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Attempts to apply heuristic research methodology in mechanical engineering on the example of rotating machines

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Abstract: On any website or in encyclopaedias such as Britannica or Wikipedia, under the entry 'heuristics', one can find numerous definitions, references, and examples from various areas of life. However, the authors of this article have been unable to find examples relevant to technology, particularly in mechanical engineering. This fact inspired us to address this topic, especially since many concrete examples from practice and everyday life seem well-suited to demonstrating the relevance of heuristic methodologies in technical sciences. According to the authors, turbomachinery appears to be of particular interest in this context. This is critical machinery, i.e., machinery whose failure threatens human life. Hence the importance of developing advanced tools to analyse them, especially across the entire operating range (both stable and unstable). With these tools, one can effectively use their intellect, intuition, and common sense in the decision-making process. A classic heuristic symbiosis is thus formed. The paper demonstrates an advanced computer system called MESWIR, developed at the Institute of Fluid-Flow Machinery of the Polish Academy of Sciences in Gdańsk (IMP PAN) generates a range of interesting diagnostic information, including multiple whirls and stochastic errors related to the unbalance vector. The research was carried out using high-speed, low-power turbines as examples. The results obtained in this way, although they do not have formal theoretical proof of their correctness, allow the right conclusions to be drawn and decisions to be made, which is the essence of decision-making heuristics.

Key words: heuristic, stochastic error, multiple whirl, rotating machine

1. INTRODUCTORY REMARKS

For many years now, it has been a legitimate question whether it is possible to build research tools for analysing the state of various kinds of objects that are similar in essence to the recognition and judgment capacity of human reason and, more broadly, to the human mentality and psyche. This concerns human qualities such as intuition, common sense, and the ability to make quick decisions in ambiguous and difficult situations. This is a rhetorical question, as we should, of course, strive for such research tools, and it is certainly a matter of the future. Another issue is the scale of the difficulty in building them. After all, how can we algorithmise and model qualities such as common sense, sensitivity to beauty, and awareness of the dangers we face, and thus capture the attributes that constitute our humanity?

The above question does not mean that we should abandon such attempts; on the contrary, it represents our scientific, technical, and even civilisational challenge. The authors of this article will present an attempt to apply heuristic methodology in mechanical engineering. An example will be the assessment of the dynamic state of rotating systems, specifically critical machinery (i.e., machinery whose failure threatens life).

2. HEURISTICS AND ENTROPY

Heuristics and entropy, in very broad terms, are two of the most esoteric and mysterious terms. They have their interpretations not only in the exact sciences and technology but also in philosophy, sociology, theology, and other fields. There are different definitions of heuristics (see Fig. 1). There is no single, universal definition. But they all relate to the human environment and its surroundings and can be most generally described as the discovery of truth through hypotheses, particularly where formal proof or verification of these hypotheses is impossible.

Heuristic methods have been used to explain a variety of physical phenomena for several decades [1–4], but their application in technology has primarily been limited to solving optimisation tasks [5–11]. Only a few examples of the application of heuristics to predict the characteristics of mechanical systems [12,13] and technical diagnostics [14] can be found in the world literature. Potentially, modelling based on probabilistic neural networks [13] should allow for estimating risk and reliability. The authors believe that the considerable potential of heuristics in this area is still untapped, as will be demonstrated later in the article.

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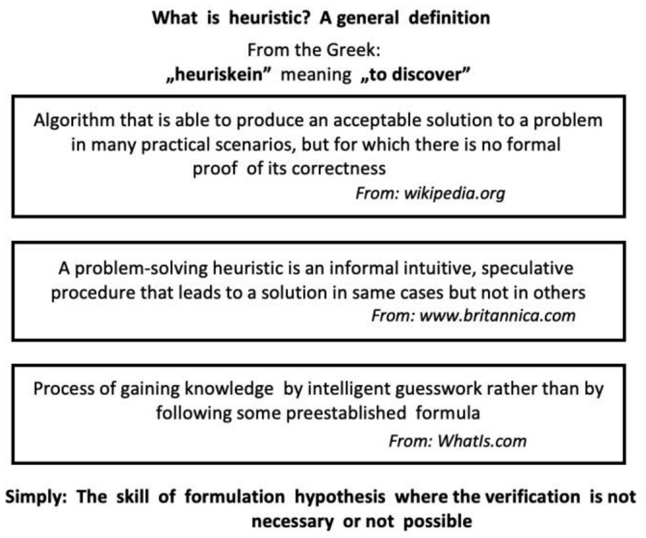


What is heuristic? A general definition

From the Greek:
„heuriskein" meaning „to discover"

Algorithm that is able to produce an acceptable solution to a problem in many practical scenarios, but for which there is no formal proof of its correctness
From: wikipedia.org

A problem-solving heuristic is an informal intuitive, speculative procedure that leads to a solution in same cases but not in others
From: www.britannica.com

Process of gaining knowledge by intelligent guesswork rather than by following some preestablished formula
From: WhatIs.com

Simply: The skill of formulation hypothesis where the verification is not necessary or not possible

Fig.1. There is no single, universally accepted definition of heuristics. The term is used in fields such as logic, artificial intelligence, computer science, and more

The definition of entropy, on the other hand, refers to the entirety of nature and even the universe, denoting the mysterious force that drives the world, nature, and people in one direction: from order to chaos, from simplicity to complexity, and ultimately from life to death. As with heuristics, there have been efforts for some time to apply entropy theory to the description of various physical phenomena, particularly in the fields of thermodynamics and astronomy [15–17]. One area where entropy theory has been increasingly applied in recent years is information theory [18–21]. Few publications attempt to use different mathematical definitions of entropy in the diagnostics of complex mechanical systems [22]. These proposals generally focus on the appropriate processing of diagnostic signals, which can aid in detecting defects [23] or assessing the technical condition of complex technical objects [24]. To date, however, research in this area has not utilised entropy theory to solve engineering problems, particularly in predicting the characteristics of complex mechanical systems.

It can, therefore, be concluded that these two concepts, heuristics and entropy, influence all our actions, although we are most often unaware of it. They thus determine the existence of our civilisation. Issues related to entropy in mechanical engineering and, in particular, in environmental protection engineering, will be elaborated on by the authors in a subsequent publication.

3. ROTOR DYNAMICS: TRADITIONAL VERSUS HEURISTIC MODELLING

Critical machinery, such as low- and high-power turbine sets, is particularly difficult to model theoretically, especially when it comes to describing their dynamic state. Often, this is high-speed machinery operating close to the stability limit and even, in the case of multi-support machinery, exceeding this limit. It is important to have the tools to assess the dynamic state of the machine in such situations, even if the assessment is approximate. In addition to this, there is the possible variability of the input data, such as random excitations or stochastic errors related, for example, to the constantly changing unbalance of the rotating masses.

The traditional approach, in which we always get the same set of results for one set of input data, although formally correct, will no longer meet our expectations. This is because we want to know what will happen to the machine when the stability limit is exceeded, or how the results will change when various types of random errors are taken into account. In these situations, we may not have proof of the mathematical correctness of the results, but they will help us form useful hypotheses about the state of the machine. This already meets the definition of heuristics (see Fig. 1). The differences between algorithmic and heuristic approaches are illustrated in Fig. 2.

Turbomachinery can, therefore, provide an excellent example of the application of heuristic methodology in mechanical engineering. Possible transitions from stable to unstable operation and random external excitations underscore this—see Fig. 3.

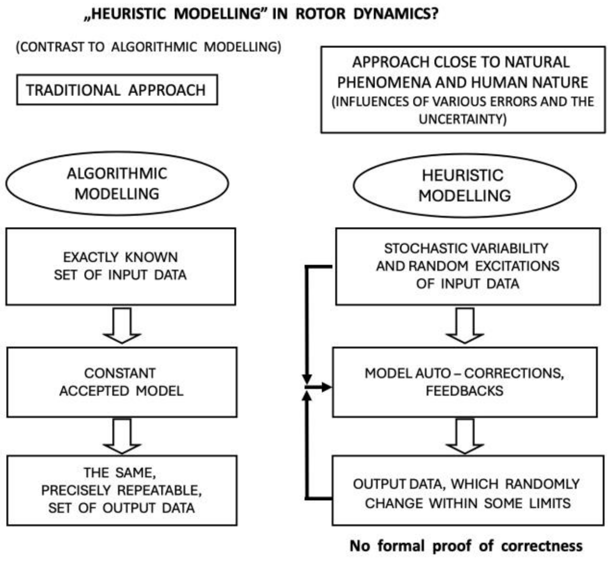


Fig.2. Differences between traditional (algorithmic) and heuristic approaches in rotor dynamics modelling

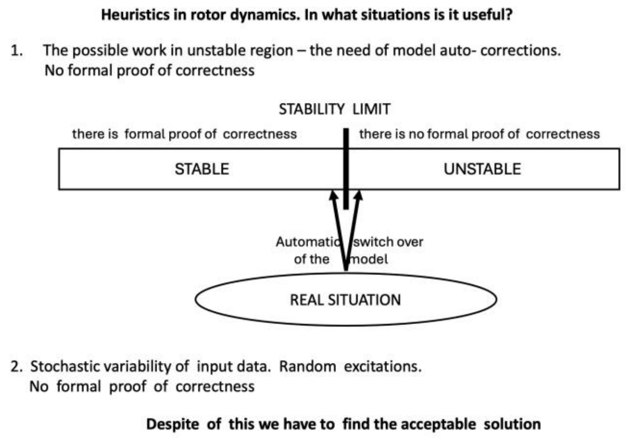


Fig.3. Situations where there is no evidence of formal correctness, but where probable and acceptable results are still desired

4. USEFUL MODELS, ALGORITHMS AND COMPUTER SYSTEMS FOR HEURISTICS

Evaluating the dynamic state of a fluid-flow machine near the stability limit, and especially after it has been exceeded, using one and the same model and computer system is essential if we want to use these tools to formulate various types of hypotheses suitable for heuristic methodology. As we will demonstrate below, only nonlinear models and systems that describe force-displacement relationships with the closest approximation possess these properties.

Figure 4 illustrates the essence of the differences between linear and nonlinear descriptions. This refers to the description of journal displacements in the lubrication gap caused by various external forces that vary over time. In the linear description, we assume not only small journal displacements but also constant stiffness ($c_{i,k}$) and damping ($d_{i,k}$) coefficients for the oil film at a selected position in the lubrication gap with coordinates ε and γ. As a result, the journal trajectory will be an ellipse, providing limited diagnostic information.

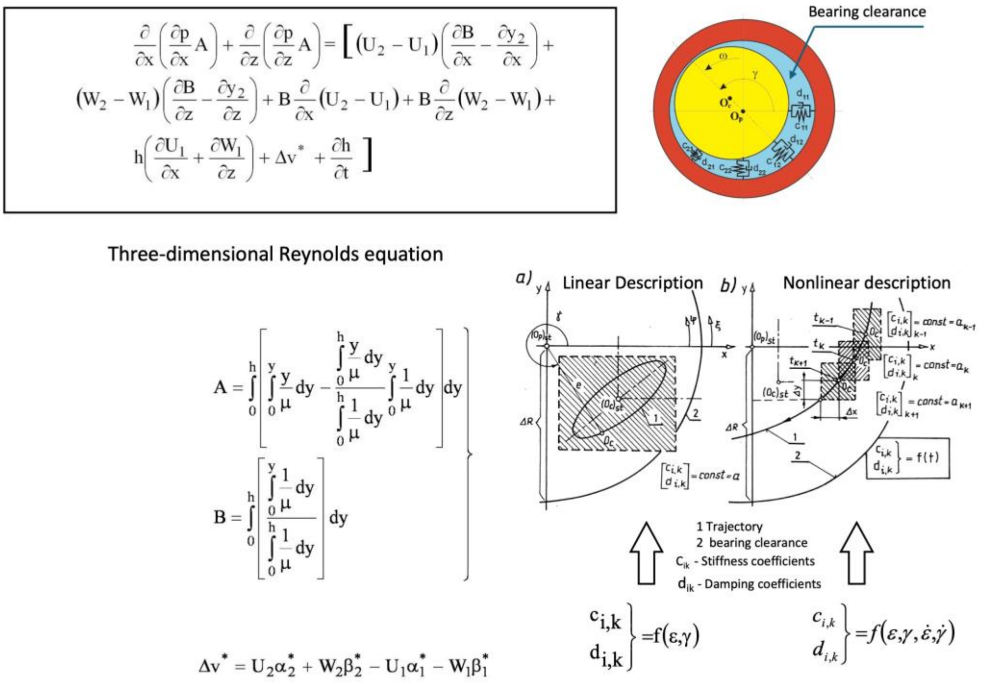
For larger displacements at each journal position (ε, γ), we need to recalculate the stiffness coefficients ($c_{i,k}$) and the damping coefficients ($d_{i,k}$). This means that these coefficients become functions of time, and the trajectory of the journal depends not only on its position but also on its instantaneous velocity (see Fig. 4). This represents a qualitative change, resulting in the journal trajectory no longer being an ellipse, but rather a curve that provides a wealth of diagnostic information. This means that the differential equations describing the dynamic states of the entire rotor-bearing-foundation system also become nonlinear.

Furthermore, incorporating the diathermic thermal model of the bearing, which accounts for heat exchange between the journal, lubricating film, and sleeve, results in a complex three-dimensional Reynolds equation (see Fig. 4).

Developing a suitable calculation algorithm and computer system that takes these assumptions into account is a challenging but achievable task. At IMP PAN in Gdańsk, such a system—called MESWIR—has been developed (see Fig. 5) [25,26]. Its most important feature is the ability to continuously describe the machine's state across its entire operating range (both stable and unstable), thereby providing motion trajectories that offer a wealth of diagnostic information. Of course, the literature on the subject includes many examples from other authors who aim to develop methods for the nonlinear description of turbomachinery [27–33].

Experimental verification is necessary for the developed computer tools to be deemed reliable. At IMP PAN in Gdańsk, such research has been conducted both on a test rig and on actual low- and high-power machinery [25,26]. This article will present only an excerpt from such research carried out on a large-size test rig (see Fig. 6). Of interest here is the operating range in which the system exceeds the stability limit and so-called oil whirls form. Such research can infrequently be carried out due to the size of the test rig and safety considerations.

For obvious reasons, such a complex model and computer system, as well as the results of its verification, cannot be described in detail in this article. We will use its properties only to analyse special cases for which it will be possible to make some hypotheses and draw approximate conclusions. This is already the aim of the present work.



$$\frac{\partial}{\partial x}\left(\frac{\partial p}{\partial x}A\right)+\frac{\partial}{\partial z}\left(\frac{\partial p}{\partial z}A\right)=\Big[(U_2-U_1)\left(\frac{\partial B}{\partial x}-\frac{\partial y_2}{\partial x}\right)+(W_2-W_1)\left(\frac{\partial B}{\partial z}-\frac{\partial y_2}{\partial z}\right)+B\frac{\partial}{\partial x}(U_2-U_1)+B\frac{\partial}{\partial z}(W_2-W_1)+h\left(\frac{\partial U_1}{\partial x}+\frac{\partial W_1}{\partial z}\right)+\Delta v^*+\frac{\partial h}{\partial t}\Big]$$

Three-dimensional Reynolds equation

$$\left.\begin{aligned}A&=\int_0^h\left[\int_0^y\frac{y}{\mu}dy-\frac{\int_0^h\frac{y}{\mu}dy}{\int_0^h\frac{1}{\mu}dy}\int_0^y\frac{1}{\mu}dy\right]dy\\B&=\int_0^h\left[\frac{\int_0^y\frac{1}{\mu}dy}{\int_0^h\frac{1}{\mu}dy}\right]dy\end{aligned}\right\}$$

$$\Delta v^*=U_2\alpha_2^*+W_2\beta_2^*-U_1\alpha_1^*-W_1\beta_1^*$$

$$\left.\begin{matrix}c_{i,k}\\d_{i,k}\end{matrix}\right\}=f(\varepsilon,\gamma) \qquad \left.\begin{matrix}c_{i,k}\\d_{i,k}\end{matrix}\right\}=f(\varepsilon,\gamma,\dot{\varepsilon},\dot{\gamma})$$

Fig.4. On the right: explanation of the essence of the linear and nonlinear descriptions of journal displacements in a bearing. On the left: three-dimensional Reynolds equation considering the nonlinear description as well as heat transfer in the lubrication gap and bearing bush [25].

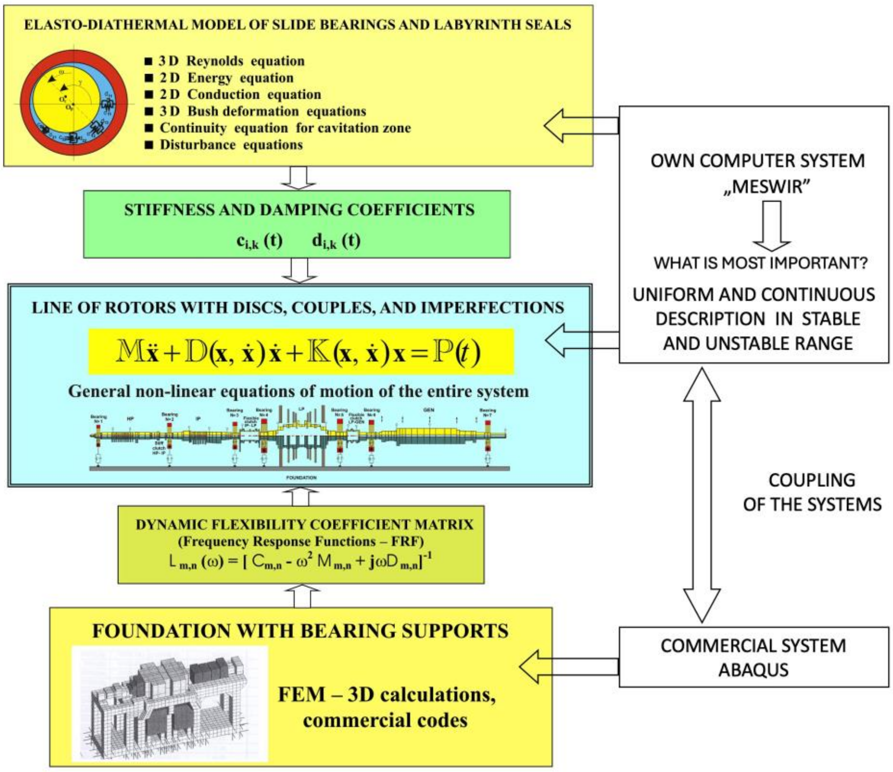


Fig.5. The MESWIR computer system, developed at IMP PAN in Gdańsk [25,26], has a key feature: it allows for a uniform and continuous description in both stable and unstable operating ranges, including strongly nonlinear ranges.

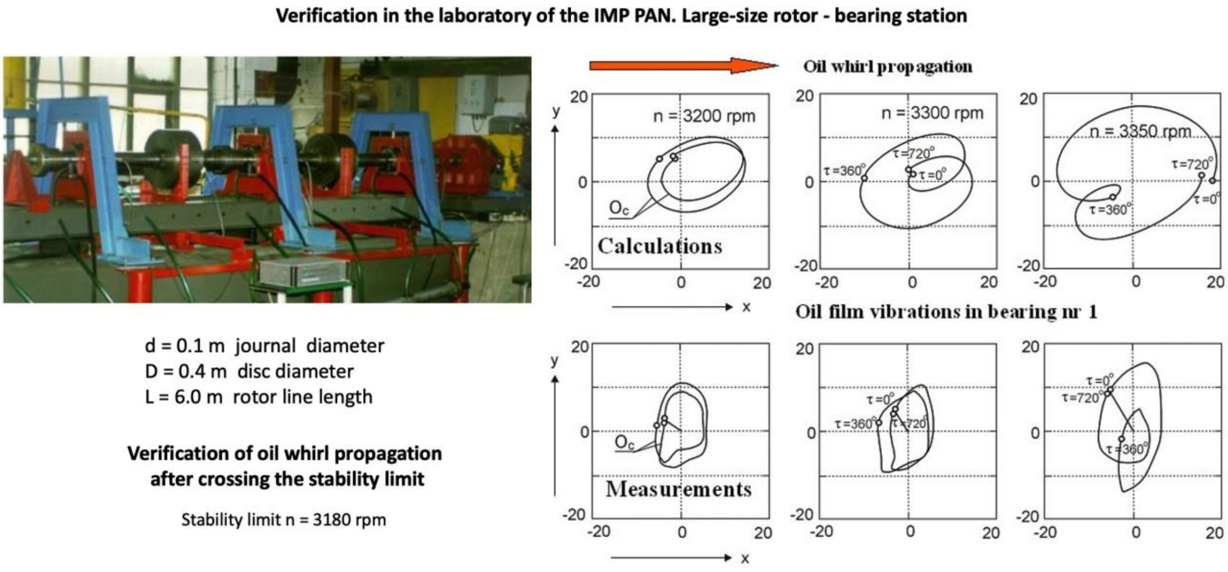


Fig.6. Verification of the MESWIR system on a large-scale test rig at IMP PAN. The scope of research immediately after crossing the stability limit—the onset of oil whirl.

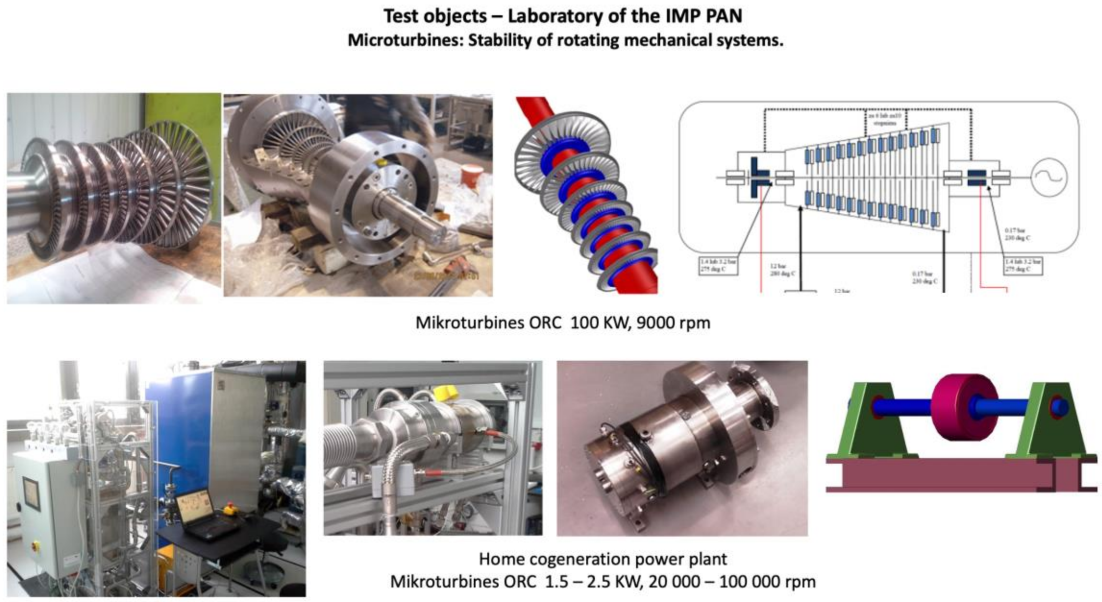


Fig.7. Objects of analysis: high-speed microturbines operating on the low-boiling agent ORC (designed and manufactured at IMP PAN). Purpose: use of waste heat in small domestic and industrial installations [34–37].

5. OBJECT OF RESEARCH

At IMP PAN in Gdańsk, work has been ongoing for many years on the development of a series of high-speed, low-power microturbines operating with organic Rankine cycle (ORC) low-boiling fluids [34–39]. Prototypes of such machinery, with power ranging from a few kilowatts to tens of kilowatts, have been designed and manufactured (see Fig. 7). The rationale for constructing such microturbines stems from the need to utilise waste heat in both small domestic installations and industrial ones. Therefore, research into expansion devices for ORC systems is also being conducted at other research centres around the world [40–42].

Small turbomachinery, due to their high rotational speeds, often operate near the stability limit or even exceed it. They are, therefore, a good example for the analyses presented in Fig. 3, i.e., analyses in which heuristic methodology can be useful.

6. RESEARCH FINDINGS

The subject of the study will be the interesting phenomenon of multiple whirls occurring in the lubrication gap of slide bearings after the hydrodynamic stability limit has been exceeded, as well as input data errors caused by random changes in the rotor's unbalance force during its rotation. These are, of course, just two selected examples that can serve as good illustrations of the use of computer system capabilities to facilitate hypothesis formulation and decision-making in difficult and ambiguous situations that can occur during machine operation.

6.1. Multiple whirls

It turns out that the initial preclamping of the bearing sleeves and their thermoelastic deformations [43–46] can cause the fluid-flow machine to return to stable operating conditions, even after the hydrodynamic stability limit has been exceeded.

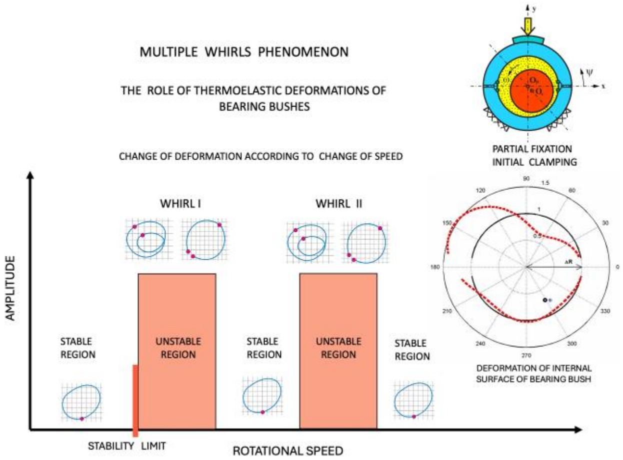


Fig.8. Explanation of the phenomenon of multiple whirls. A return to stable operating conditions despite an increase in rotor speed is possible due to the combined effects of thermoelastic deformation and initial clamping of the bearing bush.

According to vibration theory, such a situation is not possible for invariant geometrical data of the mechanical system under analysis and fixed, preset other input data. In such situations, a rotating mechanical system, once it has crossed the hydrodynamic stability limit, will not return to stable operating conditions on its own. All the more surprising and interesting, therefore, are the calculation results which, by taking into account the change in the shape of the lubrication gap during the operation of the machine, show that returns to stable operating conditions are possible. This is related to the so-called multiple-whirl phenomenon. The essence of this phenomenon is explained in Fig. 8, with an example of calculations on a real fluid-flow machine shown in Fig. 9.

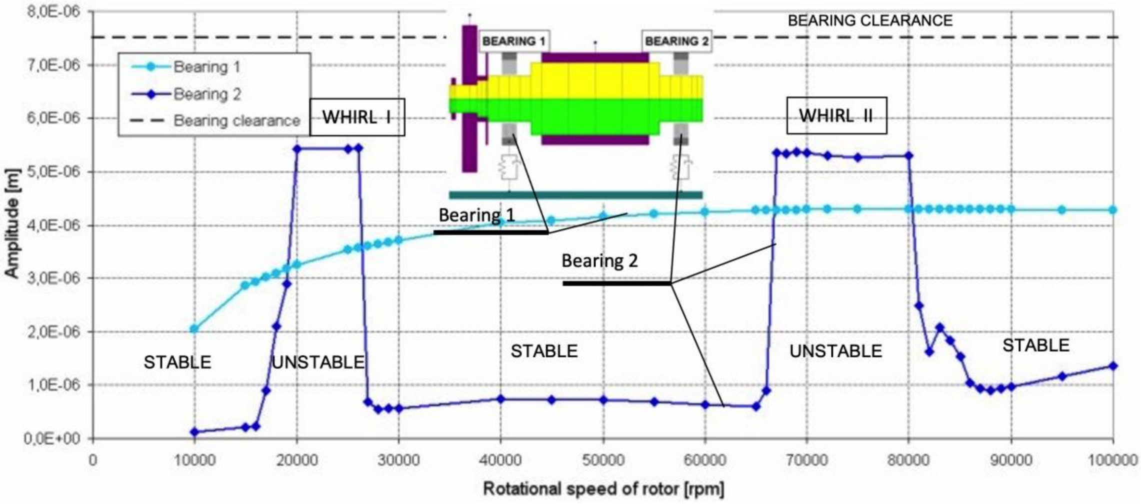


Fig.9. Results of calculations using the MESWIR system for a two-support microturbine with a power of 2.5 kW, bearing journal diameter d = 0.01 m, and rotational speed up to 100,000 rpm (Fig. 7). The phenomenon of multiple whirls is observed only in bearing No. 2.

The calculation results presented were obtained for a microturbine rotor with two hydrodynamic bearings, each with a diameter and length of 10 mm and a radial clearance of 0.0075 mm. The bearings were lubricated with a low-viscosity liquid. The outer diameter of the generator rotor was 20 mm, and its length was 50 mm. The total length of the microturbine rotor was approximately 120 mm.

It is interesting to note that bearing No. 1 of this analysed microturbine remains stable across the entire speed range, while the phenomenon of multiple whirls occurs only in bearing No. 2 (see Fig. 9). Such results could not have been predicted without the use of advanced computer systems. However, fundamental questions of a more general nature, relevant to the purpose of this work, arise here:

- Does the loss of stability in one component of a system always mean that the entire machine must be taken out of service? The answer is: No, it does not.
- Can the results obtained from computer simulations be proven theoretically and thus be considered absolutely reliable? The answer is: No, they cannot be proven.

Therefore, the decision on whether to permit a particular machine to be used must be made by a person based on their intuition, common sense, knowledge, and experience. Man-made research tools, such as advanced computer systems in this case, merely facilitate this decision. The symbiosis of research tools, human intellect, and resulting decisions is essentially what constitutes decision-making heuristics.

6.2. Random variations in rotor unbalance forces

Let us now assume that the vector determining the unbalance force of the machine's rotor, once defined, is not constant but undergoes random variations due to various types of errors, displacement of rotating parts, changes in operating conditions, contamination, etc. These are common occurrences in practice [47–50]. Similar problems in defining the exact operating parameters of a fluid-flow machine can arise when uncertainties occur in certain situations [51] or when multiple faults occur simultaneously [52]. Tools developed to analyse the dynamics of fluid-flow machinery should take this kind of situation into account, which is extremely difficult using a classical modelling approach.

Let us consider the case where the unbalance vector of the rotor centre varies stochastically during one revolution. Let these changes be within ±ΔP, and let their course be determined by a random number generator (see Fig. 10). We will use the previously mentioned MESWIR system for the analysis (see Fig. 5).

The results of the calculations are shown in Figs. 11 and 12. It is assumed here that the possible range of random variation is within ΔP = ±20% of the value of the unbalance force vector.

The calculations without the random number generator, and therefore separately for +ΔP and −ΔP, show the two appropriate trajectories (in this case the boundary ellipses), as shown in Fig. 10 (left side). Taking a random number generator into account fundamentally changes the situation (see Fig. 10, right side).

It is interesting but also dangerous to observe that the rotor vibrations now extend beyond the area of the two boundary ellipses. This is, of course, the effect of the inertial forces of the rotating masses (see Fig. 10, right side). Further interesting results are presented in Figs. 11 and 12. The effect of random changes in the unbalance vector under stable machine operating conditions (Fig. 11) and after exceeding the hydrodynamic stability limit (Fig. 12) can be seen here.

The impact of stochastic changes in the unbalance vector is much more apparent within the machine's stable operating range than beyond the hydrodynamic stability limit. This is a surprising, yet explainable, result. Oil whirls and whips already generate such large vibrations that the inertia of the rotating masses of the entire rotor nullifies the effect of finer changes caused by variations in the unbalance vector.

As in the case of multiple whirls (Section 6.1), let us pose further questions:

- Is it always the case that errors caused by rotor unbalance increase the risk of failure once the hydrodynamic stability limit has been exceeded? The answer is: No, it is not.
- Can stochastic changes in the unbalance vector cause variations beyond the region of boundary trajectories calculated without using a random number generator? The answer is: Yes, they can.

And, as before (Section 6.1), although the advanced computer system has helped us obtain so much important and interesting information, the final decision on whether to allow the machine to be used must be made by a human being.

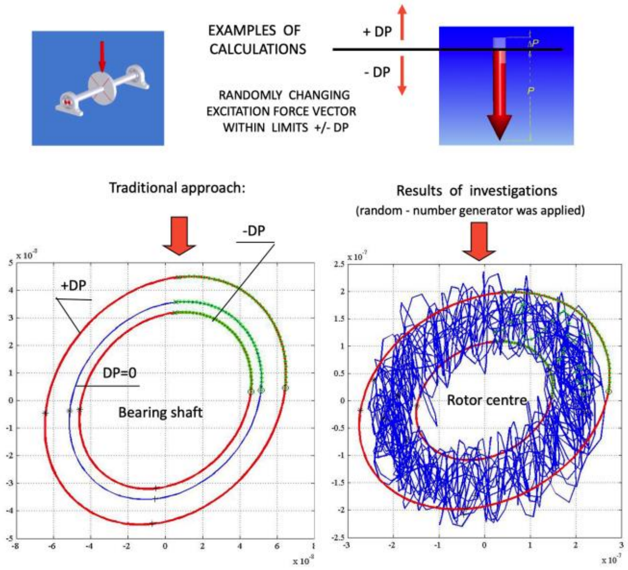


Fig.10. Examples of calculations using the MESWIR system with a random number generator to illustrate random changes in the unbalance vector. Object of research: microturbine with a power of 2.5 kW and a shaft diameter of d = 0.01 m (Fig. 7).

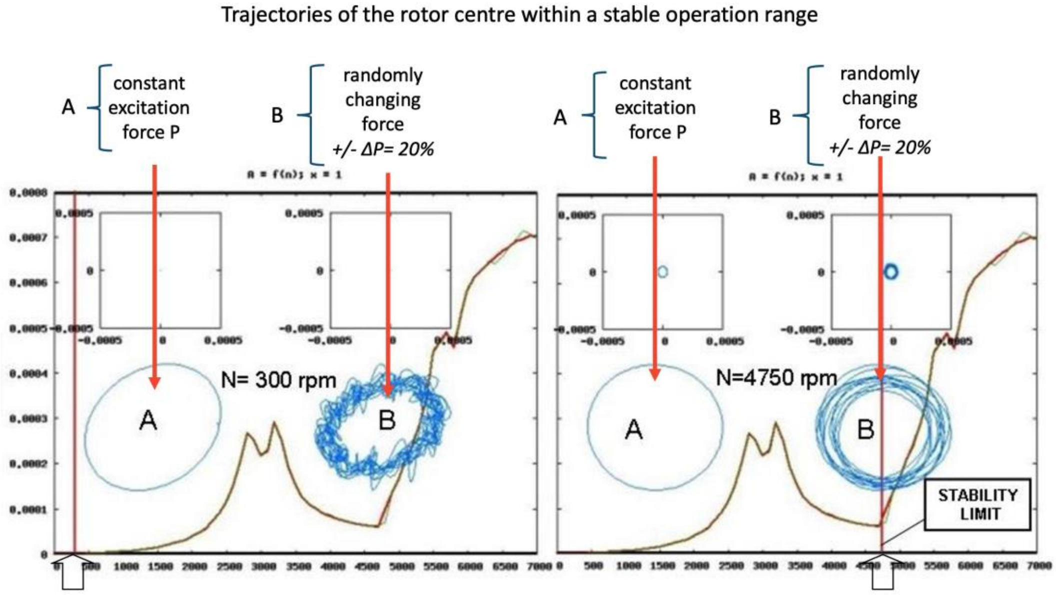


Fig.11. Trajectories of the rotor centre in a stable operating range: A - constant of the unbalance vector, B - stochastically variable unbalance vector

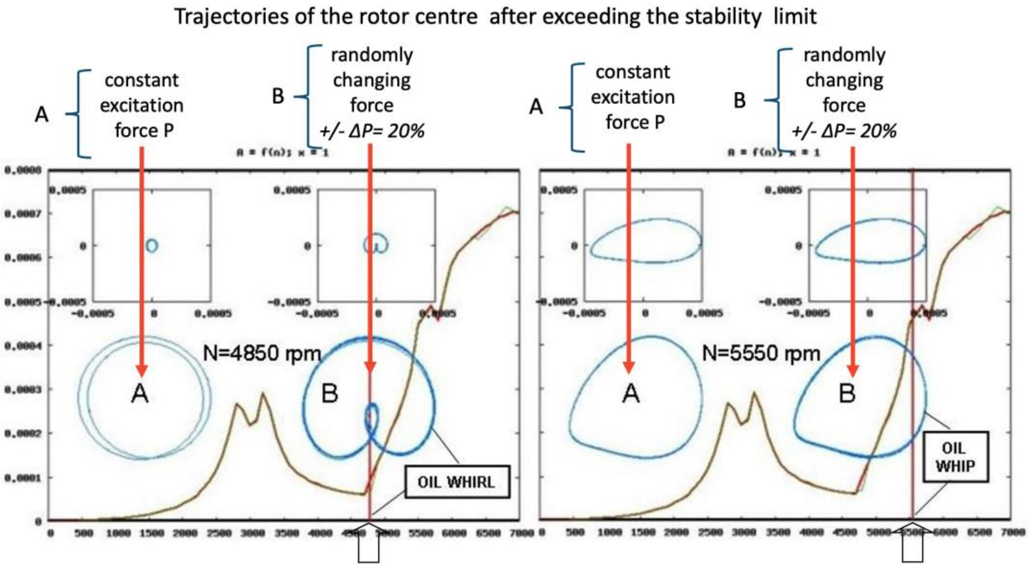


Fig.12. Trajectories of the rotor centre in the unstable operating range: A – constant unbalance vector, B – stochastically variable unbalance vector

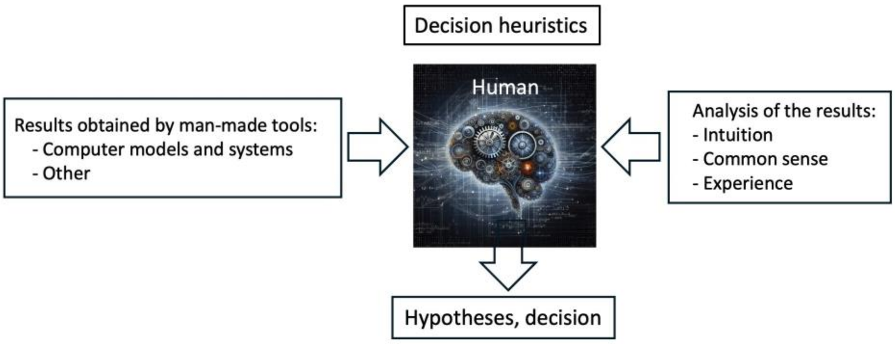


Fig.13. Decision heuristics: Only a human can formulate hypotheses and make decisions in difficult and ambiguous situations. Advanced tools can only facilitate the process.

7. CONCLUSIONS

The tools that humans have created make it significantly easier, and sometimes even possible, to make difficult and responsible decisions. For such decisions, there is often no formal proof, yet we still expect certain hypotheses and decisions to be made.

The examples provided in this article make this perfectly clear. The results generated by the computer system are both interesting and surprising. Their interpretation, however, requires a great deal of experience and knowledge, but above all, qualities that only humans possess, namely intuition and common sense. The article does not specify the decisions ultimately made for specific fluid-flow machinery. This was not the purpose of the work. The authors of the article wanted only to draw attention to the relationships between humans, tools, and decision-making.

It is about the symbiosis of man-made tools, human intellect, and brain capabilities, and ultimately the process of formulating hypotheses and making decisions (see Fig. 13). Such a process can be referred to as decision-making heuristics.

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