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An approach to optimize transportation problems with neutrosophic numbers based on a new ranking function

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A transportation problem (TP) is built on the framework of supply-demand and cost parameters which are uncertain in nature. Neutrosophic numbers are capable of handling incomplete information. This paper introduces a new solution approach to optimize TPs with neutrosophic parameters based on a new ranking function. This function utilizes the attitudinal character of a basic unit-interval monotonic function inspired from the domain of continuous ordered weighted average operators. Ranking rules are established followed by defining a neutrosophic transportation problem. A solution methodology followed by solved numerical illustrates the efficiency of the proposed method. Conclusion and future directions summarize the work.

Key words: transportation problem, neutrosophic number, ranking function, basic unit-interval monotonic function, uncertainty

1. Introduction

The classical transportation problem developed by [11] is an optimization problem over a system of equipped sources and destinations in-need which aims

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at determining how much quantity of a commodity should be transported from these sources to destinations at an optimum cost. Different formulations and number of objective functions are considered best suitable to represent the transportation scenario. In case of multiple conflicting objectives, a multi-objective transportation problem is considered [6]. To optimize one or more ratios of functions, a fractional transportation problem is considered [36] whilst a solid transportation problem contemplates three item properties [27]. However, a crisp representation of the parameters namely, cost, supply and demand is incapable of representing their uncertain and indeterminate nature and hence, another apt representation is solicited. Let us first list a set of abbreviations which will be used throughout the work tabulated in Table 1.

Table 1: List of abbreviations

DM	Decision-maker
TP	Transportation problem
NN	Neutrosophic number
IF	Intuitionistic fuzzy
AC	Attitudinal character
BUM	Basic unit-interval monotonic
OWA	Ordered weighted average
COWA	Continuous ordered weighted average
NTP	Neutrosophic transportation problem
SVNN	Single-valued neutrosophic number
MOTP	Multi-objective transportation problem
MAGDM	Multi attribute group decision making

Different sets such as fuzzy sets introduced by [48], rough sets introduced by [23] and IF sets introduced by [2] and interval forms have been used to represent uncertainty in parameters. [5] gave interval and fuzzy extensions of classical TPs. [7] and [26] represented supply, demand and cost using interval parameters. [4] found the range of the optimal cost of a TP when supply and demand are represented as intervals. [19] solved fuzzy TPs based on extension principle while [13] solved fuzzy TPs using ranking function. To handle uncertainty and hesitation, [30] and [15] used triangular IF numbers to represent transportation parameters. [21] solved a fully IFTP while [1] optimized a fully rough interval integer solid TP. Recently, weighted sum method was employed to solve a rough MOTP by [9] and for a fully rough multi-objective fractional TP by [28]. [3] proposed an approach for solving fully generalized IF TPs.

[31] introduced neutrosophic sets based on the concept of truth, falsity and indeterminacy and also extended them [32, 33] in the form of $a + a'I$ where $a, a' \in \mathbb{R}$ and I denotes indeterminacy. The concept of indeterminacy here is a literal context i.e. non-numerical [34]. Since conception, NNs have been used to represent uncertain/incomplete information residing in the supply, demand and costs parameters. Different methodologies have been worked upon by various researchers to solve TPs with neutrosophic parameters. [37] and [10] employed single-valued trapezoidal NNs to solve TPs. [20] tackled emergency TP with single-valued neutrosophic sets. [29] solved a bilevel TP in neutrosophic environment. Recently, [25] studied a MOTP with uncertain variables under neutrosophic environment. Neutrosophic hyperbolic programming strategy was employed for uncertain MOTP by [14]. In [24], a variation degree concept was used to convert a neutrosophic solid TP into an interval programming problem.

OWA operator introduced by [41] are applied to combine a finite collection of values into a single value. [39] developed new deviation measures with OWA operators. [12] employed OWA operators where the inputs consist of basic uncertain information. [8] used OWA operators to solve a fuzzy multi-objective linear programming problem while [45] solved mathematical programming problem with OWA operators as objective functions. There are other extensions of OWA operator such as the generalized OWA operator by [43] which was also extended to aggregate IF sets by [17]. An IF version of the OWA operator was given by [40]. Apart from these areas, the concept of OWA operators has also been used in MAGDM [38, 49], neural network for vegetable price forecasting [16], risk identification [35] and deep learning [22]. For the cases when the given argument is a continuous valued interval instead of a crisp argument, [44] introduced an extension of the OWA operator, the COWA operator. COWA operator has been employed in group decision-making [18], under interval-valued q-rung orthopair fuzzy set environment for quality assessment [46] and under trapezoidal fuzzy environment to handle the conflicts [50].

Motivation and contribution of the proposed work: Indeterminate supply, demand and cost parameters are the facets of a real-life transportation problem due to unavoidable reasons like fuel cost, road conditions, weather conditions, delivery area, distance etc. Vagueness and uncertainties are well exhibited by NNs and hence are a good representative. Inspired by the domain of COWA operators, concept of AC has not been applied to find the solution of TPs with uncertain supply, demand and cost parameters. We introduce for the first time their application to solve a NTP. A new ranking function and ranking rules are introduced for crisp conversion and ranking NNs. It is based on the AC which is an important measure associated with an OWA aggregation and can be supplied by the DM as per his choice. The application is not only novel in terms of literature enrichment, but a computationally less burdening crisp conversion of NNs.

The remaining paper is divided as: Preliminary definitions are mentioned in Section 2. The proposed ranking function is introduced in Section 3 with their ranking rules. Section 4 establishes the NTP with mathematical formulation and solution methodology. In Section 5, two numerical examples are formulated and solved to validate the proposed model. The results are discussed in Section 6. Conclusion and future scope finalizes the work in Section 7.

2. Preliminaries

Definition 1 (OWA operator). *[41]: An OWA operator of dimension p is a mapping $o : \mathbb{R}^p \rightarrow \mathbb{R}$ such that $o(l_1, l_2, \ldots, l_p) = \sum_{i=1}^{p} w_i u_i$ where u_i is the i^{th} largest of the l_i and w_i is the weight of u_i, which satisfies $0 \leqslant w_i \leqslant 1$ and $\sum_{i=1}^{p} w_i = 1$.*

Definition 2 (COWA operator). *[44]: A COWA operator (an extension of OWA operator) is a mapping $\mathcal{F}_T : \xi^+ \rightarrow \mathbb{R}^+$ associated with BUM function T [42], such that $\mathcal{F}_T[l, u] = \int_0^1 \frac{dT(z)}{dz}(u - z(u - l))\mathrm{d}z$ where ξ^+ is the set of intervals of the type $[l, u](l > 0)$ and $\mathbb{R}^+$ is the set of all positive real numbers. Also, $T : [0, 1] \rightarrow [0, 1]$ is a monotonic function fixed at the end points.*

Definition 3 (Attitudinal character). *[44]: The attitudinal character associated with the function T is given by $\tau = \int_0^1 T(z)\mathrm{d}z$ and $0 \leqslant \tau \leqslant 1$.*

For example, consider a notable BUM function $T(z) = z^r, r > 0$ then, $\tau = \int_0^1 z^r \mathrm{d}z = \frac{1}{r+1}$.

Definition 4 (Neutrosophic numbers). *[47]: A NN is represented by $c + c'I$ where c is the determinate part, $c'I$ is the indeterminate part. $I \in [I_l, I_u]$ denotes indeterminacy where I_l and I_u respectively denote the lower and the upper limit of the indeterminacy.*

Remark 1. *For any NN, $\overline{c} = c + c'I$ and $I \in [I_l, I_u]$ the NN gets converted into an interval of the form $[c + c'I_l, c + c'I_u] = [c_l, c_u]$. As a special case when $I_l = I_u = 0$ or $I = 0$ (case of no indeterminacy) $\overline{c}$ is nothing but a real crisp number c. For example, $5 + 9I = 5$ for $I = 0$.*

Some properties of NNs are [34]:

1. $I.I = I, I^n = I(n \geqslant 1)$.
2. $0.I = 0$.
3. $\dfrac{I}{I}$ = undefined.
4. $rI + sI = (r + s)I$.

3. Proposed ranking function and ranking rules

We now define a ranking function to rank the NNs which also takes into account the attitudinal character $\tau = \int_0^1 T(z)\,\mathrm{d}z$ associated with the BUM function T. $\tau \in [0,1]$ is a representative of the preference of the DM.

Definition 5 (Ranking function)**.** *Let $\overline{c} = c + c'I$ be a NN. For $I \in [I_l, I_u]$, $\overline{c}$ can be converted into an interval of the form $[c_l, c_u]$. The ranking function is defined as*

$$G(\overline{c},\tau) = G([c_l,c_u],\tau) = c_l(1-\tau) + c_u\tau = c_l + \tau(c_u - c_l).$$

Remark 2.

$$G(\overline{c},\tau) = \begin{cases} c_l & \text{for } \tau = 0, \\ c_u & \text{for } \tau = 1. \end{cases}$$

Thus, $c_l \leqslant G(\overline{c},\tau) \leqslant c_u$.

For example, consider the NN $\overline{c} = 9 + 4I$. Then, for $I \in [0, 0.3]$, $G(\overline{c},\tau) = G([9, 10.2],\tau) = 9 + 1.2\tau$. Now, for any preferred $\tau \in [0,1]$ this value can be calculated.

Remark 3. *The utility of the above ranking function is that not only it converts a NN into a crisp number but can be used to rank them as well. We now state the ranking rules.*

Ranking rules based on the proposed ranking function: Let $\overline{p} = p + p'I$ and $\overline{q} = q + q'I$ be two NNs. Then, for $I \in [I_l, I_u]$ we have,

1. If $G(\overline{p},\tau) \leqslant G(\overline{q},\tau)$, then $\overline{p} \leqslant_N \overline{q}$.
2. If $G(\overline{p},\tau) \geqslant G(\overline{q},\tau)$, then $\overline{p} \geqslant_N \overline{q}$.
3. If $G(\overline{p},\tau) = G(\overline{q},\tau)$, then $\overline{p} =_N \overline{q}$.

For example, consider two NNs, $\overline{p} = 9+4I$ and $\overline{q} = 11+2I$. Then, for $I \in [0, 0.3]$, $G(\overline{p},\tau) = 9 + 1.2\tau$ and $G(\overline{q},\tau) = 11 + 0.6\tau$. Let the DM gives the preferred value of $\tau = \frac{3}{4}$ ($\tau \in [0,1]$), then as

$$G\left(9+4I, \frac{3}{4}\right) \leqslant G\left(11+2I, \frac{3}{4}\right) \implies 9+4I \leqslant_N 11+2I.$$

Theorem 1. *Let $\overline{p} = p + p'I$ and $\overline{q} = q + q'I$ be two NNs. Then, $G(\lambda\overline{p},\tau) + G(\lambda\overline{q},\tau) = \lambda G(\overline{p}+\overline{q},\tau)$.*

Proof. For $I \in [I_l, I_u]$,

$$\begin{aligned} G(\lambda\overline{p},\tau) + G(\lambda\overline{q},\tau) &= G(\lambda[p_l, p_u],\tau) + G(\lambda[q_l, q_u],\tau) \\ &= G([\lambda p_l, \lambda p_u],\tau) + G([\lambda q_l, \lambda q_u],\tau) \\ &= [\lambda p_l + \tau(\lambda p_u - \lambda p_l)] + [\lambda q_l + \tau(\lambda q_u - \lambda q_l)] \\ &= [\lambda p_l + \lambda q_l] + \tau[(\lambda p_u + \lambda q_u) - (\lambda p_l + \lambda q_l)] \\ &= \lambda[(p_l + q_l) + \tau[(p_u + q_u) - (p_l + q_l)]] \\ &= \lambda G([p_l + q_l, p_u + q_u],\tau) \\ &= \lambda G(\overline{p} + \overline{q},\tau). \end{aligned}$$

4. Neutrosophic transportation problem

4.1. Mathematical formulation

Mathematically, a NTP with $\mathcal{M}$ supply points and $\mathcal{N}$ destination points is given by

$$Min\ Z = \sum_{j=1}^{\mathcal{M}} \sum_{k=1}^{\mathcal{N}} \bar{t}_{jk} x_{jk}$$

s.t.

$$\sum_{k=1}^{\mathcal{N}} x_{jk} \leqslant \overline{s}_j \ \forall\ j = 1, 2, \ldots, \mathcal{M},$$

$$\sum_{j=1}^{\mathcal{M}} x_{jk} \geqslant \overline{d}_k \ \forall\ k = 1, 2, \ldots, \mathcal{N}.$$

Here, $\bar{t}_{jk} = t_{jk} + t'_{jk} I$ is the neutrosophic transportation cost incurred while delivering one unit of the article from the j^{th} supply point to the k^{th} destination point. In the constraints, $\overline{s}_j = s_j + s'_j I$ and $\overline{d}_k = d_k + d'_k I$ are the respective neutrosophic supply and neutrosophic demand at the j^{th} supply point and the k^{th} destination point. The objective is to find how much quantity of the article $x_{jk} (\geqslant 0)$ should be allocated at the $(j,k)^{th}$ location such that the total transportation cost incurred is minimal.

4.2. Solution methodology

We present the solution methodology through the following steps to solve NTP.

1. Convert the given NTP into an interval TP by taking a suitable value of $I \in [I_l, I_u]$ (see Remark 1). The new mathematical formulation is

$$\text{Min } Z = \sum_{j=1}^{\mathcal{M}} \sum_{k=1}^{\mathcal{N}} [t_{jkl}, t_{jku}] x_{jk}$$

s.t.

$$\sum_{k=1}^{\mathcal{N}} x_{jk} \leqslant [s_{jl}, s_{ju}] \ \forall \ j = 1, 2, \dots, \mathcal{M},$$
$$\sum_{j=1}^{\mathcal{M}} x_{jk} \geqslant [d_{kl}, d_{ku}] \ \forall \ k = 1, 2, \dots, \mathcal{N}.$$

2. Apply the ranking function $G([c_l, c_u], \tau)$ as in Definition 5 on the above obtained intervals using any preferred $T(z)$ by the DM. The NTP now gets converted into crisp NTP (CNTP) with crisp supply, demand and cost parameters given by

$$\text{Min } Z = \sum_{j=1}^{\mathcal{M}} \sum_{k=1}^{\mathcal{N}} G([t_{jkl}, t_{jku}, \tau] x_{jk}$$

s.t.

$$\sum_{k=1}^{\mathcal{N}} x_{jk} \leqslant G([s_{jl}, s_{ju}], \tau) \ \forall \ j = 1, 2, \dots, \mathcal{M},$$
$$\sum_{j=1}^{\mathcal{M}} x_{jk} \geqslant G([d_{kl}, d_{ku}], \tau) \ \forall \ k = 1, 2, \dots, \mathcal{N}.$$

Some notable BUM functions $T(z)$ are:

(a) $T(z) = \begin{cases} 0 & \text{for } z = 0, \\ 1 & \text{for } z > 0. \end{cases}$

(b) $T(z) = \begin{cases} 0 & \text{for } z < 1, \\ 1 & \text{for } z = 1. \end{cases}$

(c) $T(z) = z \ \forall \ z.$

(d) $T(z) = z^r$ where $r > 0$.

(e) $T(z) = (Sin(\frac{\pi}{2} z))^r, r > 0.$

(f) $T(z) = \left(\dfrac{1 - e^{-az}}{1 - e^{-a}} \right)^r, r > 0.$

3. Depending on the type of $T(z)$ used from the above list, the obtained values of supply, demand and cost parameters may or may not be dependent on the parameter $r,\ r > 0$. In case r is present, different values of r may be taken as per the choice of the DM. In both the cases, the resulting TP is a CNTP.
4. Solve the CNTP using any standard method or any computing software to obtain the values of $x_{jk}'s$.
5. Substitute these $x_{jk}'s$ in the objective function to obtain the minimum transportation cost.

5. Model validation: Numerical examples

We solve a 3×4 and a 4×5 NTP for two different values of I. In the first numerical we take $I \in [0, 1]$ and $I \in [0, 0.6]$ in the latter. The details and data used in both the numerical examples are hypothetical.

5.1. Numerical example-1

Consider a tyre manufacturing company with three manufacturing centers (M.C.) located in Rajasthan, Delhi and Haryana. These manufactured tyres are then transported to four different locations (L.C.) *viz.* Gujarat, Bihar, Uttar Pradesh and Chandigarh. As the real-life supply, demand and cost parameters are indeterminate in nature they are taken to be neutrosophic numbers giving rise to a 3×4 NTP with neutrosophic cost coefficients given by Table 2.

Table 2: Transportation problem with neutrosophic cost coefficients

M.C. \ L.C.	Gujarat	Bihar	Uttar Pradesh	Chandigarh
Rajasthan	$8 + 3I$	$4 + 2I$	$7 + I$	$6 + 2I$
Delhi	$4 + 3I$	$6 + 3I$	$5 + 5I$	$4 + 7I$
Haryana	$2 + I$	$9 + 6I$	$6 + 3I$	$1 + 5I$

The neutrosophic source & demand constraints for $j = 1, 2, 3$ and $k = 1, 2, 3, 4$ are:

$$\sum_k x_{1k} \leqslant 10 + 2I, \quad \sum_k x_{2k} \leqslant 6 + 3I, \quad \sum_k x_{3k} \leqslant 7 + 4I,$$
$$\sum_j x_{j1} \geqslant 6 + 2I, \quad \sum_j x_{j2} \geqslant 5 + I, \quad \sum_j x_{j3} \geqslant 3 + 3I, \quad \sum_j x_{j4} \geqslant 9 + 3I.$$

We have taken a balanced TP i.e. total supply equals the total demand ($= 23 + 9I$). We now solve the NTP using the steps in Section 4.2.

Step-1 Let us take $I \in [0, 1]$, then the above TP gets converted into an interval TP with cost coefficients given by the Table 3.

Table 3: Transportation problem with interval cost coefficients

L.C. / M.C.	Gujarat	Bihar	Uttar Pradesh	Chandigarh
Rajasthan	[8, 11]	[4, 6]	[7, 8]	[6, 8]
Delhi	[4, 7]	[6, 9]	[5, 10]	[4, 11]
Haryana	[2, 3]	[9, 15]	[6, 9]	[1, 6]

Similarly, the interval source & demand constraints are given by:

$$\sum_k x_{1k} \leqslant [10, 12], \quad \sum_k x_{2k} \leqslant [6, 9], \quad \sum_k x_{3k} \leqslant [7, 11],$$
$$\sum_j x_{j1} \geqslant [6, 8], \quad \sum_j x_{j2} \geqslant [5, 6], \quad \sum_j x_{j3} \geqslant [3, 6], \quad \sum_j x_{j4} \geqslant [9, 12].$$

Step-2 The ranking function $G([c_l, c_u], \tau)$ is now applied to each of the above intervals for a suitable $T(z)$. Let the DM chooses $T(z) = z^r, r > 0$, then $\tau = \int_0^1 z^r \, dz = \frac{1}{r+1}$.

Step-3 Table 4 provides the values of $G([c_l, c_u], \frac{1}{r+1}) = c_l + \frac{1}{r+1}(c_u - c_l)$ for some notable values of r.

Table 4: Values of $G([c_l, c_u], \tau)$ for $\tau = \dfrac{1}{r+1}$

Values of r	Corresponding values of $G\left([c_l, c_u], \dfrac{1}{r+1}\right)$
$r \to 0$	c_u
$r = \frac{1}{2}$	$\frac{2c_u + c_l}{3}$
$r = 1$	$\frac{c_u + c_l}{2}$
$r = 2$	$\frac{2c_l + c_u}{3}$
$r \to \infty$	c_l

Different values of r gives rise to different TPs which have crisp supply, demand and cost parameters. The crisp cost coefficients for CNTP for $r \to 0$ s.t. $G([c_l, c_u], \frac{1}{r+1}) = c_u$ is given by Table 5.

Table 5: Transportation problem with crisp cost coefficients (for $r \to 0$)

M.C. \ L.C.	Gujarat	Bihar	Uttar Pradesh	Chandigarh
Rajasthan	11	6	8	8
Delhi	7	9	10	11
Haryana	3	15	9	6

Similarly, the crisp supplies & demands for $r \to 0$ are:

$$\sum_k x_{1k} \leqslant 12, \quad \sum_k x_{2k} \leqslant 9, \quad \sum_k x_{3k} \leqslant 11,$$
$$\sum_j x_{j1} \geqslant 8, \quad \sum_j x_{j2} \geqslant 6, \quad \sum_j x_{j3} \geqslant 6, \quad \sum_j x_{j4} \geqslant 12.$$

Step-4 Similarly, the other crisp TPs for the remaining values of r can be formulated and solved using any standard method or computing software.

Step-5 The obtained solutions x_{jk}'s and the corresponding transportation costs are tabulated in Table 6.

Table 6: Transportation costs for different values of r (Numerical example-1)

Values of r	Transportation costs (in Rs.)	Solution point
$r \to 0$	216	$x_{12} = 6; x_{13} = 5; x_{14} = 1;$ $x_{21} = 8; x_{23} = 1; x_{34} = 11$
$r = \frac{1}{2}$	163.47	$x_{12} = 5.6; x_{13} = 4.3; x_{14} = 1.4;$ $x_{21} = 7.3; x_{23} = 0.7; x_{34} = 9.6$
$r = 1$	141.75	$x_{12} = 5.5; x_{13} = 4; x_{14} = 1.5;$ $x_{21} = 7; x_{23} = 0.5; x_{34} = 9$
$r = 2$	119.1	$x_{12} = 5.3; x_{13} = 3.6; x_{14} = 1.7;$ $x_{21} = 6.6; x_{23} = 0.4; x_{34} = 8.3$
$r \to \infty$	84	$x_{12} = 5; x_{13} = 3; x_{14} = 2;$ $x_{21} = 6; x_{34} = 7$

5.2. Numerical example-2

Consider a garment manufacturing company with centers at Mumbai, Pune, Kolhapur and Goa. The manufactured garments are supplied to Chennai, Bengaluru, Hyderabad, Trichy and Amaravati. In this balanced 4×5 NTP we take neutrosophic cost, supply and demand parameters given by Table 7.

Table 7: Transportation problem with neutrosophic cost coefficients

M.C. \ L.C.	Chennai	Bengaluru	Hyderabad	Trichy	Amaravati
Mumbai	$4+6I$	$3+4I$	$2+4I$	$6+6I$	$2+2I$
Pune	$7+4I$	$17+I$	$6+3I$	$2+3I$	$4+5I$
Kolhapur	$11+4I$	$8+2I$	$9+2I$	$7+4I$	$12+2I$
Goa	$4+5I$	$5+2I$	$10+3I$	$15+4I$	$11+4I$

The neutrosophic source & demand constraints for $j = 1, 2, 3, 4$ and $k = 1, 2, 3, 4, 5$ are:

$$\sum_k x_{1k} \leqslant 7+4I, \quad \sum_k x_{2k} \leqslant 10+3I, \quad \sum_k x_{3k} \leqslant 8+3I,$$
$$\sum_k x_{4k} \leqslant 5+2I, \quad \sum_j x_{j1} \geqslant 8+2I, \quad \sum_j x_{j2} \geqslant 4+4I,$$
$$\sum_j x_{j3} \geqslant 6+2I, \quad \sum_j x_{j4} \geqslant 9+3I, \quad \sum_j x_{j5} \geqslant 3+I.$$

The above problem can be solved using the same steps as above by taking $I \in [0, 0.6]$. The obtained solutions x_{jk}'s and the corresponding transportation costs are tabulated in Table 8.

6. Discussion

Both the numerical problems have been solved for some notable values of r. For more or infinite number of values of r, we will obtain different range of values of transportation costs ranging from the minimum value of Rs.84 to the maximum value Rs. 216 for the first problem and minimum value of Rs.130 to the maximum value Rs. 230.84 for the latter.

Table 8: Transportation costs for different values of r (Numerical example-2)

Values of r	Transportation costs(in Rs.)	Solution point
$r \to 0$	230.84	$x_{11} = 2; x_{13} = 3.8; x_{15} = 3.6;$ $x_{21} = 1; x_{24} = 10.8; x_{32} = 6.4;$ $x_{33} = 3.4; x_{41} = 6.2$
$r = \frac{1}{2}$	194.44	$x_{13} = 5.2; x_{15} = 3.4; x_{21} = 1;$ $x_{24} = 10.2; x_{31} = 2; x_{32} = 5.6;$ $x_{33} = 1.6; x_{41} = 5.8$
$r = 1$	177.41	$x_{13} = 4.9; x_{15} = 3.3; x_{21} = 1;$ $x_{24} = 9.9; x_{31} = 2; x_{32} = 5.2;$ $x_{33} = 1.7; x_{41} = 5.6$
$r = 2$	161.16	$x_{13} = 5.6; x_{15} = 2.2; x_{24} = 9.6;$ $x_{25} = 1; x_{31} = 3; x_{32} = 4.8;$ $x_{33} = 0.8; x_{41} = 5.4$
$r \to \infty$	130	$x_{13} = 5; x_{15} = 2; x_{24} = 9;$ $x_{25} = 1; x_{31} = 3; x_{32} = 4;$ $x_{33} = 1; x_{41} = 5$

7. Conclusion and future work

In real-life, the parameters such as supply, demand and cost are not fixed and therefore cannot be denoted by crisp numbers. Since conception, fuzzy sets and its extensions have been used to represent uncertainty in these parameters. In order to solve the TPs in an uncertain environment, various methods have evolved to convert fuzzy numbers and their extensions into crisp numbers. In this work, the transportation parameters have been represented by NNs and are converted to crisp numbers using a proposed ranking function based on AC of BUM function. The DM can use any preferred BUM function and hence corresponding value of AC can be obtained. In our solved problem using a preferred BUM function we obtain numerous values of transportation costs within a definite range and the DM can select any value as per his choice. The introduced ranking function is also helpful in comparing any two NNs.

In an extension of this work, another formulation where only the transportation costs are NNs or only the supply-demand parameters are NNs can be taken. Instead of a balanced NTP, researchers can take up an unbalanced NTP. Moreover, different BUM functions available in the literature can be used in the ranking function to convert the intervals into crisp numbers.

References

[1] T. Anithakumari, B. Venkateswarlu and A. Akilbasha: Optimizing a fully rough interval integer solid transportation problems. *Journal of Intelligent & Fuzzy Systems*, **41**(1), (2021), 2429–2439. DOI: 10.3233/JIFS-202373

[2] K.T. Atanassov: Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, **20**(1), (1986), 87–96. DOI: 10.1016/S0165-0114(86)80034-3

[3] I. Beg, M. Bisht and S. Rawat: An approach for solving fully generalized intuitionistic fuzzy transportation problems. *Computational and Applied Mathematics*, **42**(8), (2023), 329. DOI: 10.1007/s40314-023-02467-8.

[4] R. Cerulli, C. D'Ambrosio and M. Gentili: Best and worst values of the optimal cost of the interval transportation problem. *Optimization and Decision Science: Methodologies and Applications*, Springer (2017), 367–374.

[5] S. Chanas, M. Delgado, J.L. Verdegay and M.A. Vila: Interval and fuzzy extensions of classical transportation problems. *Transportation Planning and Technology*, **17**(2), (1993), 203–218. DOI: 10.1080/03081069308717511

[6] A. Charnes andW.W. Cooper: Management models and industrial applications of linear programming. *Management Science*, **4**(1), (1957), 38–91. DOI: 10.1287/mnsc.4.1.38

[7] S. Das, A. Goswami and S. Alam: Multiobjective transportation problem with interval cost, source and destination parameters. *European Journal of Operational Research*, **117**(1), (1999), 100–112. DOI: 10.1016/S0377-2217(98)00044-7

[8] D. Dubey and A. Mehra: A bipolar approach in fuzzy multi-objective linear programming. *Fuzzy Sets and Systems*, **246** (2014), 127–141. DOI: 10.1016/j.fss.2013.07.017

[9] H. Garg and R.M. Rizk-Allah: A novel approach for solving rough multiobjective transportation problem: development and prospects. *Computational and Applied Mathematics*, **40**(4), (2021), 149. DOI: 10.1007/s40314-021-01507-5

[10] B.K. Giri and S.K. Roy: Neutrosophic multi-objective green four-dimensional fixed-charge transportation problem. *International Journal of Machine Learning and Cybernetic*s, **13**(10), (2022), 3089–3112. DOI: 10.1007/s13042-022-01582-y

[11] F.L. Hitchcock: The distribution of a product from several sources to numerous localities. *Journal of Mathematics and Physics*, **20**(1-4), (1941), 224–230. DOI: 10.1002/sapm1941201224

[12] L. Jin, Z.-S. Chen, R.R. Yager, T. Senapati, R. Mesiar, D.G. Zamora, B. Dutta and L. Martinez: Ordered weighted averaging operators for basic uncertain information granules. *Information Sciences*, **645** (2023), 119357. DOI: 10.1016/j.ins.2023.119357

[13] A. Kaur and A. kumar: A new method for solving fuzzy transportation problems using ranking function. *Applied Mathematical Modelling*, **35**(12), (2011), 5652–5661. DOI: 10.1016/j.apm.2011.05.012

[14] A. Kumar, P. Singh and Y. Kacher: Neutrosophic hyperbolic programming strategy for uncertain multi-objective transportation problem. *Applied Soft Computing* (2023), 110949.

[15] P.S. Kumar and R.J. Hussain: Computationally simple approach for solving fully intuitionistic fuzzy real life transportation problems. *International Journal of System Assurance Engineering and Management*, **7** (2016), 90–101. DOI: 10.1007/s13198-014-0334-2

[16] B. Li, J. Ding, Z. Yin, K. Li, X. Zhao and L. Zhang: Optimized neural network combined model based on the induced ordered weighted averaging operator for vegetable price forecasting. *Expert Systems with Applications*, **168** (2021), 114232. DOI: 10.1016/j.eswa.2020.114232

[17] D.-F. Li: Multiattribute decision making method based on generalized OWA operators with intuitionistic fuzzy sets. *Expert Systems with Applications*, **37**(12), (2010), 8673–8678. DOI: 10.1016/j.eswa.2010.06.062

[18] J. Li andY. Zhang: A new method for interval fuzzy preference relations in group decision making based on plant growth simulation algorithm and COWA. *Journal of Intelligent & Fuzzy Systems*, **37**(3), (2019), 4311–4323. DOI: 10.3233/JIFS-190410

[19] S.-T. Liu and C. Kao: Solving fuzzy transportation problems based on extension principle. *European Journal of Operational Research*, **153**(3), (2004), 661–674. DOI: 10.1016/S0377-2217(02)00731-2

[20] L. Lu and X. Luo: Emergency transportation problem based on single-valued neutrosophic set. *Discrete Dynamics in Nature and Society*, 2020, (2020), 1–8.

[21] A. Mahmoodirad, T. Allahviranloo and S. Niroomand: A new effective solution method for fully intuitionistic fuzzy transportation problem. *Soft Computing*, **23**(12), (2019), 4521–4530. DOI: 10.1007/s00500-018-3115-z

[22] S. Maldonado, C. Vairetti, K. Jara, M. Carrasco and J. Lopez: OWAdapt: An adaptive loss function for deep learning using OWA operators. arXiv preprint arXiv:2305.19443 (2023). DOI: 10.1016/j.knosys.2023.111022

[23] Z. Pawlak: Rough sets. *International Journal of Computer & Information Sciences*, **11** (1982), 341–356. DOI: 10.1007/BF01001956

[24] N. Qiuping, T. Yuanxiang, S. Broumi and V. Uluçay: A parametric neutrosophic model for the solid transportation problem. *Management Decision*, **61**(2), (2023), 421–442. DOI: 10.1108/MD-05-2022-0660

[25] A. Revathi, S. Mohanaselvi and B. Said: An Efficient Neutrosophic Technique for Uncertain Multi Objective Transportation Problem. *Neutrosophic Sets and Systems*, **53**(1), (2023), 27.

[26] M. Safi and A. Razmjoo: Solving fixed charge transportation problem with interval parameters. *Applied Mathematical Modelling*, **37**(18-19), (2013), 8341–8347. DOI: 10.1016/j.apm.2013.03.053

[27] E. Shell: Distribution of a product by several properties, Directorate of Management Analysis. *Proceedings of the Second Symposium in Linear Programming*. Vol. 2 (1955), 615–642.

[28] Shivani, D. Rani and A. Ebrahimnejad: On solving fully rough multiobjective fractional transportation problem: development and prospects. *Computational and Applied Mathematics*, **42**(6), (2023), 266. DOI: 10.1007/s40314-023-02400-z

[29] A. Singh, R. Arora and S. Arora: Bilevel transportation problem in neutrosophic environment. *Computational and Applied Mathematics*, **41**(1), (2022), 1–25. DOI: 10.1007/s40314-021-01711-3

[30] S.K. Singh and S.P. Yadav: A new approach for solving intuitionistic fuzzy transportation problem of type-2. *Annals of Operations Research*, **243**, (2016), 349–363. DOI: 10.1007/s10479-014-1724-1

[31] F. Smarandache: *A unifying field in logics. neutrosophy: Neutrosophic probability, set and logic*. American Research Press, Rehoboth, 1999.

[32] F. Smarandache: *Introduction to neutrosophic measure, neutrosophic integral, and neutrosophic probability*. Infinite Study, 2013.

[33] F. Smarandache: *Introduction to neutrosophic statistics*. Infinite Study, 2014.

[34] F. Smarandache: (t, i, f)-Neutrosophic Structures and I-Neutrosophic Structures (Revisited) (2015).

[35] G. Su, B. Jia, P. Wang, R. Zhang and Z. Shen: Risk identification of coal spontaneous combustion based on COWA modified G1 combination weighting cloud model. *Scientific Reports*, **12**(1), (2022), 2992. DOI: 10.1038/s41598-022-06972-4

[36] K. Swarup: Transportation technique in linear fractional functional programming. *Journal of Royal Naval Scientific Service*, **21** (5), (1966), 256–260.

[37] A. Thamaraiselvi and R. Santhi: A new approach for optimization of real life transportation problem in neutrosophic environment. *Mathematical Problems in Engineering*, 2016, (2016), 1–9. DOI: 10.1155/2016/5950747

[38] R. Verma and A. Mittal: Multiple attribute group decision-making based on novel probabilistic ordered weighted cosine similarity operators with Pythagorean fuzzy information. *Granular Computing*, **8**(1), (2023), 111–129. DOI: 10.1007/s41066-022-00318-1

[39] R. Verma and J.M. Merigo: Variance measures with ordered weighted aggregation operators. *International Journal of Intelligent Systems*, **34**(6), (2019), 1184–1205. DOI: 10.1002/int.22091

[40] Z. Xu: Intuitionistic fuzzy aggregation operators. *IEEE Transactions on Fuzzy Systems*, **15**(6), (2007), 1179–1187. DOI: 10.1109/TFUZZ.2006.890678

[41] R.R. yager: On ordered weighted averaging aggregation operators in multicriteria decisionmaking. *IEEE Transactions on Systems, Man, and Cybernetics*, **18**(1), (1988), 183–190.

[42] R.R. Yager: Quantifier guided aggregation using OWA operators. International Journal of Intelligent Systems, 11 (1), (1996), 49–73.

[43] R.R. Yager: Generalized OWA aggregation operators. *Fuzzy Optimization and Decision Making*, **3** (2004), 93–107. DOI: 10.1023/B:FODM.0000013074.68765.97

[44] R.R. Yager: OWA aggregation over a continuous interval argument with applications to decision making. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, **34**(5), (2004), 1952–1963.

[45] R.R. Yager: Solving mathematical programming problems with OWA operators as objective functions. *Proceedings of 1995 IEEE International Conference on Fuzzy Systems*. Vol. 3. IEEE. (1995), 1441–1446. DOI: 10.1109/FUZZY.1995.409869

[46] Y. Yang, Z.-S. Chen, R.M. Rodriguez, W. Pedrycz and K.-S. Chin: Novel fusion strategies for continuous interval-valued q-rung orthopair fuzzy information: a case study in quality assessment of SmartWatch appearance design. *International Journal of Machine Learning and Cybernetics*, **13** (2022), 1–24. DOI: 10.1007/s13042-020-01269-2

[47] J. Ye: Multiple-attribute group decision-making method under a neutrosophic number environment. *Journal of Intelligent Systems*, **25** (3), (2016), 377–386. DOI: 10.1515/jisys-2014-0149

[48] L.A. Zadeh: Fuzzy sets. *Information and Control*, **8**(3), (1965), 338–353. DOI: 10.1016/S0019-9958(65)90241-X

[49] L. Zhou, H. Chen and J. Liu: Continuous ordered weighted distance measure and its application to multiple attribute group decision making. *Group Decision and Negotiation*, **22** (2013), 739–758. DOI: 10.1007/s10726-012-9289-3

[50] Y. Zhou, C. Zheng, P. Wu and L. Zhou: A new compatibility model for fuzzy group decision making by using trapezoidal fuzzy preference relations with COWA operator. *International Journal of Machine Learning and Cybernetics*, **15**(3), (2024), 1055–1073. DOI: 10.1007/s13042-023-01955-x