

IDENTIFICATION OF STEP-INDEX FIBERS BY STATIC LIGHT SCATTERING

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Abstract

Accurate characterization of optical fibers is crucial for numerous applications in telecommunications, sensing, and medical diagnostics. In this study, a novel method of sizing of step-index fibers is presented on the basis of the analysis of data on light scattering. This approach integrates mathematical modeling of light scattering by step-index fibers with signal processing and correlation algorithms to extract information on the layered structure of the fiber under test. Practical measurements use of a novel optical system for laboratory-level tests. The results show a clear route to improve non-destructive and efficient fiber characterization in online industrial process control.

Keywords: optical fibers, scattering measurements, inverse scattering.

1. Introduction

In the realm of optical communication and sensing, step-index fibers play a pivotal role in facilitating efficient transmission of light signals [1–6]. The performance and reliability of step-index fibers are critically dependent on their structural and optical properties. For example, any variation in fiber diameter may lead to radiation loss and make fiber-to-fiber connections problematic. Technical verification of the characteristics of the fiber during the manufacturing process (in the fiber drawing stage) is of special importance [7, 8].

Characterization methods used for noncontact measurements of transparent fibers rely either on image projection or on scattering pattern analysis. In the first case, a beam of laser light is swept across the fiber at a constant time rate, and the fiber diameter is retrieved from a shadow registered by the detectors [7]. Non-imaging techniques, such as laser diffractometry [9–11], far-field resonance of forward scattered light [12, 13], spectral interferometry [14, 15], scattering near rainbow angle [16, 17], whispering gallery modes spectroscopy [18], and analysis of scattering effects by using standing wave illumination [19], involve estimations of fiber characteristics from a mathematical analysis of the minima and maxima of scattering patterns. Alternatively, off-line microscopic or tomographic techniques can be used to study the characteristics of fibers in detail [7, 20, 21].

Here, a non-destructive technique is demonstrated that allows characterization of step-index fibers. Compared to commercial ‘shadowgraph instruments’ commonly used in the fiber industry for real-time outer diameter measurements and feedback control purposes [7], the technique offers the ability to retrieve diameters of the internal layers, provided that their refractive indices are known. This approach is based on *static light scattering* (SLS) of a polychromatic beam of light and a step-index FG050UGA optical fiber from Thorlabs, whose refractive index tomographic scan and profile are shown in Fig. 1. The fiber has a depressed clad design, i.e. the cladding consists of two regions: i) an inner cladding with the refractive index lower than the refractive index of the core, and ii) an outer cladding that has a higher refractive index than the inner cladding region.

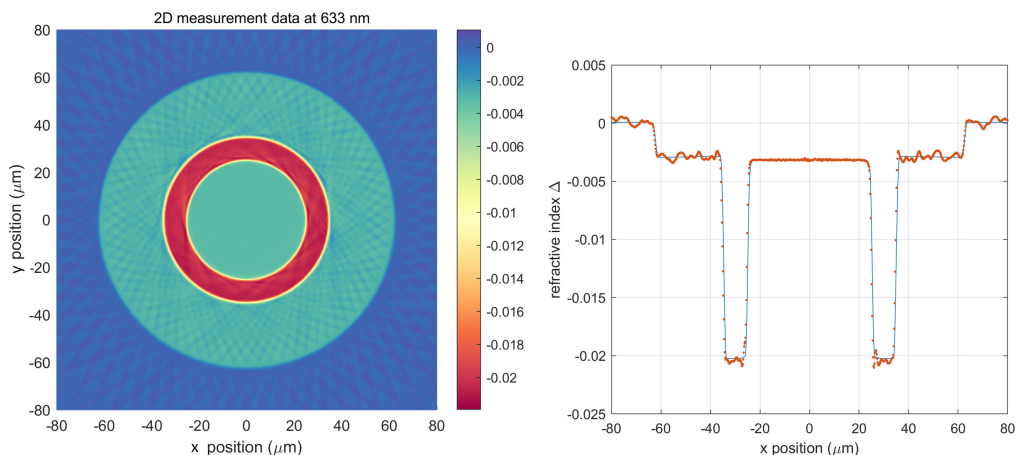


Fig. 1. Left: Tomographic image of a multimode, step-index Thorlabs FG050UGA fiber measured with an IFA-100 interferometric system; Right: the index profile inside the fiber (dots) and its average obtained by superposition of 360 tomographic projections taken at the step of 1° (solid). Refractive index units are Δn .

After this brief introduction, Section 2 summarizes the background of the SLS technique. Section 3 presents the data inversion method. An experimental setup for SLS measurements is introduced in Section 4, while measurement results are provided and discussed in Section 5. The paper ends with a conclusion section with perspectives for future improvements.

2. SLS background

SLS is particularly useful for determining the size of particles or macromolecules [22]. It is a noninvasive technique, meaning it does not require the alteration or labeling of the sample, which is advantageous for monitoring optical fibers online. SLS basically involves measurement of the intensity of light averaged over time at various spatial locations, which is shown schematically in Fig. 2. In this study, a polychromatic incident beam of light is used. This provides computational simplification when the measurement data is converted to the information on fiber properties and, importantly, reduces a detrimental sensitivity of measurement data to small changes in fiber properties [16, 23]. A polychromatic light beam can be viewed as a vector superposition of a number of mutually incoherent monochromatic fields propagating in the same direction and having a continuous distribution of angular frequencies (ω_{\min} , ω_{\max}):

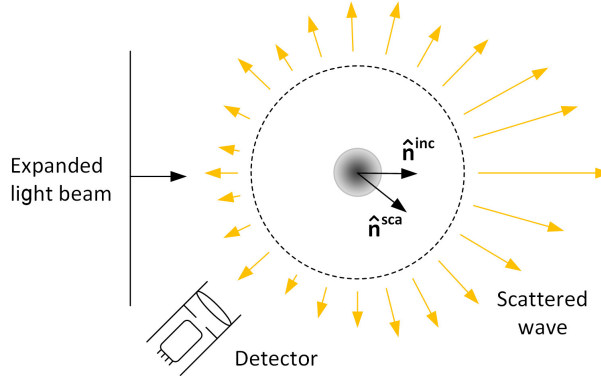


Fig. 2. Cross-section of a step-index optical fiber illuminated by a beam of well-formed light.

$$\mathbf{E}^{inc}(\mathbf{r}, t) = \sum_{n=1}^N \mathbf{E}_n^{inc}(t) \exp(ik_n^{inc} \hat{\mathbf{n}}^{inc} \cdot \mathbf{r} - i\omega_n t), \quad (1)$$

where \mathbf{r} is the position vector of the observation point (detector), $\mathbf{E}_n^{inc}(t)$ is the complex amplitude of the electric field, which fluctuates randomly in time with the period long compared to the complex temporal term $\exp(-i\omega_n t)$, and k_n^{inc} is the wave vector. The basic SLS equation transforms the incident monochromatic component \mathbf{E}_n^{inc} into the scattered one \mathbf{E}_n^{sca} upon its interaction with the fiber, using a *separation-of-variables* (SVM) solution of Maxwell's equations:

$$\begin{bmatrix} E_v^{sca}(\mathbf{r}) \\ E_\varphi^{sca}(\mathbf{r}) \end{bmatrix} \Big|_{r \rightarrow \infty} = \frac{e^{ik_0 r}}{r} \mathbf{S}(\hat{\mathbf{n}}^{sca}, \hat{\mathbf{n}}^{inc}) \begin{bmatrix} E_{0v}^{inc} \\ E_{0\varphi}^{inc} \end{bmatrix}, \quad (2)$$

where $k_0 = 2\pi/\lambda_0$ is the wave number of the surrounding medium and $\mathbf{S}(\hat{\mathbf{n}}^{sca}, \hat{\mathbf{n}}^{inc})$ is the 2×2 amplitude matrix which transforms the vector v^- and φ^- components of the incident plane wave into the v^- and φ^- components of the scattered wave. In general, the amplitude matrix depends on the fiber size, the refractive index profile, and the orientation with respect to the incident wave and the detector [24–26]. The time-averaged Poynting vector of the scattered electromagnetic field at a point \mathbf{r} in space can be obtained by applying an additivity rule:

$$\langle\langle \mathbf{S}^{sca}(\mathbf{r}, t) \rangle\rangle = \text{Re} \sum_{n=1}^N \mathbf{S}_n^{sca}(\mathbf{r}), \quad (3)$$

where $\langle\langle \dots \rangle\rangle$ denotes averaging over a long time interval and \mathbf{S}_n^{sca} is the complex Poynting vector of the n^{th} monochromatic component comprising the incident light:

$$\mathbf{S}_n^{sca}(\mathbf{r}) = \frac{1}{2} \mathbf{E}_n^{sca}(\mathbf{r}) \times [\mathbf{H}_n^{sca}(\mathbf{r})]^*, \quad (4)$$

with an asterisk denoting the complex conjugate. The scattered intensity in the far field, $I^{sca}(\mathbf{r})$, is the magnitude of the time-averaged Poynting vector. Finally, polychromatic illumination requires that the refractive index of each layer of the step-index optical fiber must be modeled as wavelength dependent:

$$m(\lambda) = \left(1 + \sum_{i=1}^3 A_i \lambda^2 / (\lambda^2 - \ell_i^2) \right)^{1/2} + i\kappa + D, \quad (5)$$

where A_i, ℓ_i are the Sellmeier coefficients [27], κ is the attenuation constant at optical frequencies, and D is an offset to account for a dopant. For typical optical fibers, it is correct to assume that a dopant does not affect the shape of the fiber's dispersion curve $m(\lambda)$ [28].

3. Data inversion

Data inversion relates the optical field back to properties of interest of the fiber. In this study, the diameters of the three refractive layers $d_1 < d_2 < d_3$, *i.e.* the core, the depressed cladding, and the cladding, are retrieved, as illustrated in the tomographic scan in Fig. 1. The inversion method involves correlations between two discrete space, real-valued signals, *i.e.*, a reference scattering signal $I_{\text{ref}}(\theta)$ and an experimental signal $I_{\text{exp}}(\theta)$. A database of reference signals is prepared only once by using the SLS theory introduced in Section 2 for various d_1, d_2 and d_3 corresponding to that which we expect to measure.

A quantitative comparison between the two signals I_{exp} and I_{ref} is made in several steps. First, I_{ref} and I_{exp} are normalized since, in general, they differ in terms of the amplitude as well as the background offset:

$$\tilde{I}_{\text{exp}} = \frac{I_{\text{exp}} - \min\{I_{\text{exp}}\}}{\sum_{i=1}^N (I_{\text{exp}}(i) - \min\{I_{\text{exp}}\})}, \quad \tilde{I}_{\text{ref}} = \frac{I_{\text{ref}} - \min\{I_{\text{ref}}\}}{\sum_{i=1}^N (I_{\text{ref}}(i) - \min\{I_{\text{ref}}\})}, \quad (6)$$

where N refers to the number of data points. The background level was assumed to be independent of θ .

A quantitative comparison between \tilde{I}_{exp} and \tilde{I}_{ref} is realized by using a correlation estimator, defined as the weighted sum of squared errors of a prediction:

$$r^2(d_{1,2,3}) = \min_j \left\{ \frac{1}{N} \sum_{i=1}^N W(i) (\tilde{I}_{\text{exp}}(i+j) - \tilde{I}_{\text{ref}}(i))^2 \right\}, \quad \text{subject to } j \in (\theta_1, \theta_2). \quad (7)$$

The term $\tilde{I}_{\text{ref}}, d_{1,2,3}$ denotes a normalized reference signal, which has been calculated for particular values of d_1, d_2 and d_3 , while W is a vector of weighting coefficients that are equal to the variance of the experimental signal recorded without the presence of an optical fiber (*i.e.* the read noise of a detector). As \tilde{I}_{exp} may undergo small angular shifts due to experimental uncertainties, the solution is found among a series of \tilde{I}_{exp} shifted within the limits (θ_1, θ_2) .

Finally, d_1, d_2 and d_3 are retrieved by minimizing the global residual (7):

$$\arg \min_{d_{1,2,3}} r^2(d_{1,2,3}), \quad d_{1,2,3} \in H \subset \mathbb{R}^k \quad (8)$$

where H is a *search region* in a $k = 3$ -dimensional domain determined by constraints and limits on the variables to be retrieved.

4. Experimental setup

An optical setup for SLS measurements of optical fibers is presented in Fig. 3. The fiber runs vertically through the middle of a 3-axis measurement stage. An integrated controller provides a fine rotation 0–180° around the fiber's main axis together with its vertical transitions along

its length. A 14-bit CCD camera (Pixelfly, PCO AG) acquires weak optical signals and can be positioned at any angle $\theta = 0\text{--}180^\circ \pm 0.01^\circ$ with respect to the incident light. The illumination optics uses thermally stabilized red polychromatic light with the center wavelength of 630 nm and the spectral width (FWHM) of 14.5 nm. The polarization of the beam is adjusted parallel to the axis of the optical fiber. The beam is collimated, expanded, and filtered so that its cross section is about 7 mm^2 . The fiber is side-illuminated at a normal angle.

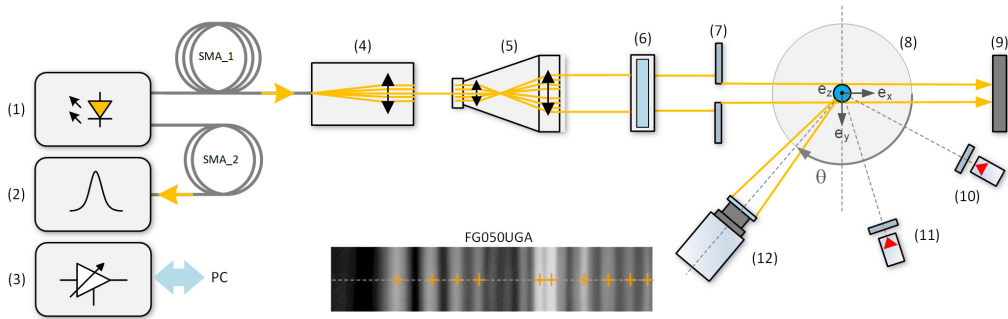


Fig. 3. Optical setup: (1) polychromatic illuminator, (2) spectrometer, (3) measurement platform controller, (4) fixed focus collimator, (5) beam expander, (6) linear polarizer, (7) spatial filter, (8) 3-axis measurement stage, (9) beam block, (10, 11) fiber alignment optics, (12) CCD 1392×1040 px camera.

In general, the fiber axis, O_f , does not coincide with the axis of rotation of a detector, O_d , as shown in Fig. 4. In particular, the CCD views at angle α_c rather than α_f which causes an image shift and affects the estimates. Fiber alignment optics uses two laser diodes and a CCD to determine the Δx_f and Δy_f misalignments by analyzing pairs of CCD images recorded at different camera viewing angles.

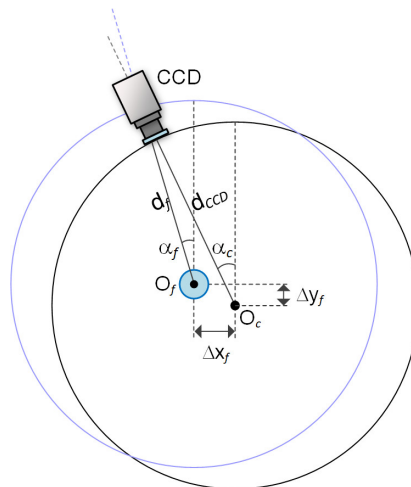


Fig. 4. An illustration of misalignment of the camera optical center (O_c) and the fiber axis (O_f).

A MATLAB-based software combines multiple images taken at successive angular positions. Each measurement is followed by a calibration procedure which removes unwanted data bias caused by the fiber misalignment effects. Then, the CCD capture is subjected to a simple denoising procedure by using an average over rows of the CCD image.

5. Results and discussion

For the purposes of inverse studies, a database of reference signals was prepared for various operating conditions: $d_3 = 120, 120.5, \dots, 130 \mu\text{m}$, $d_2 = 65, 65.5, \dots, 75 \mu\text{m}$, $d_1 = 45, 45.5, \dots, 55 \mu\text{m}$. The database contains $21^3 = 9261$ reference signals. Figure 5 compares the SLS measured signal (solid) with the reference signal (dotted line) estimated by the inversion algorithm. The plot shows scattering in the vicinity of so-called ‘primary rainbows’, caused by light that has been subjected to one internal reflection [29]. The fiber under test was roughly characterized by the vendor with an outer cladding diameter of $125 + 1/-2 \mu\text{m}$, the core diameter $50 \pm 1 \mu\text{m}$. The core was made of pure silica glass, characterized by a refractive index of $n = 1.457$ at 633 nm [30], while the cladding was fluorine doped to decrease the refractive index. The closest agreement

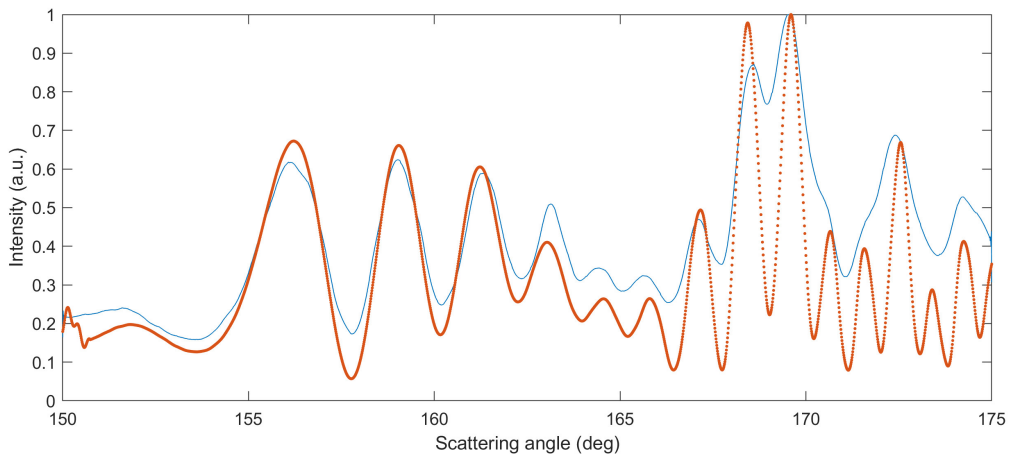


Fig. 5. SLS measured signal (solid) versus reference signal (dots) as estimated by the inversion algorithm; both normalized. The best match achieved for: $d_3 = 124.0 \mu\text{m}$, $d_2 = 70.0 \mu\text{m}$, $d_1 = 50.0 \mu\text{m}$. Experimental data at: 633 nm , $\text{FWHM} = 14.5 \text{ nm}$.

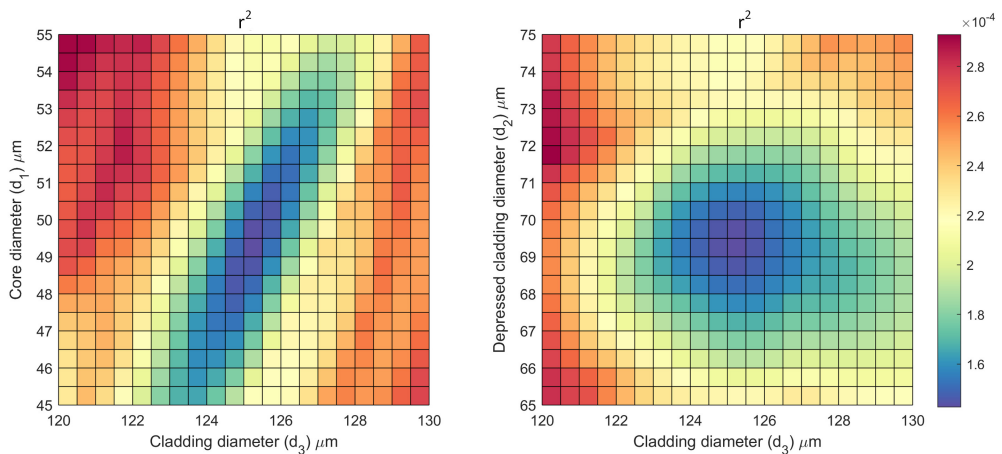


Fig. 6. Diagrams of the r^2 function where (left) the diameter d_2 of the depressed layer is fixed to $70.0 \mu\text{m}$ (left), and the core diameter d_1 is fixed to $50.0 \mu\text{m}$ (right).

between the measurement and the reference signals was achieved for $d_3 = 124.0 \mu\text{m}$, $d_2 = 70.0 \mu\text{m}$, and $d_1 = 50.0 \mu\text{m}$. Notably, there are slight deviations between amplitudes of compared signals as the assumed and real experimental conditions differ; however, the angular positions of the extremes are in close agreement, which confirms the SLS modeling assumptions from Section 2.

Figure 6 shows how the r^2 function evolves as the d_3 and d_1 vary while d_2 is kept constant (left panel) and d_3, d_2 vary, d_1 – constant (right panel). Note that both diagrams show well-defined local minima that correspond to the estimated diameter values. Here the ‘stability’ of the r^2 plots is due to the fact that, for polychromatic light beams, the scattered signal does not reveal high-frequency ripples or resonance effects, typical for coherent illumination [29]. The r^2 function is also pretty sensitive to measured diameters. This manifests the ability of the method to retrieve the diameters of interest with much better accuracy.

6. Conclusions and perspectives

Practical tests have demonstrated that the SLS technique supported by the data inversion algorithm based on the correlation between signals can be successfully applied to noninvasive measurements of step-index optical fibers. The method is able to retrieve internal diameters of fiber layers (core, inner- and outer cladding) and can be easily adopted to characterize step-index fibers of other complicated structures (e.g., straight core/cladding, trench-assisted, with nanostructure ring, *etc.*) or capillary tubes with or without fluid [31].

Practical measurements using polychromatic light are rather nonstandard as the vast majority of SLS-based techniques use laser beams. This improves the stability of the inverse problem (which relates the measurements back to the fiber properties of interest), since the SLS measurement data is detrimentally sensitive to small changes in fiber properties with monochromatic illumination [16,23]. Nevertheless, the detection of weak rainbow signals for polychromatic light sources is a challenge.

The method is rather abstractive in terms of precise quantitative understanding of what is happening in the scattered field; therefore, one cannot derive accurate formulas for converting measurements to fiber properties of interest. Deep studies of scattering in layered fibers show that a simple physical interpretation of scattering for such complicated cases is decidedly non-trivial [29].

The next steps will be focused on assessing measurement errors of the method by means of statistical considerations. The primary concern here will be deviations of the fiber shape from perfectly axial. A promising perspective for the present study is that some machine learning algorithms that are capable of reducing the numerical effort by using various parallelization schemes can be easily adopted for efficient signal correlation.

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