Przegląd Filozoficzny – Nowa Seria
R. 32: 2023, Nr 1 (125), ISSN 1230–1493
DOI: 10.24425/pfns.2023.146782

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Syntax, Semantics and Tarski's Truth Definition

Keywords: *definition, language, logic, metalanguage, morphology, satisfaction, A. Tarski*

Roughly speaking, syntax operates inside a language, i.e. concerns relations holding between linguistic expressions, but semantics takes into account how units of language are related to the extralinguistic reality, i.e. semantical parlance goes outside language as such. More specifically:

> Syntax is the theory of the construction of sentences out of words. In linguistics, syntax is distinguished from morphology, or the theory of the construction of words out of minimal units of significance, only some of which are words (Higginbotham 1996, p. 561).

Syntax in the above understanding belongs to linguistics. Rudolf Carnap introduced the concept of logical syntax (Carnap 1937, p. 1; emphases in bold and italic follow the original):

> By the **logical syntax** of a language, we mean the formal theory of the linguistic form of that language – the systematic statement of the formal rules which govern it together with the development of the consequences which follow from these rules.
> A theory, a rule, a definition is to be called *formal* when no reference is made in it either to the meaning of the symbols (for example the words) or to the sense of the expressions (e.g. sentences), but simply and solely to the kinds and order of the symbols from which the expressions are constructed.

Carnap (1937, p. 9) contrasts logical syntax and linguistic syntax. The latter is "not pure in its method and does not succeed in laying down an exact system

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of rules" – it concerns natural languages. The former considers languages as calculi. If a calculus **C** is the subject of logical-syntactic investigations, it plays the role of the object language, but its analysis is performed in the metalanguage, called the syntax language.

In the field of semantics, we have similar distinctions as in the case of syntax. Carnap (1958, p. 11) distinguished descriptive semantics (empirical studies on semantic aspects of historical languages, particularly related to changes of meaning) and pure semantics (analysis of languages as semantical systems constituted by rules; Carnap employed the adjective "semantical"). Furthermore, there is a contrast between semantics as the theory of meaning (this understanding appeared in Bréal 1897) and as the theory of relations of what language is about (referential relations, like truth or designation). Charles Morris (1938) distinguished semantics *sensu stricto* (the theory of referential relations) and semantics *sensu largo* (frequently called "semiotics") consisting of syntax, semantics *sensu stricto* and pragmatics. Carnap (1958, pp. 8–11) developed this division by pointing out that pragmatics investigates linguistic signs from the point of view of their users, semantics abstracts from the user and concentrates on expressions and their designata, and, finally, logical syntax abstracts from referential relations and investigates relations between expressions; consequently, the metalanguage does not need to be syntactic. To some extent, one can speak about grammatical semantics (Michel Bréal was a philologist) and logical semantics developed by Alfred Tarski and Rudolf Carnap in his "post-syntactic" works (particularly in Carnap 1939, Carnap 1942, 1958; there is practically nothing on semantics in Carnap 1934). Yet, as it is documented by a typical survey of the philosophy of language (see, e.g., several papers in Stalmaszczyk 2022), the border between grammatical (linguistic) and logical semantics is very vague, much less explicit than that between grammatical syntax and logical syntax, because syntactic differences between formal and natural languages are much more transparent than in the case of semantic matters associated with both kinds of linguistic systems. However, the situation is more complicated, or perhaps even obscured to some extent, because the adjective "formal" is applied to semantics of formal as well as natural language (syntax is formal by definition, although formality in the context of syntactic relations requires further explanations)[1].

The standard view of the development of contemporary logic is as follows. Its founding fathers, like Gottlob Frege and Bertrand Russell, formulated axiomatic logical systems, supplemented by various semantic *sensu largo* and philosophical remarks or even theories, like Frege's treatment of concepts or

[1] These problems have a long history going back to antiquity (see: Schneider, Stekeler-Weithofer 1995, Stetter 1998 for encyclopaedic presentations). The topic of the present paper does not require entering into the huge variety of contemporary syntactic and semantic theories.

Russell's theory of descriptions. Logic became more formal, due to David Hilbert's formalistic programme in the foundations of mathematics (the textbooks Hilbert, Ackermann 1928 and Łukasiewicz 1929 represent this approach, which became standard in almost all later works). This line was supplemented by the development of metamathematics in Hilbert's school and metalogic in Poland (particularly in the Warsaw group of logic). Carnap's *Logische Syntax der Sprache* (Carnap 1934) can be viewed as a summary of this approach and, due to Carnap's historical perspective, one might speak about the syntactic stage in the development of logic, according to which semantic concepts should be eliminated by syntactic ones, e.g., "is true" by "is provable" – consequently, semantic notions could be employed in informal comments on logic. Kurt Gödel's incompleteness theorems demonstrated essential limitations of syntactic methods as represented by proof-theoretic procedures[2]. Tarski (1933) succeeded in giving a truth-definition formulated in metalogical terms. Although the Vienna Circle was initially sceptical about semantics, many of its members (notably Carnap) became convinced about the legitimacy of semantic methods around 1935. As Carnap says:

> Tarski, both through his book [on truth] and in conversation, first called my attention to the fact that the formal methods of syntax must be supplemented by semantical concepts, showing at the same time that these concepts can be defined by means not less exact than those of syntax (Carnap 1958, p. X).

Semantics triumphed in the late 1930s. The present systematisation of mathematical logic divided it into proof theory (a counterpart of logical syntax), model theory (a counterpart of logical semantics) and recursion theory (this part of metamathematics is a hybrid of syntax and semantics).

How did Tarski use the syntax/semantics distinction in his semantic theory of truth (hereafter: **STT**)? An outline of this theory can be as follows (see Tarski 1933; Tarski 1944; Tarski 1969; a systematic presentation of (**STT**) and its problems is given in Woleński 2019; I use fragments of this book below). Assume that **L** is a formalised language. We look for a definition of truth for sentences of **L**. In fact, this task can be formulated as expressed by the demand "define truth predicate for **L**", i.e. the expression "is true in **L**"; consequently a truth definition should generate the set of truths in **L**. Tarski considered the concept of satisfaction as the basic primitive of semantics. In particular, truth is defined in (**STT**) as a special case of satisfaction. Typically, satisfaction (or non-satisfaction) characterises so-called open formulas, i.e. formulas having free variables, like (a) "*x* is a natural number", which is satisfied by the number

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[2] The given dates show that the related history was very dense and complex. For instance, Carnap 1934 (the Bible of the syntactic paradigm) appeared after Tarski 1933, the main semantic document.

3, but not by the number ½. If we substitute x by 3 in (a), we obtain a true sentence (b) "3 is a natural number", but replacing x by ½ in (a) produces a false sentence (c) "½ is a natural number". Another way of passing from (a) to sentences consists in using quantifiers, e.g. like in (d) "$\exists x$ (x is a natural number)" (a true sentence) or (e) ($\forall x$ is a natural number) (a false sentence). These considerations suggest three statements: (i) a sentence is a formula without free variables (note that (b) and (c) do not contain free variables) – this view requires an earlier definition of a sentential formula of **L**; (ii) sentences are true or false, but open formulas not – the latter are satisfied or not; (iii) sentences are a special case of open formulas – this stipulation is well-motivated by (i). Thus, (at least some) links between satisfaction and truth are generated by pure logic.

Thus, what we are looking for is a truth definition related to satisfaction. Before giving it, two remarks are in order. Firstly, due to semantic paradoxes, particularly the Liar antinomy (I omit details), the definition in question must be formulated in **ML**, i.e. a metalanguage with respect to **L**. Thereby, the expression "is true in **L**" (the truth predicate for **L**) belongs to **ML**, not to **L** – as a result self-referential "semantic" sentences, like "This sentence is true", are excluded from the set of well-formed formulas. Secondly, any satisfactory truth definition has to logically entail every instance of the following scheme:

(**T**) "A" is true if and only if A^*,

where the symbol "A" is a metalinguistic name of the sentence A (this name belongs to **ML**, but the sentence itself to **L** (symbolically, "A" $\in$ **ML**, $A \in$ **L**) and the symbol A^* refers to the way of embedding A into **ML**, e.g. via translation (this requirement express the so-called convention **T**). An example from natural language illustrates the issue. Assume that German is the object language, but English is the metalanguage. So, the sentence "Schnee is weiss" is true in German if and only if snow is white, where the expression "Schnee is weiss" is the metalinguistic name, which by definition belongs to English, but the phrase "Snow is white" is the English name of the German sentence in question. Observe that (**T**) cannot be formalised by the formula "$\forall A$("A" is true if and only if A^*)", because A cannot be quantified for its occurrence inside the quotes in (**T**); otherwise speaking, the expression "A" is not free in (**T**) – in fact, it is not a variable at all. Consider now two collections of ideas (questions marks indicate that something is to be done):

(I) (General case): open formulas,
satisfaction by some objects from **U**;
non-satisfaction by some objects from **U**;

(Special case): closed formulas (sentences), satisfaction by?;
non-satisfaction by?

The above "proportion" suggests that we need a generalisation of the concept of satisfaction for obtaining the required truth definition based on it.

(**STT**) assumes that **L** is formalised but interpreted. This means that we have a universe **U** which provides references for terms (individual constants and individual variables) and predicates (subsets of **U** for unary predicates and relations defined on **U** for binary, ternary, etc. predicates)[3]. Inspecting the special cases discussed above leads to the conclusion that although satisfaction depends on valuation of free variables, truth and falsehood do not, because sentences contain no free variables. Consider a formula $\exists x P(x)$. It is true, if if $P(x)$ is satisfied by at least one $\mathbf{a} \in \mathbf{U}$. It means that the sentence $\exists x P(x)$ cannot be showed to be false by taking any object from **U**. The sentence $\forall x P(x)$ is true, if $P(x)$ is satisfied by any $\mathbf{a} \in \mathbf{U}$ – this means that this sentence cannot be demonstrated as false by any object from **U**. Thus, we arrive at an intuition that a sentence is true, relatively to its interpretation in **U**, if and only if it is satisfied by any object from **U**; consequently, *A* is false if and only if it is satisfied by no object occurring in **U**. It can be proved that *A* is satisfied by all objects from **U** if and only if it is satisfied by one object if and only if it is satisfied by the empty sequence of objects. Sentences can contain predicated of arbitrary arity. Hence, the general definition of truth required infinity sequences of objects (it secured that we have at disposal sufficiently long finite sequences; in particular **a** is converted into <**a**> – a one-termed sequence). This leads to the following definition (**SDT** – semantic definition of truth):

(**SDT**) a sentence *A* is true in **U** if and only if *A* is satisfied by every infinite sequence of objects from **U** (= is satisfied by at least one such sequence, = is satisfied by the empty sequence); a sentence *A* is false if and only if it is satisfied by no sequence of objects from **U**.

(**SDT**) satisfies the convention **T**, i.e. entails any equivalence falling under the scheme. This constraint constitutes the so-called condition of material adequacy of (**SDT**).

Few additional comments about (**SDT**) are in order. Firstly, a philosophical remark. Tarski himself considered (**SDT**) a modern version of Aristotle's view on truth – many other commentators, including myself, share this opinion. However, Tarski seem to think that the content of (**T**) links (**STT**) with Aris-

[3] My presentation of (**STT**) is semiformal, like in Tarski 1933 and Tarski 1944. In particular, the concept of model is not used explicitly. For contemporary accounts in which truth is defined as relativised to a language **L** and a model **M**, see, e.g., Grzegorczyk 1974, Ch. II.

totelian (classical) theory of truth, that is, e.g., the sentence "Warsaw is the capital of Poland" is true, provided Warsaw is the capital of Poland. However, it is disputable whether (**SDT**) has any connection with the tradition of truth as *adaequatio rei et intellectus*. In particular, one should not interpret sequences of objects as facts or pieces of reality. Eventually, the satisfaction by all sequences (one sequence, the empty sequence) could be understood to mean that truth is dependent on the fixed domain, but not on varying interpretations of variables, but this issue requires further debates. Secondly, Tarski very strongly insisted that language should be interpreted. He wrote:

> It remains perhaps to add that we are not interested here in "formal" languages and sciences in one special sense of the word "formal", namely sciences to the signs and expressions of which no material sense is attached. For such sciences the problem here discussed [i.e. the problem of truth] has no relevance, it is not even meaningful. We shall always ascribe quite concrete and, for us, intelligible meanings to the signs which occur in the languages we shall consider. The expressions which we call sentences still remain sentences after the signs which occur in them have been translated into colloquial language. The sentences which are distinguished as axioms seem to us to be materially true, and in choosing rules of inference we are always guided by the principle that when such rules are applied to true sentences the sentences obtained by their use should also be true (Tarski 1956b, pp. 166–167).

This quotation invokes a problem of how formalised languages are related to natural speech and suggests that this relation is conventional, i.e. no sharp a priori border can be drawn. Carnap's conception was different to some extent. He wrote:

> I emphasize that the distinction between semantics and syntax i.e. between semantical systems as interpreted language systems and purely formal, uninterpreted calculi, while for Tarski there seems be no sharp demarcation (Carnap 1958, p. XI).

Being formalised does not mean being formal (in the sense of artificially constructed), but rather a product of formalising something which can be even informal. In Tarski 1944, we can find an idea of a language having a specified structure. I will return to this concept later, but here I note an interesting evolution in Tarski. Tarski (1933) was rather sceptical about a possibility of defining semantic concepts for natural language, due to mixing language and metalanguage, which resulted in semantic paradoxes. He became more friendly to daily speech as subjected to strict semantic analysis, provided that there it would deal with fragments formalised or at least reformed towards systems with specified structure. Anyway, a too fast identification of being formalised with being purely formal caused some misunderstandings concerning (**STT**).

The above presentation of (**STT**) contains very little about syntactic aspects of this theory. One can eventually say that the distinction of **L** and **ML**, conditions imposed on **T**-equivalences, concerning the difference between

"*A*" and *A** or exclusion of self-referentialities have syntactic import. Now it is time to say something more about Tarski's views about syntax. They are represented by the following quotations:

> The metalanguage in which we carry out the investigations contains exclusively structural-descriptive terms, such as names of expressions of the language, structural properties of these expressions, structural relations between expressions, and so on, as well as expressions of a logical kind among which (in the present case [i.e. (**STT**)] we find all the expressions of the language studied. What we call metatheory is, fundamentally, the *morphology of language* – a science of the form of expressions – a correlate of such parts of traditional grammar as morphology, etymology and syntax (Tarski 1956b, p. 251).

> The semantics of any formalized language can be established as a part of the morphology of language based on suitably constructed definitions, provided, however, that the language in which the morphology is carried out has a higher order than the language whose morphology it is. […]. It is impossible to establish the semantics of a language in this way if the order of the language of its morphology is at most equal to that of the language itself (Tarski 1956b, pp. 273–274, 276).

> The statements which establish the essential properties of semantical concepts must contain both the designations of the objects referred to (thus the expressions of the language itself) and then terms used in the structural descriptions of the language. The latter terms belong to the so-called morphology of language and are the individual designations of the language, of structural properties of expressions, of relations between expressions, and so on (Tarski 1956a, p. 403).

Tarski in his writings on the concept of truth and general semantics worked in the frameworks of a system similar to the simple theory of types in its linguistic version. Roughly speaking, types in this understanding are identified with orders, finite or infinite, of languages. For instance, if **L** is of the n^{th} order, **ML** has the $n + 1$ order. The third quotation expresses Tarski's theorem that the concept of truth is not definable for languages having the infinite order. It is so, because if n is infinite, $\mathbf{ML}_{n+1}$ would have to define its own truth predicate, which is impossible, because of the danger of semantic paradoxes.

Doubtless, Tarski used the word "morphology" as a word referring to syntax in the traditional meaning – it is suggested by the context "a science of the form of expressions – a correlate of such parts of traditional grammar as morphology, etymology and syntax". Although he did not employ the phrase "logical morphology", his description of the science dealing with forms of expressions corresponds with Carnap's account of logical syntax. Clearly, structural properties of expressions refer to their syntactic attributes. On the other hand, Tarski was not so much interested in building a general logical syntax in Carnap's sense. Tarski's main task consisted in the characterisation of **ML** suitable for (**STT**) and other semantic concepts, and eventually for dealing with other metamathematical problems. The last point is expressed in the following way:

> discussion is conducted throughout the book within an appropriate *metasystem*. In the *metalanguage*, i.e., the language of the metasystem, we have at our disposal various logical, set-theoretical, and metalogical symbols and notions. […] The metasystem and its language are not assumed to be formalized. The set-theoretical notions occurring in the metasystem are sometimes employed in a way which is usually described by the phrase "in the sense of naive set-theory". […] Among metalogical notions of the metasystem we find, in particular, symbolic designations of all expressions occurring in formal languages to which the discussion refers. No symbols, i.e., expressions appearing in our metalogical discussion, should be interpreted as belonging to formal languages themselves. […] In this work we use the terms "formalism" and "formal language" […] interchangeably. In other contexts it may be useful to differentiate between the meanings of these two terms. Formal languages would then be constructed as structures with a different list of fundamental components; the list would include some notions referring to the intrinsic structure of sentences, such as the vocabulary of a language (Tarski, Givant 1987, p. 1).

The position taken in the last quotation is openly pragmatic. If **MTh**, expressed in a language **ML**, is a metatheory of a theory **Th** formulated in **L**, the former must contain everything that is needed for analysis of the metamathematical properties of **Th**. In the case of (**STT**) understood as a general truth theory, **ML** cannot contain semantic terms other than satisfaction for the danger of circularity (Tarski explicitly says that his construction preserves traditional conditions of the correctness of definitions as non-circularity and not being formulated with *idem per idem*.) Note that the concept of satisfaction is defined by set-theoretical terms (in particular, sequences of objects from a given **U**) and logical concepts (see Grzegorczyk 1974, pp. 264–286 for details of such a construction). Even if there is some amount of conventionality in this approach, it seems admissible. Eventually, one can speak about an extended syntax, i.e. a "morphological" vocabulary in which no semantic terms occur, or in which they are defined by non-semantic notions.

The present-day logic replaced the framework of the type theory and operates via the division of first-order logic and higher-order logic[4]. The Hilbert thesis (or the first-order thesis) says that an arbitrary mathematical theory presented as a formal (i.e. formalised) calculus can be expressed in a suitable first-order language[5]. Since, according to Hilbert, any formalisation should (and can) be reduced to such that uses finitary means, it can be treated, in the terminology of the present paper, as syntactic. Furthermore, all metama-

[4] Note that some versions of the type theory, but rather local than global, contrary to Russell's view, are still developed – see Bell 2022, Ch. 4.

[5] A first-order language is a language in which quantifiers bind individual variables; higher-order languages admit quantification over predicate letters, e.g. the formula $\forall x \exists y P(x, y)$ (for any x, there is y, such that $P(y, x)$, e.g. for any natural number x, there is a natural number y, such y is greater than x) is first-order, but $\forall x \exists P(x, y)$ (for any x, there is P, such that $P(x)$, e.g. for any x, there is a property P, such that x is P; briefly, every object possesses a property) – is second-order. To avoid some misunderstandings, the Hilbert thesis does not mean that any mathematical theory is reducible to pure first-order logic, i.e. quantifiers theory.

thematical properties should be finitistically defined and metamathematical theorems proved by finitary methods, i.e. constructively in this sense[6]. There is no place in Hilbert's programme for formal semantics as something different from logical syntax. Hence, although semantic concepts are admissible in informal and heuristic arguments, they must be constructively reducible to syntactic ones in metamathematically justified mathematical discourse. Gödel's results demonstrated that this programme cannot be carried out. Although he proved the completeness theorem for first-order logic (every logical truth is provable), the proof was not entirely constructive (finitary). However, there was still a hope to find a proof acceptable from the finitistic point of view. The incomplete theorems (arithmetic is incomplete, if consistent, the proof of the consistency of arithmetic cannot be carried out in arithmetic itself) show limitations of Hilbert's programme. Moreover, it is remarkable that both are constructively provable and the method of arithmetisation shows how to embed the syntax of any first-order theory into this theory itself.

Remarks made in the last paragraph lead to the problem of the relation between incompleteness theorems and Tarski's undefinability theorems[7]. In fact, Gödel used the concept of truth in his informal arguments. Hence his results can be taken as a demonstration that there are true, but unprovable arithmetical statements. Thus, a set of arithmetical truths cannot be reduced to the set of provable theorems of arithmetic. In this sense, the concept of truth of arithmetic is not defined inside it. However, Gödel seems to maintain that this concept exceeds mathematical resources. Using philosophical jargon, one can say that the concept of truth (and thereby semantics as such) appears as transcendental with respect to mathematics. On the contrary, Tarski defined the predicate "is true" in a mathematically satisfactory way, but demonstrated that it is not definable in precisely described circumstances. Moreover, the Tarski undefinability theorem has no constructive proof. Clearly, Gödel's results can be interpreted as showing that semantics is not reducible to syntax in an intuitive sense, but Tarski's result locates the issue on a definite level of metalogical investigations.

Let **Th** be a first-order formalised theory such that it contains arithmetic of natural numbers. In order to construct its semantics, we must use second-order logic. Due to the arithmetic character of **Th**, it is enough to employ weak second-order arithmetic with the axiom of arithmetical comprehension (see Murawski 1999, pp. 97–210; Cieśliński 2017, pp. 18–19) as **MTh**[8]. Using

[6] There is a continuous controversy concerning the scope of the predicate "is finitary". A common view is that finitary means cannot go beyond first-order arithmetic of natural numbers. This limitation is sufficient for my aims in the present paper.

[7] A detailed presentation of this question is contained in Woleński 2019, pp. 255–269.

[8] I neglect the axiomatic approach to the concept of truth (see Halbach 2015 for a detailed presentation of this turn). Perhaps I only remark that since axiomatic truth theories consider "is

Tarski's way of speaking, arithmetisation of **Th** syntax, together with standard logical terms, constitutes a part of morphology of **MTh**. In order to define **VER**(**Th**) (the set of truths of **Th**), we must add to morphology the concepts of model and interpretation, which exceed the resources of arithmetisation. Having these tools, one might show that the concept of satisfaction cannot be defined in **Th** – this immediately leads to the undefinability of **VER**(**Th**) in **Th** – both notions are definable in **MTh**. Moreover, the method of proving Tarski's theorem can be employed in proofs of incompleteness theorem in a non-constructive way. This scheme of reasoning precisely shows the sense in which semantics of **Th** is not reducible to its syntax as well as definability of **VER**(**Th**) in **MTh**. There is another way to show that semantics essentially transcends syntax. If we construct the Kleene–Mostowski arithmetical hierarchy, the concept of truth does not belong to any level of this hierarchy (see Murawski 1999, pp. 284–295), but provability does. This shows that the gap between truth and provability is essential. Another way of expressing this fact consists in saying that although the set of provable arithmetical sentences is recursive, the set of arithmetical truths is not. Still one circumstance should be noted. Assume that **Th** is formalised in **MTh**. If so, due to Tarski's quoted remarks, the latter must be at least partially informal. Thus, informal semantics or even pragmatics appears as inevitable in metalogical investigations. Finally, considerations in the present paper are limited to classical logic and its metatheory – in fact, (**SDT**) implies the principle of bivalence, the core ingredient of classical logic. It is unclear how to adapt the above argument to non-classical logics and theories, e.g. intuitionism.

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true" as an object-language predicate, they cannot use the language/metalanguage distinction as a device saving from paradoxes. A typical way of avoiding semantic antinomies consists in weakening the equivalence $Tp \leftrightarrow p$ by rejecting its part $p \rightarrow Tp$ (see Turner 1999). Tarski himself (see Tarski 1936a) preferred introducing the concept of truth by definition.

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Syntax, Semantics and Tarski's Truth Definition

Keywords: *definition, language, logic, metalanguage, morphology, satisfaction, A. Tarski*

Until Tarski's semantic truth definition, the concept of truth was used informally in metalogic (metamathematics) or even proposed to be eliminated in favour of syntactic concepts, as in Rudolf Carnap's early programme of philosophy via logical syntax. Tarski demonstrated that the concept of truth can be defined using precise mathematical devices. If **L** is a language for which the truth definition is given, it must be done in the metalanguage **ML**. According to this construction, semantics for **L** must be performed in **ML**. The most important example concerns the arithmetic of natural numbers. According to Tarski's theorem of undefinability, the set of truths of this theory cannot be defined in it – such a definition can be formulated in the metatheory. This fact illuminates the relation between syntax and semantics. If **Th** is a rich theory and presented as a syntactic theory (a calculus), its semantics is not reducible to its syntax. According to Tarski's view, related to his work in the simple theory of types, semantics for **L** can be always constructed in the morphology of **ML**, provided that **L** is of the finite order. Two problems arise: what does the word "morphology" mean and how to formulate these ideas, when the framework is based on the distinction between first-order logic and higher-order logic. As far as the issue concerns morphology, it is possible to consider it as an extended syntax, i.e. vocabulary which does not refer to semantic concepts or defines such notions by not-semantic, e.g. set-theoretical, categories. If the hierarchy of logical types is replaced by the distinction of logics of various orders, in particular between first-order and higher-order (it is sufficient to use second-order), it is possible to show that semantics of first-order rich theories cannot be defined inside them.