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**Research paper**

# Random shear resistance of a headed-stud connector in composite steel-concrete beam

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**Abstract:** In the traditional, standard, computational approach, the shear resistance of a single headed-stud connector, ensuring the composite connection between a steel beam and a reinforced concrete slab resting on this beam, is determined by comparing the load capacity $P_{Rs}$ – determined by the destruction of the steel connector itself, and the load capacity $P_{Rc}$ – conditioned by the destruction of the concrete surrounding this connector. In a single implementation, the smaller of these both values, i.e. $P_R = \min(P_{Rs}, P_{Rc})$, is authoritative for the designer. If, however, both combined strengths are treated as the random variables and a statistically homogeneous sample grouping potentially possible implementations of this type is taken into account, then the design resistance $P_{R,d} = [\min(P_{Rs}, P_{Rc})]_d$, representative for the verification of ultimate limit state for the considered connection, will be quantitatively different from the value $P_{R,d} = \min\left(P_{Rs,d}, P_{Rc,d}\right)$ recommended for use in this regard in the standard EN 1994-1-1. In this paper a detailed algorithm for the correct specification of this value is presented in detail. The dependence of such value on the mutual relationship between the coefficients determining the statistical variability of the strength of the connector steel as well as the strength of the concrete from which the floor slab was made is also demonstrated. The proposed approach is based on the fully probabilistic design format, according to which the appropriate level of the probability of reliable work of the analyzed connection is ensured. The presented considerations are illustrated with a numerical example. On its basis, the degree of simplification of such evaluation is estimated, as well as its consequences, resulting from the use of a conventional standard model in this respect.

**Keywords:** design value, headed-stud connector, probability-based design, shear resistance, steel-concrete composite connection

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# 1. Introduction

The shear connectors, named also the shear stud concrete anchors [1], are usually used in composite steel–concrete beams to tie the steel members to reinforced concrete slab resting on these members and, due to that, resist shear forces generated between both interacting components. The shear flow inducing between these components, adjacent to each other, is a natural consequence of the requirement for composite action. If there were no connectors in this location, the beam and the slab would bend separately as illustrated schematically in Fig. 1a. The presence of shear connectors here prevents the slip between the considered materials and achieves a much stiffer and significantly stronger beam (Figs. 1b–c).

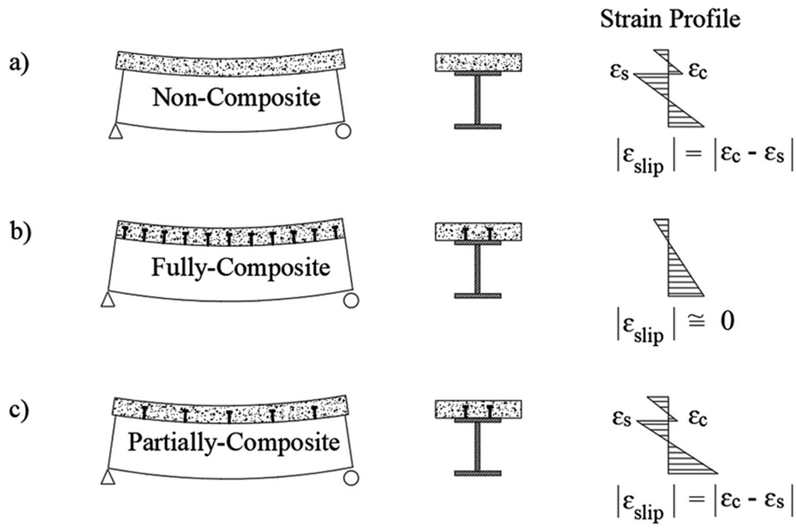


Fig. 1. Full and partial composite connection of a steel beam with a reinforced concrete slab

In this paper a selected type of such shear connector, shaped as the headed-stud one, is subjected to detailed analysis (Fig. 2) [2].

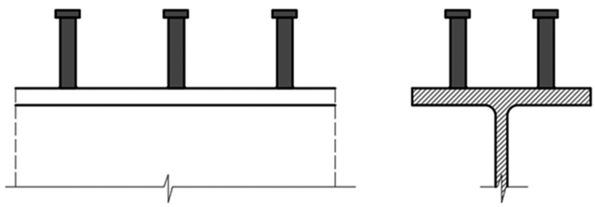

Fig. 2. The typical shear headed-stud connector welded to the upper flange of a steel beam

The shearing force acting to such connector is usually not introduced through its base but is subjected directly onto its shank. The increased load of this type produces concrete crushing in front of this connector and, due to that, the shear of its steel shaft is transferred exclusively via bending. As a result, the connection shear resistance can be reached either due to the failure of concrete or due to the failure of the steel connector above the weld (Fig. 3).

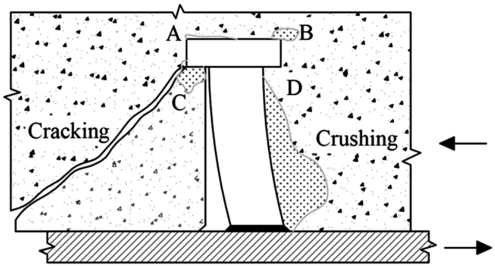


Fig. 3. Mechanism of the shear force transferring through the shank of a headed-stud connector. The letters A, B, C and D mark concrete slab zones identified as critical for the composite connection carrying capacity exhaustion

An analogous mechanism of composite connection failure, typical for reinforced concrete slabs with profiled steel sheeting, supported on the steel floor beams, is shown in detail in Fig. 4. This case frequently leads to the concrete pull-out failure (Fig. 5).

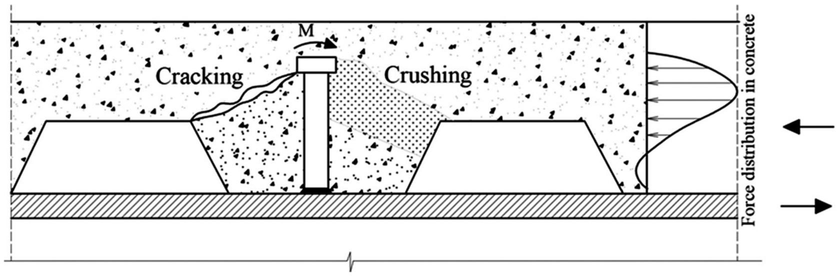


Fig. 4. Scheme of the load capacity exhaustion for a composite connection in the case of a reinforced concrete floor slab with profiled steel sheeting

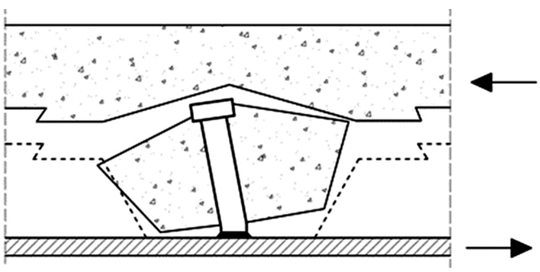

Fig. 5. The concrete pull-out failure in composite floor slab with profiled steel sheeting

The structural behaviour of headed-stud connectors used in typical steel-concrete composite beams has been recently the subject of many detailed analyses published in professional literature [3–31]. Various formal models were developed in this regard, including in particular:

the model of a stud being a steel bending member with constrained boundary conditions, resting on an elastic foundation [7] (Fig. 6), as well as the alternative model of the stud rigid-plastic failure, with the generation of one or maximum two plastic hinges in its shaft (Fig. 7) [22].

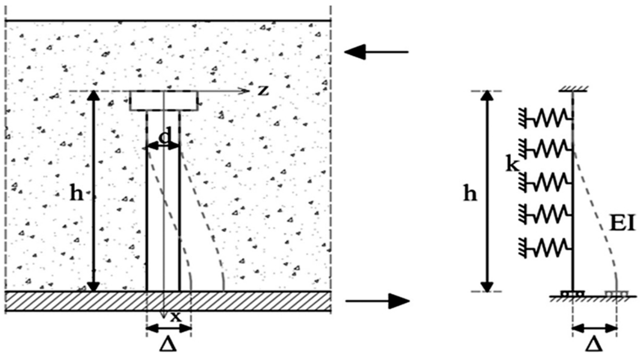


Fig. 6. The headed-stud connector modelled as a steel bending member resting on an elastic foundation (according to [7])

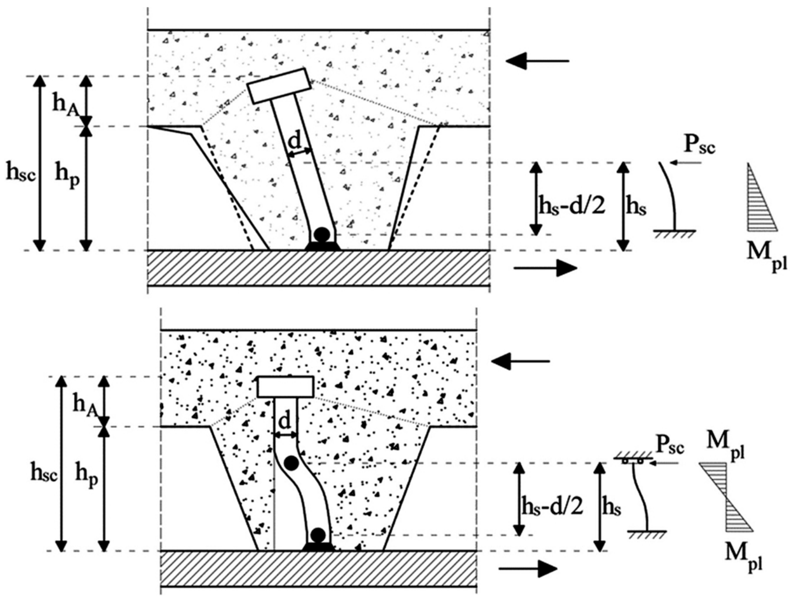


Fig. 7. The rigid-plastic failure of a headed-stud connector with one (upper Figure) or with two (bottom Figure) plastic hinges generated in its shaft (according to [22])

In the traditional design approach, recommended to practical use in this regard in the standard EN 1994-1-1 [32], if the considered connector is placed in a concrete solid slab, then the design value of a shear stud resistance $P_{R,d}$ may be determined as the lesser of two design values, $P_{Rs,d}$ and $P_{Rc,d}$ respectively. These values are specified there by two separate equations, one of which represents the failure of the stud itself (so called "steel failure") while the other – failure of the surrounding concrete. Consequently, the following occurs:

$$P_{R,d} = \min\left(P_{Rs,d}, P_{Rc,d}\right) \tag{1.1}$$

where, in particular:

$$P_{Rs,d} = \frac{P_{Rs,k}}{\gamma_\upsilon} = 0.8 f_u \frac{\pi d^2}{4} \frac{1}{\gamma_\upsilon} \tag{1.2}$$

and also:

$$P_{Rc,d} = \frac{P_{Rc,k}}{\gamma_\upsilon} = 0.29\alpha d^2 \sqrt{f_{ck} E_{cm}} \frac{1}{\gamma_\upsilon} \tag{1.3}$$

In the above mentioned formulae the quantities $P_{Rs,k}$ and $P_{Rc,k}$ are the characteristic values of the random resistances $P_{Rs}$ and $P_{Rc}$, respectively. This means that the coefficient $\gamma_\upsilon = 1.25$ can be interpreted here as a partial safety factor specified for the considered connection. Furthermore, $f_u$ [Mpa] – is the ultimate tensile strength of the steel from which the connector was made, $f_{ck}$ [Mpa] – is the characteristic "cylinder" compression strength of the surrounding concrete, $E_{cm}$ [GPa] – is the secant modulus of elasticity of such the concrete, $d$ [mm] – is the diameter of the shank of a considered connector, $\alpha$ – is the coefficient taking into account the effective slenderness of such the connector. In further analysis it is assumed that $\frac{h_{sc}}{d} > 4$, where $h_{sc}$ [mm] is the stud length measured after welding. This allows to set that $\alpha = 1.0$.

The basic advantage of such computational model is its simplicity, however, in the opinion of the authors, it is not fully correct formally, due to the applied rules of mathematical inference. For this reason, a novel, alternative procedure, recommended by us to specify the sought design value of a headed-stud connector shear resistance in more accurate way [33–35], is presented and widely discussed in this paper. It is based on a fully probabilistic design format.

## 2. The new random variable defined as a minimum of two independent random variables

In the presented analysis both the resistance $P_{Rs}$ and the resistance $P_{Rc}$, compared with each other when specifying the conclusive load capacity of the considered composite connection, are interpreted as two statistically independent random variables. This interpretation seems to be justified because not only each of these quantities depends on different factors but also the failure modes corresponding to each of them can be analysed as formally separate cases. The small correlation between both random variables considered above, being a consequence of their common dependence on the stud diameter, is neglected in further considerations.

When the only one specific composite connection is taken into account, with unambiguously established basic mechanical characteristics of the materials used, measured in situ, then a conclusive, deterministic, value of the headed stud shear resistance $P_R$, determined exclusively for this connection, is usually calculated as a smaller value from the pair of numbers, $P_{Rs}$ and $P_{Rc}$, computed in advance. This means that:

$$P_R = \min(P_{Rs}, P_{Rc}) \tag{2.1}$$

Let $P_{Rs} = X$ and $P_{Rc} = Y$. Then:

$$P_R = Z = \min(X, Y) \tag{2.2}$$

Let us now consider, instead of a single well-defined deterministic implementation, a statistically homogeneous sample grouping all potentially possible random implementations of this type. In this approach, the quantity $Z$ must be interpreted as the new random variable, constituting a minimum of two other random variables, $P_{Rs}$ and $P_{Rc}$, respectively.

To identify a cumulative distribution function (*cdf*) specified for such new random variable $Z$, marked in further calculations by the symbol $F_Z(z)$, it is necessary to integrate the joint probability density function (*pdf*), continuous by assumption and specified jointly for random variables $X$ and $Y$. The integration limits are in this case limited to the area in which the minimum $x$ and $y$ is smaller than $z$ [36]. This is also a complementary area to the area in which both $x$ and $y$ are greater than $z$ (Fig. 8). Hence:

$$\begin{aligned} F_Z(z) &= P(Z \le z) = P[\min(X,Y) \le z] = \\ &1 - P[\min(X,Y) > z] = 1 - \int_z^\infty \int_z^\infty f_{XY}(x,y)\mathrm{d}x\mathrm{d}y \end{aligned} \tag{2.3}$$

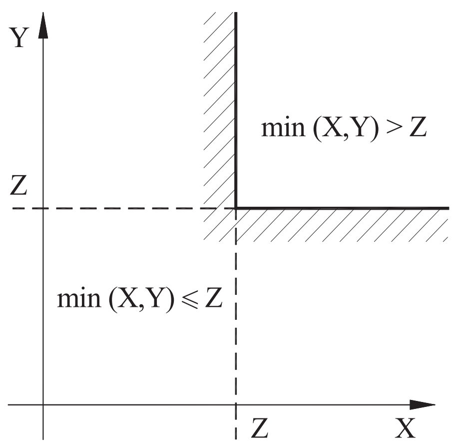


Fig. 8. The integration area used to determine the *cdf* function specified for the random variable $Z$

Thus, an appropriate *pdf* function, marked by the symbol $f_Z(z)$, may be specified by the formula:

$$f_Z(z) = \frac{\mathrm{d}}{\mathrm{d}z} F_Z(z) = f_X(z) + f_Y(z) - f_X(z)F_Y(z) - f_Y(z)F_X(z) \tag{2.4}$$

If the form of a continuous function $f_Z(z)$ is known in advance then two basic probabilistic moments of a random variable $Z$ may be calculated in conventional way. These are as follows:

– a mean value $\mu_Z = E(Z)$ as the first raw moment:

$$\mu_Z = \int_{-\infty}^{\infty} z f_Z(z) \mathrm{d}z \tag{2.5}$$

– a variance $\sigma_Z^2 = var(Z)$ as the second central moment:

$$\sigma_Z^2 = \int_{-\infty}^{\infty} f_Z(z) \left(z - \mu_Z^2\right) \mathrm{d}z \tag{2.6}$$

# 3. Characteristics of a random variable $Z$ assuming that it is described by log-normal probability distribution

As it has been mentioned previously, the random variable $Z$ is in these considerations the measure of a headed-stud shear resistance. For this reason, the log-normal probability distribution is assumed here for its description. This assumption seems to be well-justified because such distribution is specified only for $z \geq 0$ (i.e. in the range $0 \leq z < \infty$). This also means that the random variable $\ln Z$ is characterised by the normal probability distribution which is described in the range $-\infty < \ln Z < \infty$. According to such specification the following occurs:

$$f_Z(z) = \frac{1}{\sqrt{2\pi} z \sigma_{\ln Z}} \exp\left\{-\frac{[\ln(z) - \mu_{\ln Z}]^2}{2\sigma_{\ln Z}^2}\right\} \tag{3.1}$$

Because it is true that:

$$\mu_{\ln Z} = \ln \breve{\mu}_Z \tag{3.2}$$

where $\breve{\mu}_Z$ is a median value of the random variable $Z$, then simultaneously:

$$\breve{\mu}_Z = \exp(\mu_{\ln Z}) \tag{3.3}$$

This value is quantitatively different than the analogous mean value $\mu_Z = E(Z)$ calculated from the formula:

$$\mu_Z = \exp\left(\mu_{\ln Z} + \frac{\sigma_{\ln Z}^2}{2}\right) \tag{3.4}$$

Moreover, a variance $\sigma_Z^2 = var(Z)$ is equal to:

$$\sigma_Z^2 = \left[\exp\left(\sigma_{\ln Z}^2\right) - 1\right] \mu_Z^2 = \left[\exp\left(\sigma_{\ln Z}^2\right) - 1\right] \cdot \exp\left(2\mu_{\ln Z} + \sigma_{\ln Z}^2\right) \tag{3.5}$$

which means that:

$$\sigma_{\ln Z} = \sqrt{\ln\left(\frac{\sigma_Z^2}{\mu_Z^2} + 1\right)} \tag{3.6}$$

This allows to calculate the appropriate standard deviation:

$$\sigma_Z = \sqrt{var\,(Z)} = \mu_Z\sqrt{\exp\left(\sigma_{\ln Z}^2\right) - 1} \tag{3.7}$$

and also the corresponding coefficient of variation:

$$v_Z = \frac{\sigma_Z}{\mu_Z} = \sqrt{\exp\left(\sigma_{\ln Z}^2\right) - 1} \tag{3.8}$$

# 4. Specification of the representative values for the random headed-stud connector shear resistance

A conventional safety condition specified for the log-normally standardized random variable $\ln\left(\frac{\breve{Z}}{z}\right) = \ln\left(\frac{\breve{\mu}_Z}{z}\right)$ is usually given in the following form:

$$\beta_R = \frac{\ln\left(\breve{Z}/z\right)}{v_Z} \geq \beta_{R,\mathrm{req}} = \alpha_R\beta_{\mathrm{req}} \tag{4.1}$$

In this formula $\beta_R$ is the partial reliability index specified for a considered headed-stud connector shear resistance while $\beta_{R,\mathrm{req}}$ means the target value of such the index, setting the required safety level depending on the acceptable failure probability. Thus the failure in our analysis is interpreted as the random event corresponding to the situation when the calculated value of shear resistance, related to the specific connector considered in the analysis, turns out to be smaller than the design value of such resistance, determined independently as a proper quantile of the log-normal probability distribution describing this random variable. This design value is then in such an approach the smallest value of the random shear resistance being possible to accept due to the ensured safety level. Obviously, index $\beta_{R,\mathrm{req}}$ is here only a part of a conventional global reliability index $\beta_{\mathrm{req}}$ commonly used to verify the global safety condition type $E_d \leq Z_d = P_{R,d}$ (symbol $E_d$ denotes in this case the design value of a conclusive, most unfavourable, action effect related to the combination of the loads applied to the considered steel connector). According to the standard EN 1990 [37], for the ordinary safety requirements, it is usually assumed that $\beta_{\mathrm{req}} = 3.8$, which is associated with the acceptable failure probability set at the level $p_{\mathrm{f,ult}} \approx 7.2 \cdot 10^{-5}$. Moreover, using the computational format recommended in this code a fixed value $\alpha_R = 0.8$ can be assumed in the analysis. This leads to the specification that $\beta_{R,\mathrm{req}} = \alpha_R\beta_{\mathrm{req}} = 0.8 \cdot 3.8 = 3.04$.

Condition (4.1) is then fully equivalent to the formula:

$$P\left(Z < Z_d\right) \leq p_{\mathrm{f,ult}} = \Phi\left(-\beta_{R,\mathrm{req}}\right) = \Phi\left(-\alpha_R\beta_{\mathrm{req}}\right) \tag{4.2}$$

In which the symbol $\Phi(.)$ means the *cdf* function of a standardized normal probability distribution. In other words, it is a well-known Laplace function with values compiled in the conventional statistical tables. If it is accepted that $\beta_{R,\text{req}} = 3.04$ then, based on (4.2), the maximum acceptable value of a failure probability is set at the level $p_{\text{f,ult}} \approx 1.18 \cdot 10^{-3}$.

In a situation when the ultimate limit state is reached the equality $Z = Z_d$ has taken place in the formula (4.2). This allows to transform the formula (4.1) to a form:

$$\beta_R = \frac{\ln\left(\breve{Z}/Z_d\right)}{\nu_Z} = \beta_{R,\text{req}} = \alpha_R \beta_{\text{req}} \tag{4.3}$$

which leads to the following specification:

$$Z_d = \breve{Z} \exp\left(-\alpha_R \beta_{\text{req}} \nu_Z\right) = \breve{Z} \exp\left(-3.04 \nu_Z\right) \tag{4.4}$$

Based on the equations (3.3) and (3.4) it can be written that:

$$\breve{Z} = \breve{\mu}_Z = \exp\left(\ln \mu_Z - \frac{\sigma^2_{\ln Z}}{2}\right) = \frac{\mu_Z}{\exp\left(\frac{\sigma^2_{\ln Z}}{2}\right)} = \frac{E(Z)}{\exp\left(\frac{\sigma^2_{\ln Z}}{2}\right)} \tag{4.5}$$

This allows to describe the formula (4.4) in an alternative way:

$$Z_d = \mu_Z \exp\left(-\alpha_R \beta_{\text{req}} \nu_Z - \frac{\sigma^2_{\ln Z}}{2}\right) = E(Z) \exp\left(-3.04 \nu_Z - \frac{\sigma^2_{\ln Z}}{2}\right) \tag{4.6}$$

The representative, characteristic value of a random headed-stud connector shear resistance is recommended here to be determined in a conventional way, as a 95% quantile of the log-normal *pdf* function $f_Z(z)$. This leads to the formula:

$$Z_k = \breve{Z} \exp\left(-1.645 \nu_Z\right) = E(Z) \exp\left(-1.645 \nu_Z - \frac{\sigma^2_{\ln Z}}{2}\right) \tag{4.7}$$

Taking into account the representative values $Z_d$ and $Z_k$, calculated due to the application of the formulae (4.6) and (4.7), respectively, it is possible to determine a minimum value of the partial safety factor $\gamma_{\upsilon,\min}$ for which a randomly implemented headed-stud connector shear resistance will not be underestimated. This factor can be evaluated by the following ratio:

$$\gamma_{\upsilon,\min} = \frac{Z_k}{Z_d} = \exp\left[(3.04 - 1.645)\, \nu_Z\right] = \exp\left(1.395 \nu_Z\right) \tag{4.8}$$

It is clearly visible that this value depends on the value of a coefficient of variation $\nu_Z$. Simple comparison of the results obtained from (4.8) with a constant value $\gamma_\upsilon = 1.25$ recommended to use in this regard in the standard [32] is given in Fig. 9. As one can see, in case when the variability $\nu_Z$ is large enough (i.e. for $\nu_Z > 0.17$) a constant value $\gamma_\upsilon = 1.25$ turns out to be insufficient to ensure the required safety level.

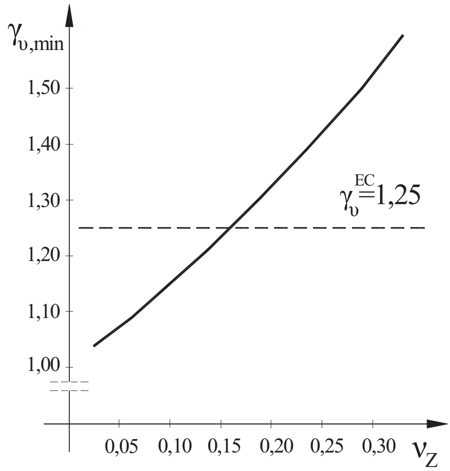


Fig. 9. The minimum values of a partial safety factor $\gamma_\upsilon$ ensuring the required safety level relating to the random headed-stud connector shear resistance and their relation to a constant value $\gamma_\upsilon = 1.25$ recommended for application in the standard [32]

## 5. Proposed procedure for specification of the design value of random headed-stud connector shear resistance

In the probability-based approach recommended by the authors in this paper the sought design value of a random headed-stud connector shear resistance may be calculated directly from the formula (4.6). A mean value $\mu_Z = E(Z)$ used in such formula is then determined on the basis of (2.5) while a coefficient of variation $\nu_Z$ – according to (3.8). To do this, a variation $\sigma_Z^2 = \mathrm{var}(Z)$ should be specified previously, as shown in (2.6). This allows for the specification of the value of the standard deviation $\sigma_{\ln Z}$, based on (3.6). Alternatively, in the formula (4.6) the variance $\sigma_{\ln Z}^2$ can be effectively eliminated by the substitution:

$$\sigma_{\ln Z}^2 = \ln\left(\nu_Z^2 + 1\right) \tag{5.1}$$

The procedure mentioned above is useful for practical application due to the assumption that the boundary *pdf* functions, both $f_X(z)$ and $f_Y(z)$, taking into account in (2.4) to determine the joint *pdf* function $f_Z(z)$, are characterized by the log-normal probability distribution.

## 6. Numerical example

A numerical procedure discussed above is illustrated in this chapter by an exemplary evaluation of the design value of a random headed-stud connector shear resistance. Let the considered stud has the diameter of size $d = 16$ mm and the length measured after welding equal to $h_{sc} = 70$ mm. Furthermore, it is assumed that this stub is made of steel for which $f_u = 400$ MPa.

As far as the parameters of the surrounding concrete are concerned these are as follows: $f_{ck}$ = 20 MPa and $E_{cm}$ = 30.5 GPa (which is typical for the concrete C20/25). Such data, after their applying to the conventional standard formula (1.1), lead to the evaluation that $P_{R,d}^{EC}$ = 46 kN.

Using the novel probability-based approach, however, reveals the relationship between the sought design value of a random headed-stud connector shear resistance $P_{R,d} = Z_d$ and the coefficient of variation $v_Z$, as it is presented in the formula (4.6). This, after taking into account (2.4) and (2.6), is transformed into two appropriate dependences including: the first one – dependence on the degree of variability $v_X = v_s$, relating to the strength of the steel the considered connector is made of, and the second one – dependence on the degree of variability $v_Y = v_c$, corresponding to the surrounding concrete parameters. The sought design values of a random headed-stud connector shear resistance $P_{R,d} = P_{R,d}(v_c)$, obtained in the considered example for subsequent values of the variability $v_c$, with an assumption that the variability $v_s$ is constant and set at the level $v_s = 0.10$, are shown in detail in Fig. 10.

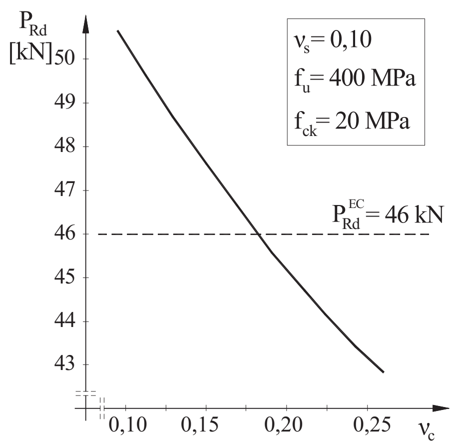


Fig. 10. The design values of a random headed-stud connector shear resistance obtained for the input data considered in the example

The detailed analysis of the structure of formula (1.3) leads to the conclusion that the value of a coefficient of variation $v_c$ is a measure of the influence of not only the random variability of the compressive strength of the concrete surrounding the headed-stud connector considered in this article but also the random variability of the modulus of longitudinal elasticity of such the concrete.

It is clearly visible that the evaluation of the design value specified for the random shear resistance of the composite connection considered in the example, obtained by the application of the conventional standard approach, on the one hand leads to the assessments which are underestimated when the variability of the parameters describing the concrete properties is sufficiently small, and, on the other hand, to those that are significantly overestimated if only the variability of these parameters is large enough.

# 7. Concluding remarks

The probability-based calculation procedure, presented by the authors in this paper, indicated the existence of an important dependence between the design value specified for random headed-stud connector shear resistance $P_{R,d}$, determined by calculations, and the known a priori values of the specific coefficients of variation, including both $v_s$ – which is relating to the variance of the resistance of steel the considered connector is made of, and $v_c$ – which is associated with the variance of the strength of surrounding concrete. Let us note that the relationship of this type cannot be detected using only the conventional design approach, based on the recommendations given in the standard EN 1994-1-1 [32]. It seems, therefore, that the approach proposed by us allows for obtaining the sought assessments with a greater degree of credibility in this regard. Such new estimates may be considered more accurate because they are identified in conjunction with the maximum acceptable level of failure probability. This level is usually set as $p_{\mathrm{f,ult}} \approx 1.18 \cdot 10^{-3}$, which corresponds to the specification that $\beta_{R,\mathrm{req}} = 3.04$.

To unambiguously determine a conclusive headed-stud connector shear resistance for a single random implementation a smaller value of randomly drawn pair of two numbers, including $P_{Rs}$ and $P_{Rc}$, respectively, should be identified. It is crucial that the design value of this resistance, specified for the statistical population of such random minima, is not quantitatively equivalent to the deterministic minimum of the appropriate design values, $P_{Rs,d}$ and $P_{Rc,d}$, calculated separately, as it is incorrectly recommended in the standard [32]. This means that:

$$P_{R,d} = [\min(X,Y)]_d = [\min(P_{Rs}, P_{Rc})]_d \neq \min\left(P_{Rs,d}, P_{Rc,d}\right) \tag{7.1}$$

The results of a numerical example, presented in this paper, allow to conclude that for a fixed value of the variability $v_s$, set at the level $v_s = 0.10$, and with a sufficiently high homogeneity of surrounding concrete (which is equivalent to the specification that the level of the variability $v_c$, relating to its strength, is low enough), the design value of a random headed-stud connector shear resistance, determined using the probability-based procedure recommended in this article, is higher than the analogous design value resulting from a simple deterministic comparison of the values $P_{Rs,d}$ and $P_{Rc,d}$. In such a situation the simplified standard design procedure gives the safe, though sometimes overly conservative, estimates of the sought connector resistance. However, if the concrete surrounding the considered connector is identified to be less homogeneous (for instance, for the data used in the presented example the variability of the concrete strength in such case should meet the condition $v_c > 0.17$) then the calculated design value of its random shear resistance, specified by conventional standard formula, turns out to be dangerously overestimated.

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# Losowa nośność na ścinanie łącznika sworzniowego z główką w belkach zespolonych typu stal–beton

**Słowa kluczowe:** łącznik sworzniowy z główką, probabilistyczny format obliczeń, wartość obliczeniowa, wytrzymałość na ścinanie, zespolenie stali z betonem

## Streszczenie:

W tradycyjnym, normowym, ujęciu obliczeniowym nośność na ścinanie pojedynczego łącznika sworzniowego z główką, zapewniającego zespolenie belki stalowej z opartą na tej belce płytą żelbetową, określana jest przez porównanie ze sobą nośności $P_{Rs}$ – determinowanej zniszczeniem samego łącznika stalowego, oraz nośności $P_{Rc}$ – warunkowanej zniszczeniem betonu otaczającego ten łącznik. W pojedynczej realizacji miarodajną dla projektanta jest mniejsza z tych wartości, a zatem $P_R = \min(P_{Rs}, P_{Rc})$. Jeżeli obie zestawione ze sobą wytrzymałości potraktować jako zmienne losowe i rozpatrywać statystycznie jednorodną próbę grupującą realizacje potencjalnie możliwe to obliczeniowa nośność $P_{R,d} = [\min(P_{Rs}, P_{Rc})]_d$, reprezentatywna dla weryfikacji stanu granicznego nośności rozpatrywanego połączenia, będzie ilościowo różna od wartości $P_{R,d} = \min\left(P_{Rs,d}, P_{Rc,d}\right)$ zalecanej do stosowania w normie EN 1994-1-1. W niniejszej pracy pokazano szczegółowy algorytm poprawnego specyfikowania tej wartości. Wykazano jej zależność od wzajemnej relacji pomiędzy współczynnikami określającymi statystyczną zmienność wytrzymałości stali łącznika i wytrzymałości betonu, z którego wykonano płytę stropową. Zaproponowane podejście opiera się na w pełni probabilistycznym formacie obliczeń, w którym gwarantuje się odpowiednie prawdopodobieństwo niezawodnej pracy analizowanego połączenia. Rozważania zilustrowano przykładem numerycznym. Na jego podstawie oszacowano stopień uproszczenia, a także jego konsekwencje, wynikające z zastosowania w tym zakresie konwencjonalnego modelu normowego.

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