man begun to think about the ratio of its reliability and the expanses. There is extensive bibliography on this occasion and in physics the individual trend is defined — the mechanics of demolition [2, 3].

2.1. Microdefective model of demolition

The mechanical demolition in consequence of power force may be defined as a result of some deformation and rupture of structural relations of the construction material. The research of this process is occurred based on physical speculative models, imitating real solid bodies. Comparatively little number of models may be studied and analyzed depending on aims of research (Fig. 3).

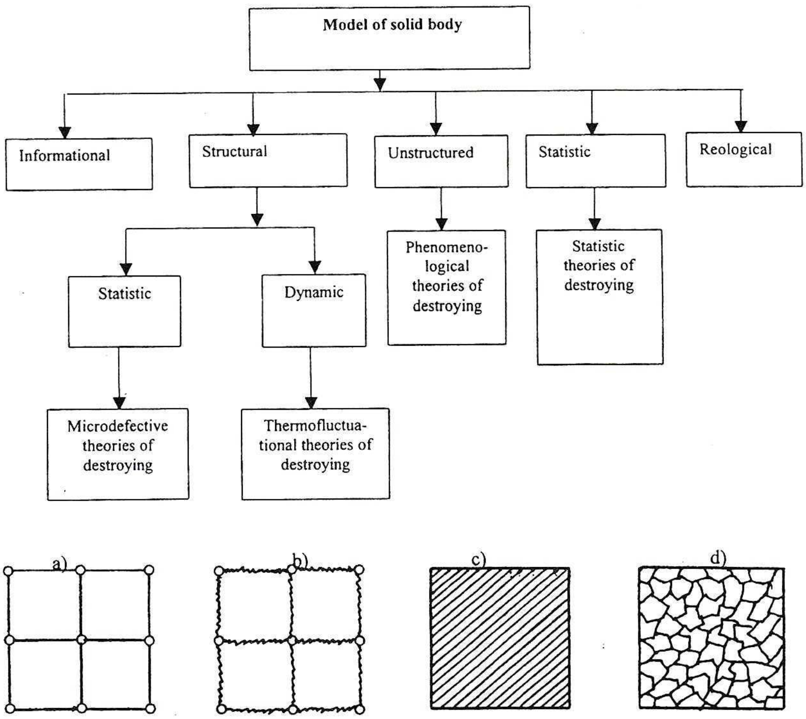

Fig. 3. Models of solid body

a) structural, statically; b) structural, dynamical; c) unstructured; d) statically

Rys. 3. Model ciała stałego

a) strukturalny statyczny; b) strukturalny dynamiczny; c) pozbawiony struktury; d) statyczny

For the first time this task for the fragile materials was decided by Griffith [3]. Based on analytical research of Kozlov G.V. and Ingles [2], in which the task about distribution of stresses around elliptical cuttings under preset boundary conditions was considered, Griffith A.A. showed, that the density of fragile solid bodies is defined by the strength according with the break of the structural ties, considerably weakened for the account of microdefects.

These microdefects (cracks), inevitably, in different reasons, are presented in initial material. In sedimentary rocks, for example, their presence is connected with the genesis and following metamorphism.

A.F. Ioffe [4], making experiences with the rock salt, determined, proceeding from the supposition of A.A. Griffith, that the most dangerous are microcracks, which are situated on the surface of loaded object. Dissolving these defects in the water, he could receive the durability on the rupture of crystals of rock salt near to theoretical.

His conclusions relatively weakening influence of microdefects on the durability of firm bodies A.A. Griffith embodied in the harmonious theory formed the base of further researches in the sphere of so called microdefective theories of durability.

The criteria, which was used, can define whether the rupture of solid body will happen or not, has the following view [5]

$$(\sigma_1 - \sigma_3)^2 + 8R_p(\sigma_1 - \sigma_3) = 0, \quad \text{if} \quad 3\sigma_3 + \sigma_1 > 0, \tag{1}$$

$$\sigma_3 = R_p, \quad \text{if} \quad 3\sigma_3 + \sigma_1 < 0, \tag{2}$$

Putting in the expression (1) $\sigma_3 = 0$, we get the correlation between the durability limit on one-axial compression and the solidity limit on one-axial tension,

$$R_c = -8R_p \tag{3}$$

that is quite good coordinated with experienced data for rocks [6, 7].

The mathematic model of durability, developed by A.A. Griffith at the beginning of 20-th years of last century, is quite required to modern conceptions about the mechanism of destruction of solid bodies and coordinates with experienced data good enough. Just because of this reason, it is given much attention to it in special researches, where it takes further development.

Microdefective theories are common in fact that they considered the destruction as the result of disintegration of solid body along one main crack. In such way, for example it is destructed the rock slopes, slopes, carriers sides and the similar objects. In rock developments the fragile destruction is occurred in other way, it realizes in the form of the system of cracks. For describing of such mechanisms of crack formation, it is necessary to introduce in working hypotheses the special preconditions.

In deformation process of solid bodies, it is existed two mechanisms with the help of which the destruction of materials is occurred — the plastic stream (shift) and fragile cracking (alienation) [8]. Now, it may be considered as the established fact that the destruction just only with the help of alienation or only shift is impossible in principle. If the plastic deformation occurred by the tangent stresses, looses the material and prepares it to rupture, the breach of solidity is occurred under the influence of normally stretched stresses.

2.2. Models of plastic deformations

The mechanism of plastic deformation of solid bodies in the results of deformation of shift was discovered in 1934 by Jh. Tailor who firstly came to conclusion of existing the linear defects in crystal lattice — dislocations [9]. The dislocations are born at the tips of cracks or other concentrations of stresses; they can interact with each other and considerably reproduce during this process (the so called source of Frank-Ryd).

In solid bodies like rocks, the plastic deformations are begun under comparatively small loads. At big external efforts the plastic deformations is already become predominant. Irreversible shifts are occurred in most crystals along the weakest surfaces, especially, if they have directions close to the surfaces of maximum tangent stresses. It finds its reflection in formation of stripes of sliding (the so called lines of Chernov-Luderce) on the polished lateral surfaces of deformed rock samples.

There are displacements of atoms in results of application of external forces in crystals, shifting atoms not only on the integer of positions, but also on some distortion is occurred in crystal lattice. Therefore, alongside with plastic deformation it is existed also elastic one. It is established that plastic deformation in the result of shift is irreversible and is proceeded without changing in the volume of material.

2.3. Thermo fluctuation model of destruction

The second important stage in the development of models about durability, after the account of atomic construction, was consisted in the calculation of influence of thermal movement in solid body on the process of destruction.

The account of thermal movement of atoms made serious changes in merely "mechanical" statement of the task: in this case not only static ensemble of connected atoms is resisted to the external forces but some system, which is situated in hesitated movement (Fig. 3).

The systematic analyses of temperature temporary dependence of durability was begun by S.N.Zhurkov in 1952 [9—11]. In general, the thermofluctational model and appropriate to it theory of the solid bodies durability are obliged to the workings of this scientific school.

The experiments with the stretching of solid bodies with different structures (monocrystals, polycrystals, polymers, compositional materials), fulfilled in different conditions, showed that dependence of durability τ from current tension σ and temperature T is always described by the empiric formula of the same type,

$$\tau = \tau_0 \exp (u_0 - \gamma\sigma)/kT \tag{4}$$

where k is the constant of Boltsman; u_0, τ_0 and γ are some constants of experienced material.

The fundamental researches of Ya.I. Frenkel [13—14] helps to determine unequivocally the physical meaning and numerical significance of constants, entered in the dependence (4):

τ_0 — the period of atom hesitations near the displacement of balance ($\tau_0 \approx 10$ c); u_0 — the energy of relations between atoms; γ — the definition connected with structural peculiarities of deforming body.

Thus, the main equation (mathematic model) of thermo fluctuation theory of durability has real physical meaning and reflects the appropriateness of the processes, occurred in loaded solid body on the atomic level. The reason of destruction is energetic fluctuation of atoms during thermal movement.

Except describing above theory, it is existed one more theory of thermo fluctuation durability, which is actively develop nowadays [16, 17]. In the frame of this theory, the mechanism of origin of fracture occurs in the result of the energy pumping from environment in destructive fluctuation of the durability — dilaton. It leads to warming-up and thermal extension of dilatons to critical size, disintegration of dilatons and forming in the solid body the microfractures. During the break on the boundary of dilaton the pressure drop is occurred. Finally, dilaton is not only the source of local destruction, but is simultaneously the dot fluctuation source of dislocations. Thus, the elementary mechanisms of destruction and plastic deformation are interconnected and working simultaneously.

Dilaton theory of durability helps to explain the reason of destruction of structures without defects in the reason of internal, inherent to any ensemble of atoms instability.

2.4. Unstructured model of destruction

The researches of solid bodies on the bases of structural models have allowed to understand the mechanism, laying in the base on their durability, and have stated the main conceptions of the theory of destruction. However, real solid bodies are essentially differed from these idealized notions that lay on the base of above-mentioned theories. The engineering practice demands the presence of concrete formula, which can help to estimate the durability of projected constructions. Just this circumstance has promoted to develop the practical theories of durability, on the base of which the unstructured models of continuous deformed solid body are laid. On present moment from all these notions presented models on Fig. 3 this class was studied best of all.

Unstructured model of continuous solid deformed body from the sight of connections between the external influence and received result is presented as the so called "black box". It is accepted to call "the black box" any system with stochastic structure, which, received at the entrance the external signal (influence), transform it and give at the exit the result (the result of experience) as some quantity, correlatively connected with external influence. Thus the researches are not interested in what way the examined system transforms the external influence in final result. The theories of destruction, based on the research of unstructured models, have received the name phenomenological models. Often they are called engineering models, meant their practical orientation.

The engineering theories of durability proceed from the supposition that the destruction of solid bodies is occurred in case when the definite combination of components of stress (deformations) reaches the critical level. In most general view the condition of destruction for main components of stress may be showed in the view

$$F(\sigma_1, \sigma_2, \sigma_3) \leq k \tag{5}$$

where F is some function, connecting in the whole one the ratio of the component of stress, and which in limiting condition is equal to the criteria of destruction k. K is depended usually from the basic durable characteristics of material: the limits of durability during compression, stretching and shift.

The further development of theories of durability was continued in general in the side of creation of analytical criterion, which would allow to define as exactly as possible the utmost condition in any constructive materials, including fragile materials, unequally resisting to the efforts of stretching and compression, which are in absolute majority rocks.

As applied to such materials O. More [17] supposed the theory of durability the meaning of which is concluded in the following: the destruction is occurred in the case, if the tangent tensions, which are the functions of normal, achieve the definite level. Thus, the durability of rocks practically does not depend from the quantity of average in size of stress σ_2.

The special put experiences for finding-out the degree of influence of average in quantity of normal stress σ_2 showed that the mistake from unaccountance of σ_2 [19, 20] do not exceed 10—15% and is in the limits of precision of measured parameters. For fragile anizotropical rocks, the influence of σ_2 increases [21].

Therefore, the precondition, accepted by More, is confirmed generally by the experiences and essentially simplifies the research of rocks conductivity in the compound intense condition.

Studying the problems of deformation, in 1913 Mizes and in 1914 Guber independently came to the conclusion that it is better to take not the all quantity of the potential energy, but only the part which is used for changing of the form. The theories of durability of O. More, Tresk-Sen-Ventan and Guber-Mizes are the most used for solving of elastic- plastic tasks in mechanics of rocks.

The researches of rock destruction in the conditions of "rigid", or controlled, tension allowed to formulate a lot of the theories of durability, taking into account the heterogeneity

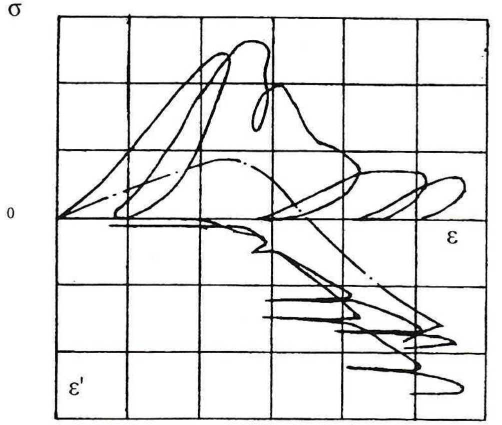

Fig. 4. The graphics of deformation the sample of rocks out of limit of durability

Rys. 4. Wykres deformacji próbki skalnej poza granicą wytrzymałości of materials, showed in the process of destruction [4, 21—23]. For the accountant of this circumstance, reflected in the parameters of dropping site of curving on the graphics of deformation (Fig. 4), in the condition of durability it is often entered the so called function of reducing the durability. Then the condition of durability (5) may be written in the following way:

$$F(\sigma_1, \sigma_2, \sigma_3) \leq k(x, y, z) \tag{6}$$

where k (x, y, z) is the criteria of durability, the quantity of which is different in different points of regions of destruction.

2.5. Statistic models of destruction

All above considered theories of durability are proceeded from the supposition about ideal structure of solid body, which either has the structure, or is continuous, homogeneous. The real constructive materials and especially rocks are not so perfect.

For heterogeneous solid rocks, determined model of continuous environment is insufficient. As the places of concentration of stresses on rocks are local and related with the heterogeneity, which are placed in the material by the casual mode, so the statistical treatment of durability has the significant meaning.

The idea about statistical nature of durability was expressed by the Soviet physicians A.P. Akexandrov and S.I. Zhukov in 1933 [24]. The general approach to the description of durability of heterogeneous environments was proposed by I.M. Lifshits and L.N. Rosentsveig [25] based on the method of John. Gibbse. The most successful working out in this direction is, in our opinion, the statistical theories of durability of S.D. Volkov and L.G. Sedrakyan.

Therefore, the modern notions about the destruction of solid bodies in its development have three stages. On the phenomenological stage, it was considered that the destruction occurs at approach of some combinations of components the tensor of stress (deformations) of definite limiting quantities.

On the structural level, it was shown as the overcoming of attraction between atoms by the enclosed stress, significantly amplified by the definite defects of structure.

The modern thermofluctuation stage considers destruction as the process, although dependent from the parameters of defective structure, but carrying out by the fluctuations of thermal movement.

The staged character of development of notions about destruction reflects in the existence of appropriate approaches to the solving of problems of durability, which exist mutually supplemented and enriched each other. On a measure of modification of these methods and approaches, all three trends will allow, probably, in due course to develop certain general theory of durability of solid bodies.

The supposed review of models of destruction of rock environment allows to determine the tactic approaches of research of elastic plastic conditions in geomechanics. The base for it is two circumstances:

— theoretically and practically the proved fact of rock destruction in the results of activity of two internal factors — shift and alienation;

— hypothesis based on the model of O. More that destruction of material depends from the quantity of the maximum and minimum normal stresses, which functionally connected with the tangent stresses, working in the same point of researched region.

2.6. About phenomenological model of destruction

The supposition about little influence of intermediate in quantity stress on the destruction of rocks is the only fact in the theory of More, which in other cases does not demand the checking, as it is completely based on the experimental data. However, the analytical criteria of durability is obtained by way of selection of appropriate empirical expression and its usage limits in essence by the regions of intense conditions, in which was fulfilled the experiments. That is why, it is produced interesting the conclusion of analytical criteria of the theory based on the analyses of the process of destruction in local region of solid body, which may be represented in the following way.

The number of theories of durability was received from general functional dependence (6), uniting in one whole the ratio of intensity of stresses σ_i and the components of globular tensor I:,

$$\sigma_i^2 + aI^2 + bI = c \tag{7}$$

where a, b, c are some parameters, defined from tests in simplest intensive conditions:

$$\sigma_i = \frac{\sqrt{2}}{2}\sqrt{(\sigma_1 - \sigma_2) + (\sigma_2 - \sigma_3) + (\sigma_3 - \sigma_1)^2} \tag{8}$$

$$I = \sigma_1 + \sigma_2 + \sigma_3 \tag{9}$$

Following to the hypothesis of More, it is supposed that the durability of material practically depends not only from those members of the statements (8) and (9), which define the difference and the sum of the maximum and minimum components of stress. Then from (7) under a = 0, receive the following statement:

$$\frac{1}{2}(\sigma_1 - \sigma_3)^2 + b(\sigma_1 + \sigma_3) = c \tag{10}$$

Parameters of b and c are defined from (10) in the results of tests of rocks in simple intensive conditions: during one-axial compression in limiting condition we obtain

$$\frac{1}{2}R_c^2 + bR_c = c \tag{11}$$

during one-axial stretching

$$\frac{1}{2}R_p^2 - bR_p = c \tag{12}$$

Solving in common the equations (11) and (12), we find, that

$$b = \frac{1}{2}R_c(1-\psi); \qquad c = \frac{1}{2}R_c^2\psi \tag{13}$$

where

$$\psi = \frac{R_p}{R_c}$$

Substituting the value of parameters (13) in ratio (10), we obtain the following condition of durability

$$(\sigma_1 - \sigma_\gamma)^2 - (R_c^2\psi - (1-\psi)R_c(\sigma_1 + \sigma_3) = 0 \tag{14}$$

or in general case of intensive condition

$$(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2 - R_c^2\psi - (1-\psi)R_c(\sigma_x + \sigma_y) = 0 \tag{15}$$

From the expression (14) we will obtain

$$\sigma_1 - \sigma_3 = 2k \tag{16}$$

where

$$k = 0{,}5\sqrt{R_c^2\psi + (1-\psi)R_c(\sigma_1 + \sigma_3)} \tag{17}$$

Notice, that under axial-symmetrical distribution of tensions we have $\sigma_1 + \sigma_3 = \sigma_r + \sigma_\theta =$ = const. It is followed, that the expression (16) is in the essence the modification of the condition of durability Tresk-Sen-Venan. In general case of intensive condition the right part of condition (16) depends from the quantities of components of stress. For materials, which have the identical resistance to the compression and stretching, $\psi = 1$. In this case, from the expression (16) we obtain the theory of durability of Kulon.

Designate accordingly maximum tangent stress and quantity, characterized the form of intensive condition, as $\tau = \frac{\sigma_1 - \sigma_3}{2}$ and $\sigma = \frac{\sigma_1 + \sigma_3}{2}$. Then expression (14) with the account of received designations will have the form

$$4\tau^2 - 2\sigma(1-\psi)R_c - R_c^2\psi = 0 \tag{18}$$

Dependence (18) in the system of coordinates <<τ – σ>> represents the equation of parabola, which is convex and constantly curved, that corresponds to the demands of postulate of Drukker [26] and corresponds to the modern representations about the nature of destruction of solid bodies [27].

The destruction of fragile materials, as it was underlain, is described good enough by the theory of Griffith. Murel showed [28], that in the system of coordinates <<τ – σ>> the basic equation of the theory of Griffith might be represented in the following way

$$4\tau^2 - 2R_c\sigma - 0{,}25R_c^2 = 0 \tag{19}$$

If in dependence (18) we put, according to Griffith, $\psi = 1/8$ and compare it with the dependence (19), the analytical expressions of two theories of durability, received on the base of different physical notions about the nature of destruction, are practically identical.

On Fig. 5 it is shown the theoretical curved of dependence (14) in the system of stretch coordinates $\left(X = \frac{\sigma_1}{R_c};\ Y = \frac{\sigma_3}{R_c}\right)$ and the results of tests of rocks, obtained by A.N. Stavrogin [23].

In spite of some scattering of experimental dots, inevitable in tests of so structurally heterogeneous materials, which are rocks and concrete, it is followed from figure, that obtained mathematical model durability (15) is adequately enough describe the process of their

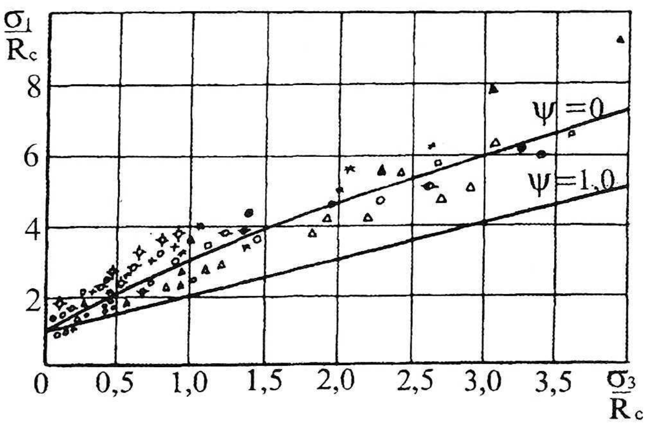

Fig. 5 The comparison of analytical criteria with the results of tests of rocks and concrete [17]
- — sandstone; - — argillite; - — talcohotith; - - - — marble 1; + — diabase; * — diorite; ¤ — alcvorit D-12;
- - — concrete

Rys. 5. Porównanie analitycznych kryteriów z wynikami testów skał i betonu [17]
- — piaskowiec; - — argilit; - — talkotyt; - - - — marmur; + — diabaz; * — dioryt; ¤ — aleworyt D-12;
- - — beton

destruction during volume compression and may be recommended for solving of elastic-plastic tasks in geomechanics.

From the expression (14) it may be obtained the formula for bringing of tough condition from the simple one-axial. It has the following view:

$$\sigma_e = \frac{(\psi - 1)(\sigma_1 + \sigma_3) + \sqrt{(1-\psi)^2(\sigma_1 + \sigma_3)^2 + 4\psi(\sigma_1 - \sigma_3)^2}}{2\psi} R_c \tag{20}$$

Her is σ_e is the so called stress, which is equivalent to the one-axial.

Using the formula (20), it is possible to estimate the degree of danger of destruction of rock environment for any dot of homogeneous rock mass in the surroundings of development, compared the quantity σ_e with the limit of durability on one-axial compression R_c.

3. Scaled effect in rocks

However, the real rock mass, as the natural system, has the high degree of heterogeneity, in which dependently from the size of researched region it is possible to allocate (Fig. 6) four scale levels of heterogeneity [29, 30]:

1. Microscopic: the environment is considered on the level of crystals, the size of the elements of heterogeneity is 10^{-8}—10^{-5} m.

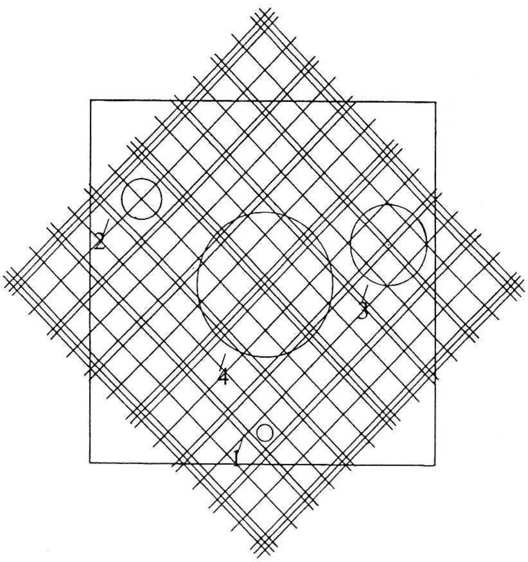

Fig. 6. The levels of heterogeneousness in rock mass
1 — microscopic, 2 — submicroscopic, 3 — macroscopic, 4 — megascopic

Rys. 6. Poziomy niejednorodności masywu skalnego
1 — mikroskopowy, 2 — submikroskopowy, 3 — makroskopowy, 4 — megaskopowy

2. Sub microscopic: the environment is considered on the level of elementary volume of litilogical difference, the size of elements of heterogeneity is 10^{-5}—10^{-2} m.

3. Macroscopic: the environment is considered on the structural level, the sizes of elements of heterogeneity are 10^{-2}—10^{-1} m.

4. Megascopical: the environment is considered as the complex structural structure, having the texture and is under influence of gravitation and tectonically forces, the size of elements of heterogeneity is more 1.0 m.

The first two levels are studied by physics of solid body, and two other are the example of the research of the mechanics of rocks.

The formula (20) is the mathematical model of destruction of rocks on macroscopic level. Under the transfer from macroscopic to megascopical level the scale of object is sharply changes. The durability of the same materials with the increasing of the size of tested samples reduces. This phenomenon has the name the scaled effect and is explained that the probability of presence in big volume of rock the hidden defects of structure is higher then in the little one. The transfer from the limit of durability if rock samples to the durability of rock mass is conducted with the help of the so called coefficient of structural weakening

$$k_c = \frac{R_c}{R_c^M} \tag{21}$$

The numerous researches are devoted to the questions of scaled effect in the mechanics of rocks, metal science [30—46].

As a rule, the recommendations about the size of structural weakening are obtained in the testing way. However, they can be find also analytically, for example, on the base of analysis of stochastic model of structure of rock mass. The accounted scheme of such task is shown on Fig. 7. The statement of the task is concluded in the following.

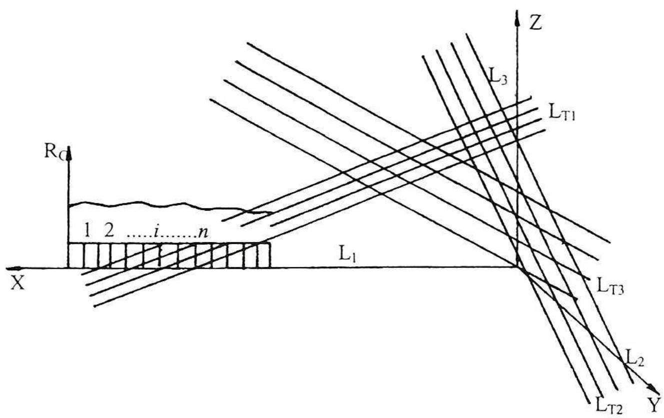

Fig. 7. The accounted scheme for the solving of task about the structural mechanical weakening of rock mass

Rys. 7. Model numeryczny rozwiązywania zadania dotyczącego mechanicznego strukturalnego osłabienia masywu skalnego

The base is the fact that the rock mass is the heterogeneous environment, having casualty distributed microdefects of different nature and macrodefects in the view of the system of cracks, very weak contacts and so on.

If to test the large amount of standard samples with microdefects, in results it is formed some statistical set, helping from the probable positions to estimate the researched mechanical system — the rock mass. The principal quality defining the degree of order of such system is dispersion.

For homogeneous environment, the dispersion is equal to zero, for heterogeneous is differed from zero and its difference more when the degree of heterogeneity is higher. So, the presence in rock mass microdefects is taken into account automatically in ordinary testing which are correctly selected and prepared rock samples.

The influence of macroefects in considered system with probability structure might be accounted in the following way. Let us mentally break the whole rock mass into densely adjoin to each other microblocks, having the sizes and shape of standard rock samples (Fig. 7). These microblocks can be tested separately on the one-axial compression and the results of these tests form the general set of data. There are the systems of differently directed cracks in rock mass. Fir example, their number in coal rocks is not exceed six [47—50]. Let us suppose that microblocks, crossed by the cracks, have the zero durability in one-axial compression and in ordinary conditions do not take part. However, they are in mass and must be showed in general set. The addition in the initial statistical line the auxiliary zero size of durability will change the primary dispersion, which quantitatively reflect the changing of mass durability owing to the presence of some its volume, breaking by the cracks.

Received in such a way dispersion of meanings of durability is the complex quantity, reflected fully enough the micro- and macrostructure of mass.

The solving of the task based on the usage of statistic theory of durability of L.G. Sedrekayan [46]. The final formula are obtained for different laws of distribution. In case of normal law of Gauss the dependence has the view

$$k_c = 1 - \sqrt{0{,}5\eta \exp(-0{,}25\eta)} \tag{22}$$

where η — the variation of durability of rock mass, defined by the formula.

$$\eta = \sqrt{\frac{l_m + l_0}{l_m}(\eta_0^2 + 1) - 1} \tag{23}$$

There, l_m is the average distance between cracks, l_0 is the character size of the standard sample, η_0 is the variation of working, consisted on the base of laboratory tests.

On Fig. 8 it is shown the graphic interpretation of dependence (22). It coincides with numerous results of laboratories and natural measurement.

So, from all objects showed on Fig. 1, the geological environment is investigated fully enough. Its mathematical model in the frames of formulated hypothesis and proved assumptions in sufficient degree is adequate to the real object.

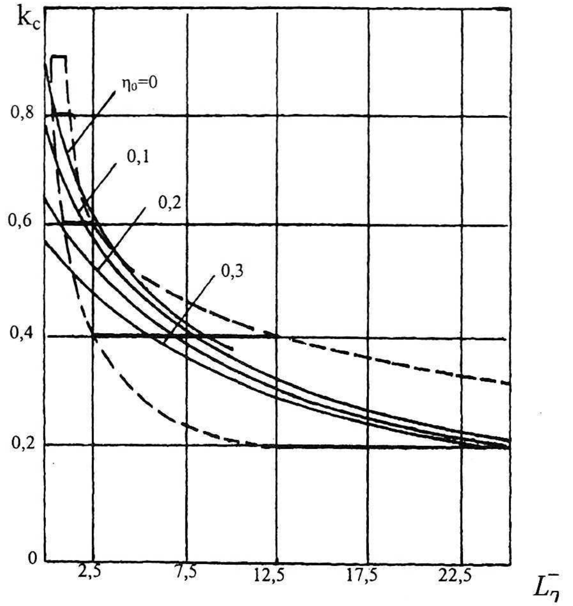

Fig. 8. The dependence of coefficient on structural weaking from the distance between cracks and coefficient of variation of development

Rys. 8. Zależność współczynnika strukturalnego osłabienia od odległości pomiędzy pęknięciami i współczynnika zmienności rozwoju

4. Some managerial solutions

On Fig. 9 it is shown the scheme to the solving of elastic-plastic task about the stress of deformed condition of rock mass around the horizontal long single development with round outline of cross section. The solving of task of flat deformation is fulfilled with the attraction of equation of balance, combination of deformations and above considered condition of durability (14). With that it was supposed that the rock mass in the vicinity of the development deforms in the regime of "rigid" loading.

The distribution of tension around the working, obtained in the results of solving the task, is shown on Fig. 10.

The formula for determination of the region size of inelastic deformations has the view

$$R_L = R_0 \exp\left(\sqrt{\frac{\gamma H - p_0}{2R_c k_c}} - \frac{1}{2}\right) \tag{24}$$

where R_0 is the radius of working, p_o is the reaction of support.

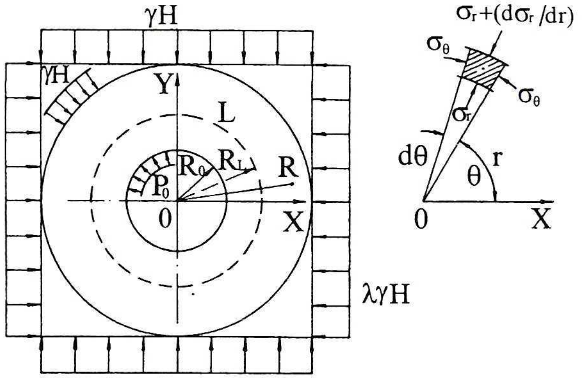

Fig. 9. The accounted scheme to the solving of task about elastic-plastic distribution of stress around the horizontal working

Rys. 9. Model numeryczny rozwiązywania zadania dotyczącego sprężysto - plastycznego rozkładu naprężeń wokół poziomego wyrobiska

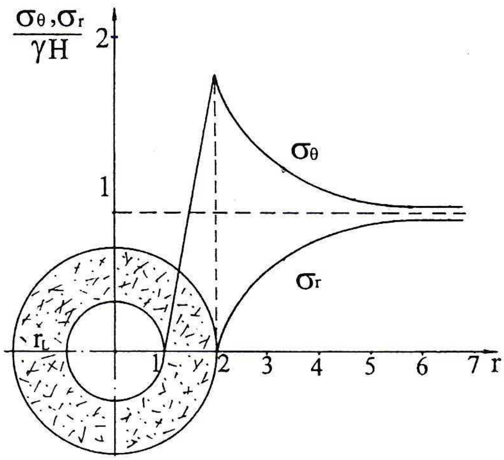

Fig. 10. The distribution of stress around the development at formation of zone of inelastic deformations

Rys. 10. Rozkład naprężeń wokół wyrobiska oraz rozwój strefy nieelastycznych deformacji

On Fig. 11. it is shown the geometrical interpretation of the dependence (24) and is carried out its comparison with data of natural measurement, which showed good convergence of accounted and experimental sizes.

The considered mathematical model of underground development is used in more complex geomechanical researches as test task.

The stability of development in real rock mass cannot be researched on the base of above considered simple model. For its solving it needs the attraction of numerical methods of mathematical research, as the method of final elements (MFE), the method of boundary

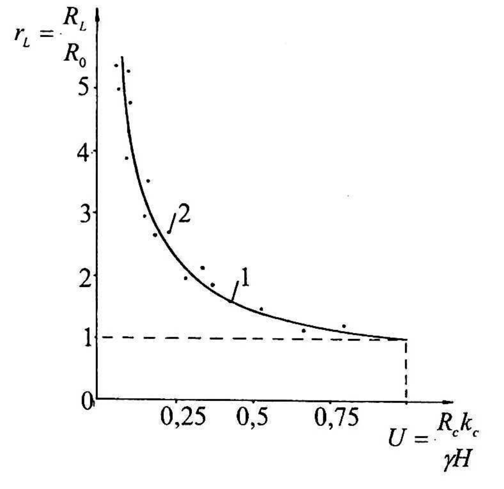

Fig. 11. The sizes of zones inelastic deformations in dependence from geomachanical parameter of conditions of location of the development
1 — theoretical curve, 2 — data of natural measurements

Rys. 11. Rozmiar stref nieelastycznych deformacji w zależności od geomechanicznych parametrów warunków lokalizacji wyrobiska
1 — krzywa teoretyczna, 2 — dane pomiarowe

elements (MBE) and the method of discrete elements (MDE). The choose of the research, method is determined, first of all, by the character of the object and the model of researched environment: continuous weightless, continuous heavy, loose and so on. Often during the solving of the task, it has to use for different regions of research space the different methods or their combination. The possibilities of modern computers help to do it correctly enough.

In computer mathematical models the solving of geomechanical tasks is begun from the reflection of geometrical specifications of the object and geological environment. They are the models of the Fig. 1b and c.

Dependently from the aim of research it can consider the flat or volume, elastic or elastic plastic, reonomic or scleronomic computer models of the object. The researches show that the contour of development usually has some casual outline, in this or that degree, approaching to the projecting (Fig. 12). This difference from the ideal geometrical contour has the significance in the case of solving the elastic task and must be taken into account in boundary conditions.

In case of inelastic decisions, that are characterized, for example, for most deep mines, the difference of the shape of development from ideal practically does not influence on the parameters of elastic-plastic conditions. The meaning has just the ratio of sizes (the maximum and minimum) and the character of loading on the boundaries of examined region.

The existing software allows with enough accuracy to set on the plane or in the space any configuration of development and to tie it to the definite geodesic system of coordinates. All known systems of surveying visualization work on this base.

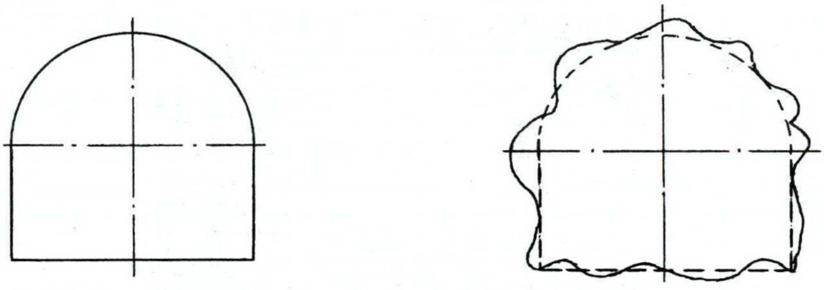

Fig. 12. Projected (a) and real (b) contour developments

Rys. 12. Projektowany (a) i rzeczywisty (b) kontur wyrobiska

The geological environment with all revealed on the stage of prospecting failures also can be shown by the volumetrical or flat (in sections) computer model with usage of modern geoinformational technologies. The imposing of geological and surveying models gives the possibility to receive the computer model of underground object.

On Fig. 13 it is shown the distribution of stresses and the sizes of zone of inelastic deformations for single development, situated in the structurally heterogeneous rock mass.

The task of flat deformation was solved by the help of iterations based on the method of final elements. Obtained results are differed from the test task on 10—15%, that is comparable with the accuracy of initial information.

On Fig. 14 and 15 of the same task of flat deformation it is shown accordingly the fields of distribution of equivalent tensions in case of rock heaving of ground and setting in development of roof bolting.

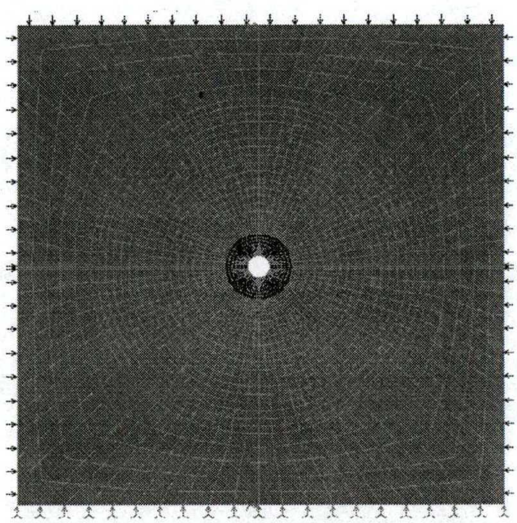

Fig. 13. Definition of the region of inelastic deformations near single round development (test task)

Rys. 13. Definicja regionów nieelastycznych deformacji w pobliżu pojedynczego okrągłego wyrobiska (zadanie testowe) a)

b)

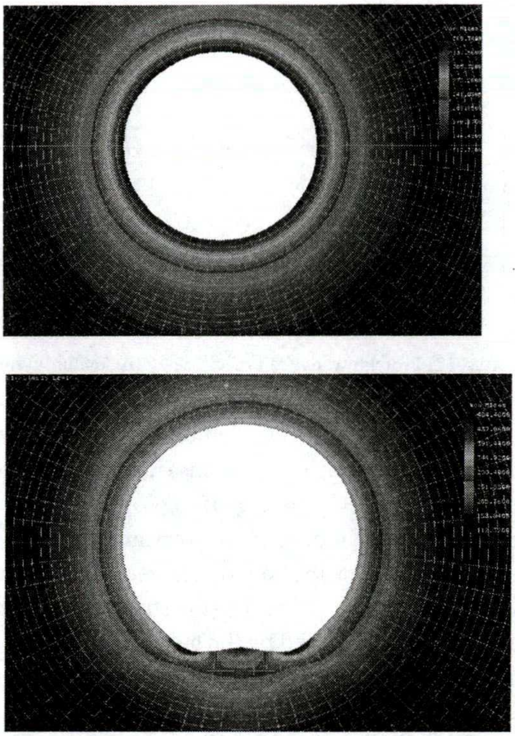

Fig. 14. The pictures of distribution of stresses on the contour of round single development, situated in homogeneous mass (elastic task)

a — before the heaving of contour in ground; b — after the heaving of contour in ground

Rys. 14. Rysunki rozkładu naprężeń na konturze okrągłego pojedynczego wyrobiska usytuowanego w jednorodnym masywie (zadanie sprężyste)

a — przed falowaniem konturu w gruncie; b — po falowaniu konturu w gruncie

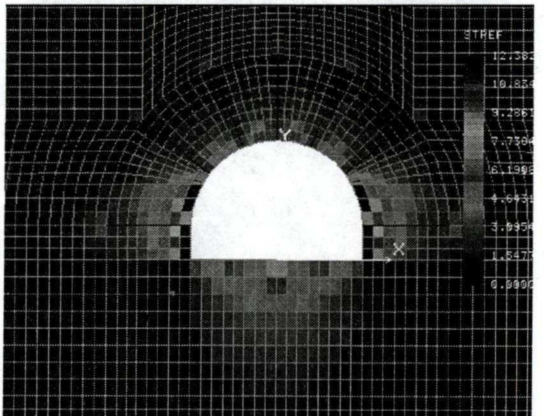

Fig. 15. The picture of distribution of the quantity σ_e/R_c near the development during the installation of 7 anchor

Rys. 15. Rysunek rozkładu wielkości σ_e/R_c w pobliżu wyrobiska w czasie instalacji 7 kotw

5. The system of geomechanical monitoring

So, there is enough of the basis to assert that the intensive deformed condition of heterogeneous mass near the single development can be described by the above stated mathematical models in the sufficient measure adequate to the original object. This statement in equal degree is concerned both to single and to the systems of developments.

Let us underlie one more time, that the original computer model of heterogeneous rock mass, having the system of developed in the space developments, is formed on the base of initial geological information and the project of building of underground object. The net of prospective wells has some minimal sizes. The information about geological defects in rock mass, situated between the wells, has the indefinite character, and sometimes is absent at all. In this connection during transferring of the working face of developments under the ground it is necessary to have the method helping to scan the heading and receive the additional information about not revealed structural heterogeneities of rock mass and abnormal concentration of tensions, capable to result in the sudden outburst of rock, coal, gas.

This recommended on practice method is the scanning of rock mass by the acoustic hesitations. The developed theory of the wave distribution in heterogeneous environment, the continuous testing of appropriate devices in outburst-prone working face, allowed to develop the set of approaches, providing the additional information about power and structural heterogeneous ahead moving working face of development. The entering of information in computer geomechanical model of underground object allows to improve it in process of reception of additional information.

And, at last, we must remember about fact that the rock environment has the stochastic properties, i.e. physical mechanical characteristics, which are appropriated to the arbitrary isolated rock volume, will correspond to the real with some deviations, which are very often

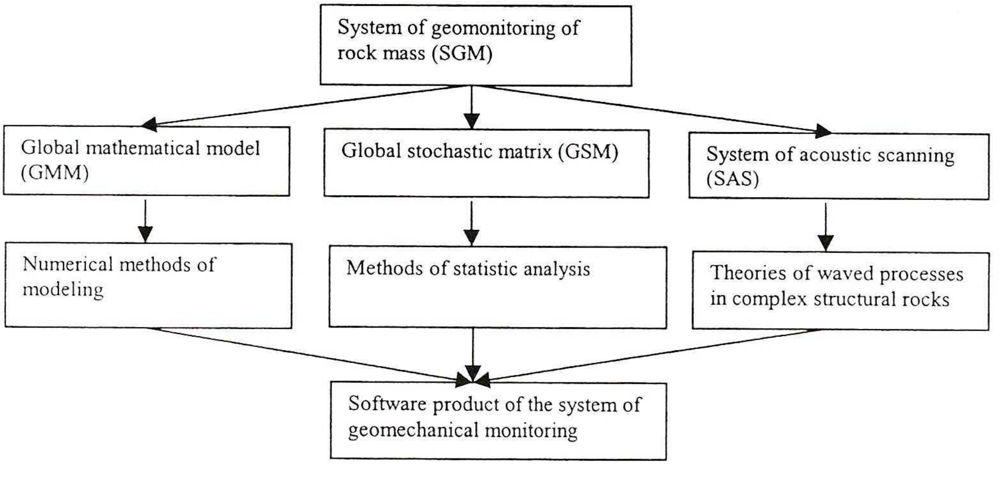

Fig. 16. General scheme of the system of geomonitoring and main tasks of the researches

Rys. 16. Ogólny schemat systemu geomonitoringu i głównych zadań badawczych significant. In the whole the mastered rock mass is the statistical set of properties and attributes, which are submitted to the definite law of distribution, more often to the lognormal law.

So, some statistical matrix must be put on the determined computer model, the parameters of which are also specified in the process of development of rock environment.

6. Generalization

In view of stated, the general scheme of the system of geomechanical monitoring of underground object has the form represented on Fig. 16.

This system is based on the set of modern mathematical means of modeling with the usage of results of operative acoustic information, received in the process of scanning of researched region. The software product, as the reflection of the set of mathematical models, with the account of continuously acting information allow to do this system of geomechanical monitoring the self-adopting system with feedback, having high level of reliability.

REFERENCES

[1] Blechman I.I., Myshkis A.D., Panovko YA.G., 1990 — Mechanics and applied mathematics: logics and features of application of mathematics. Nauka, 356 p.

[2] Timoshenko S.P.,Gud'er Jh., 1975 — The theory of elasticity. Nauka, 576 p.

[3] Ioffe A.F., Kirpichev M.V., Levitskaya A.I., 1924 — Deformation and durability of crystals. The magazine of Russian physical chemical society no 22, pp. 286—293.

[4] Stavrogin A.N., Protosenya A.G., 1985 — The durability of rocks and stability of mining developments on large depths. Nedra, 271 p.

[5] Timoshenko S.P., 1957 — The history of science about the resistance of materials. Inostrannaya literature, 536 p.

[6] The accounts on durability in mechanical engineering. Under red. S.D. Ponomarev. Gostechizdat, 1956, 885 p.

[7] The properties of rocks and methods of their determination. E.I. Ilnitskaya, R.I. Teder, E.S. Vatolin, M.F. Kuntish. Nedra, 1969, 392 p.

[8] Davidenkov N.N., 1957 — The dynamic conception of the solid bodies durability. Vestnic AN SSSR no 11, pp. 78—85.

[9] Zhurkov S.N., 1957 — The kinetic conception of durability of solid bodies. Vestnic AN SSSR no 11, pp. 78—85.

[10] Zhurkov S.N., 1968 — The kinetic conception of durability of solid bodies. Vestnic AN SSSR no 3, pp. 46—52.

[11] Zhurkov S.N., 1980 — To the question about physical nature of durability. Phisica tverdogo tela no 22, pp. 3344—3349.

[12] Regel V.R., Slutsker A.I., Tomashevskiy E.E., 1974 — The kinetic nature of durability of solid bodies. Nauka, 560 p.

[13] Frenkel Ya.I., 1945 — The kinetic nature of liquids. Izd. AN SSSR 422 p.

[14] Frenkel Ya.I., 1958 — Introduction in the theory of metals. Phismatgiz., 368 p.

[15] Kusov A.A., 1979 — The photon model of destruction of loaded atomic chain. Fizika tverdogo tela 21, no 10. pp. 3095—3099.

[16] Zhurkov S.N., — The dilaton mechanism of durability of solid bodies. In col.: Phisick of durability and plasticity. Izd. AN SSSR, pp. 5—11.

[17] Mor O., 1915 — By what the limit of durability and temporary resistance of materials is caused. In is.; New ideas in techniques. Petrograd: Obrazovanie no 1, pp. 1—50.

[18] Beron A.I., Chirkov S.E., 1969 — The research of durability of rocks in conditions of three-axed unequal compression. Nauchnye soobsheniya IGD im.Skochinskogo, pp. 33—38.
[19] Chirkov S.E., 1976 — The durability of rocks during three-axed unequal compression. Fiziko-technicheskie problemyu razrabotki poleznih iskopaemych no 1, pp. 11—17.
[20] Kuznetsov G.N., Bud'ko I.N., 1971 — The research of limited conditions of fragile material in different conditions of three-axed compression. In Book: The problems of mechanics of rocks. Novosibirsc, Izd. IGD SO AN SSSR, pp. 217—266.
[21] Annin B.D., 1966 — One flat elastic plastic task in exponential condition of liquidity. Mechanics of solid body no 3, pp. 122—123.
[22] Baklashov I.V., Kartoziya B.A., 1986 — Mechanical processes in rock mass. Nedra, 272 p.
[23] Stavrogin A.N., Protosenya A.G., 1979 — The elasticity of rocks. Nedra, 301 p.
[24] Glushko V.T., Vinogradov V.V., 1980 — The destruction of rocks and forecasting of display of rock pressure. Nedra, 214 p.
[25] Lifshits I.M., Rosentsveig L.N., 1946 — To the theory of elastic properties of polycrystals. Magazine of experimental and theoretical physics. vol. 16, pp. 967—979.
[26] Druccer D., 1957 — About the uniqueness of solutions in the theory of elasticity.- Mechanics. Translated articles. no 4, pp. 12—80.
[27] Pisarenko G.S., Lebedev A.L., 1969 — The resistance of materials to the deformation and destruction in complex intensive condition. Naukova dumka, 209 p.
[28] MacClinton F., Argon A., 1970 — The deformation and destruction of materials. Mir, 443 p.
[29] Ershov L.V., Iofis I.M., Haiman I.B., 1983 — The mathematical models on mass of rock. Izd. MGI, 85 p.
[30] Rats M.V., 1973 — The structural models in engineering geology. Nedra, 216 p.
[31] Chechulin B.B., 1963 — The scale factor and statistical nature of metals. Metallurgizdat, 120 p.
[32] Chirkov S.E., 1969 — The influence of scale factor on the durability of coals. Nauka, 113 p.
[33] Koifman M.I., 1962 — About the influence of sizes on the durability of rocks. The research of physical mechanical properties of rocks according to the tasks of rock pressure control. Izd. AN SSSR, pp. 6—14.
[34] Melikidze I.G., 1959 — About the influence of form and sizes of samples on their mechanical properties. Mining magazine no 9, pp. 38—43.
[35] Grisikova N.P., 1959 — The dependence of the limit of durability on compression from the size and form of the samples. Mechanical properties of rocks. Ugletechizdat.
[36] Il'nitskaya E.I., 1972 — The influence of scale factor on durable properties of rocks. Physical mechanical properties, pressure and destruction of rocks. Izd. AN SSSR, pp. 17—24.
[37] Kuntish M.F., 1964 — The research of methods of definition of main physical mechanical characteristics of rocks, using in solving of tasks of rock pressure. Diss. IGD im. A.A. Skochinskiyi, 166 p.
[38] Pen'kov A.M., Vopilkin A.L., 1958 — The account of support pillar in extraction of rock salt. Izd. AN USSR, 218 p.
[39] Theil A.O., 1973 — Okryteriach zniszenia zespotov skalnych w switle polozouch badan mettoda besposrednego szinania. Pozpz. Hydrotehn. z. 32, s. 277—290.
[40] Baron L.I., Kurbatov B.M., 1959 — To the question about the influence of scale factor in testing of rocks on compression. The scientific researches at working of coal and ore deposits. Gosgortehizdat, pp. 128—136.
[41] Koifman M.I., 1963 — The main scale effect in rocks and coals. The problems of mechanization of mining workings. Izd. AN SSSR, pp. 39—56.
[42] Volkov S.D., 1956 — About scale effect in complex intensive condition// Physics of metals and metal science. vol. II, is. 3, pp. 428—440.
[43] Matveev B.D., 1955 — About the dependence of results of mechanical tests of rocks from sizes of their samples. The questions of destruction and pressure of rocks. Ugletehizdat, 132—139.
[44] Ryzhov P.A., 1973 — Mathematical statistics in mining. Vysshaya shkola, 287 p.
[45] Karasev B.V., 1971 — Statistical approach to the research of nature and some laws of distribution of substance on Earth. The ways of searching the Earth. Nauka, pp. 131—151.
[46] Sedrakayan L.G., 1968 — The elements of statistic theory of deformation and destruction of fragile materials. Ereven: Aistan, 247 p.
[47] Ivanov G.A., Sarbeeva L.I., 1940 — Klivazh (separations) in coals and coaling rocks and the ways of its practical usage. P.II. Gostoptehizdat, 280 p.

[48] Malinin S.I., 1970 — The geological basics of forecasting of rock conduction in rock developments by prospecting data. Nedra, 184 p.
[49] Michailov A.E., 1956 — The field methods of searching of cracks in rocks. Gosgeoltehizdat. 132 p.
[50] Establishment of laws of display of sudden outbursts of coal and gases with the purpose of development of forecasting of outburst-prone zones in the fields of working and building mines of Donbass. V.S. Vereda, B.K. Yurchenko, V.G. Surovtsev, V.A. Yampolskiyi. Donetsk: PO Ukrgeologiya, 1968, 358 p.

ALEXANDER N. SHASHENKO

MATEMATYCZNE MODELE W GEOMECHANICE

Słowa kluczowe

Modelowanie matematyczne, geomechanika

Streszczenie

W niniejszym artykule rozważane są następujące modele matematyczne: model otoczenia geologicznego, geometryczny model obiektu, model rozmieszczenia nacisków i przemieszczeń w wyrobisku (wyrobisko główne, prace przygotowawcze i przodek), model interakcji sygnałów skanujących (fal) w warunkach strukturalnej niejednorodności mas skalnych.